Eternal domination on digraphs and orientations of graphs

BGW 2019

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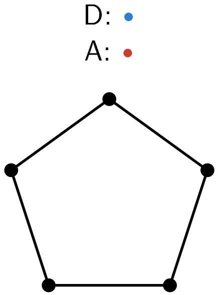

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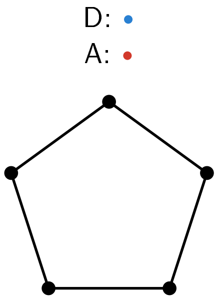

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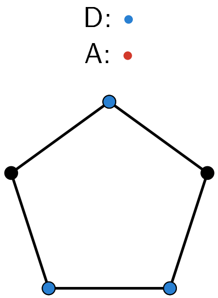

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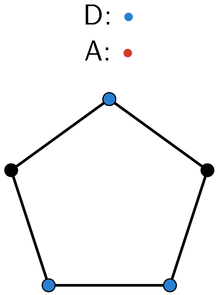

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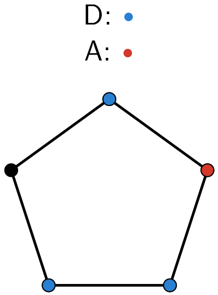

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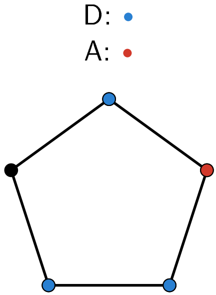

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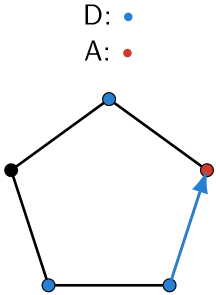

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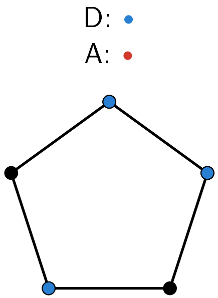

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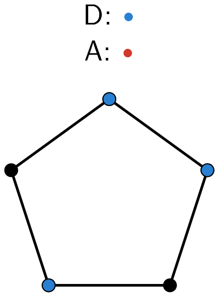

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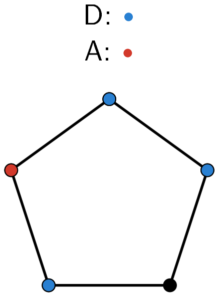

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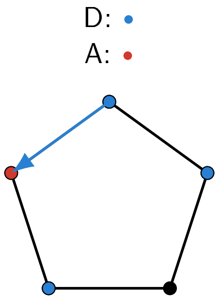

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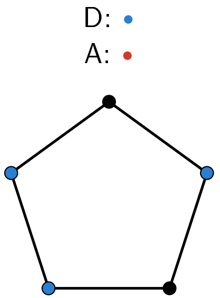

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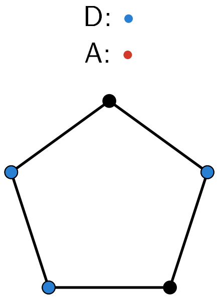

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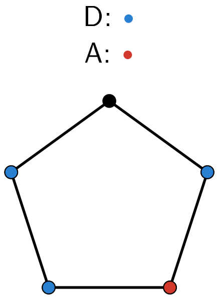

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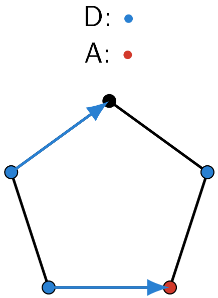

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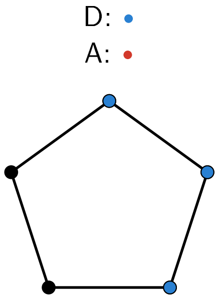

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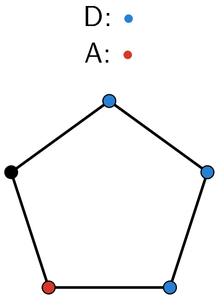

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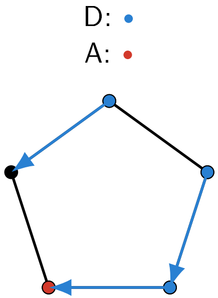

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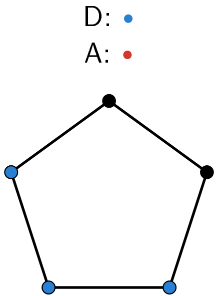

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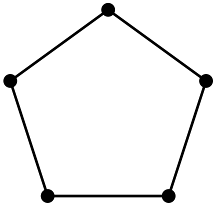

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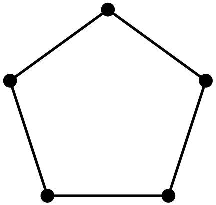
Example:

eternal domination on C_5 with 2 guards



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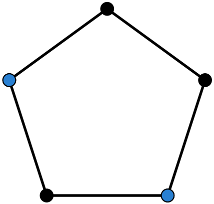

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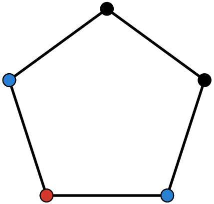

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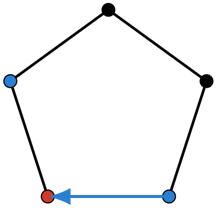

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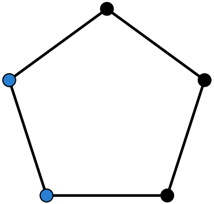

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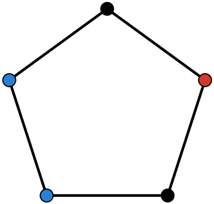

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Example:

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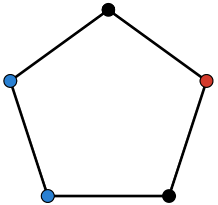

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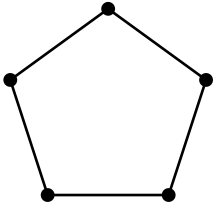

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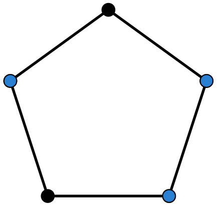

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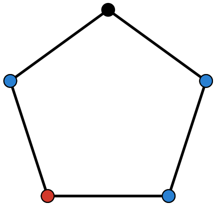

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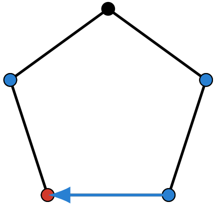

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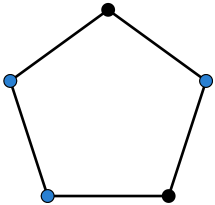

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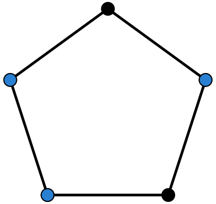

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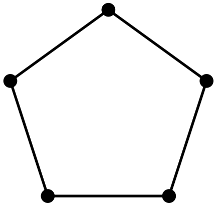

$\gamma^{\infty}(C_5) = 3$

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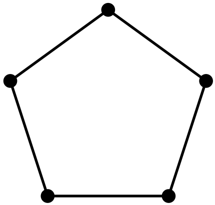
m-eternal domination on C_5 with 1 guards → A wins



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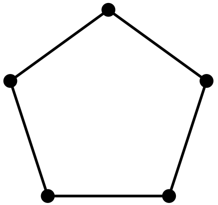

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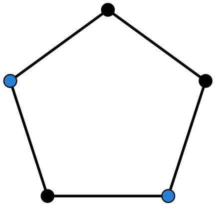

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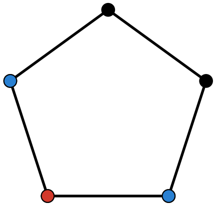

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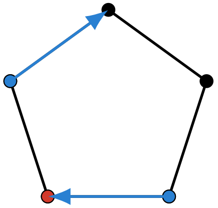

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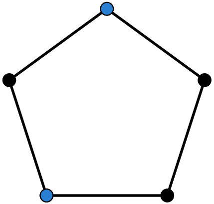

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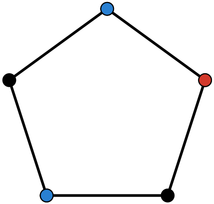

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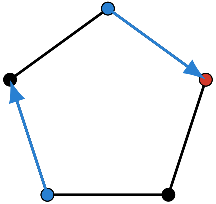

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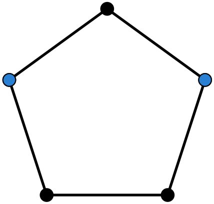

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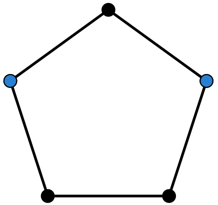

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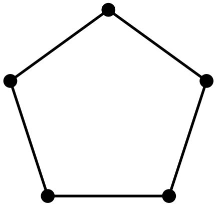

$\gamma^{\infty}(C_5) = 3$

$\gamma^{\infty}_{m}(C_5) = 2$

State of the art

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- Introduction of the eternal domination: Burger et al, 2004.

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- Studies on classes of graphs such as cliques, complete bipartite graphs, cycles and grids.
- General bounds for the two parameters:

Theorem (Burger 04, Goddard 05, Klostermeyer 07)
$\gamma(G) \leq \gamma_m^\infty(G) \leq \alpha(G) \leq \gamma^\infty(G) \leq \binom{\alpha(G)+1}{2}$ where γ is the domination number and α the independent set number.

Theorem (Burger et al 04)
$\gamma^\infty(G) \leq \theta(G)$ where θ is the clique covering number.

Eternal and m-eternal domination on digraphs

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The guards move along the direction of the arcs.

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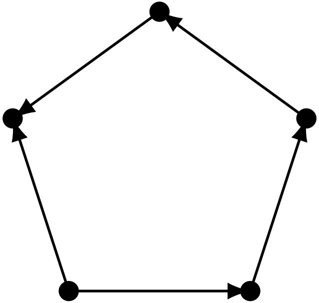

D:

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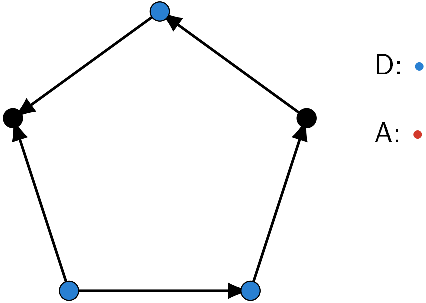

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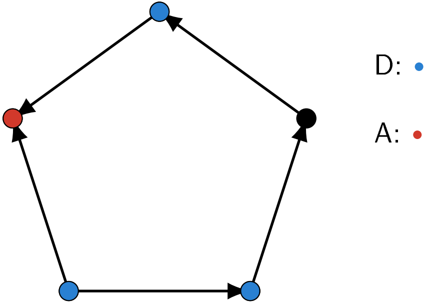

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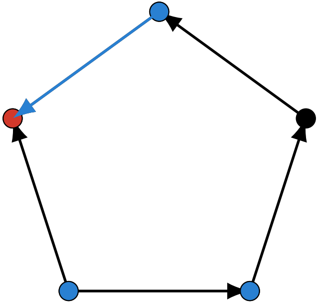

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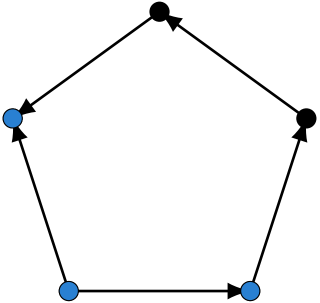

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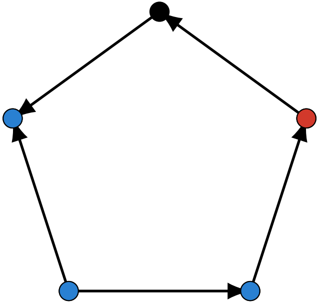

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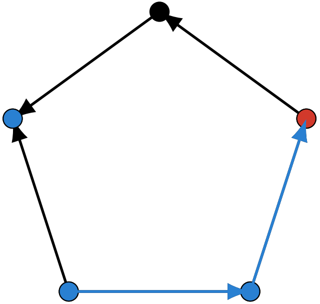

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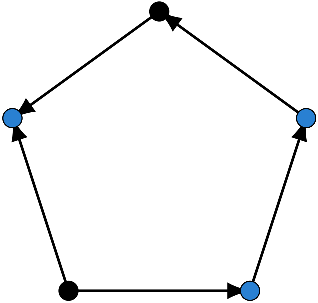

D:

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General bounds

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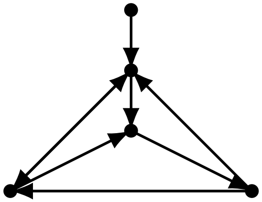

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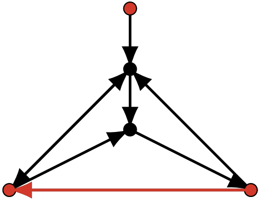

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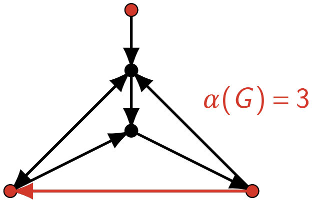

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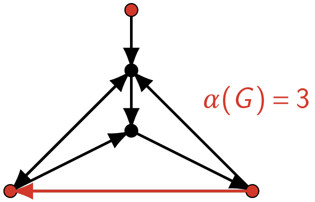

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Principle: Orient G to minimize its (m-)eternal domination number.

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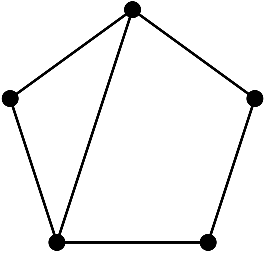

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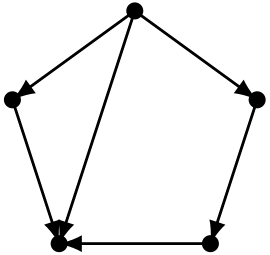

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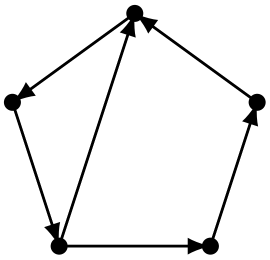

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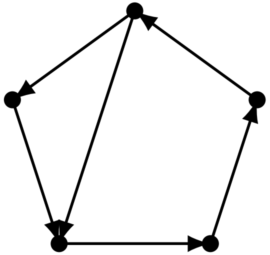

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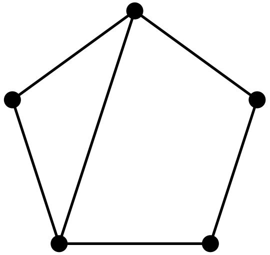

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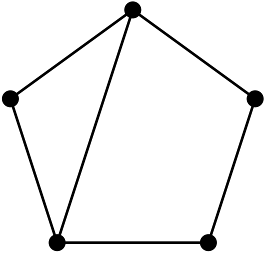

Proposition

For G a graph with at least one edge,
$\gamma(G) \leq \alpha(G) < \overrightarrow{\alpha}(G) \leq \overrightarrow{\gamma^{\infty}}(G)$.

Forests

Forests

Theorem

Let G be a graph with order n. Then, $\overrightarrow{\gamma^{\infty}}(G) = n$ iff $\overrightarrow{\gamma^{\infty}_{m}}(G) = n$ iff G is a forest.

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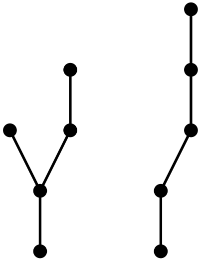

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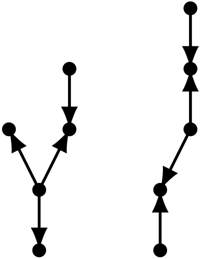

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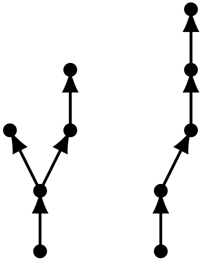

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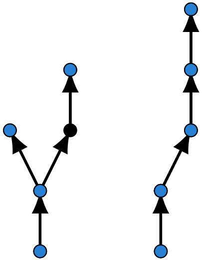

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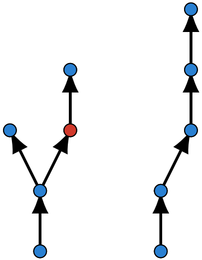

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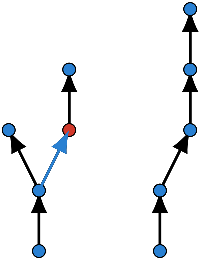

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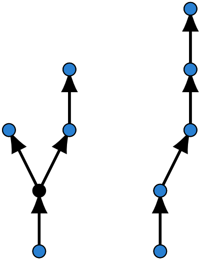

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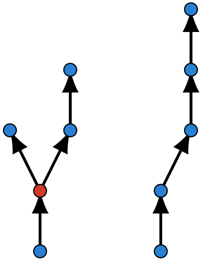

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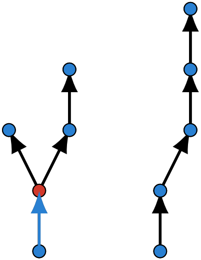

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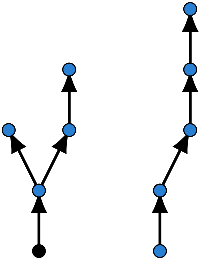

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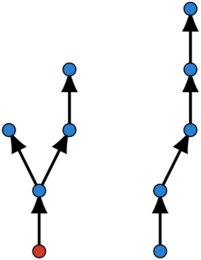

Cycles

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$\overrightarrow{\gamma^{\infty}}(C_n) = n-1$ and $\overrightarrow{\gamma_m^{\infty}}(C_n) = \lceil \frac{n}{2} \rceil$ for every $n \geq 3$.

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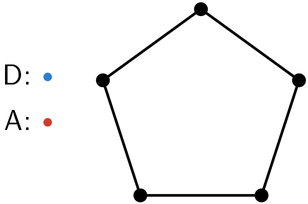

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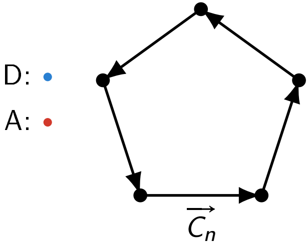

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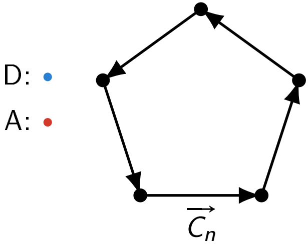

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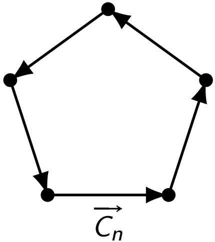

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eternal domination on $\overrightarrow{C_n}$ with $n-1$ guards

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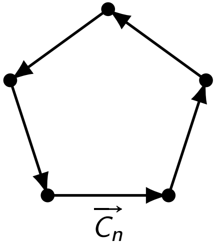

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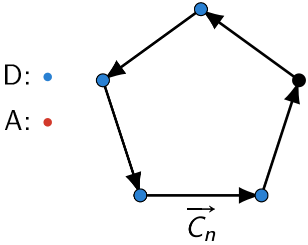

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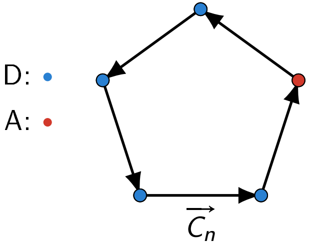

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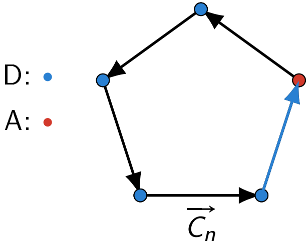

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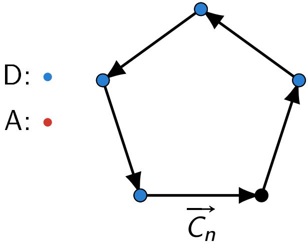

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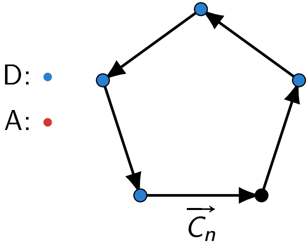
Proof:

eternal domination on $\overrightarrow{C_n}$ with $n-1$ guards : D wins



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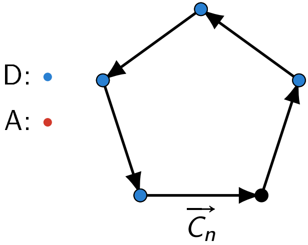

$\overrightarrow{\gamma^{\infty}}(C_n) \geq n-1$

$\overrightarrow{\gamma^{\infty}}(C_n) \leq n-1$

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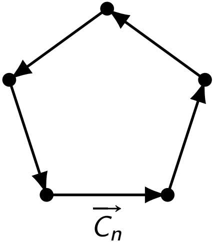
$\gamma(\overrightarrow{C_n}) = \lceil \frac{n}{2} \rceil$

D: •

A: •



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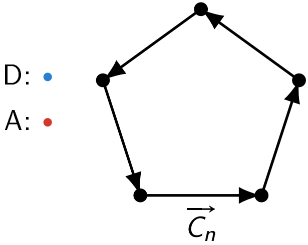

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D: •

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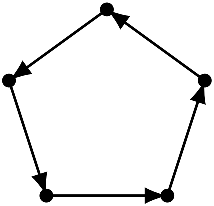

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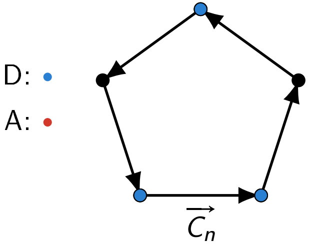

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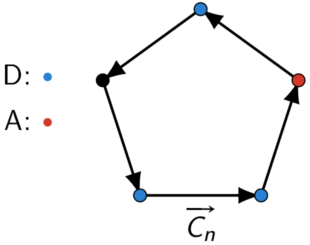

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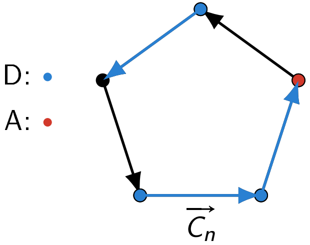

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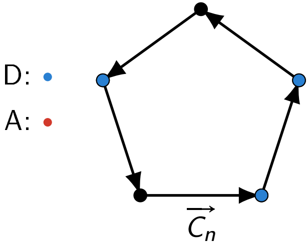

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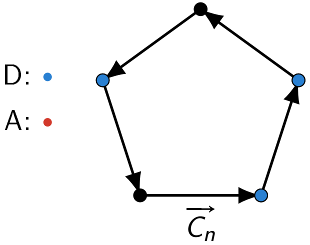

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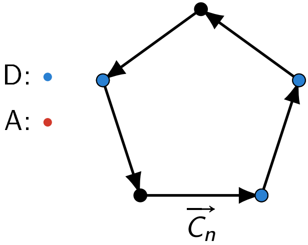

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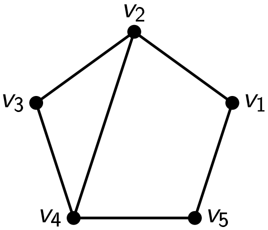

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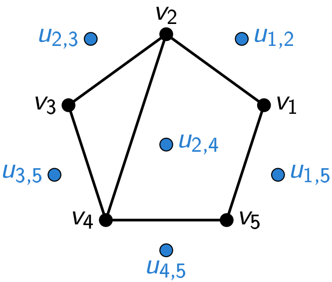

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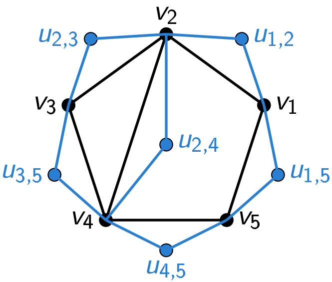

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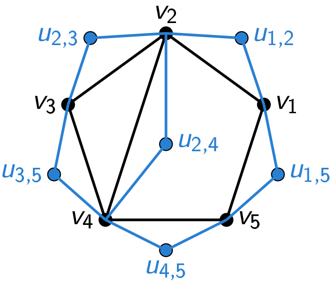

Lemma

Let G be an undirected graph with m edges. Then,
$\overrightarrow{\gamma^{\infty}}(C(G)) = \gamma^{\infty}(G) + m$, *and* $\overrightarrow{\alpha}(C(G)) = \alpha(G) + m$.

Proof of $\overrightarrow{\gamma^{\infty}}(C(G)) = \gamma^{\infty}(G) + m$

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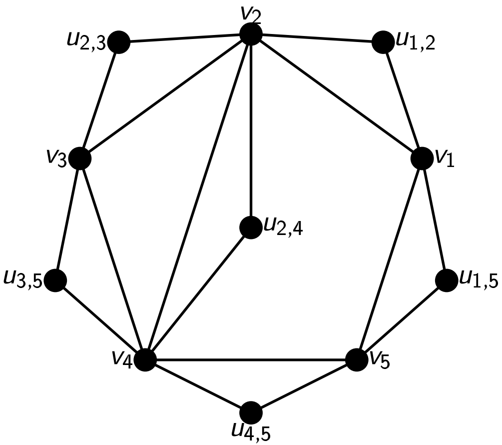

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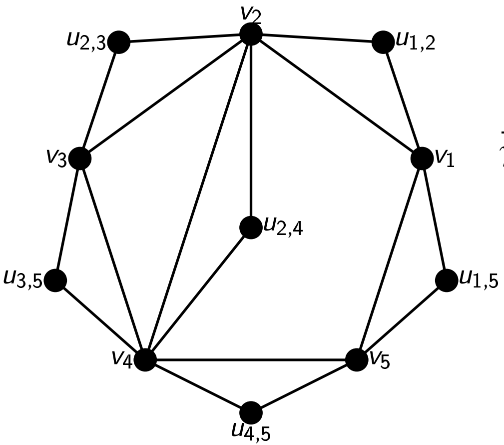

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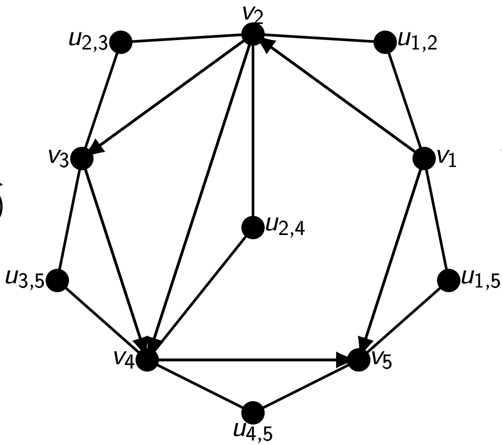

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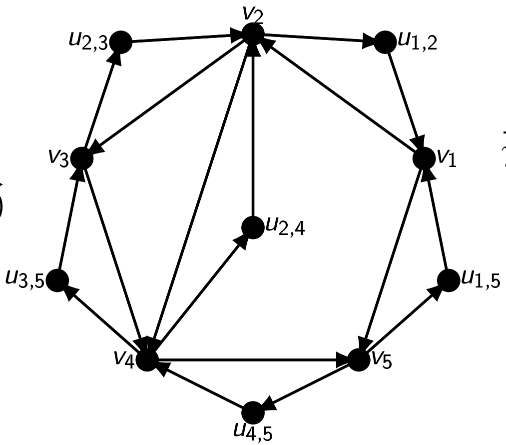

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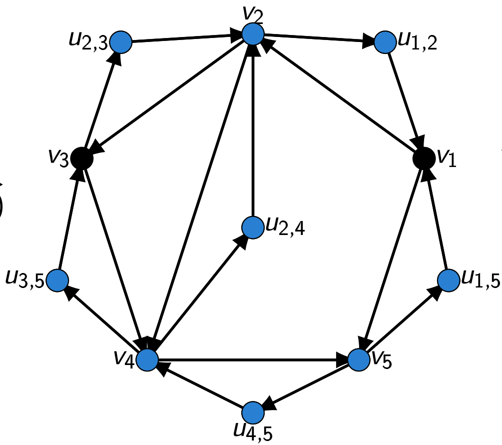

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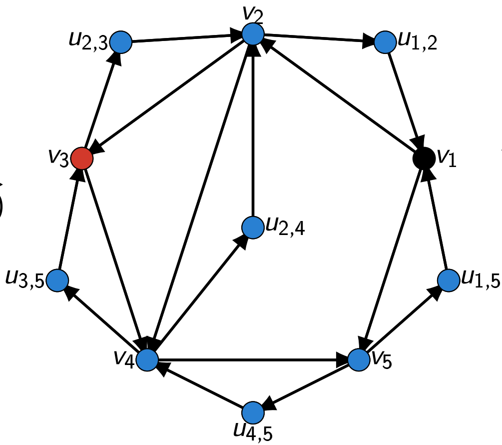

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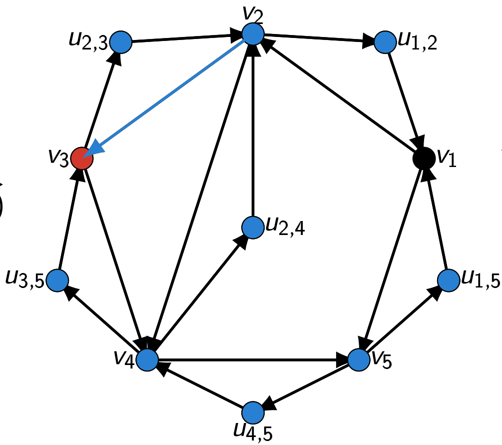

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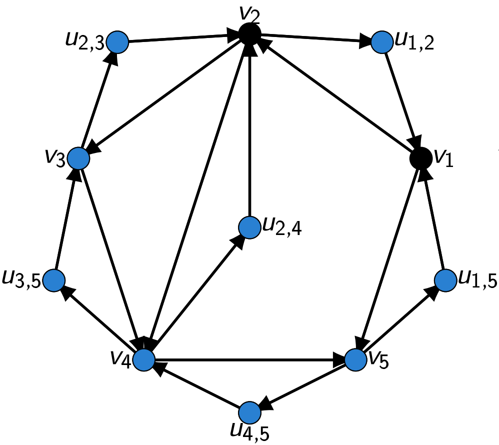

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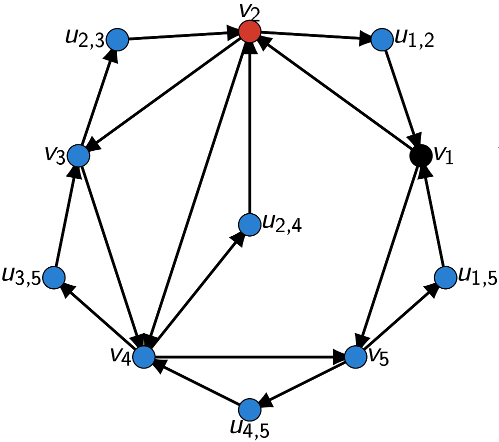

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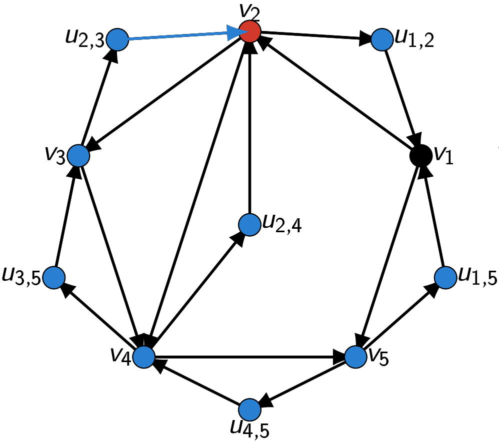

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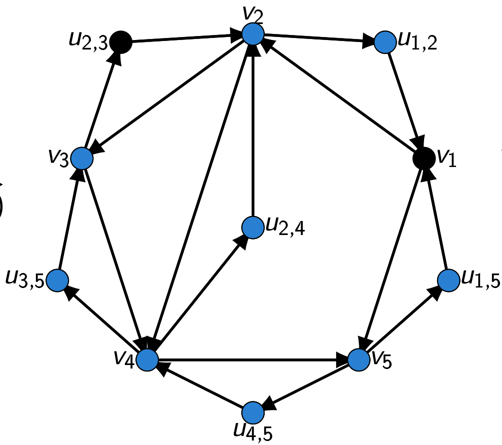

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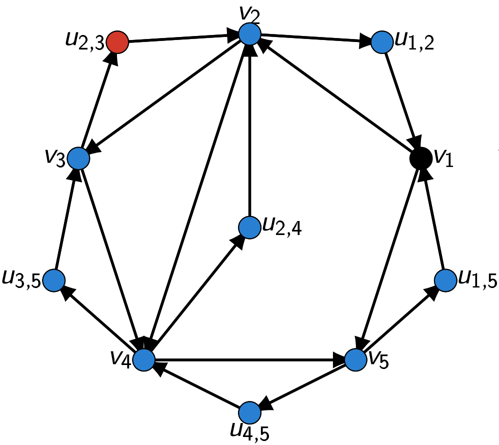

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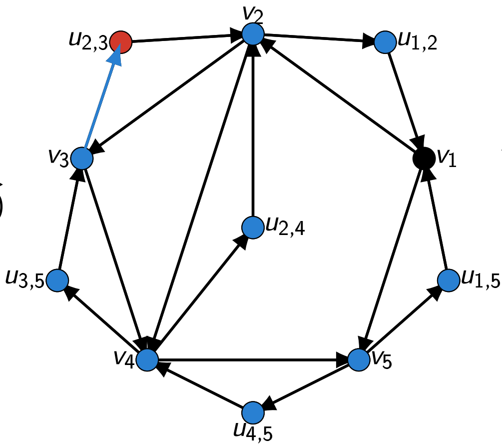

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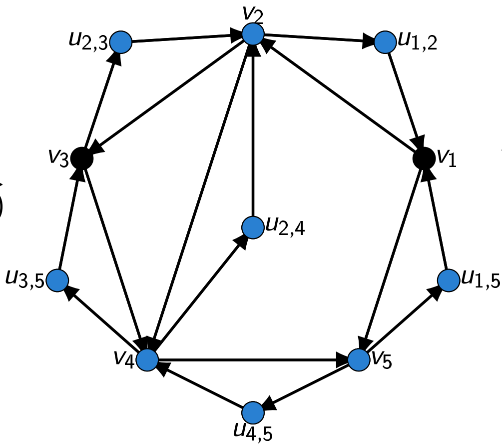

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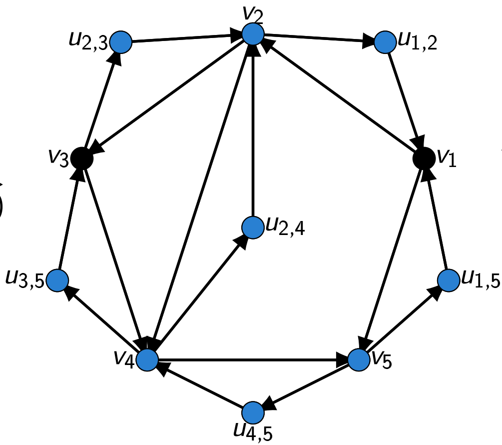

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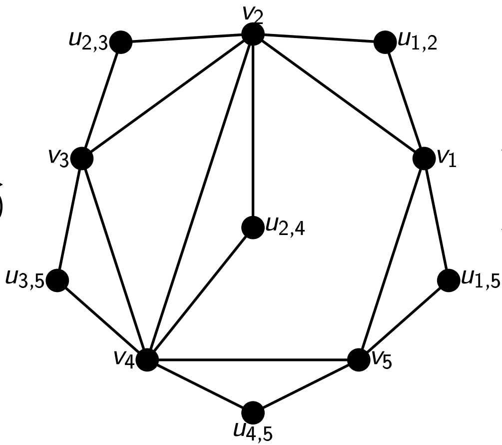

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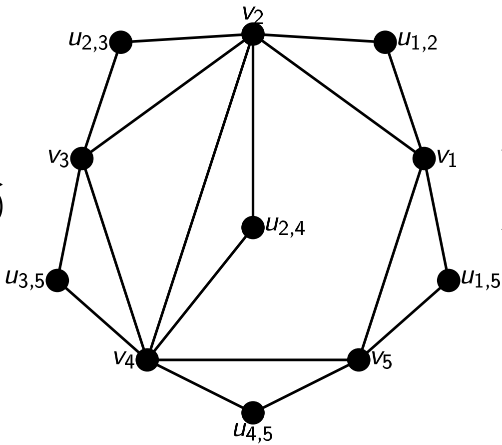

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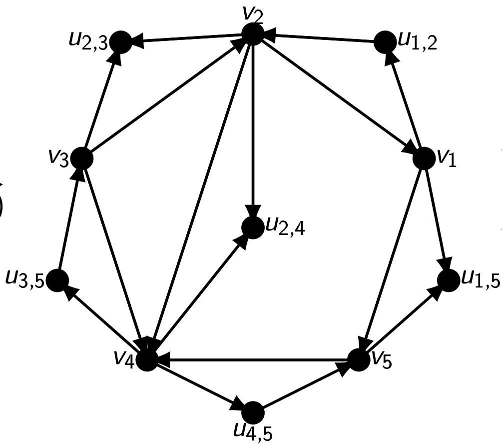

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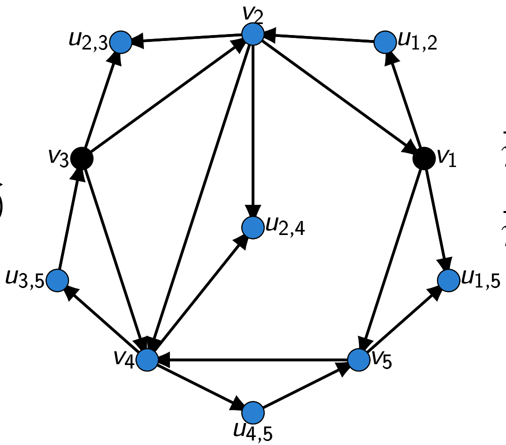

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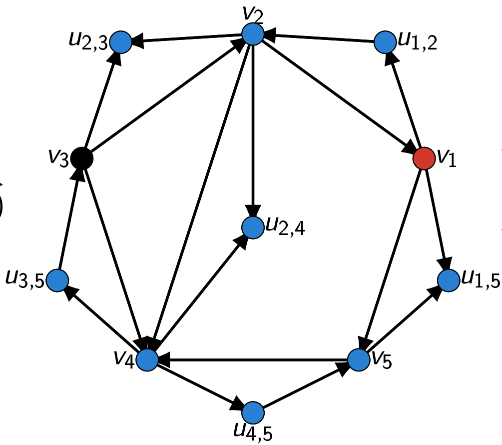

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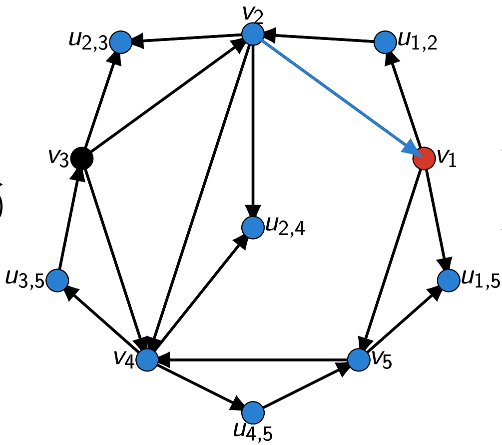

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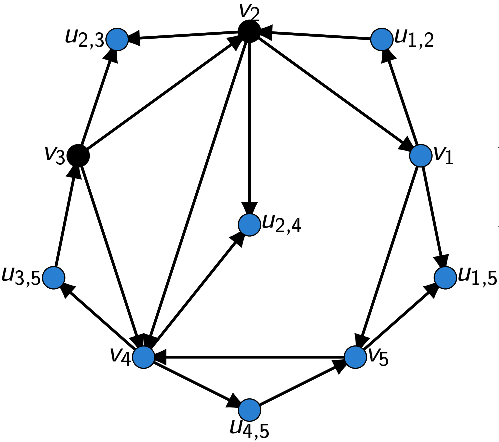

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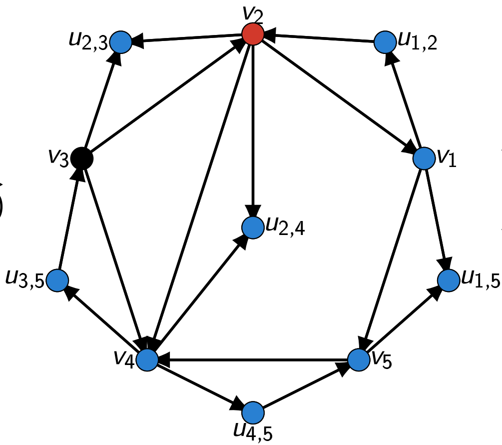

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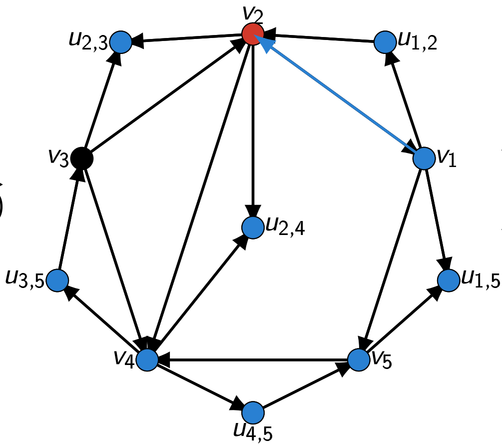

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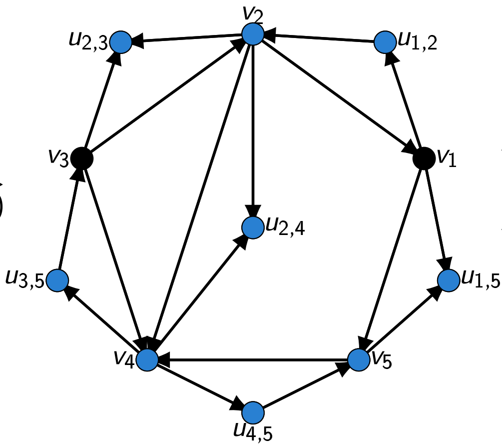

$\overrightarrow{\gamma^\infty}(C(G)) \leq \gamma^\infty(G) + m$

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Proof of $\overrightarrow{\gamma^\infty}(C(G)) = \gamma^\infty(G) + m$

eternal domination on G with $\overrightarrow{\gamma^\infty}(C(G)) - m$ guards: D wins

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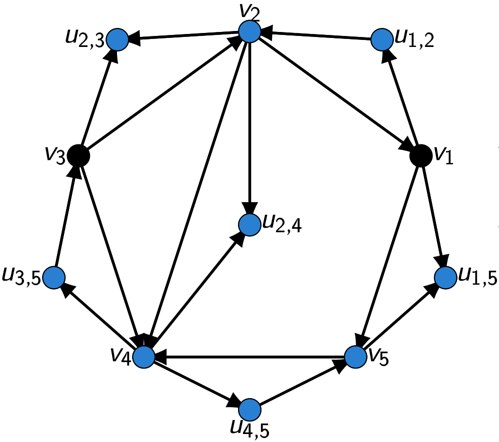

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