Graph domination and reconfiguration problems

Thesis presentation

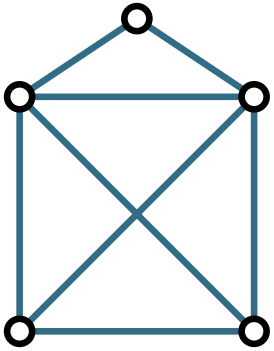
Under the supervision of Hamamache Kheddouci and Nicolas Bousquet

Alice Joffard

November 25, 2020

Graph theory

Graph theory



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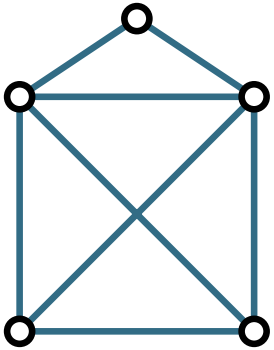

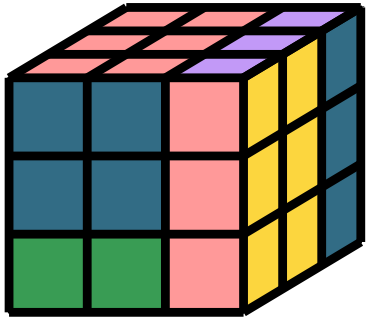

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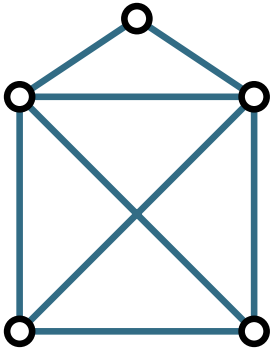

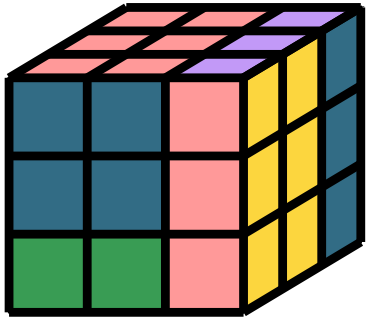

Graph: (V,E)

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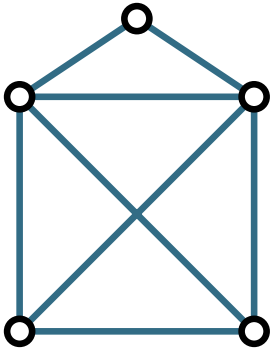

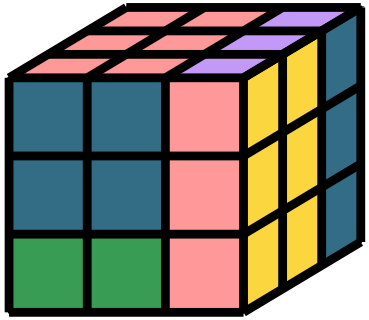

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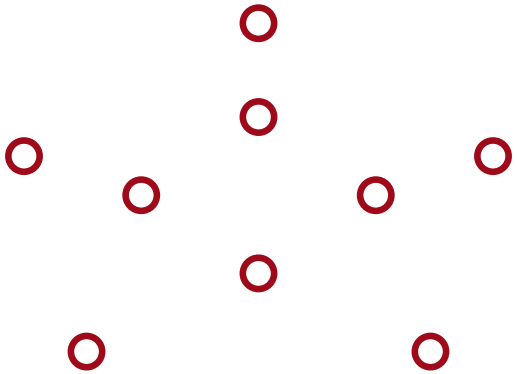

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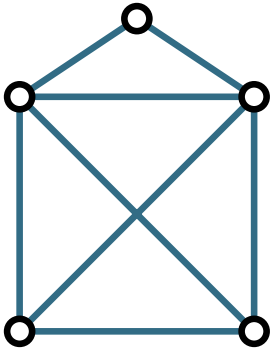

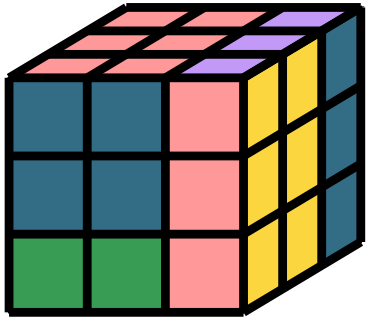

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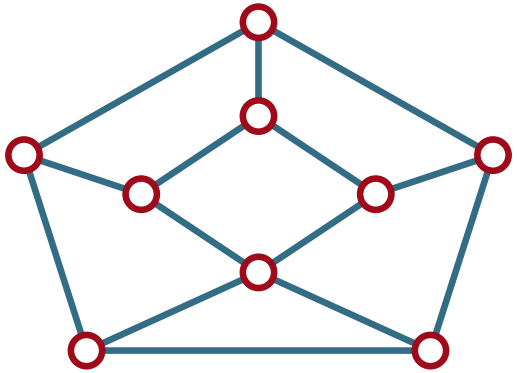

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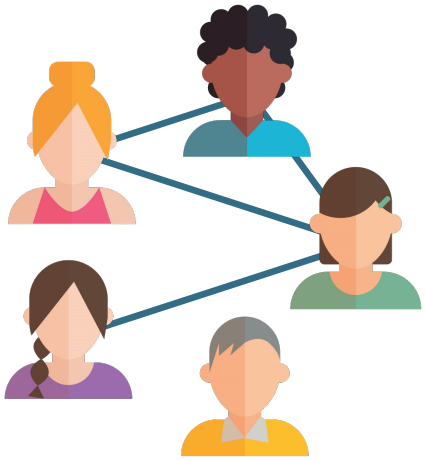

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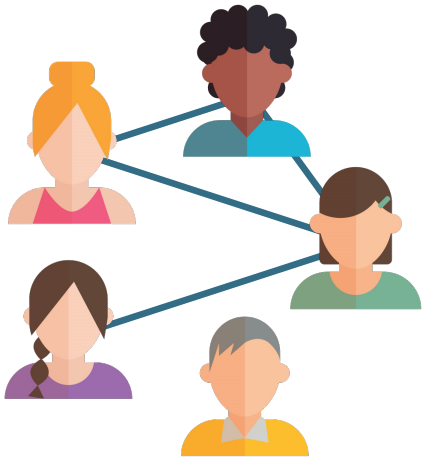

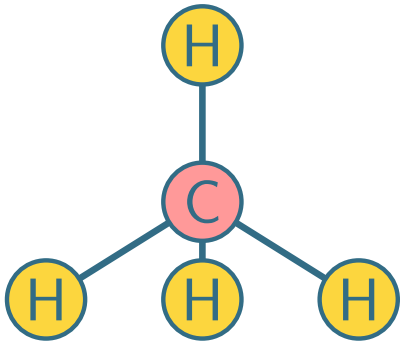

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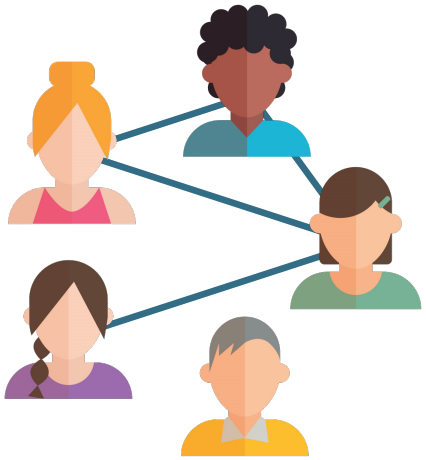

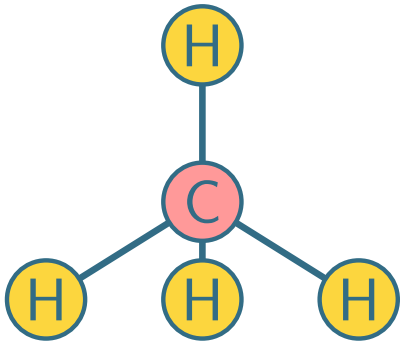

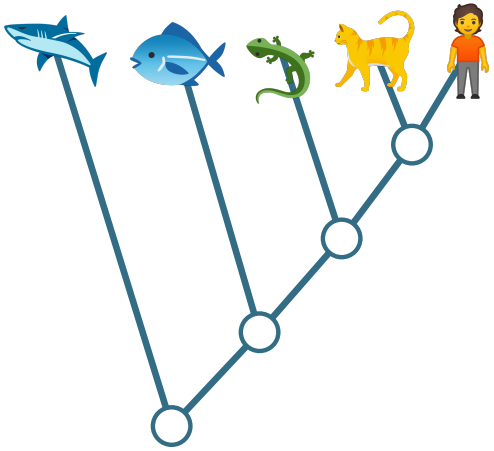

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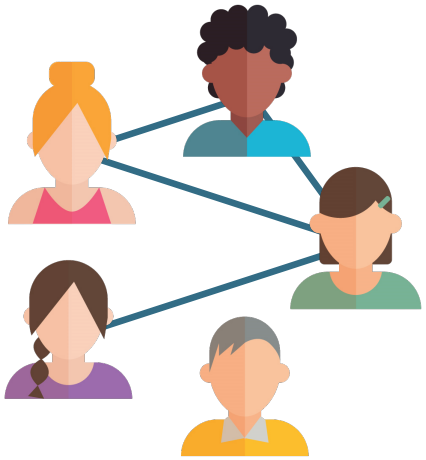

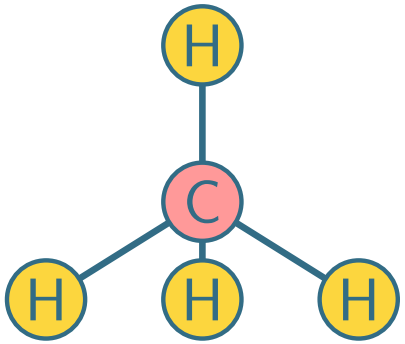

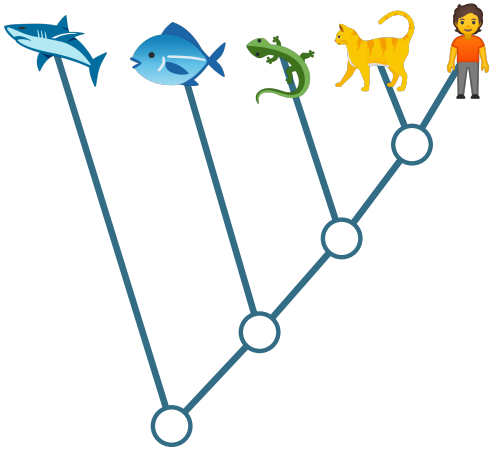

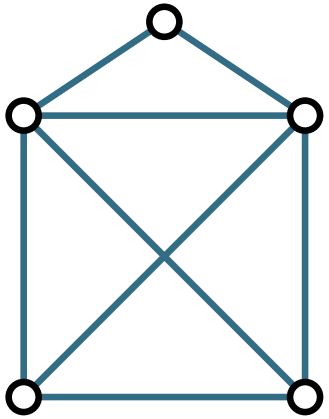

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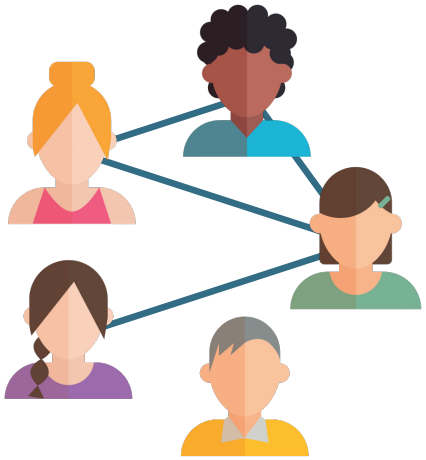

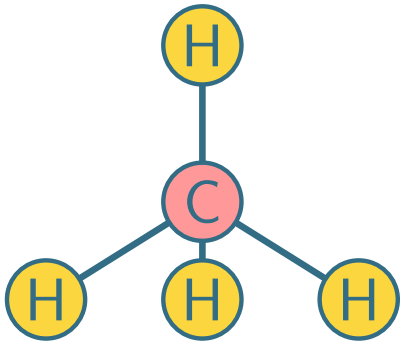

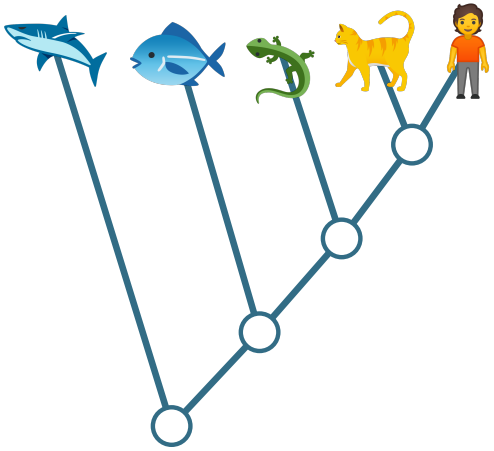

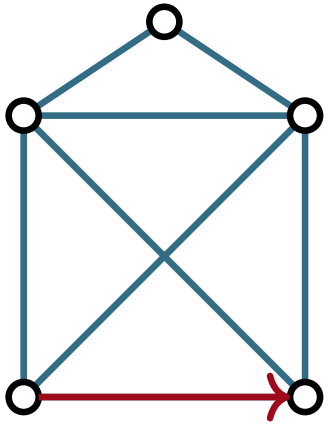

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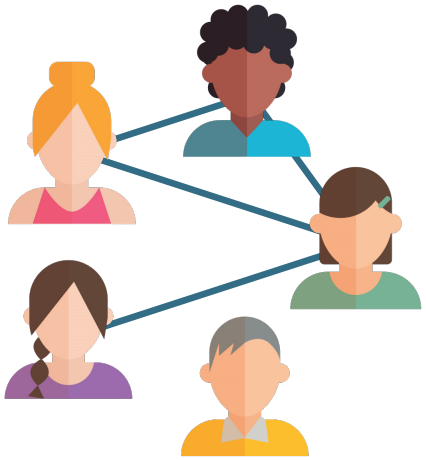

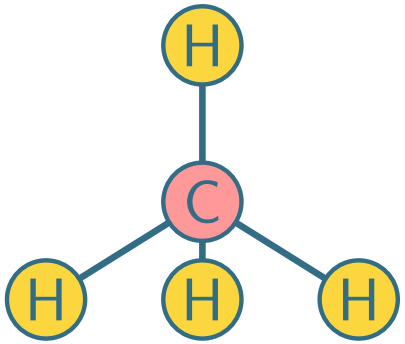

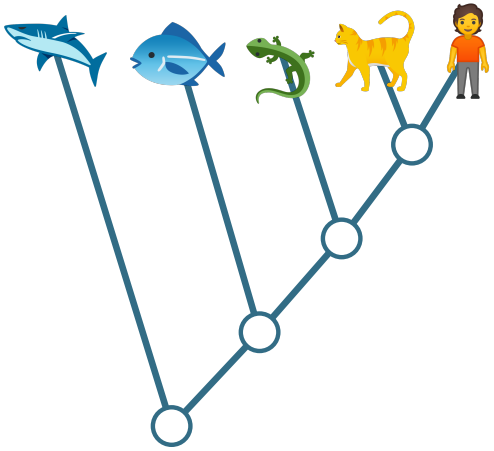

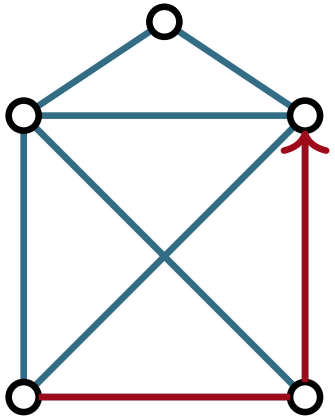

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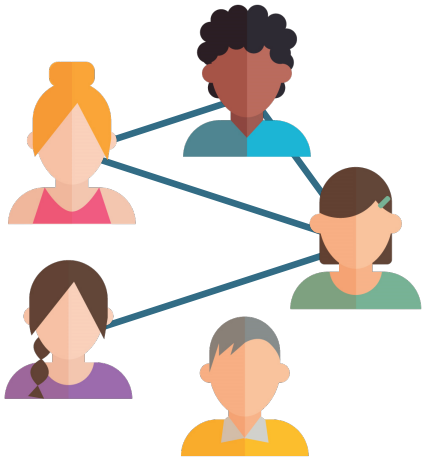

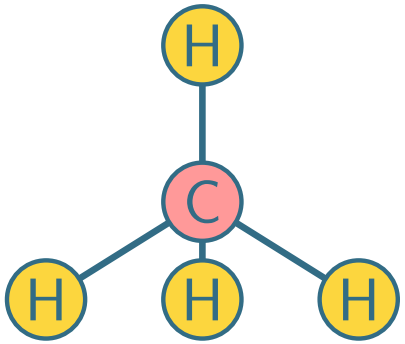

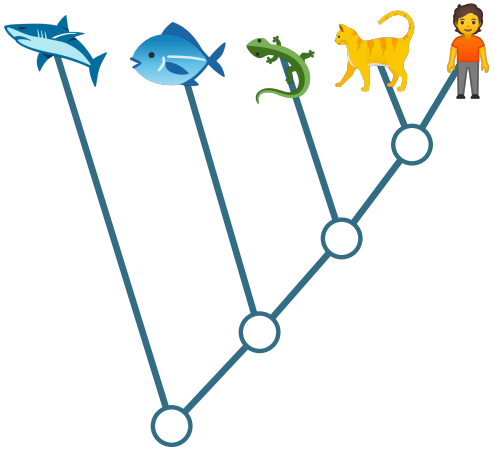

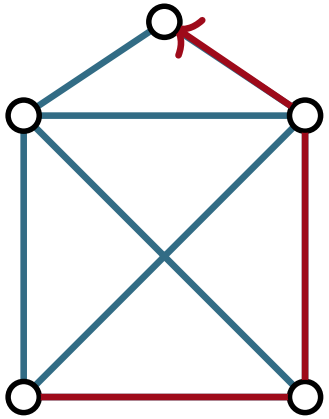

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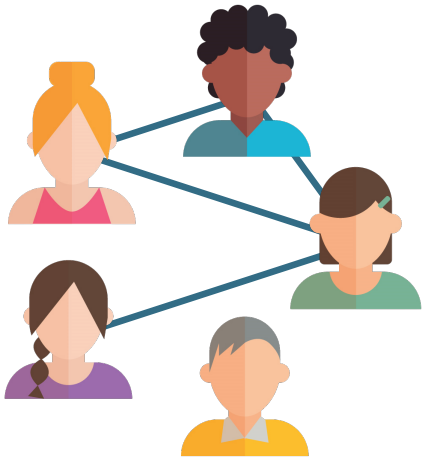

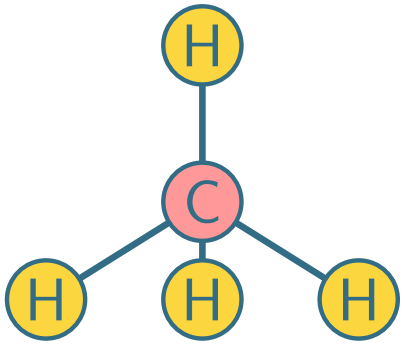

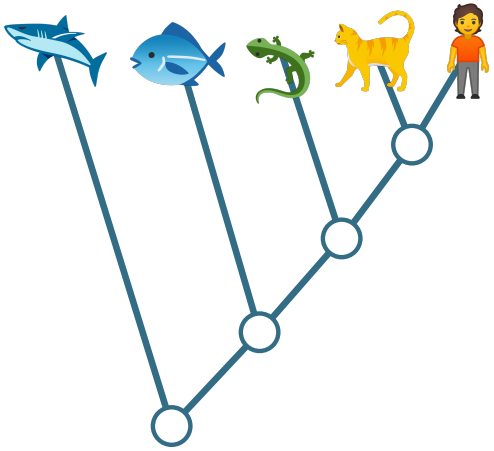

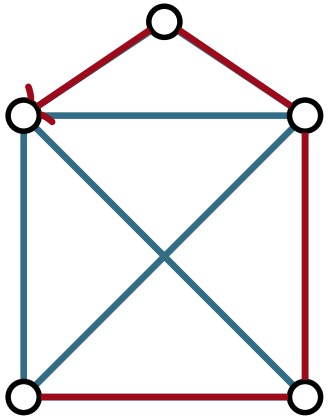

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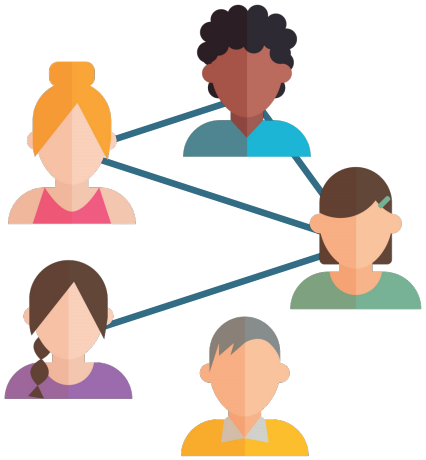

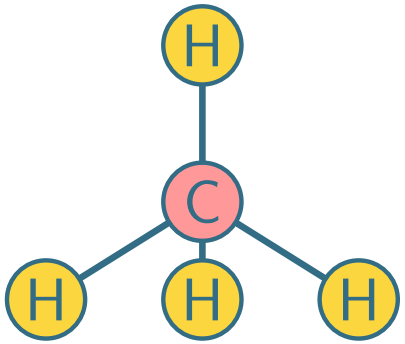

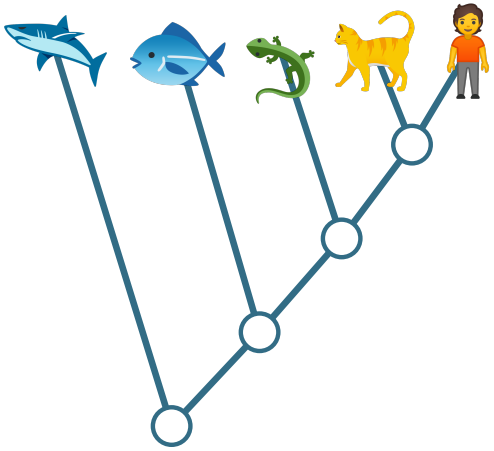

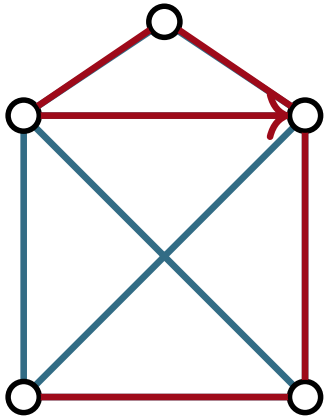

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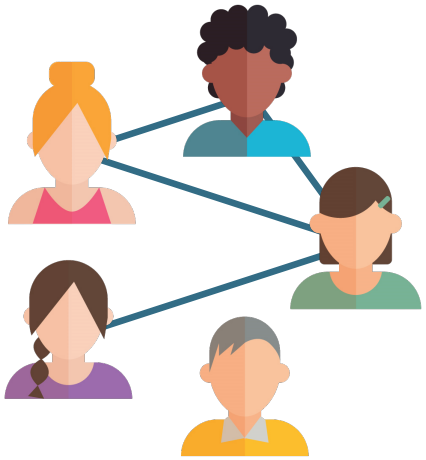

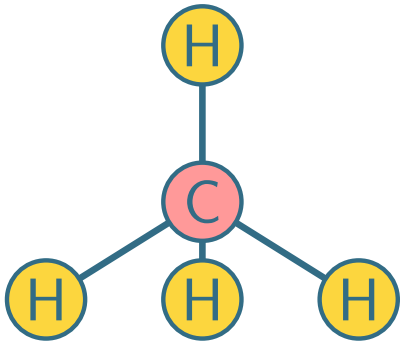

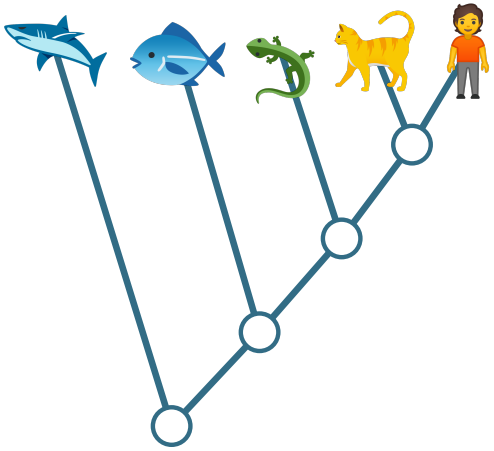

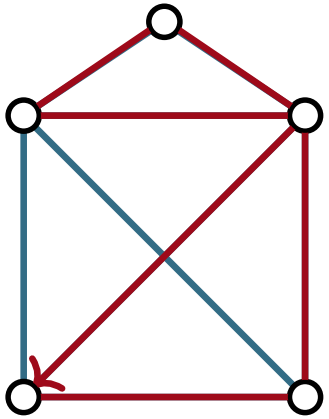

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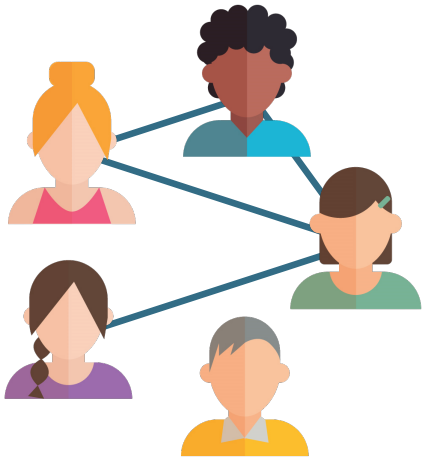

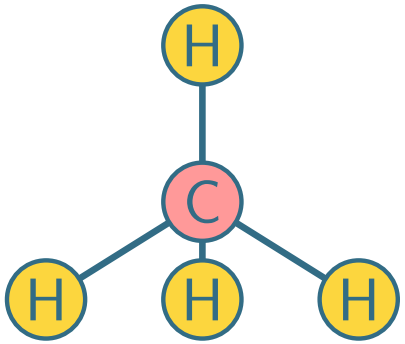

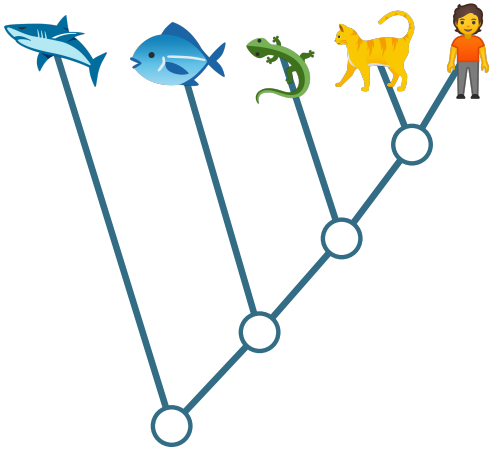

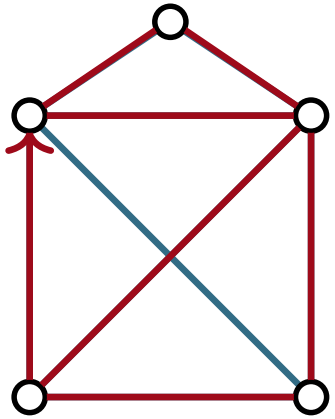

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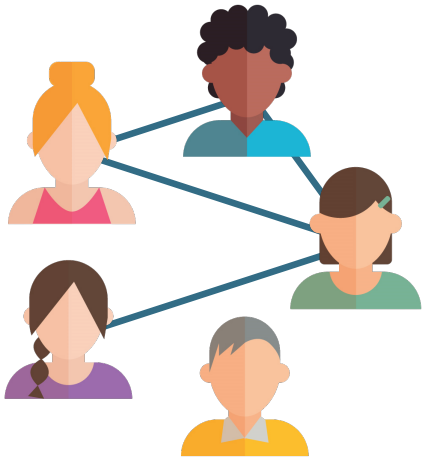

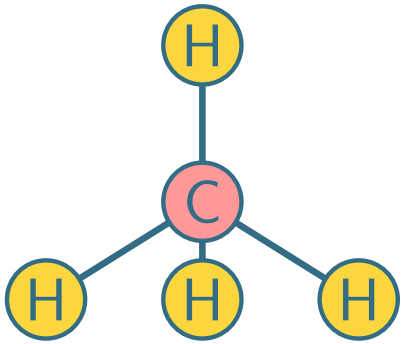

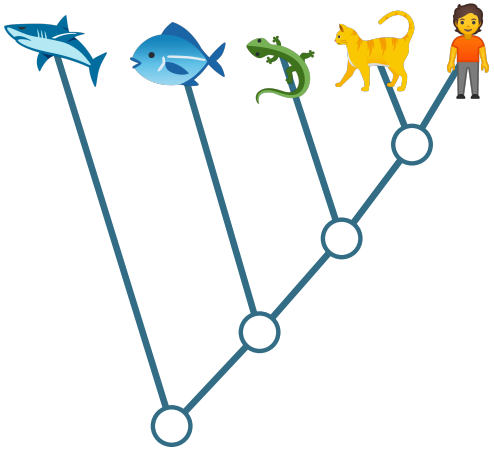

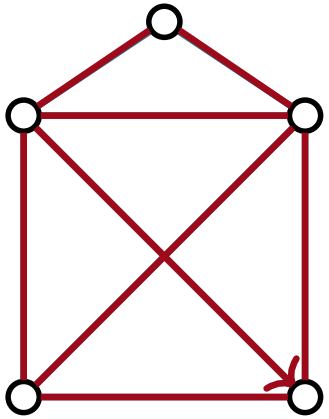

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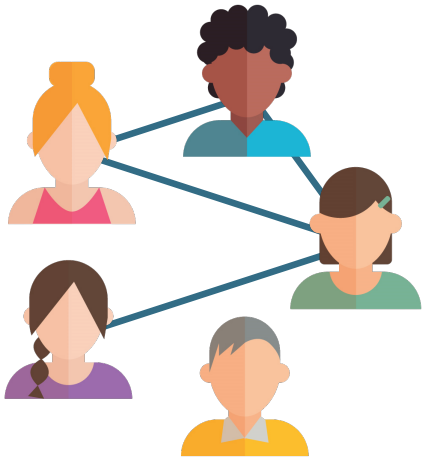

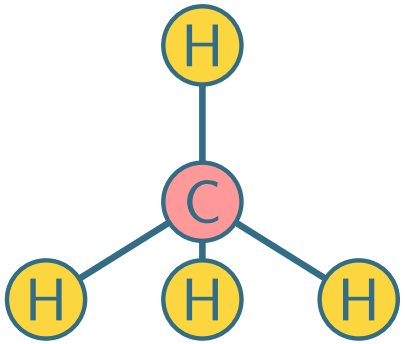

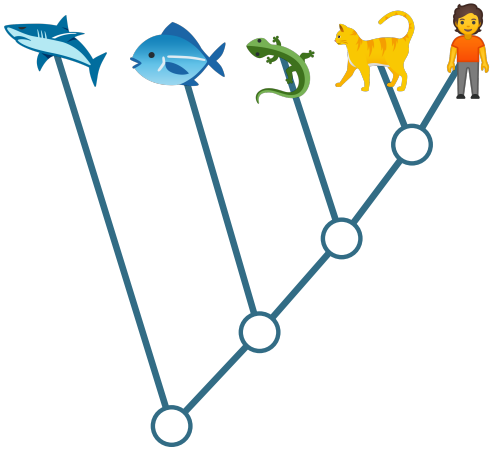

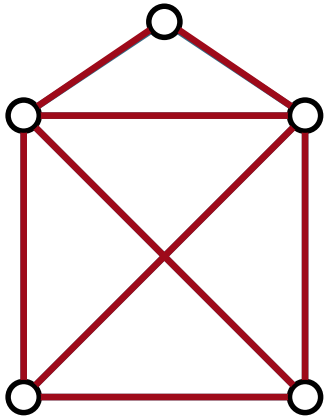

Graph domination

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Dominating set D of G: for any $v \in V$, $v \in D$ or there exists $u \in D$ with $uv \in E$

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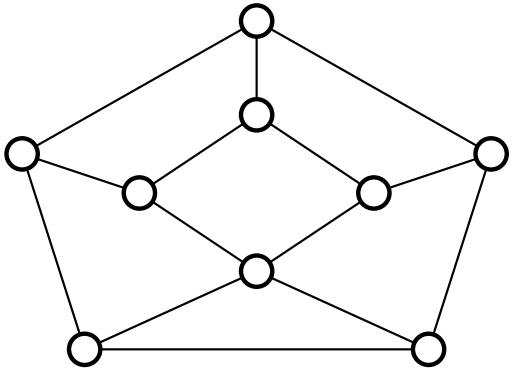

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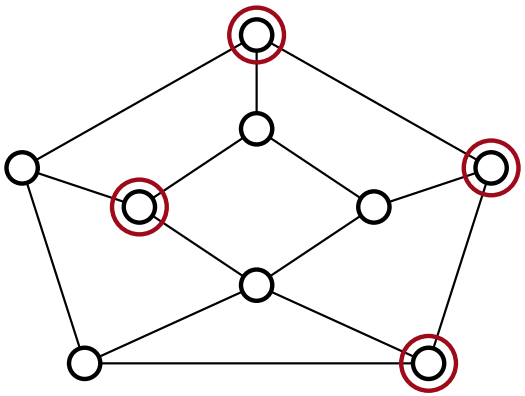

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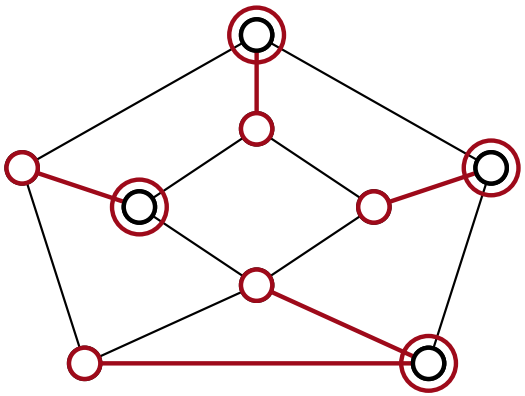
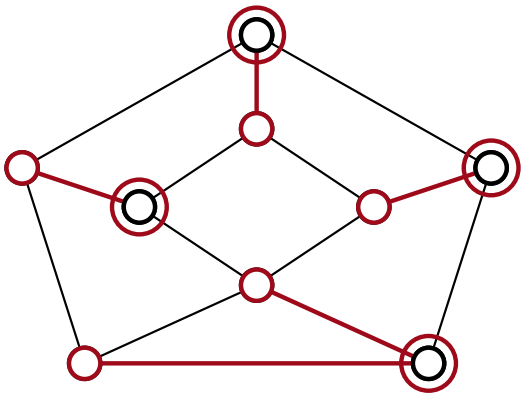


Graph domination

Dominating set D of G: for any $v \in V$, $v \in D$ or there exists $u \in D$ with $uv \in E$



Applications:

Graph domination

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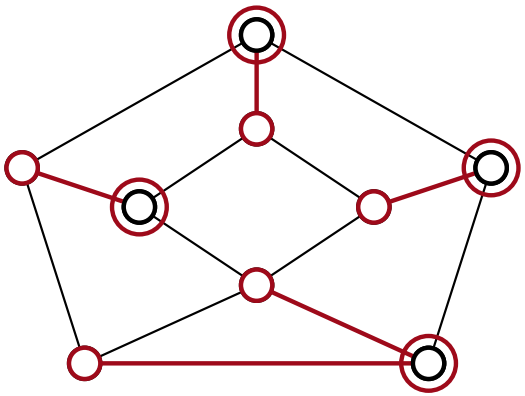
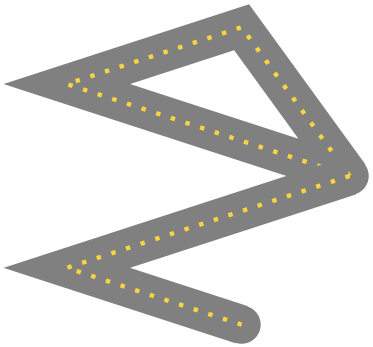


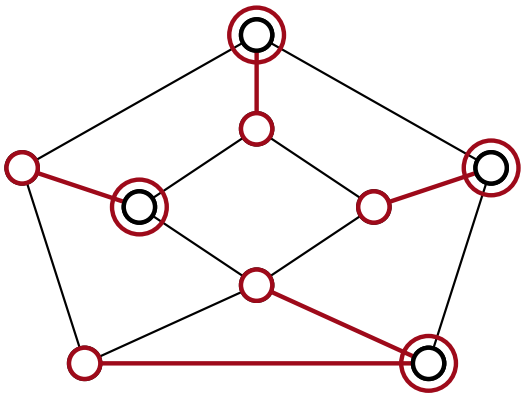
Applications:



Network monitoring

Graph domination

Dominating set D of G: for any $v \in V$, $v \in D$ or there exists $u \in D$ with $uv \in E$



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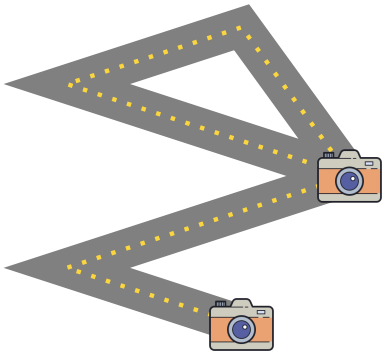

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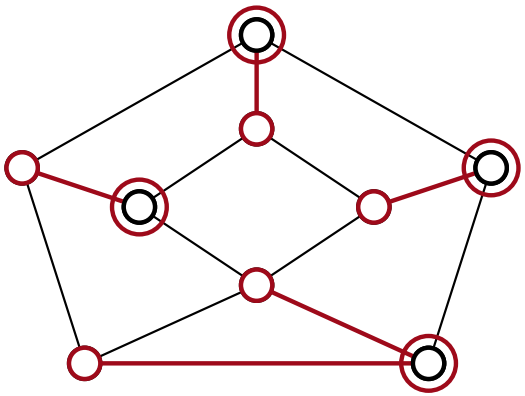

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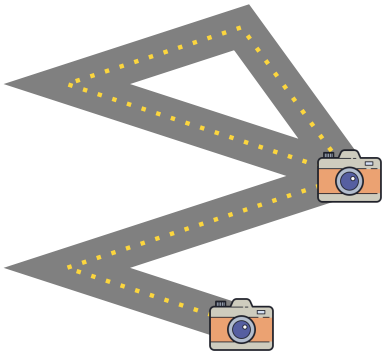

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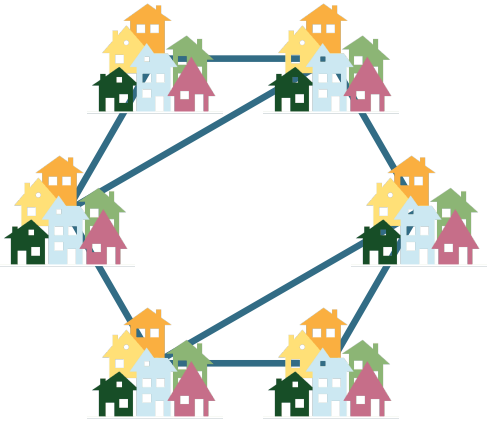

Facility location

Graph domination

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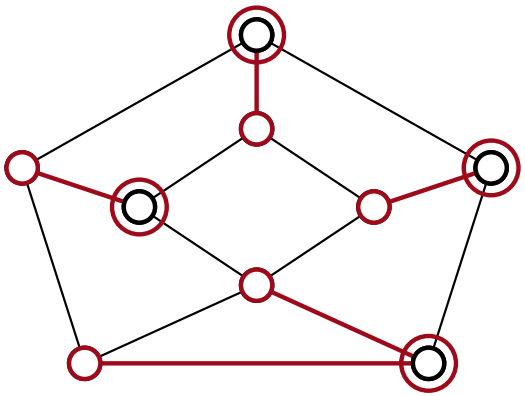

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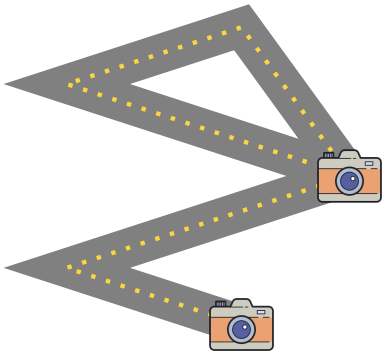

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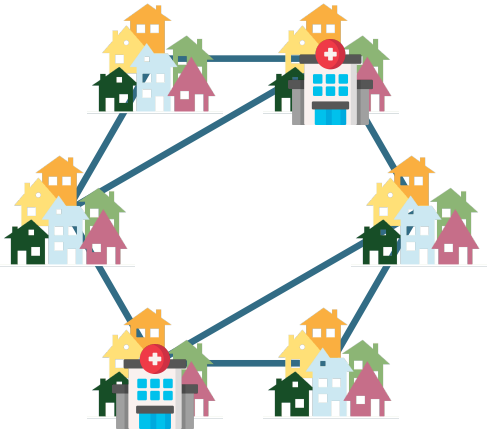

Facility location

Minimum dominating set

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γ: minimum size of a dominating set of G

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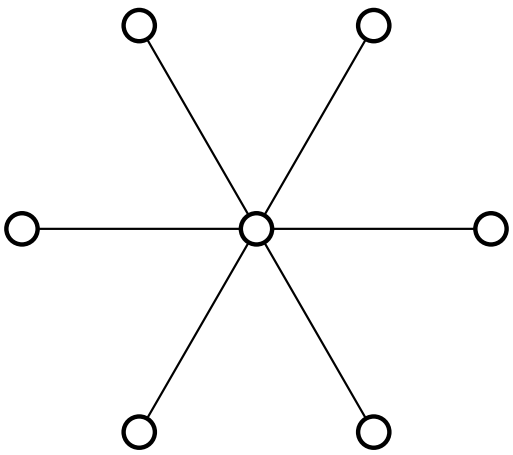

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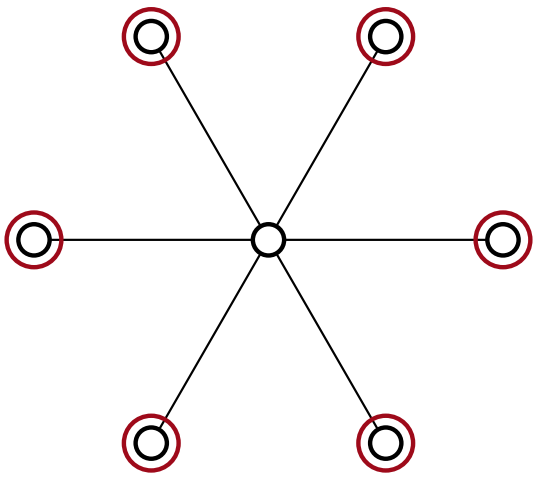

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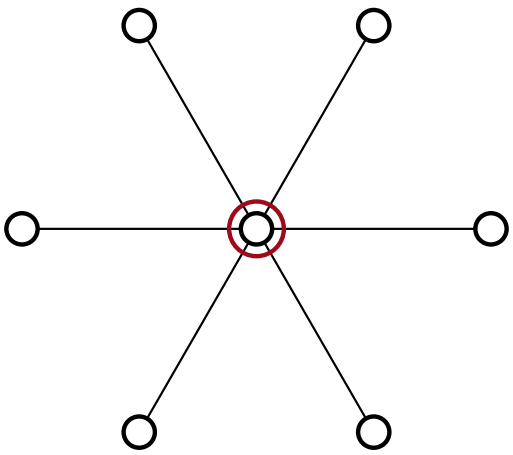
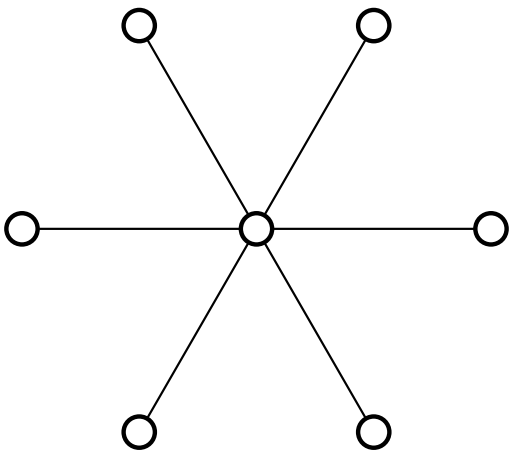


$\gamma = 1$

Minimum dominating set

γ: minimum size of a dominating set of G



DOMINATING SET

Minimum dominating set

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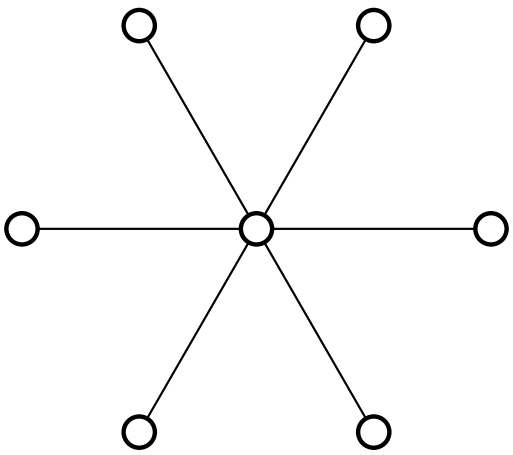
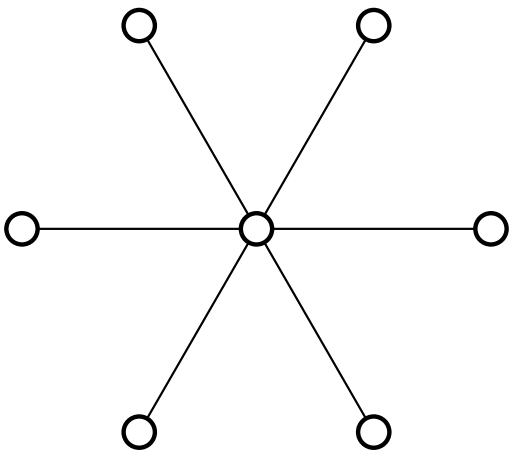


DOMINATING SET

INPUT: A graph G, an integer k

Minimum dominating set

γ: minimum size of a dominating set of G



DOMINATING SET

INPUT: A graph G, an integer k

OUTPUT: TRUE iff $\gamma(G) \leq k$

Minimum dominating set

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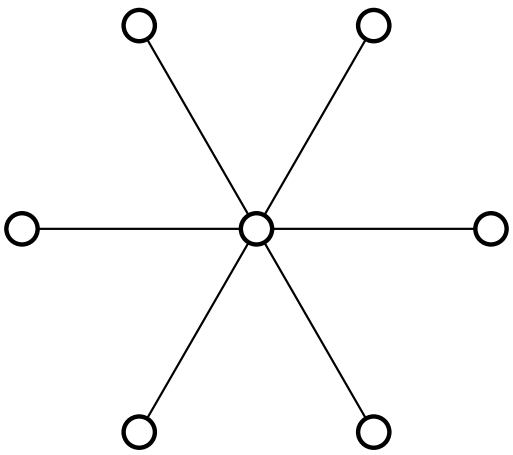

DOMINATING SET

INPUT: A graph G, an integer k

OUTPUT: TRUE iff $\gamma(G) \leq k$

DOMINATING SET is NP-complete (Garey & Johnson 79)

Reconfiguration

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Can we transform a solution of a problem into another by applying a sequence of changes that maintain a solution ?

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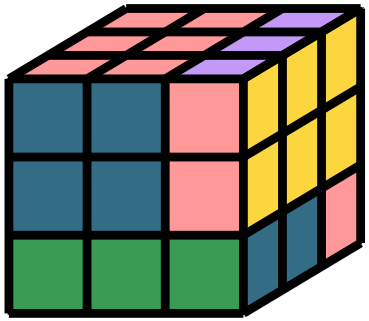

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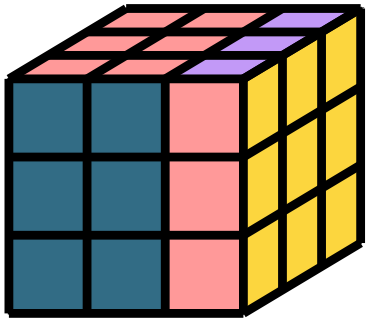

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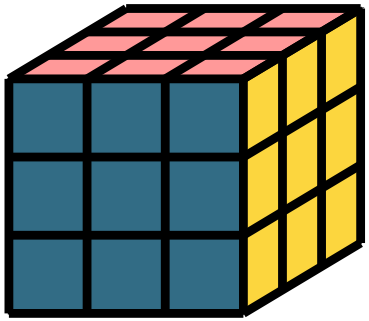

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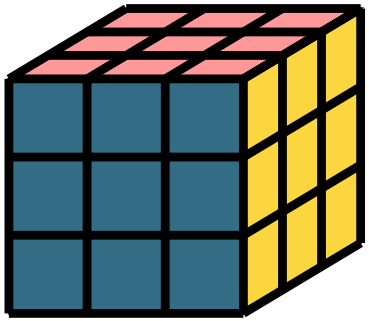

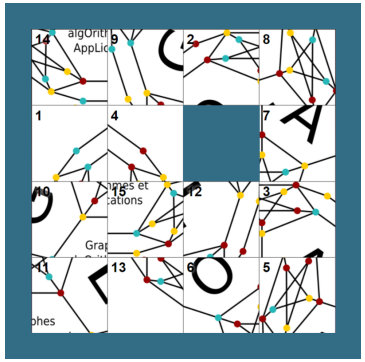

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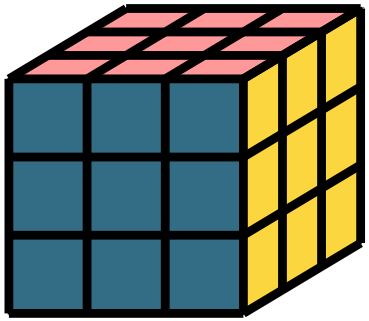

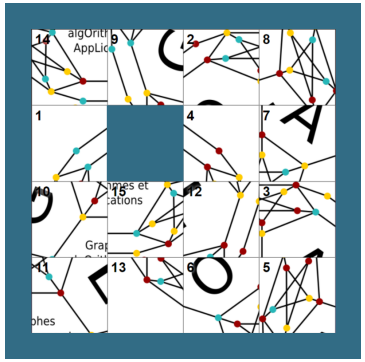

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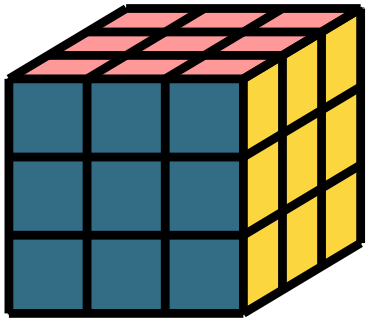

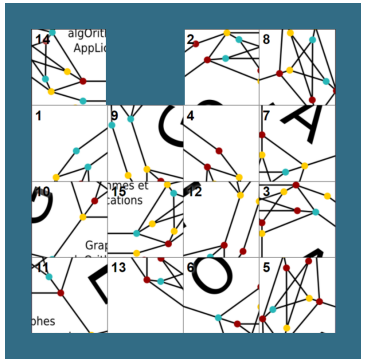

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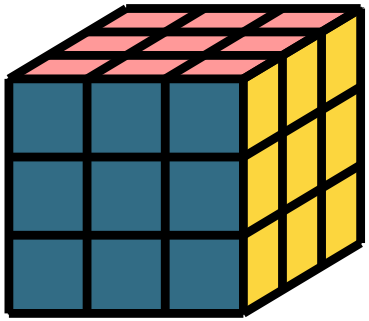

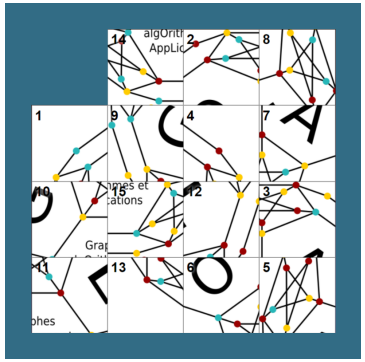

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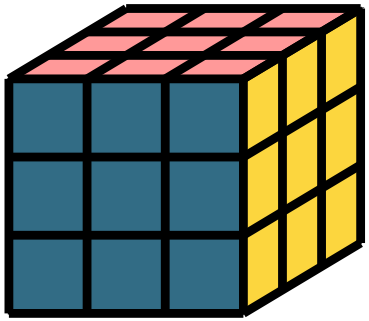

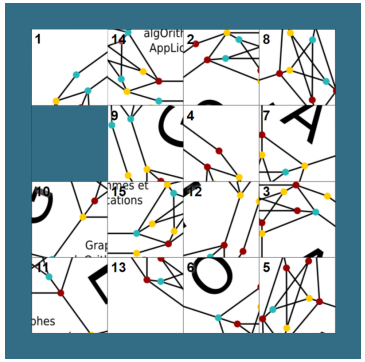

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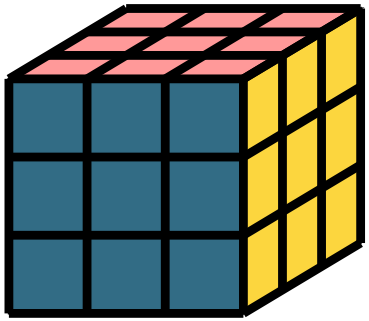

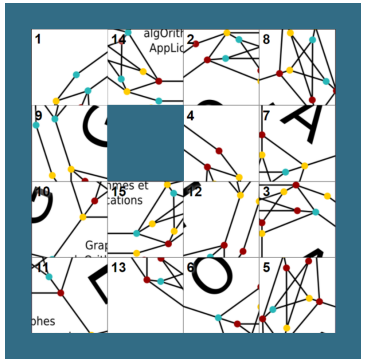

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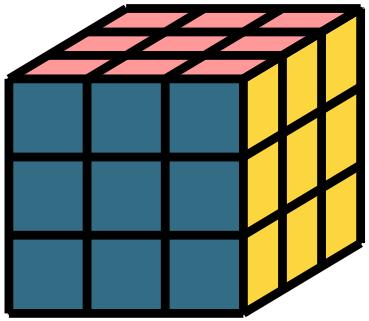

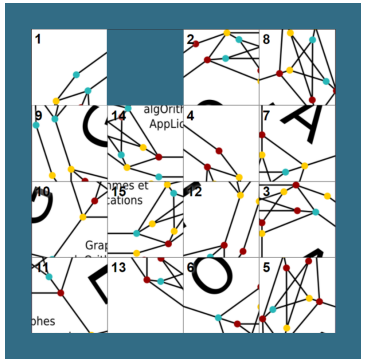

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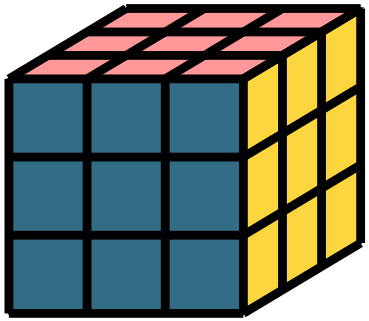

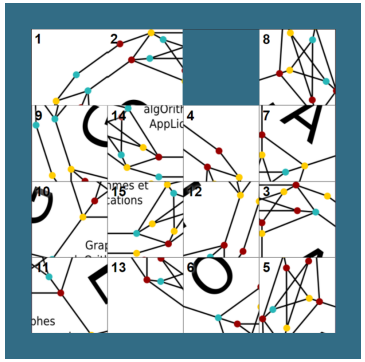

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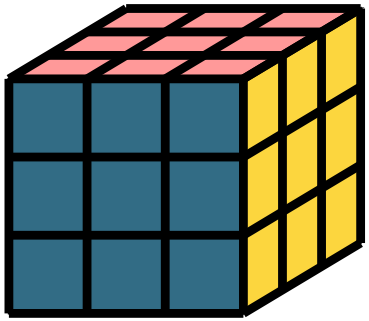

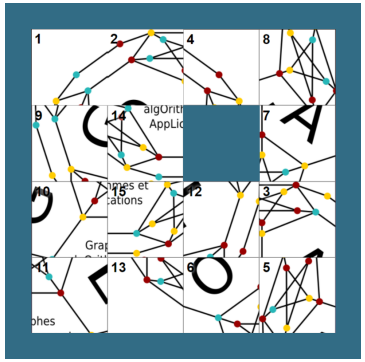

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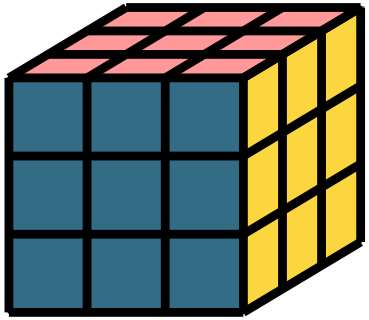

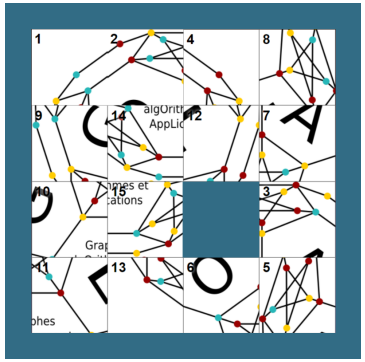

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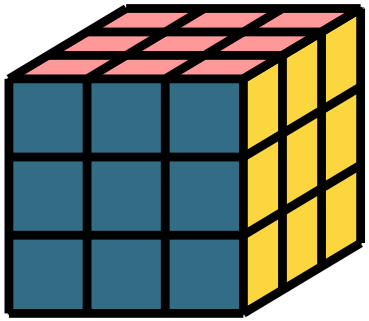

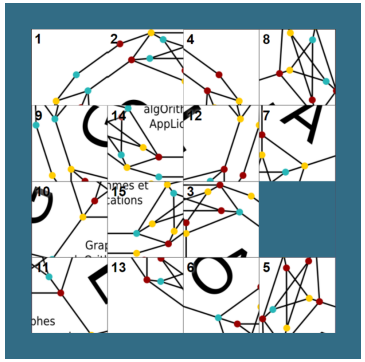

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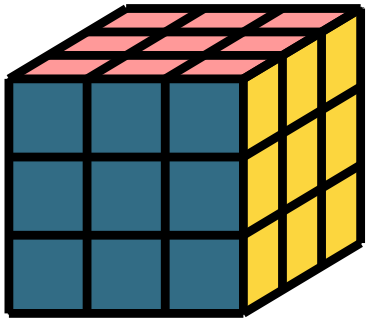

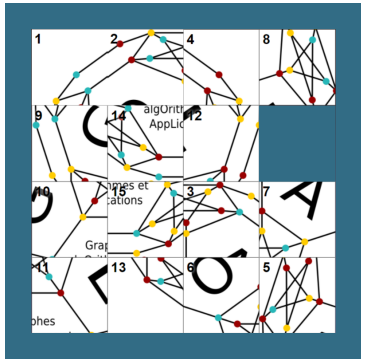

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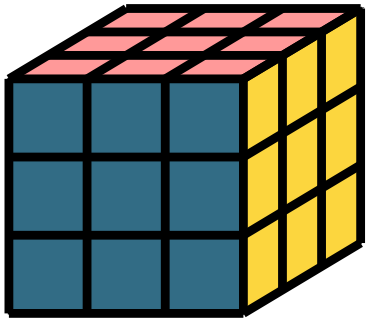

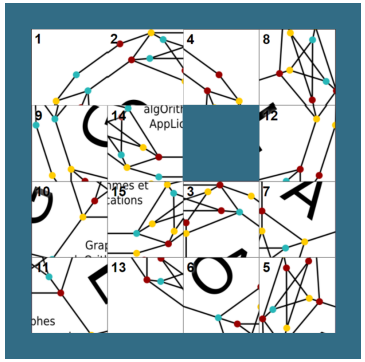

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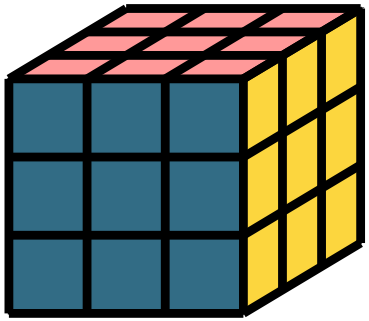

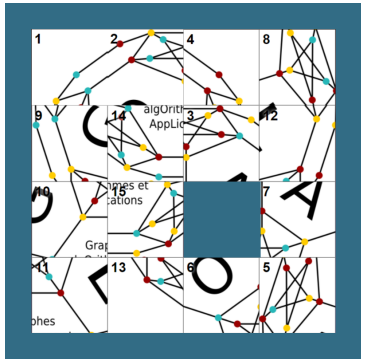

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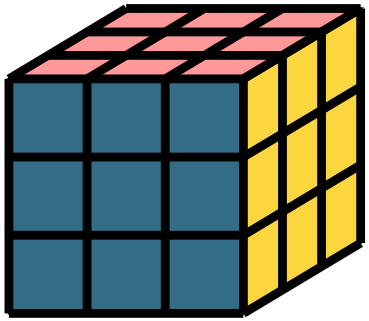

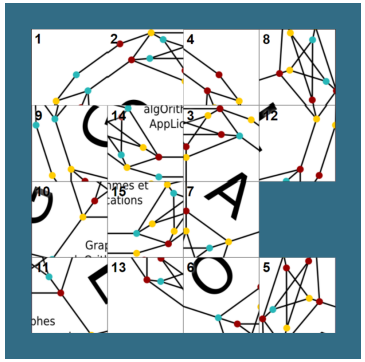

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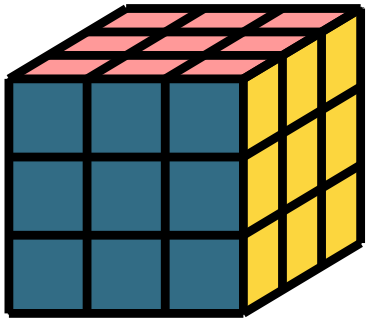

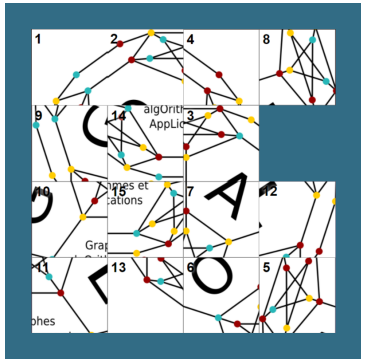

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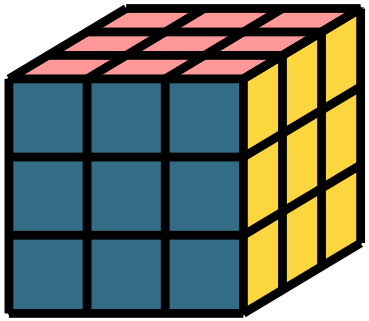

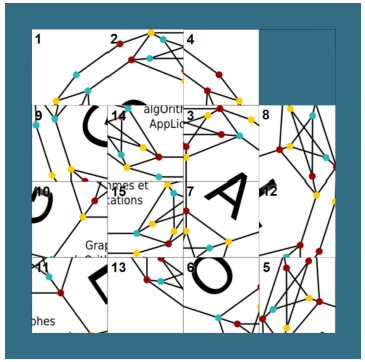

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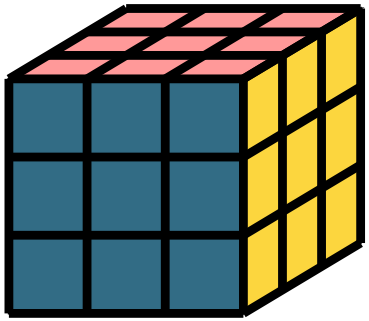

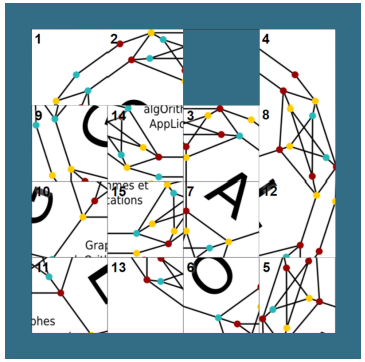

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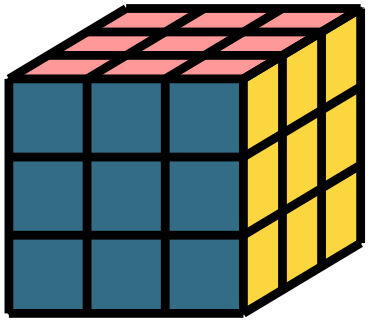

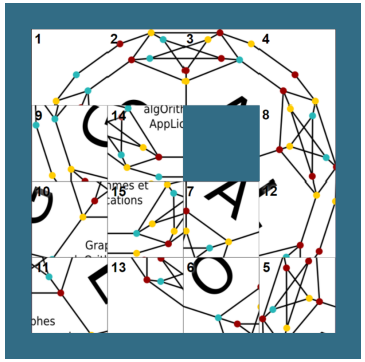

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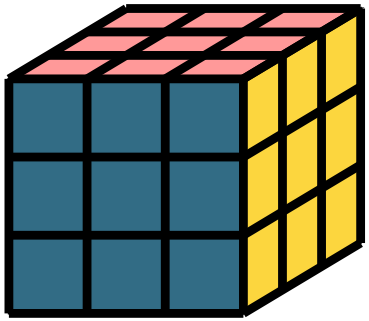

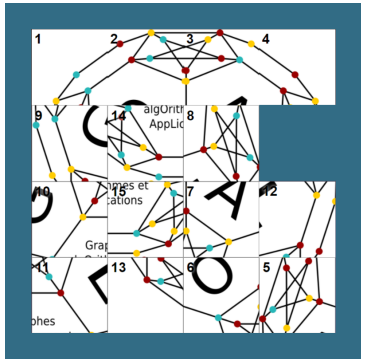

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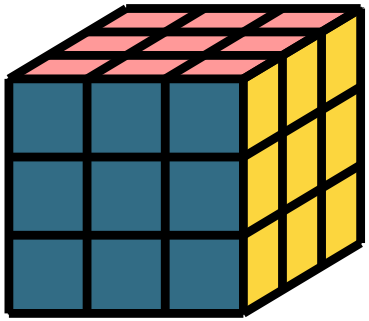

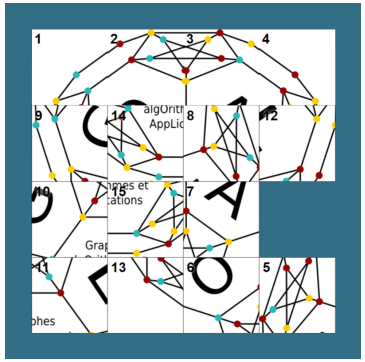

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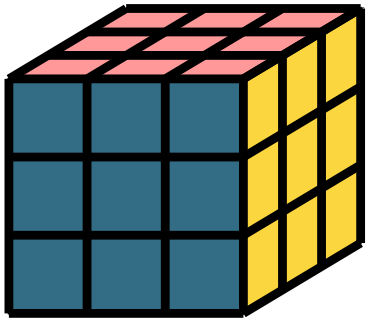

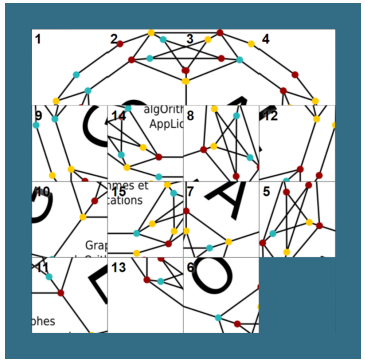

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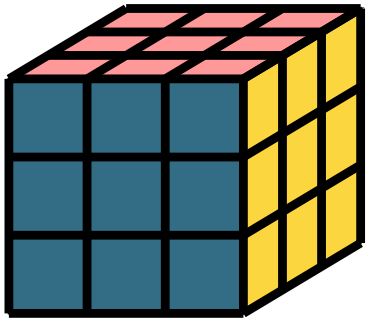

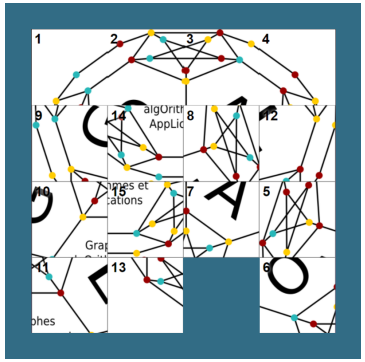

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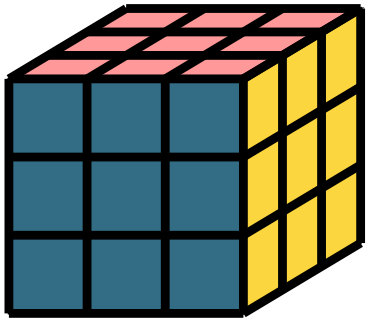

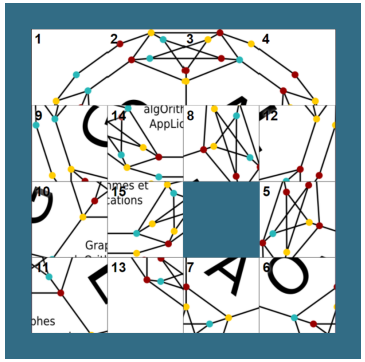

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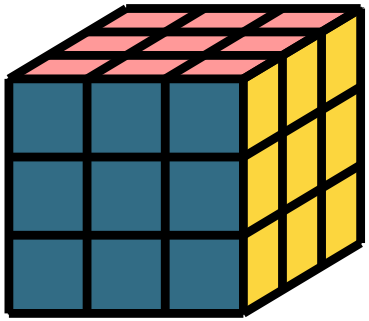

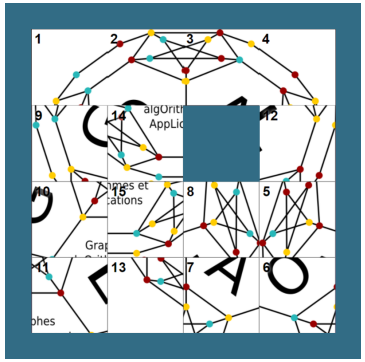

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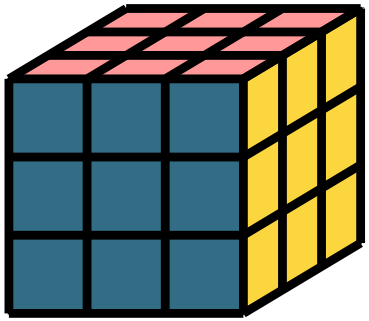

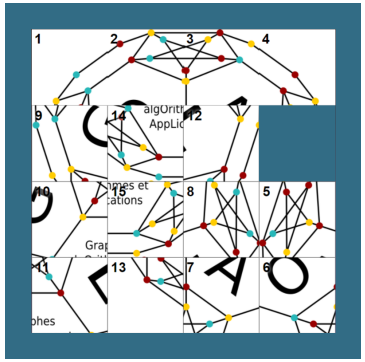

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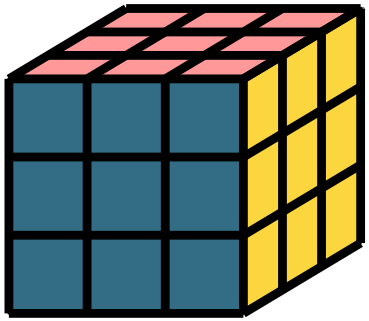

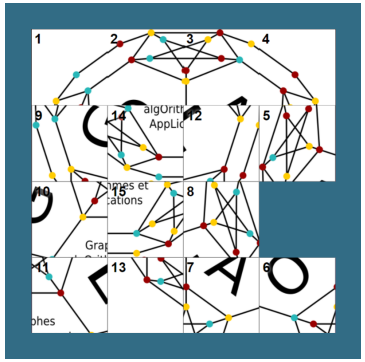

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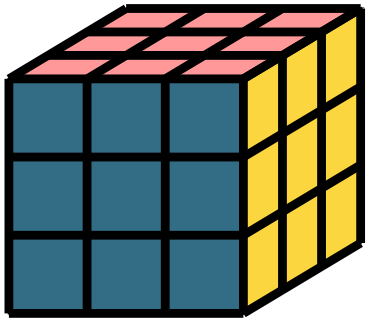

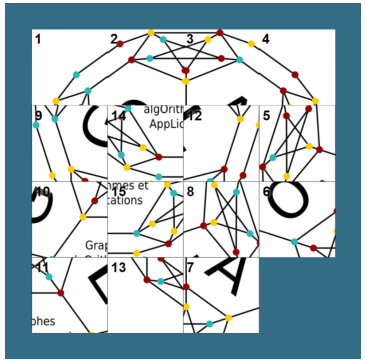

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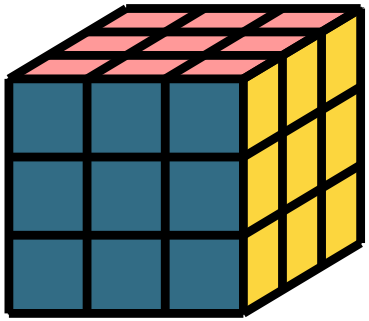

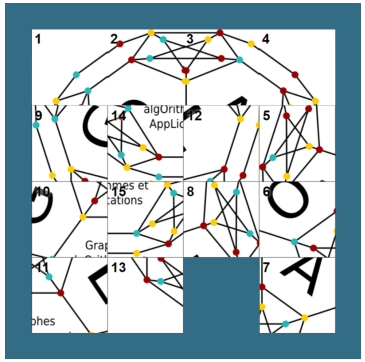

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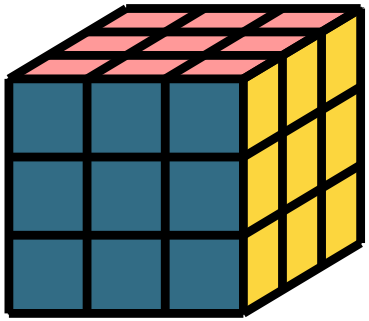

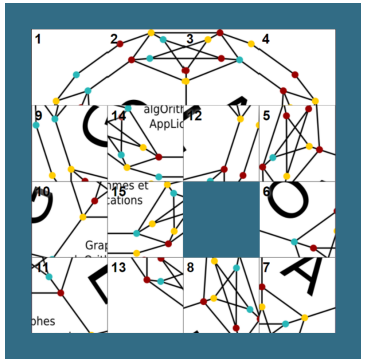

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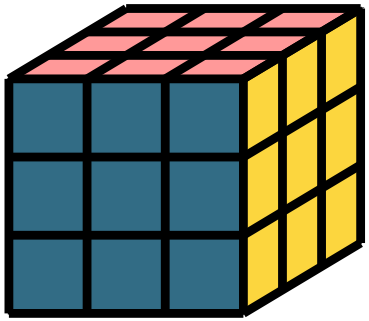

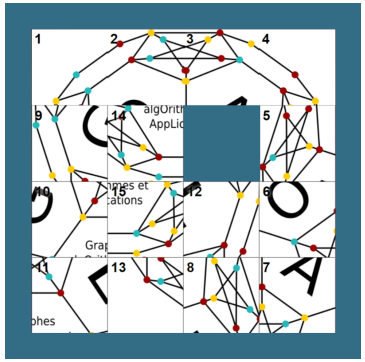

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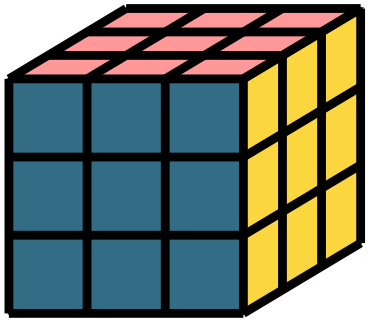

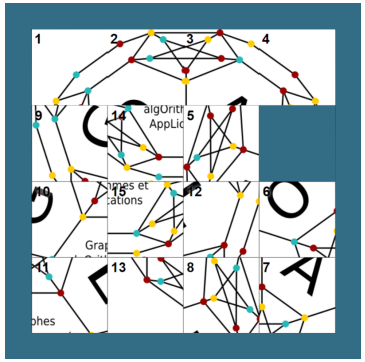

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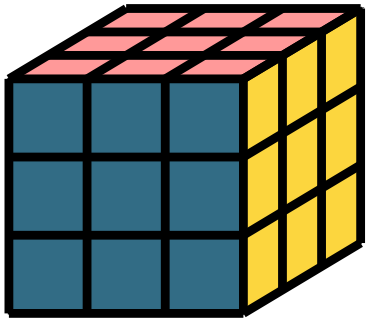

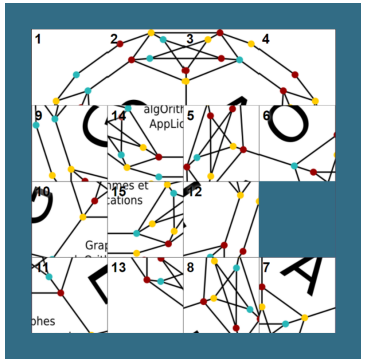

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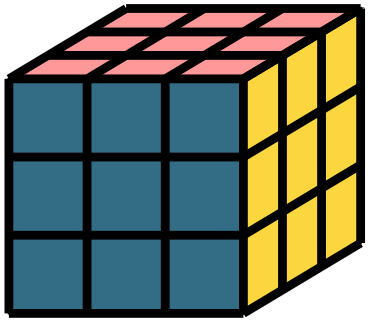

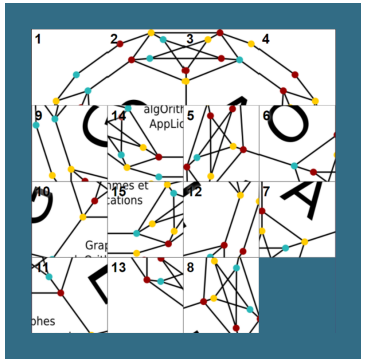

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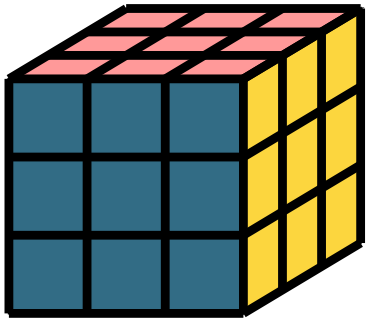

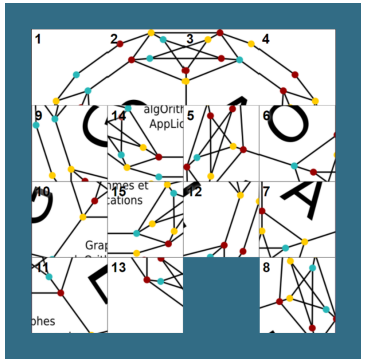

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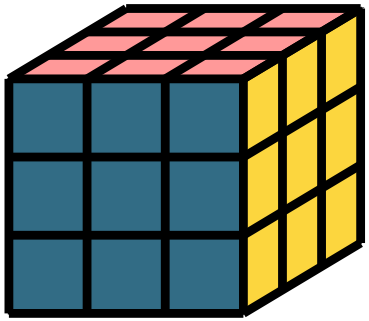

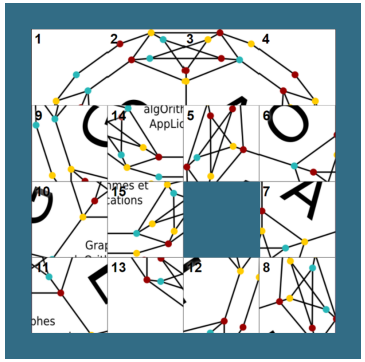

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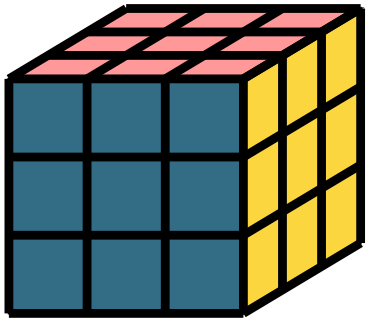

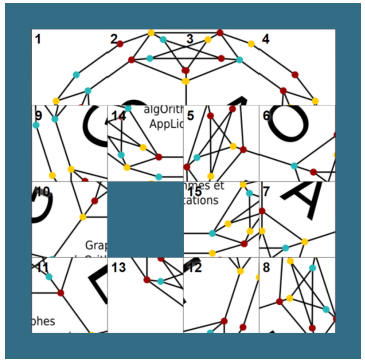

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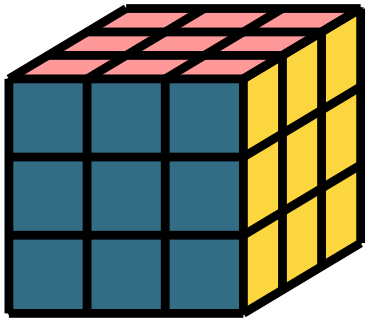

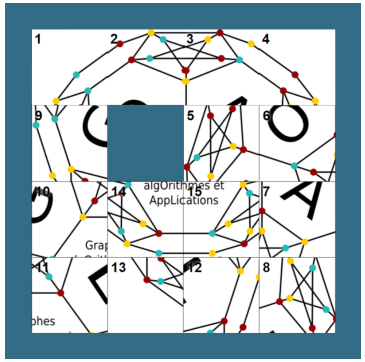

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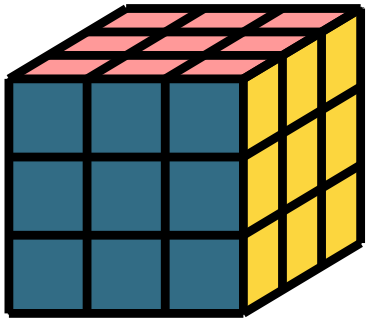

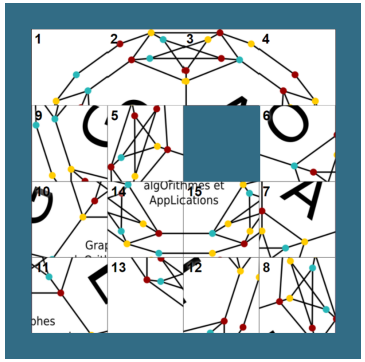

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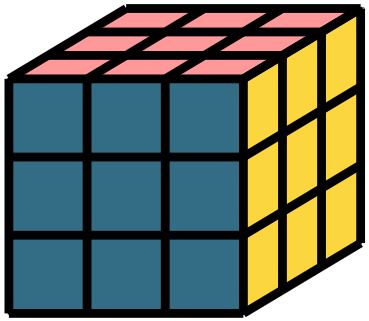

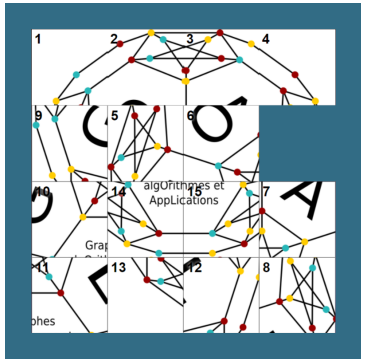

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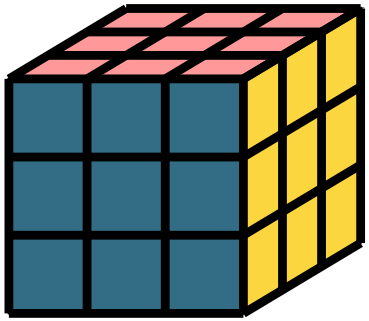

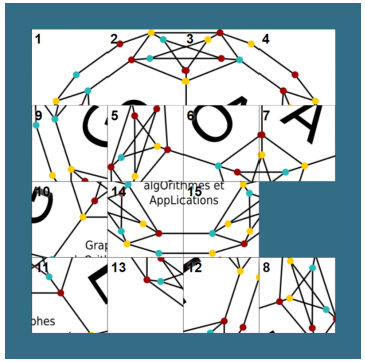

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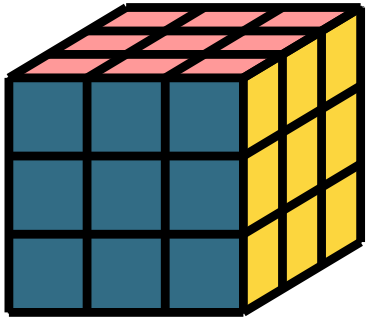

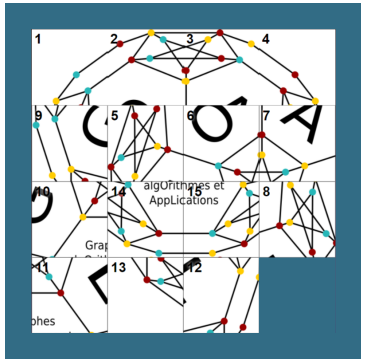

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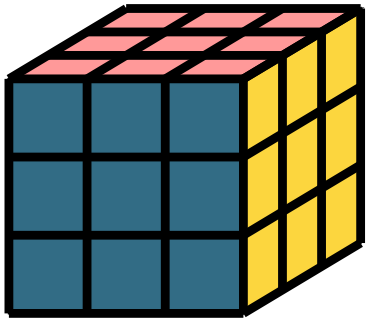

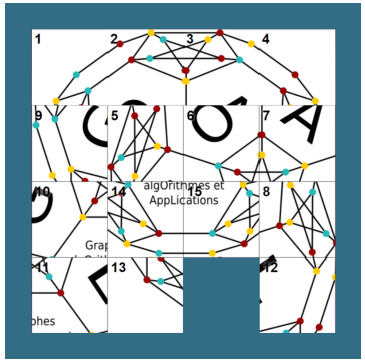

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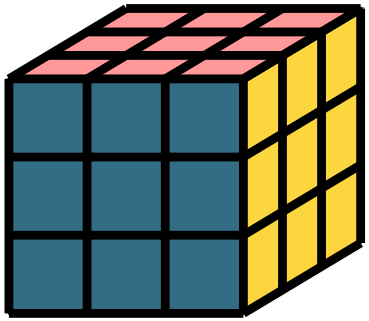

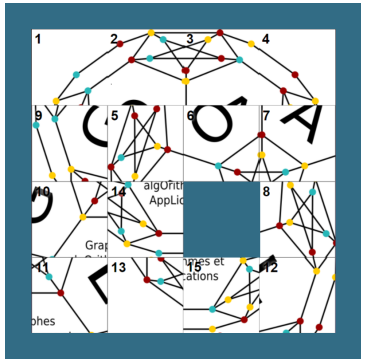

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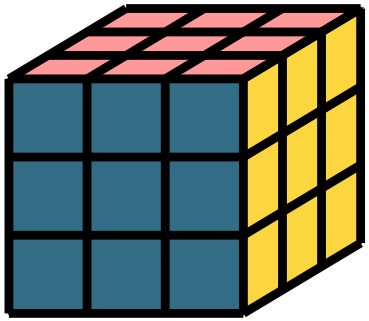

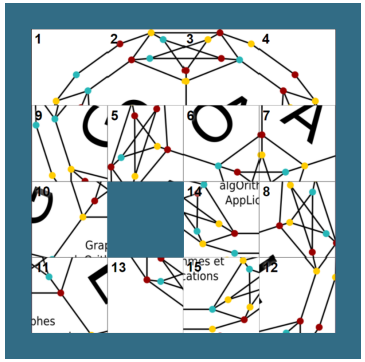

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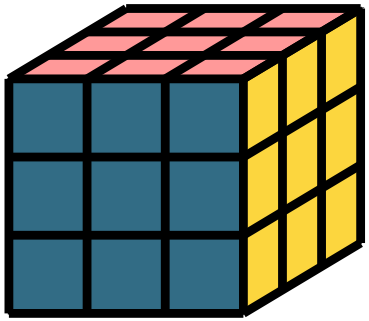

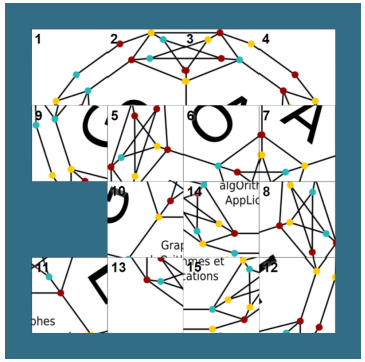

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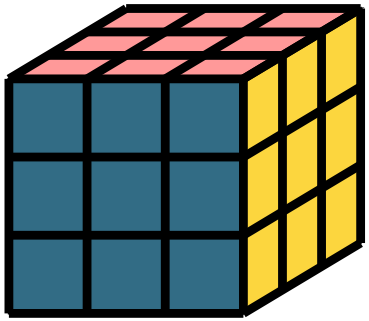

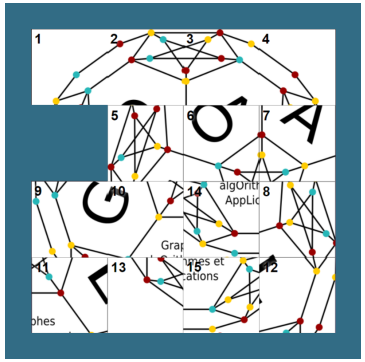

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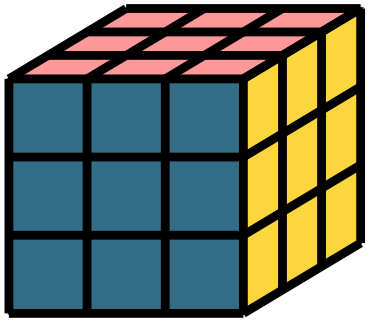

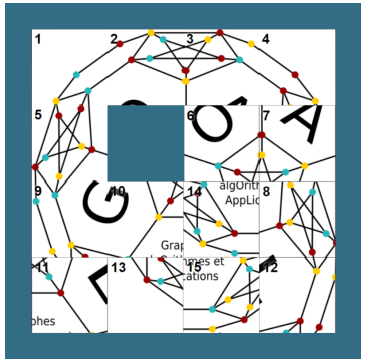

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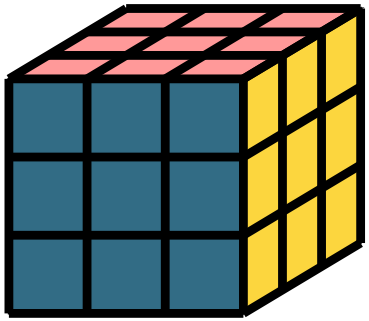

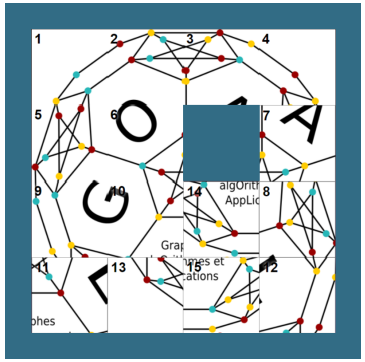

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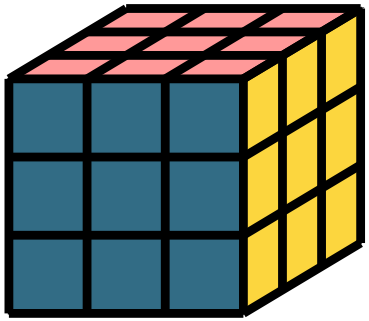

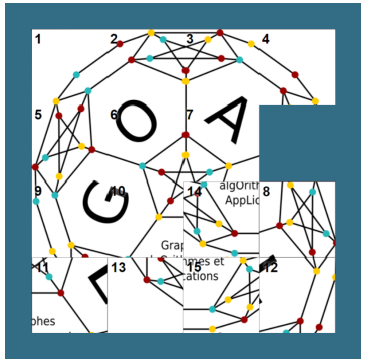

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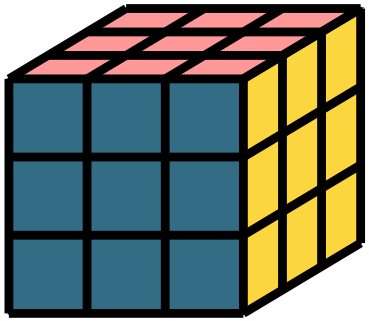

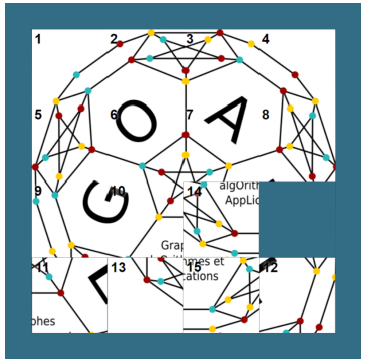

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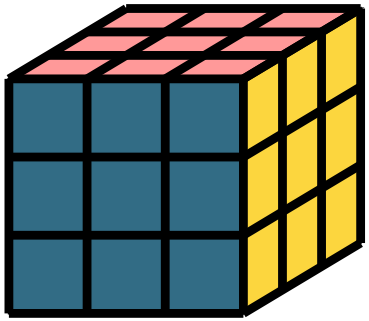

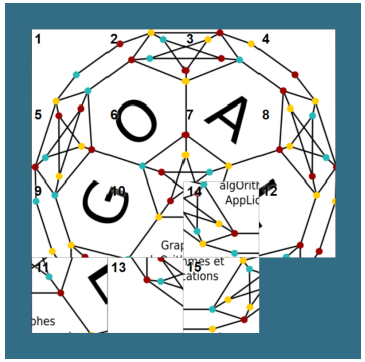

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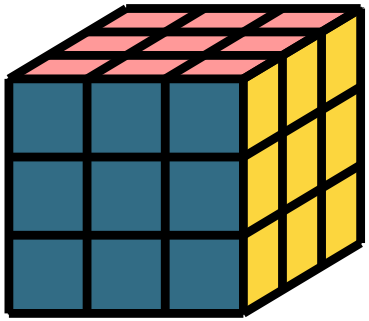

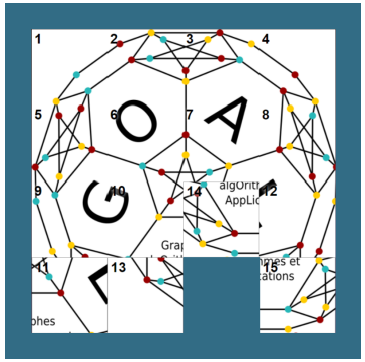

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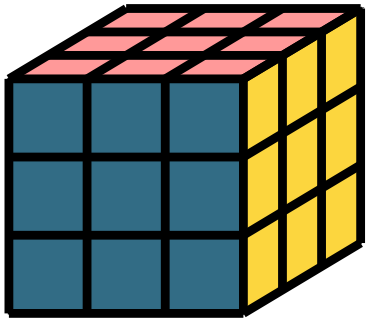

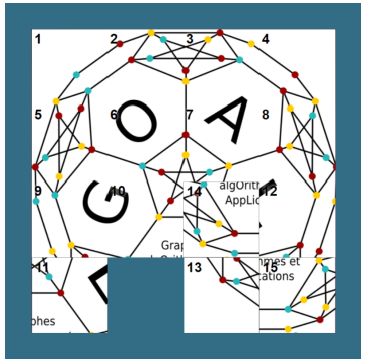

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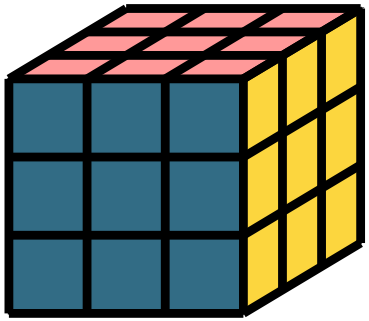

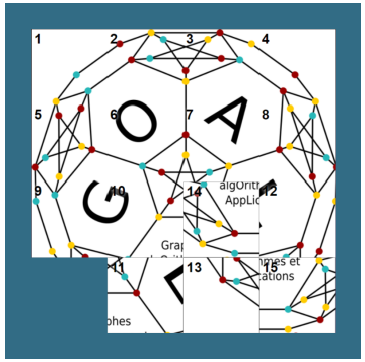

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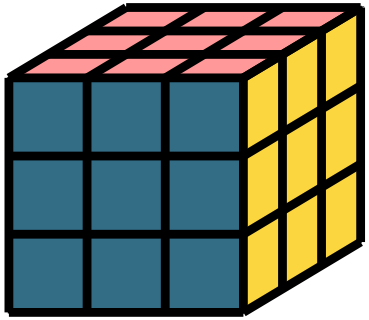

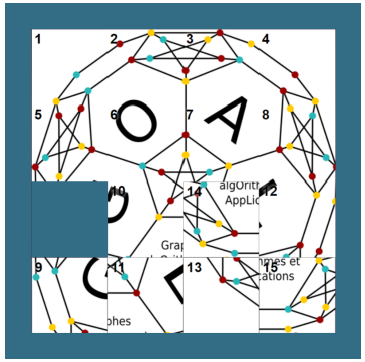

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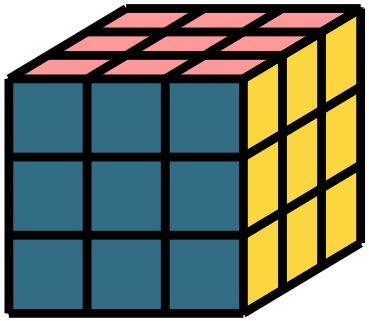

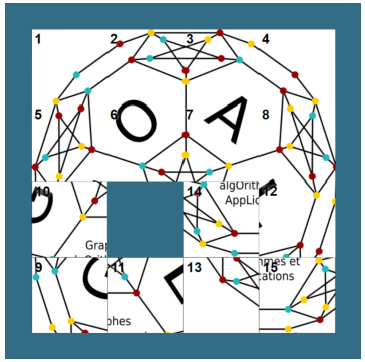

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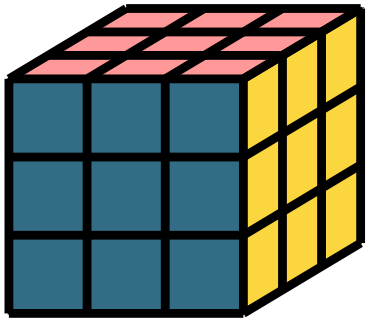

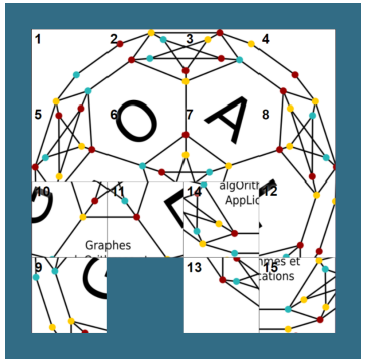

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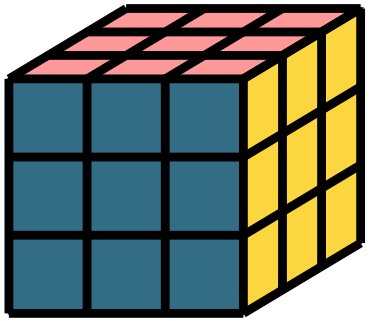

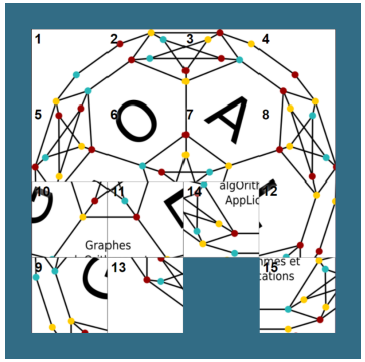

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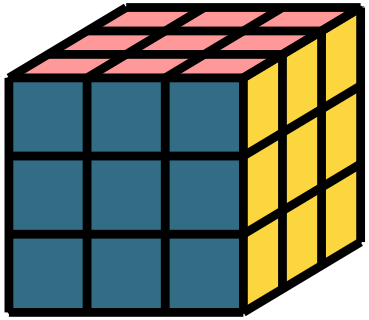

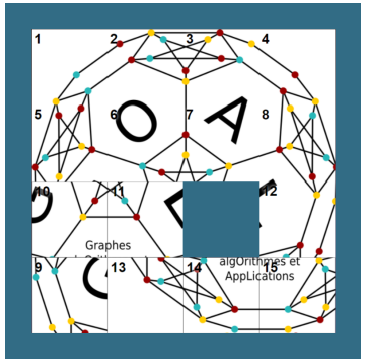

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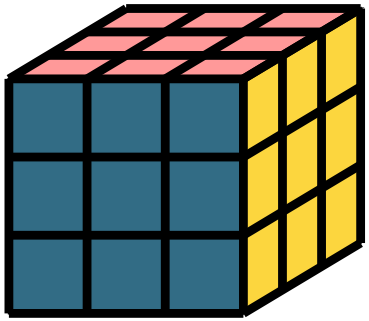

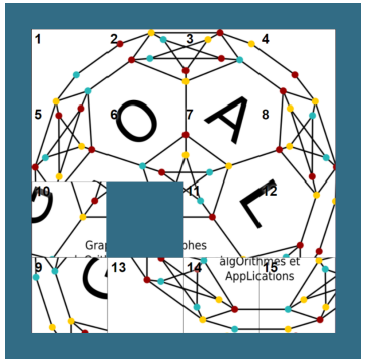

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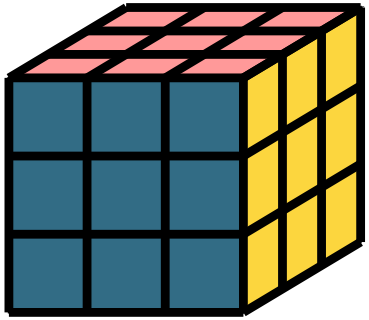

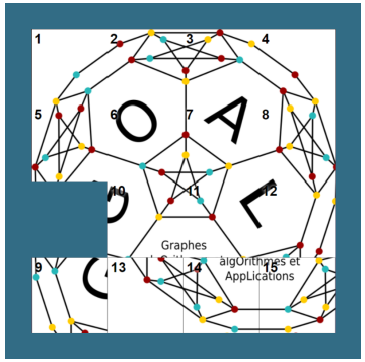

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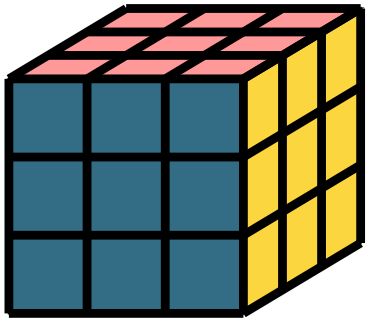

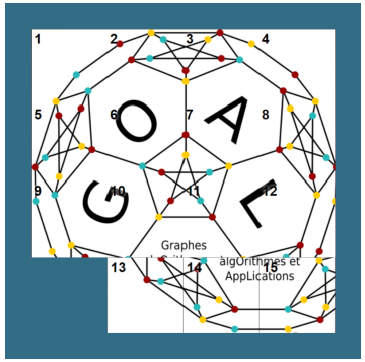

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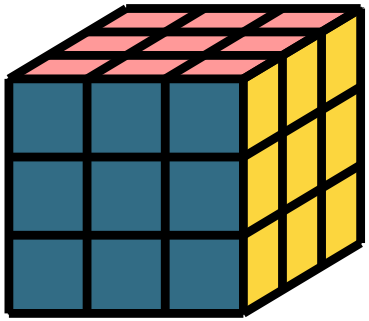

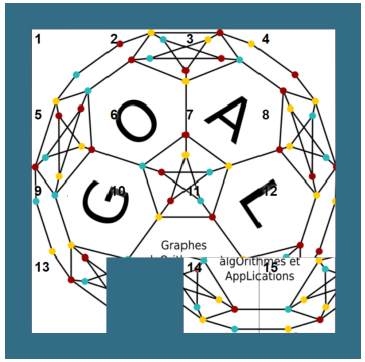

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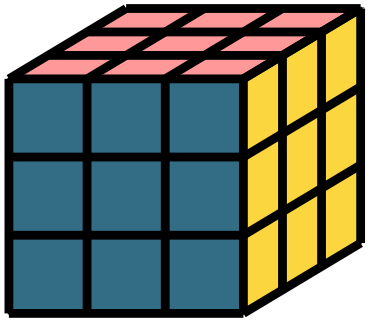

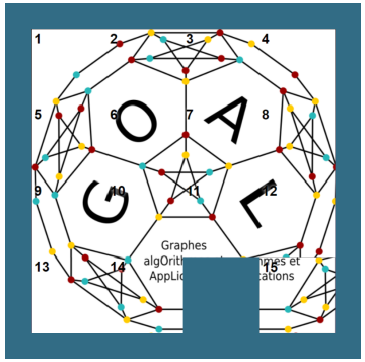

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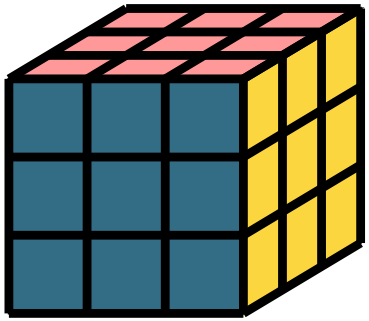

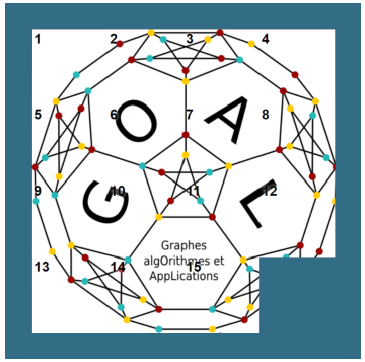

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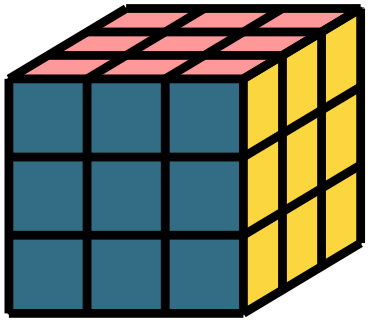

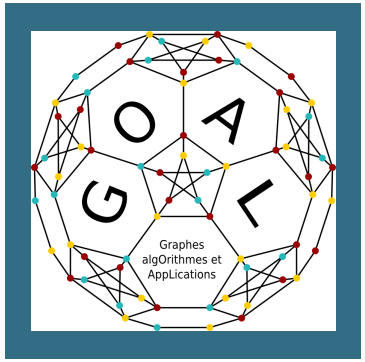

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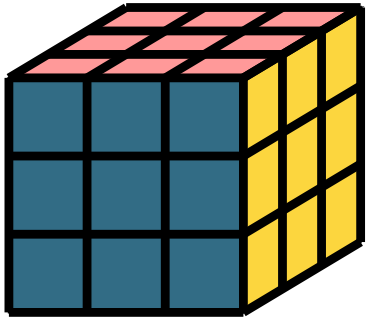

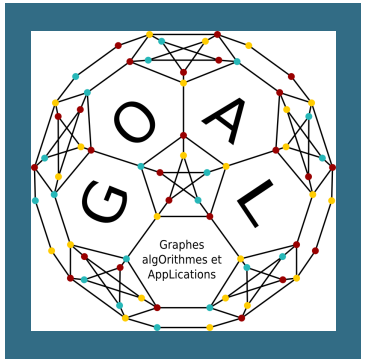

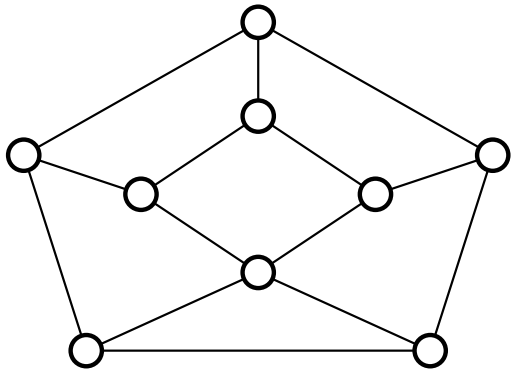

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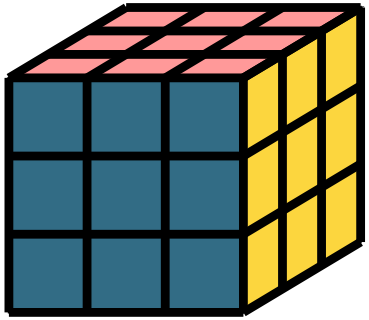

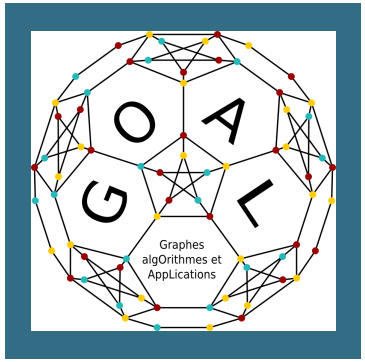

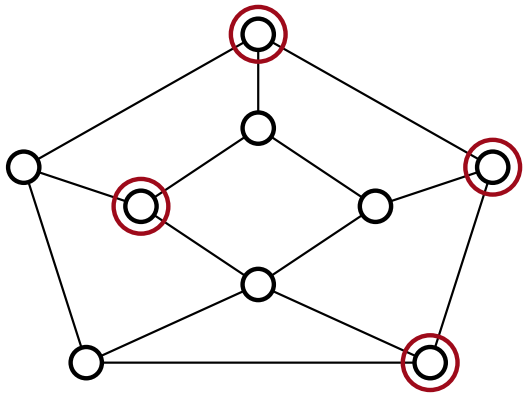

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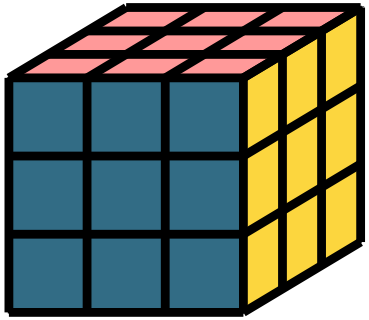

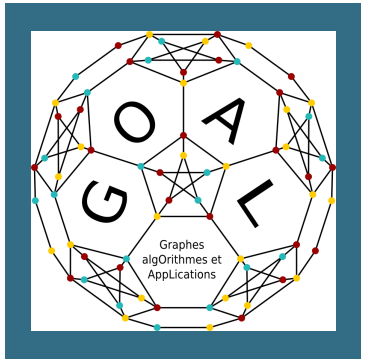

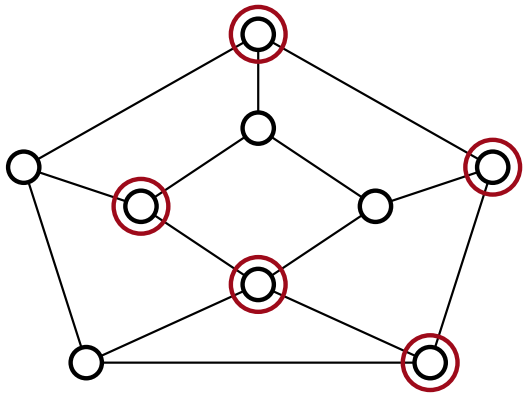

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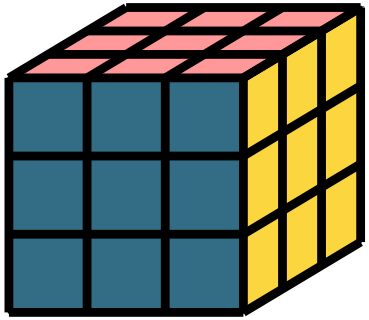

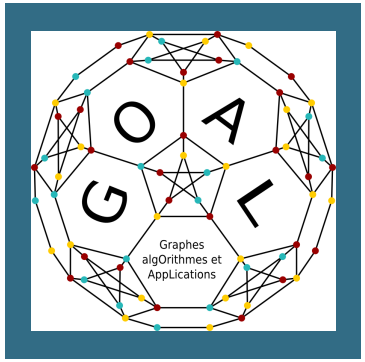

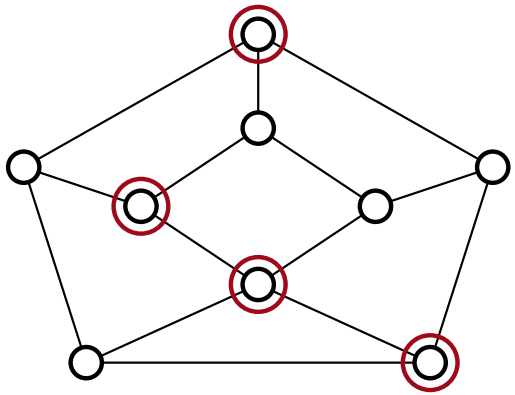

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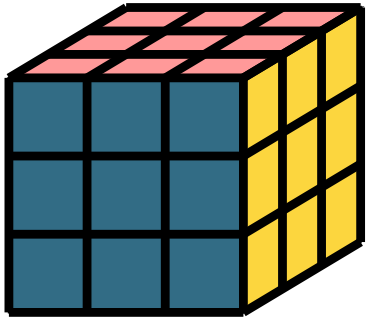
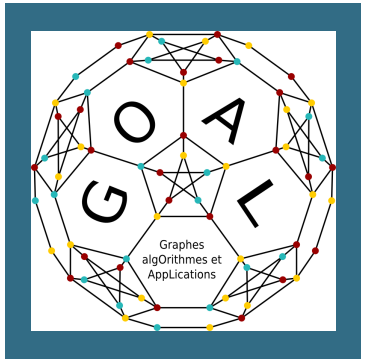
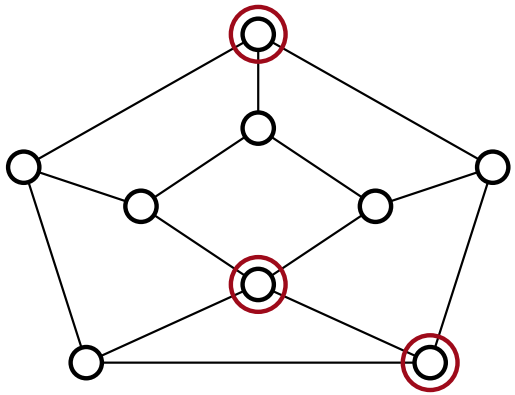
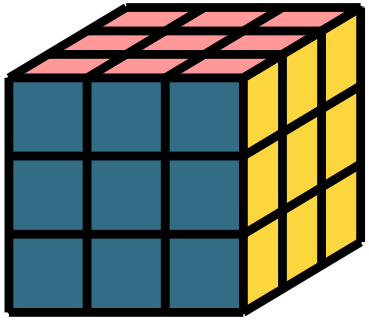


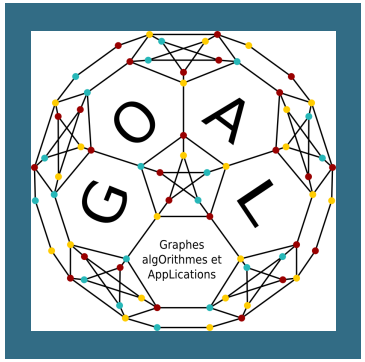


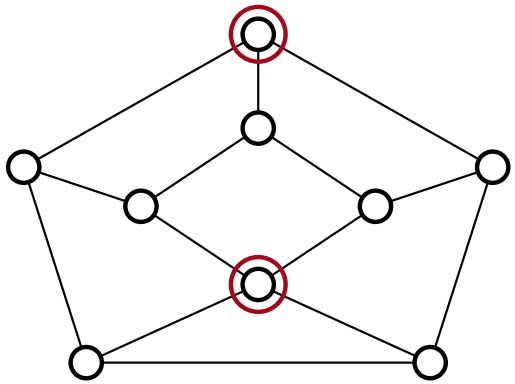


Reconfiguration

Can we transform a solution of a problem into another by applying a sequence of changes that maintain a solution ?







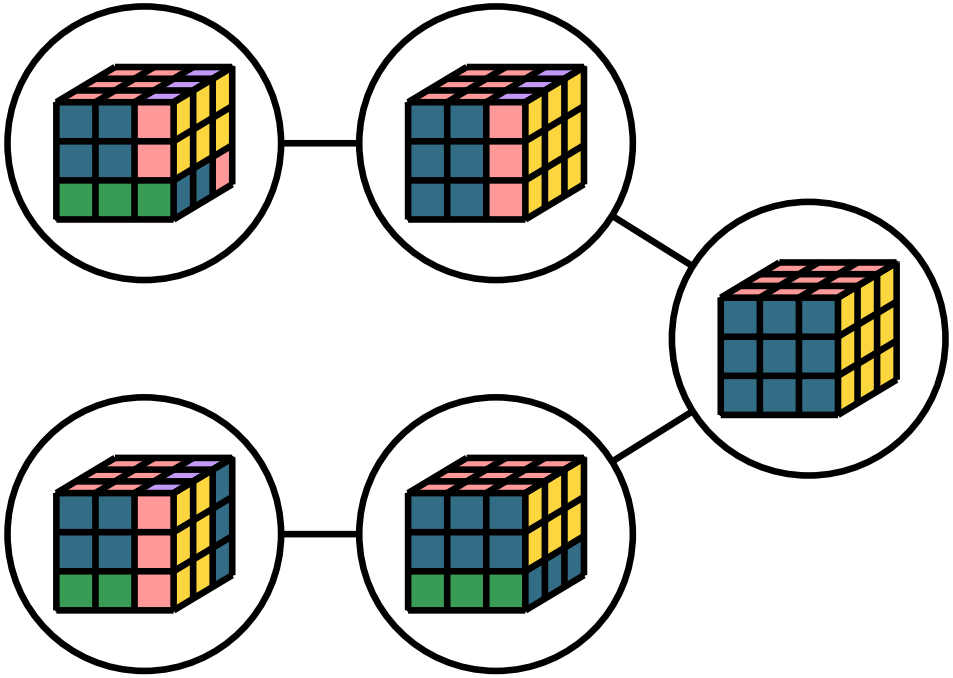
Reconfiguration

Reconfiguration

Reconfiguration graph $\mathcal{R}$: the vertices are the solutions, two solutions are adjacent if they are different by a single change

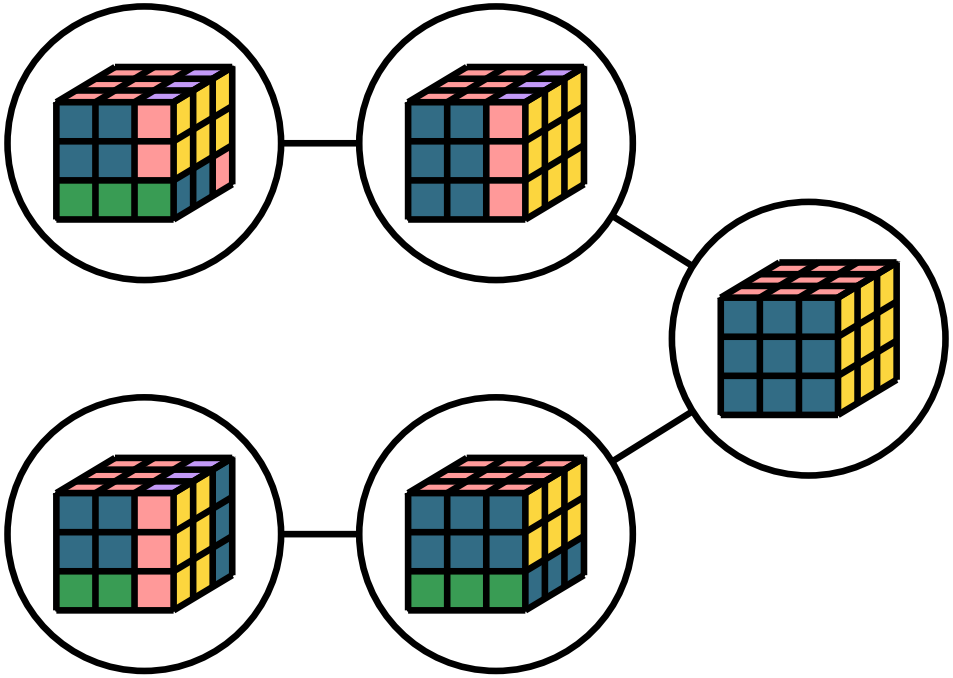
Reconfiguration

Reconfiguration graph $\mathcal{R}$: the vertices are the solutions, two solutions are adjacent if they are different by a single change



Reconfiguration

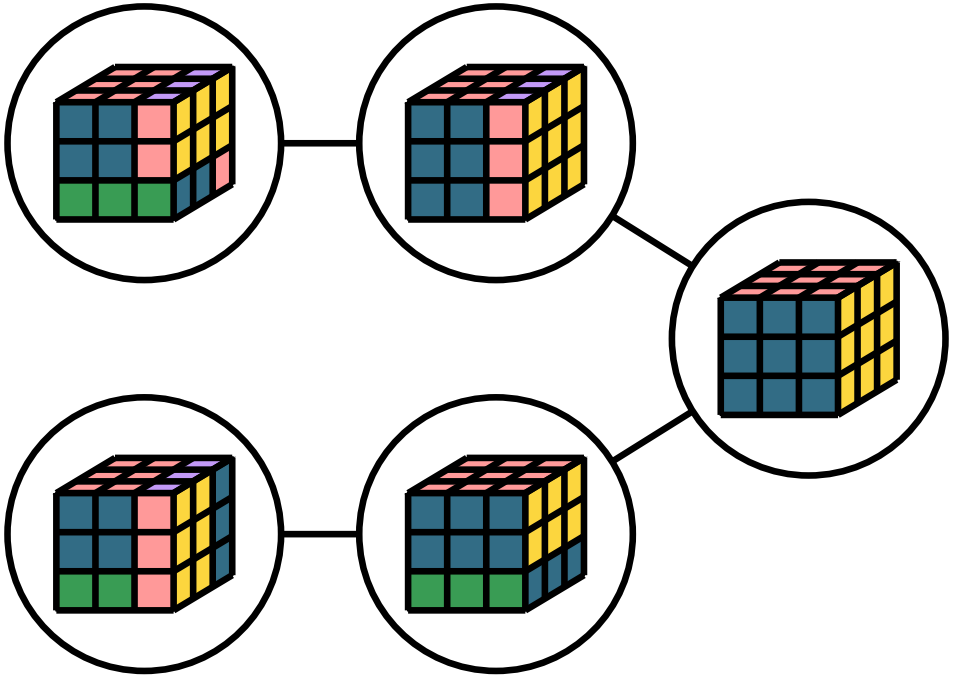
Reconfiguration graph $\mathcal{R}$: the vertices are the solutions, two solutions are adjacent if they are different by a single change



Questions:

Reconfiguration

Reconfiguration graph $\mathcal{R}$: the vertices are the solutions, two solutions are adjacent if they are different by a single change

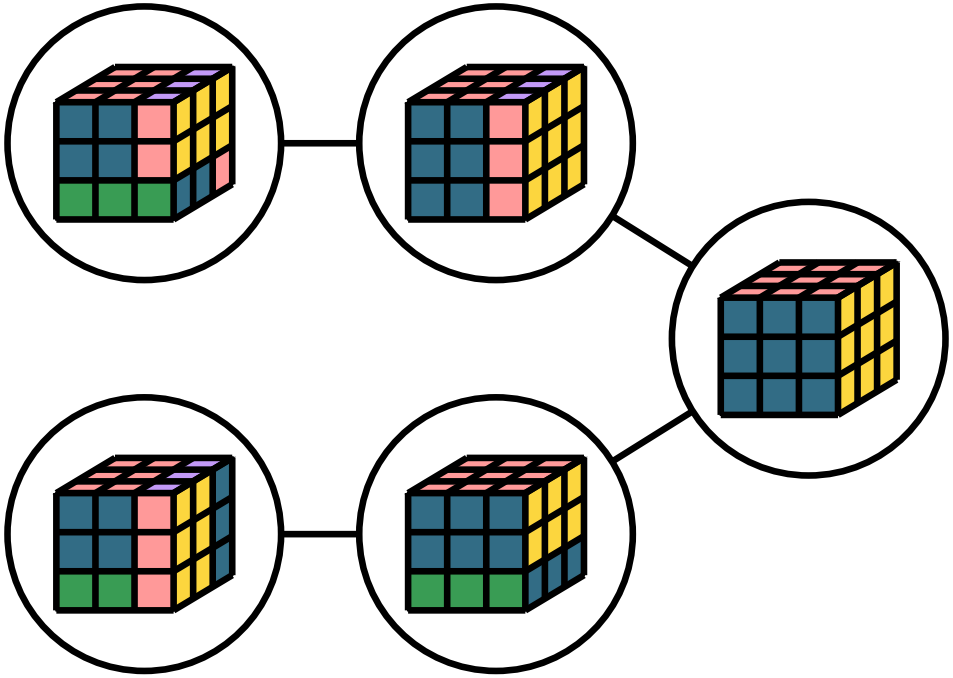


Questions:

- Given $\mathcal{S}_s$ and $\mathcal{S}_t$, is there a path from $\mathcal{S}_s$ to $\mathcal{S}_t$?

Reconfiguration

Reconfiguration graph $\mathcal{R}$: the vertices are the solutions, two solutions are adjacent if they are different by a single change

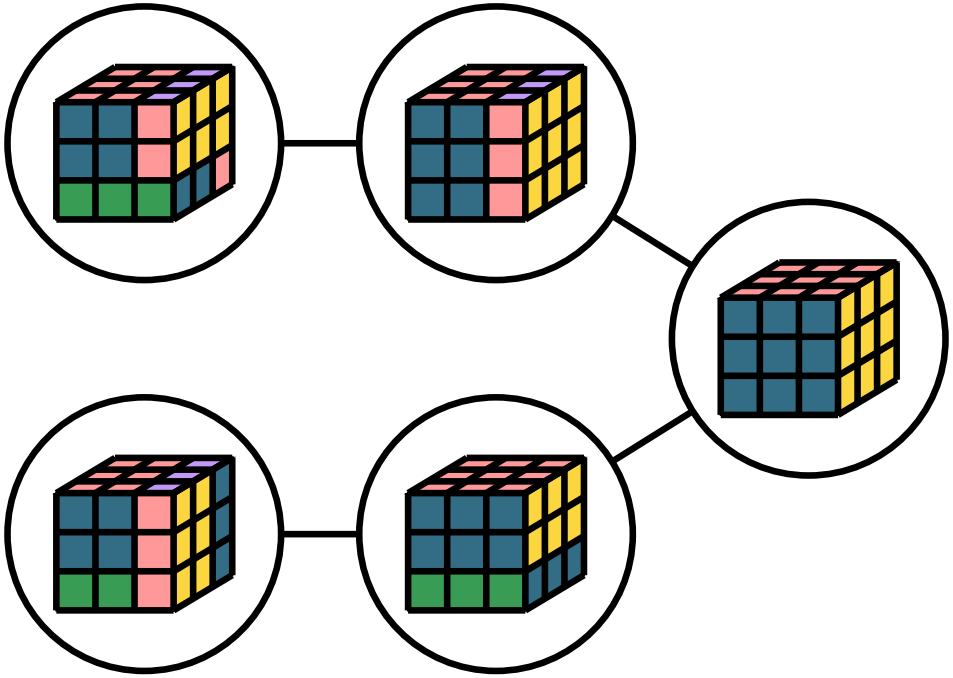


Questions:

- Given $\mathcal{S}_s$ and $\mathcal{S}_t$, is there a path from $\mathcal{S}_s$ to $\mathcal{S}_t$?
- Given $\mathcal{S}_s$ and $\mathcal{S}_t$, what is the minimum length of such a path ?

Reconfiguration

Reconfiguration graph $\mathcal{R}$: the vertices are the solutions, two solutions are adjacent if they are different by a single change



Questions:

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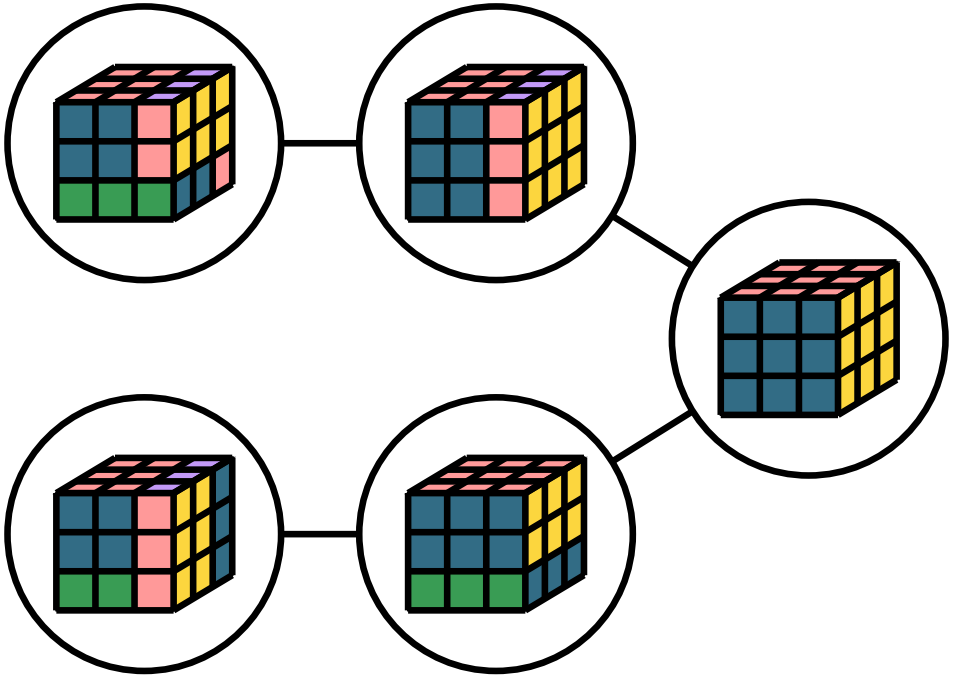
- Given $\mathcal{S}_s$ and $\mathcal{S}_t$, what is the minimum length of such a path ?
- Is $\mathcal{R}$ connected ?

Reconfiguration

Reconfiguration graph $\mathcal{R}$: the vertices are the solutions, two solutions are adjacent if they are different by a single change

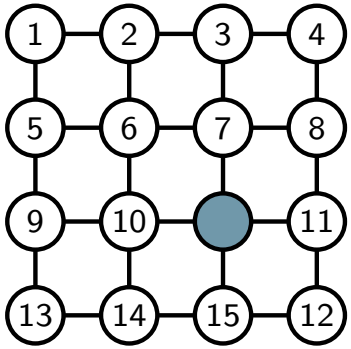


Questions:

- Given $\mathcal{S}_s$ and $\mathcal{S}_t$, is there a path from $\mathcal{S}_s$ to $\mathcal{S}_t$?
- Given $\mathcal{S}_s$ and $\mathcal{S}_t$, what is the minimum length of such a path ?
- Is $\mathcal{R}$ connected ?
- What is the diameter of $\mathcal{R}$?

Reconfiguration of tokens on a graph

Reconfiguration of tokens on a graph



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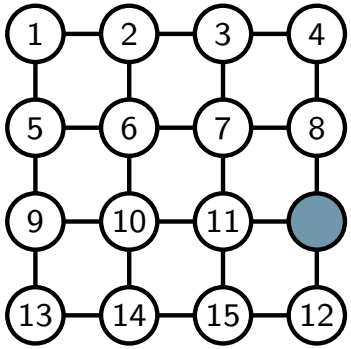
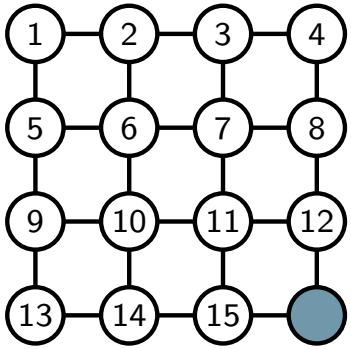


Reconfiguration of tokens on a graph



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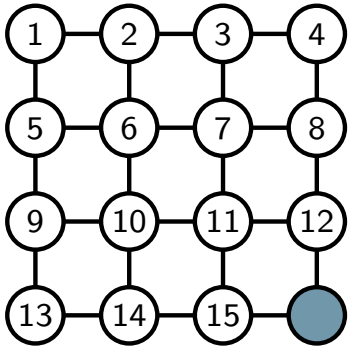

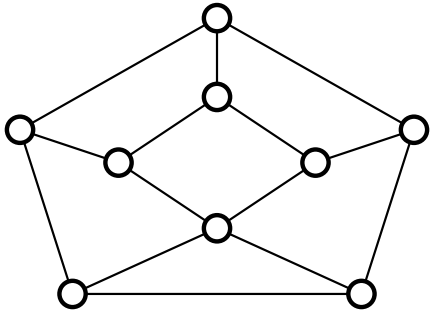

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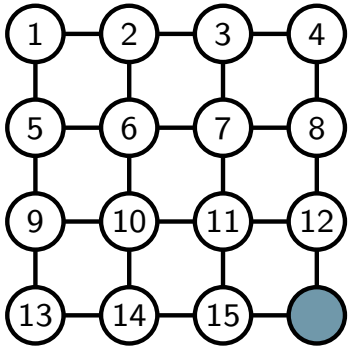
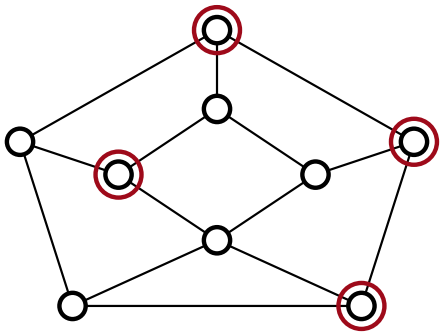
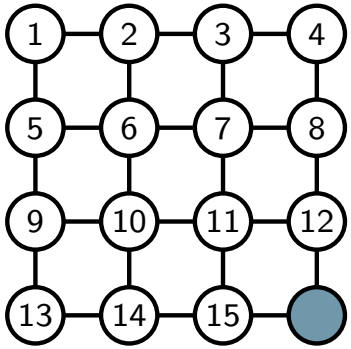
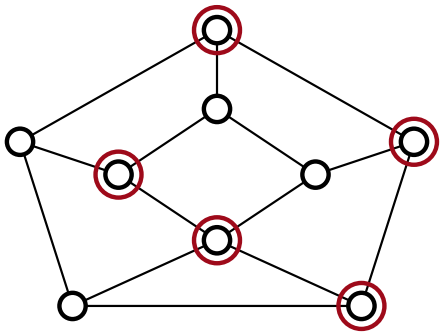




Reconfiguration of tokens on a graph





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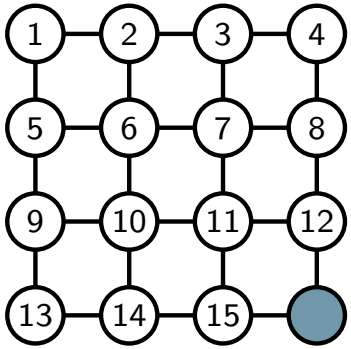

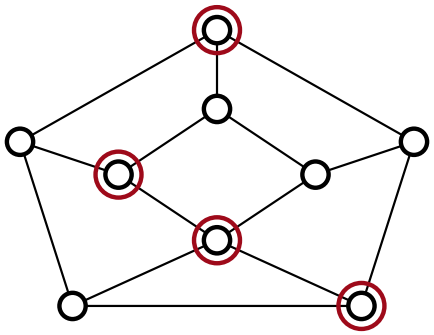

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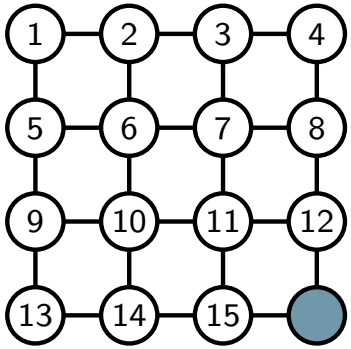

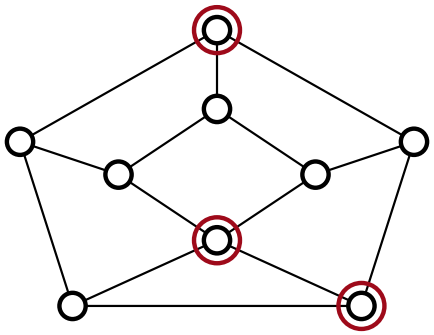

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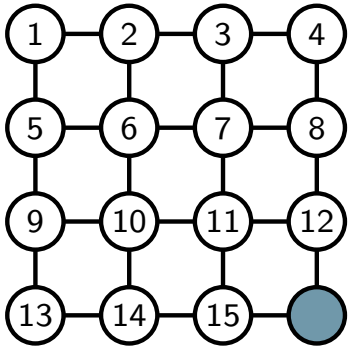

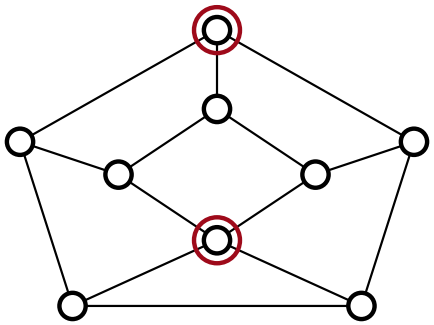

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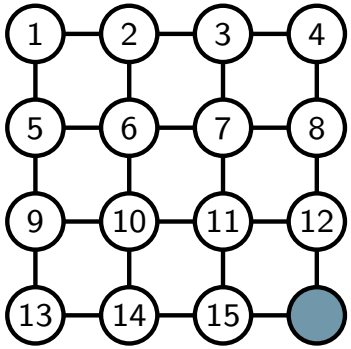
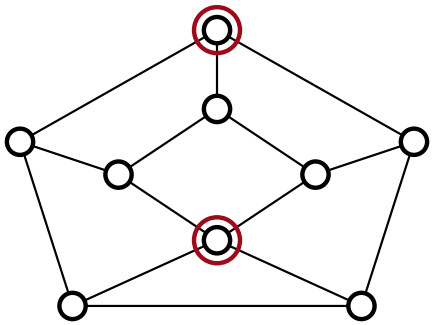
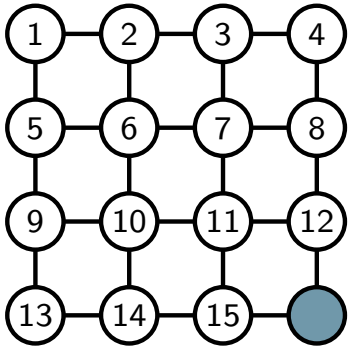
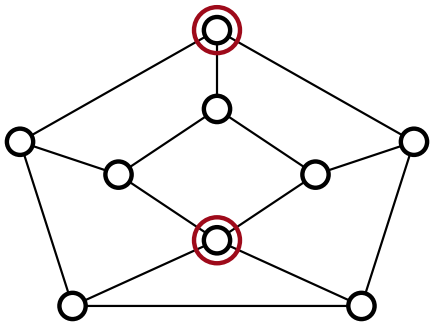




Adjacency rules:

Reconfiguration of tokens on a graph

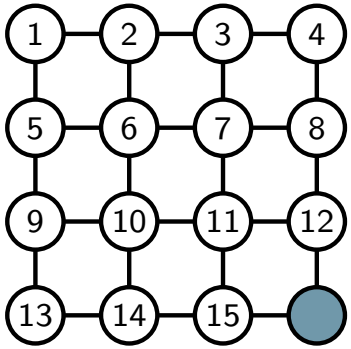


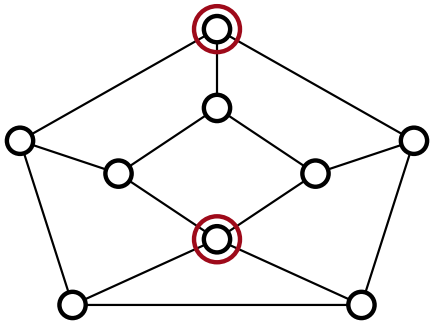


Adjacency rules:

- Token Addition-Removal (TAR):

Reconfiguration of tokens on a graph





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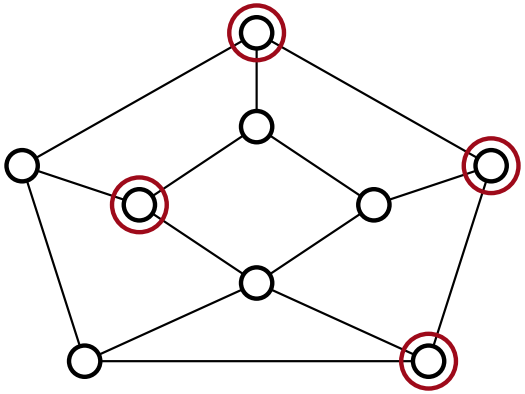
- Token Addition-Removal (TAR):



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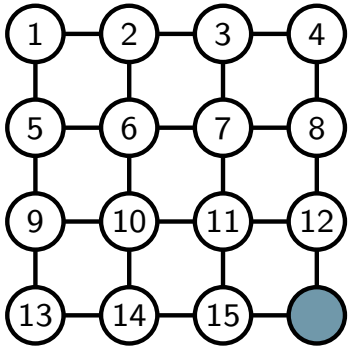

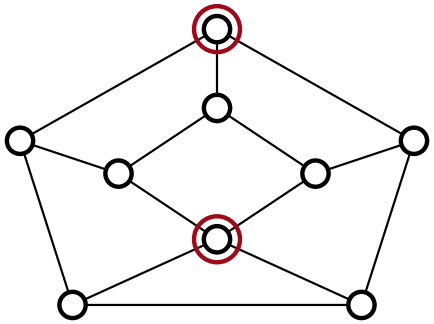

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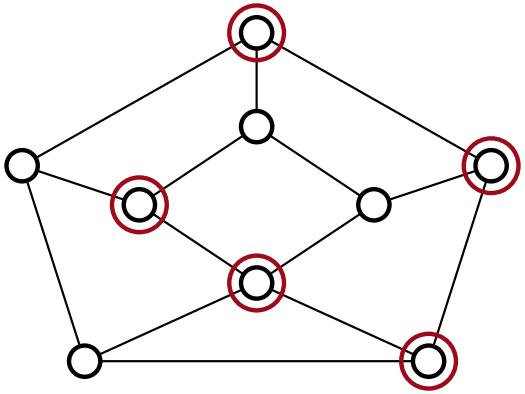
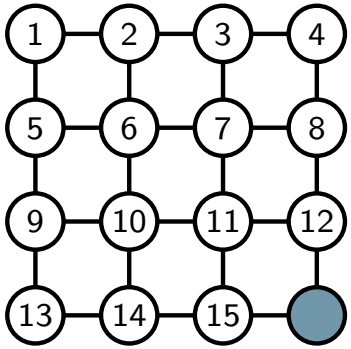
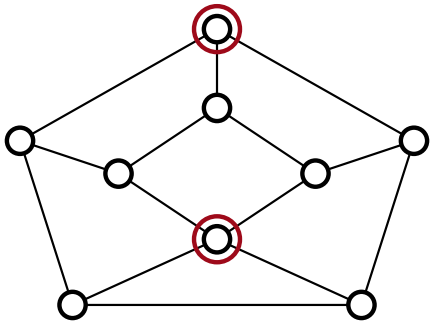


Reconfiguration of tokens on a graph





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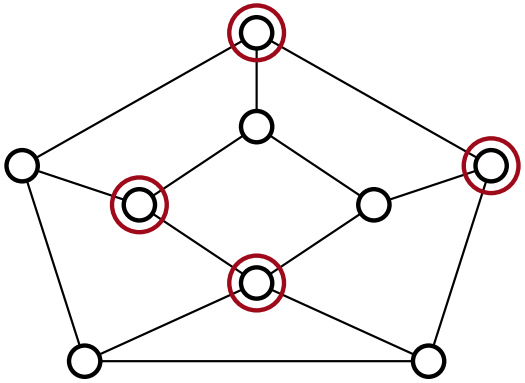

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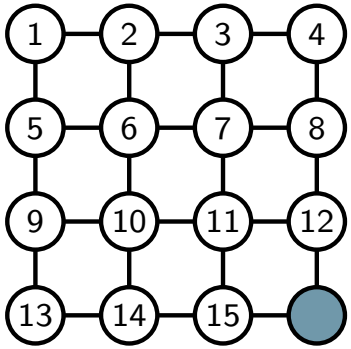

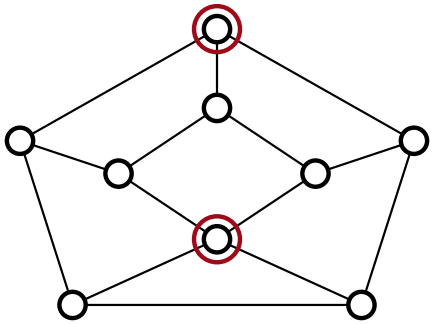

Adjacency rules:

- Token Addition-Removal (TAR):
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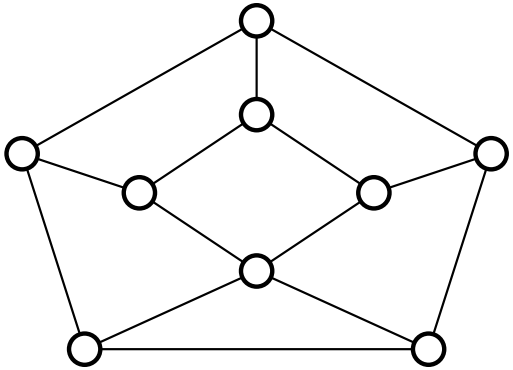
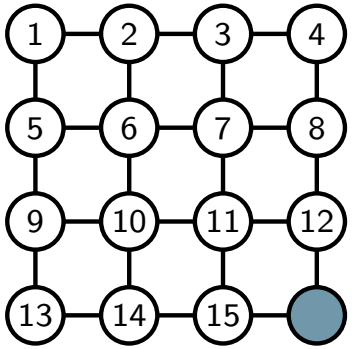
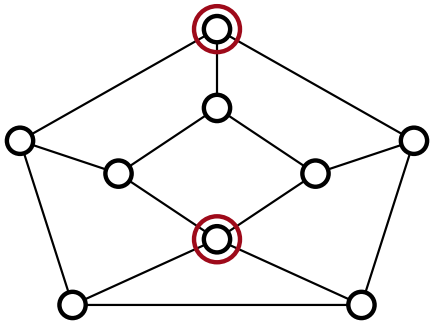


Reconfiguration of tokens on a graph





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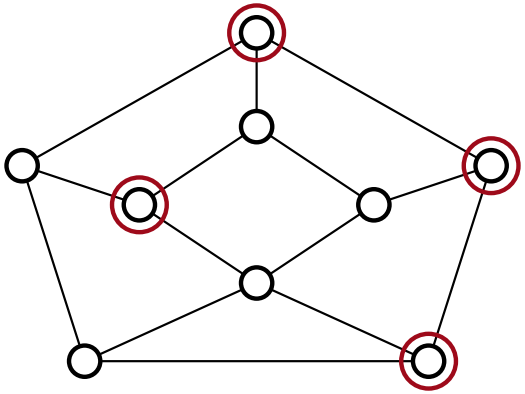

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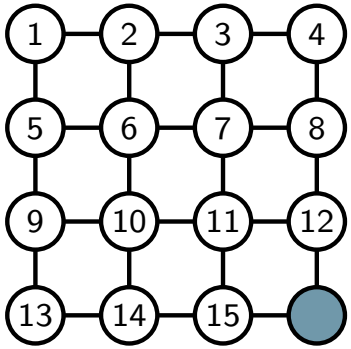

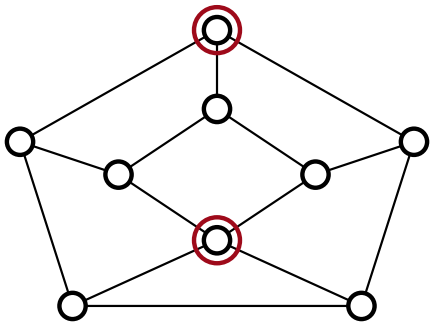

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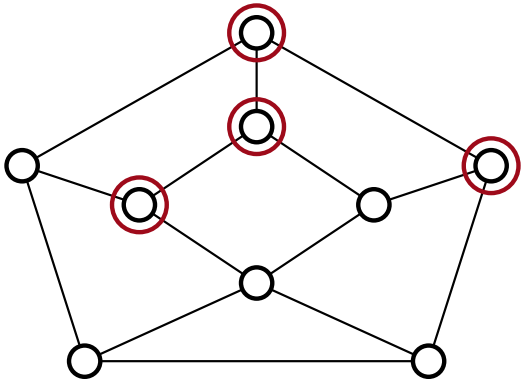

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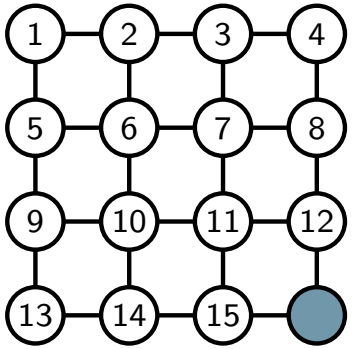

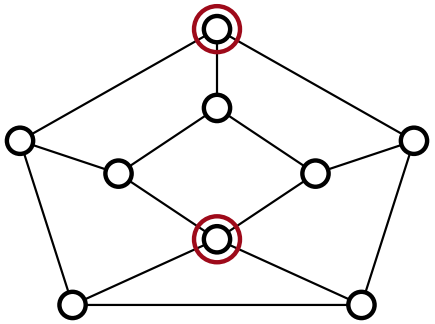

Adjacency rules:

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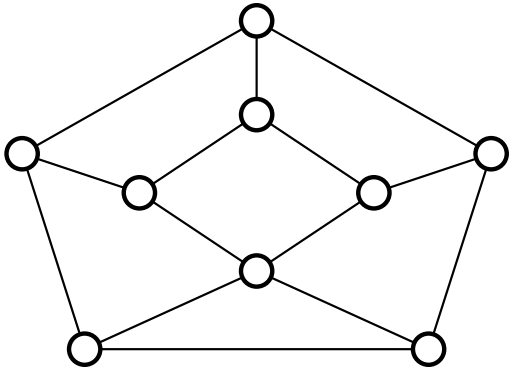
- Token Sliding (TS):



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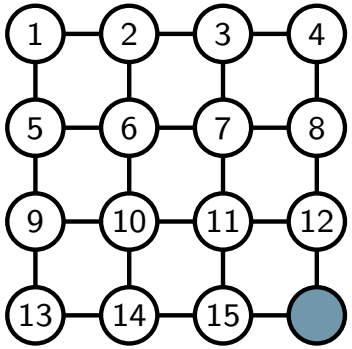

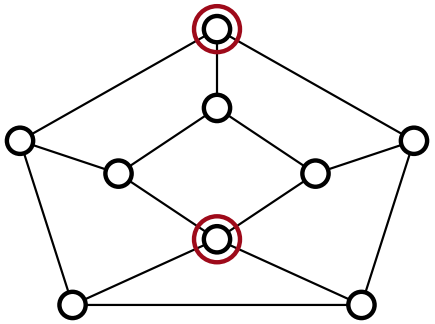

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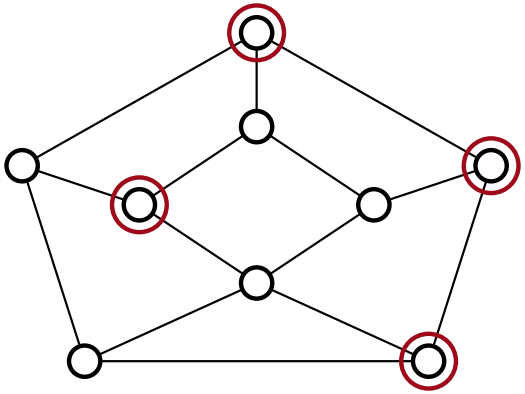
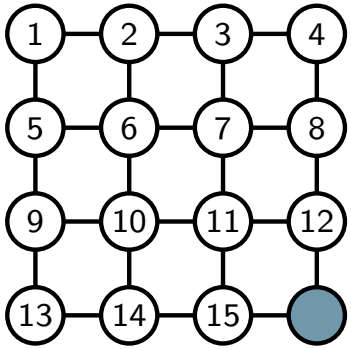
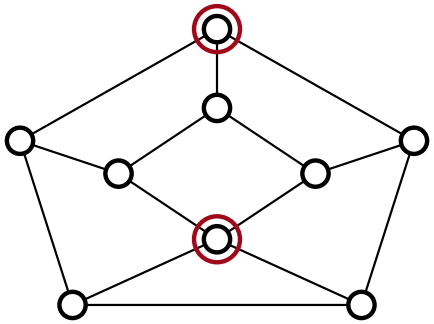


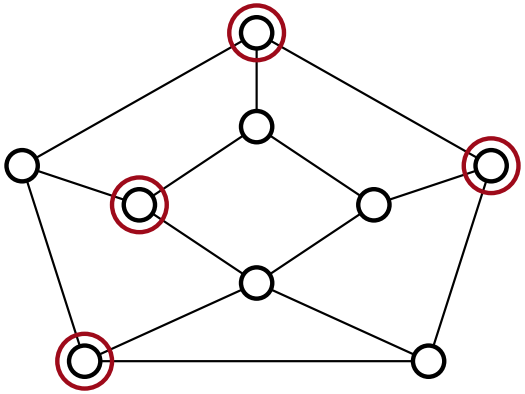
Reconfiguration of tokens on a graph





Adjacency rules:

- Token Addition-Removal (TAR):
- Token Jumping (TJ):
- Token Sliding (TS):

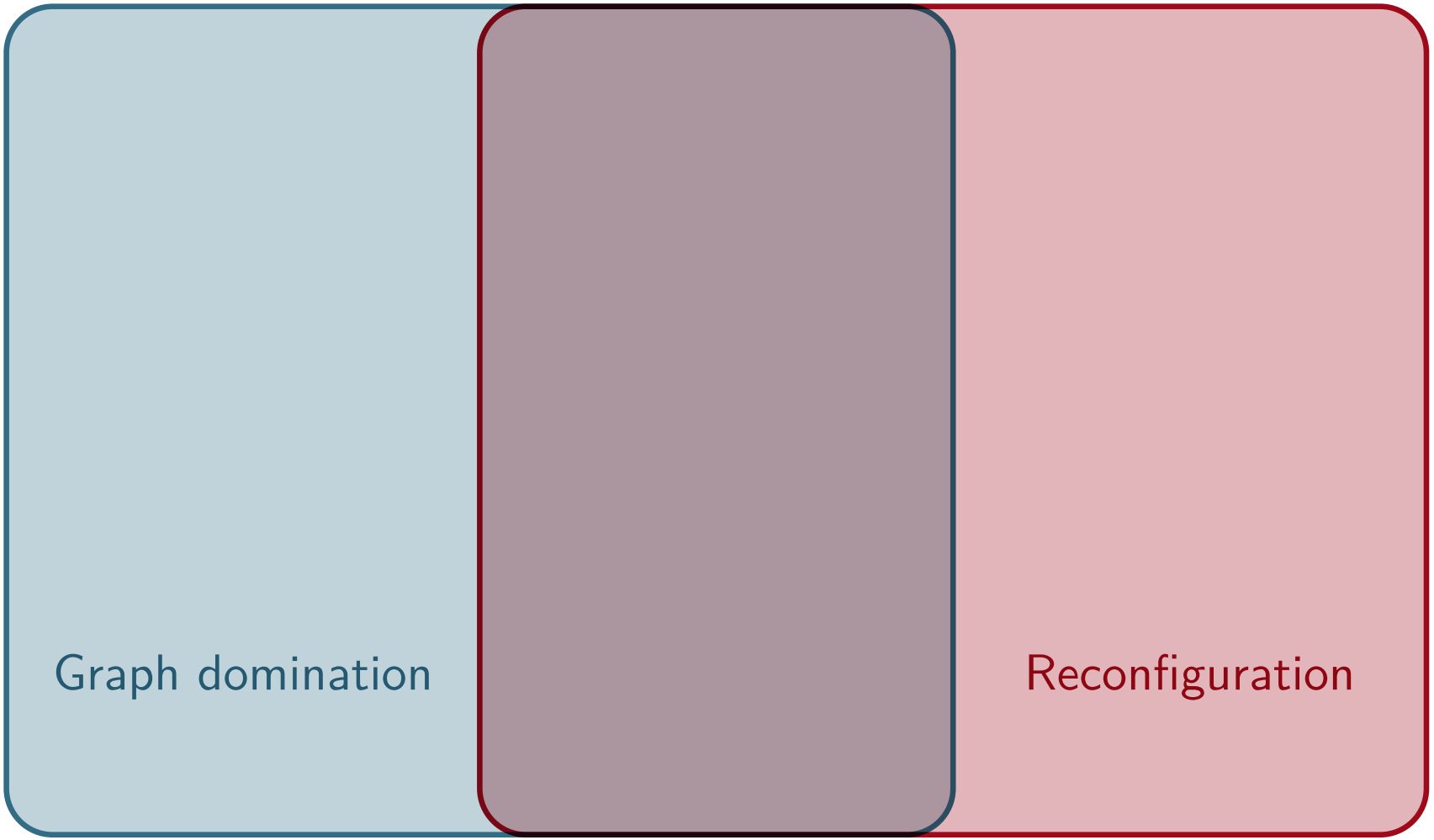


Outline

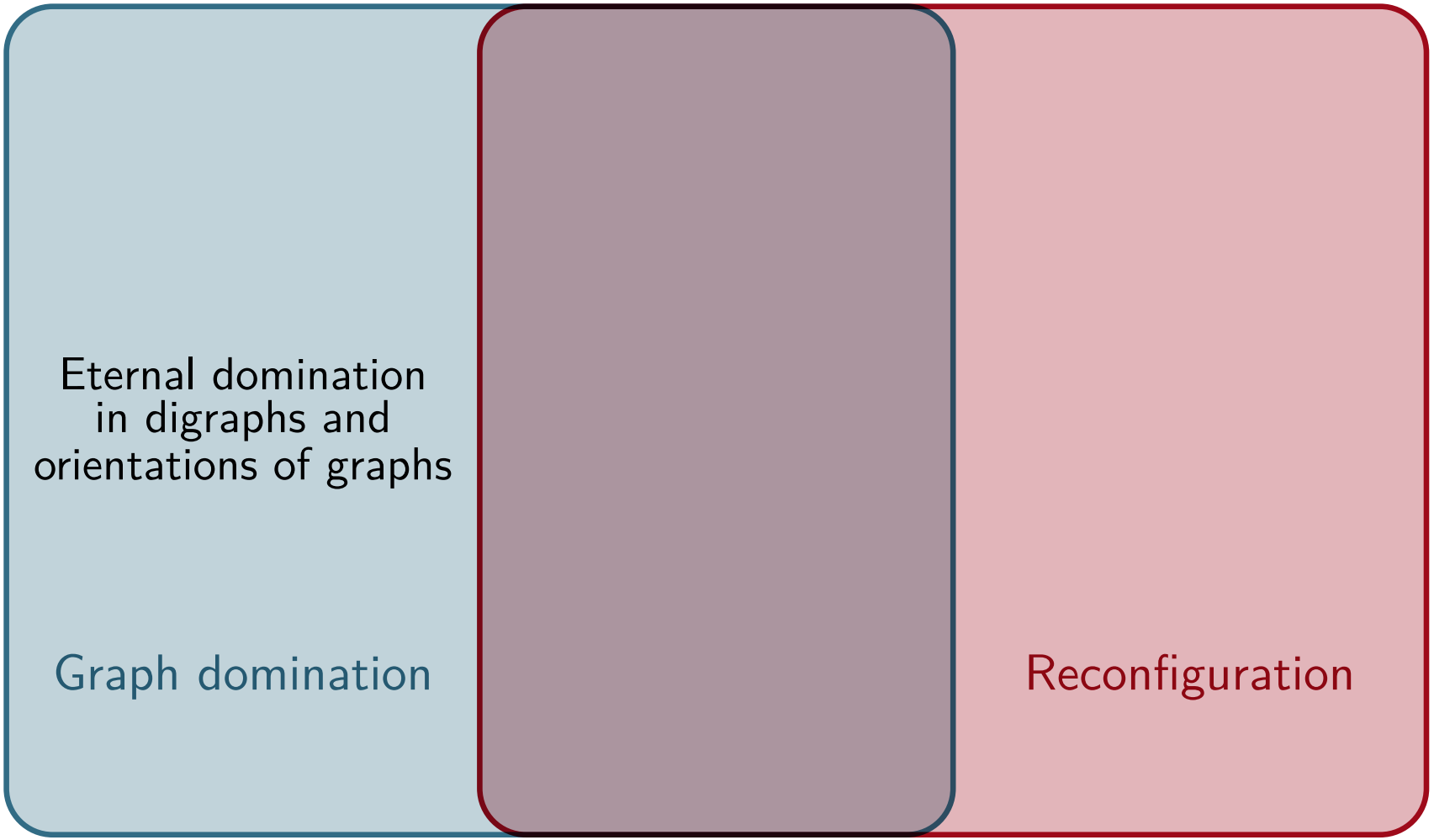
Outline

Graph domination

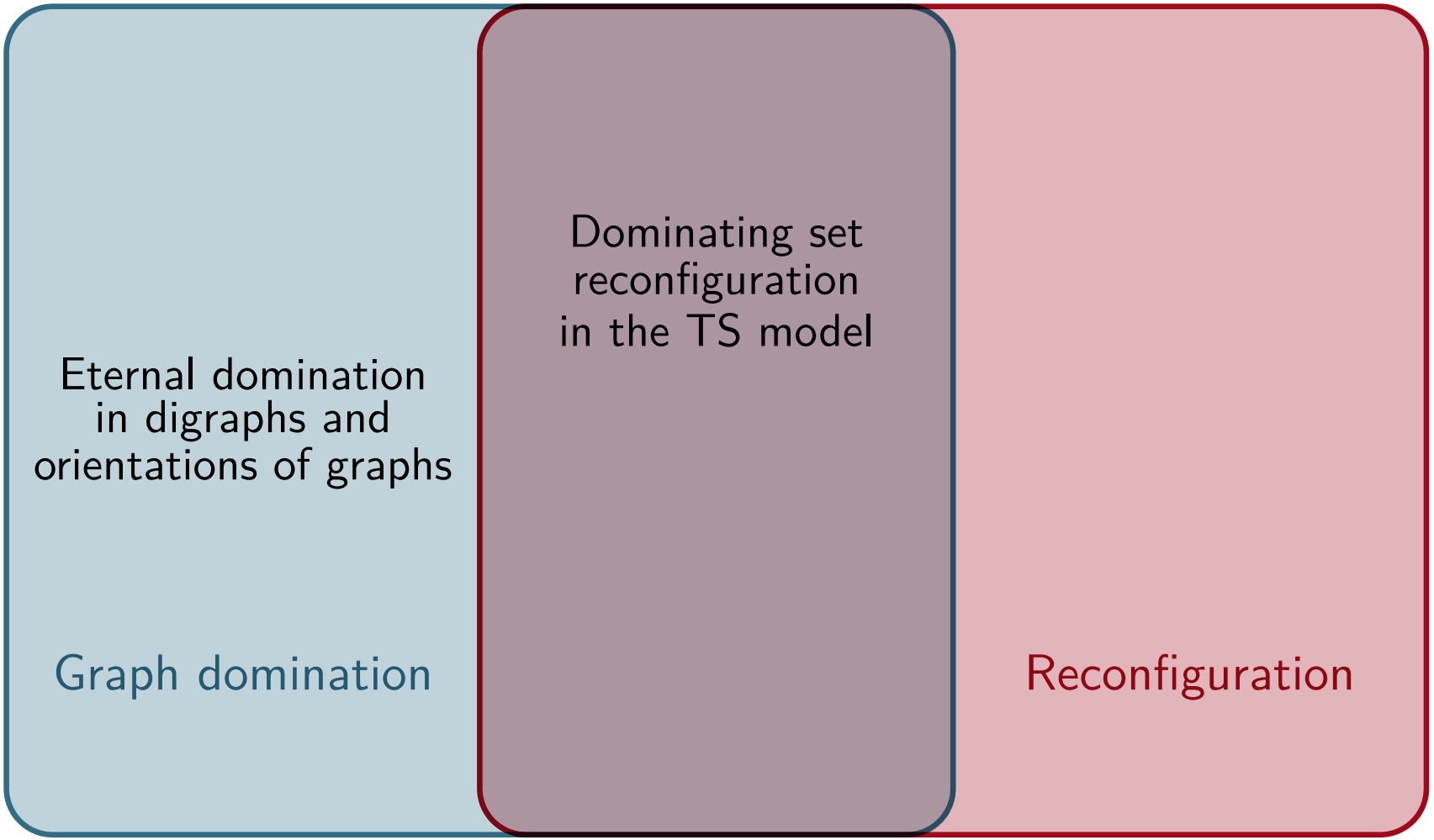
Outline



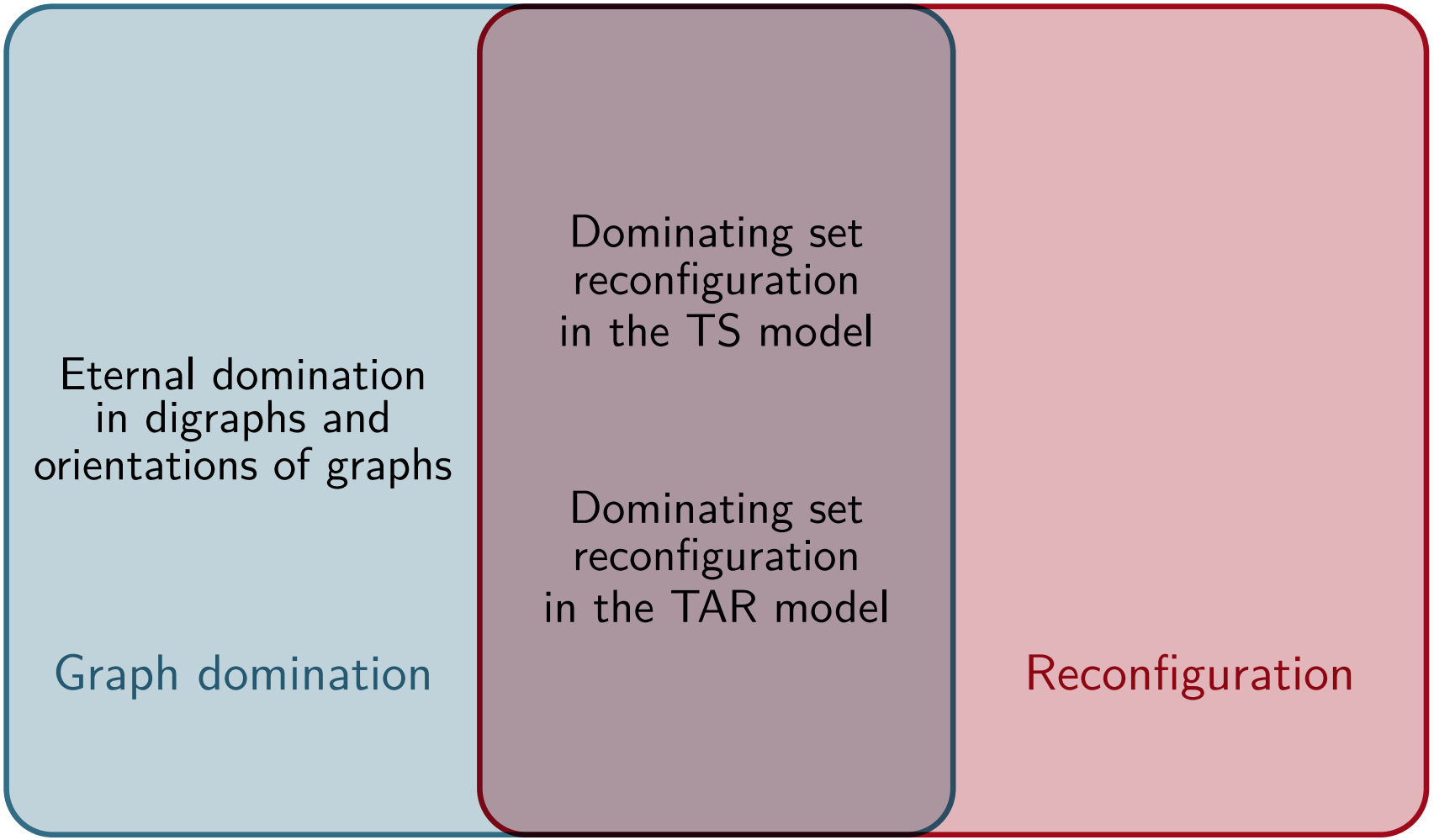
Outline


Eternal domination in digraphs and orientations of graphs
Graph domination
Reconfiguration

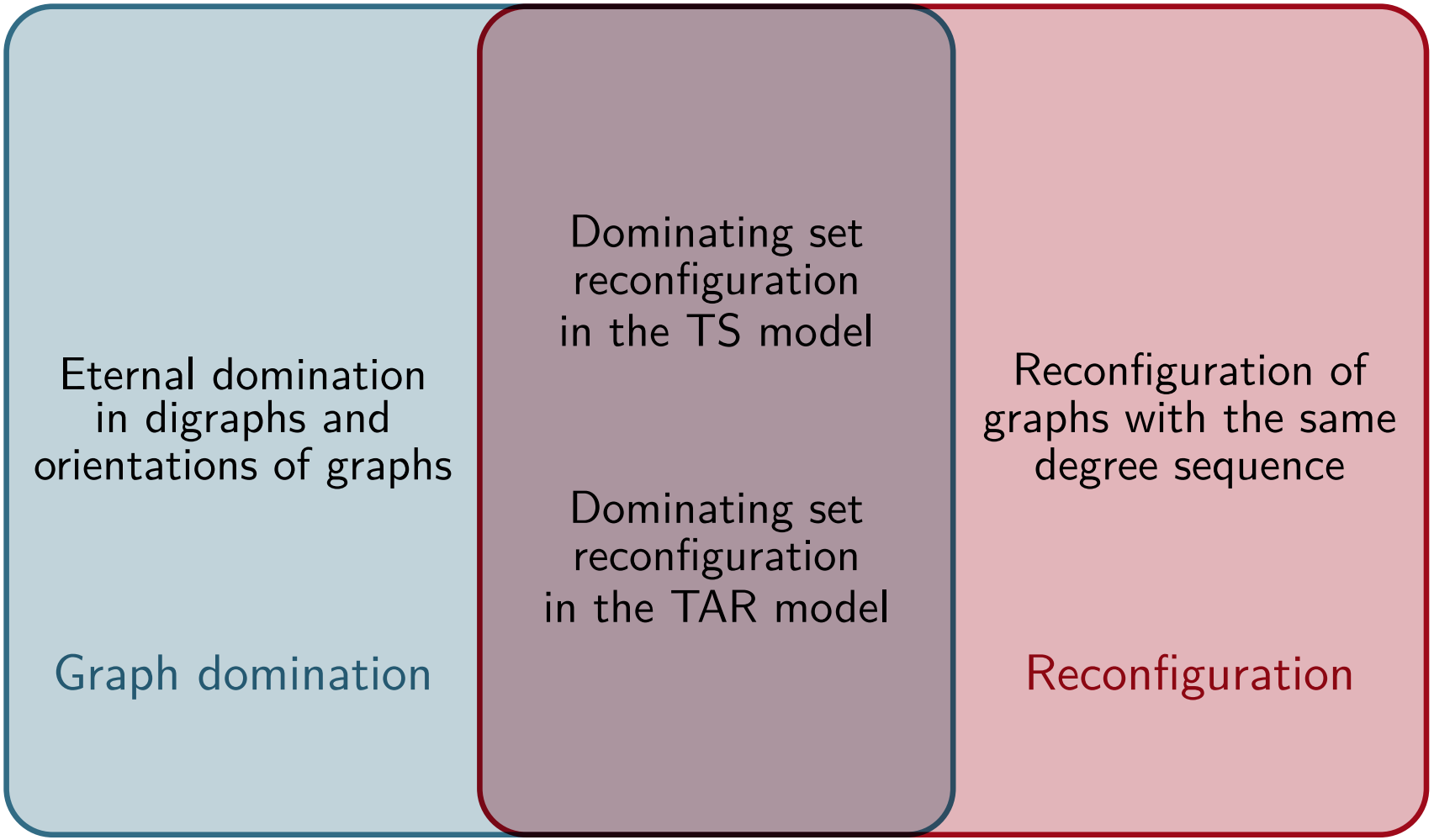
Outline


Eternal domination in digraphs and orientations of graphs
Dominating set reconfiguration in the TS model
Graph domination
Reconfiguration

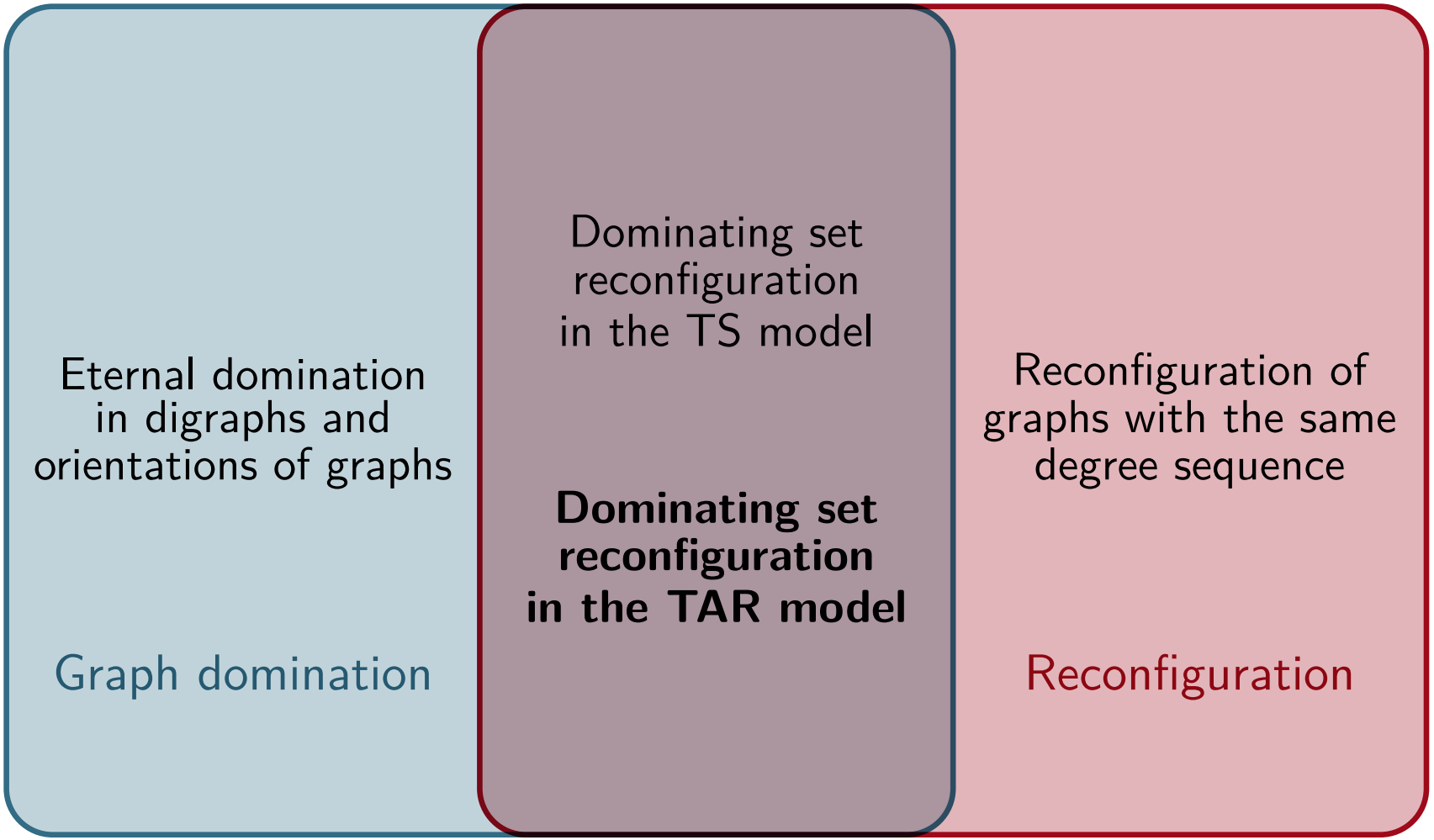
Outline


Eternal domination in digraphs and orientations of graphs
Graph domination
Dominating set reconfiguration in the TS model
Dominating set reconfiguration in the TAR model
Reconfiguration

Outline


Eternal domination in digraphs and orientations of graphs
Graph domination
Dominating set reconfiguration in the TS model
Dominating set reconfiguration in the TAR model
Reconfiguration of graphs with the same degree sequence
Reconfiguration

Outline


Eternal domination in digraphs and orientations of graphs
Graph domination
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Dominating set reconfiguration in the TAR model
Reconfiguration of graphs with the same degree sequence
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Reconfiguration of dominating sets in the TAR model

Reconfiguration of dominating sets in the TAR model

Can we transform D_s into D_t with
a sequence of vertex additions/removals
keeping G dominated ?

Reconfiguration of dominating sets in the TAR model

Can we transform D_s into D_t with
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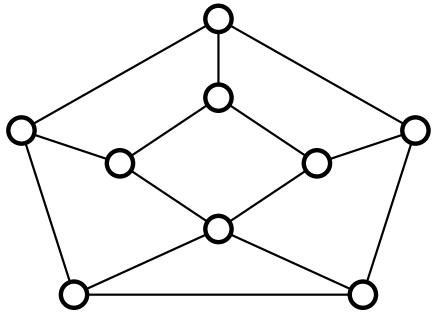

Reconfiguration of dominating sets in the TAR model

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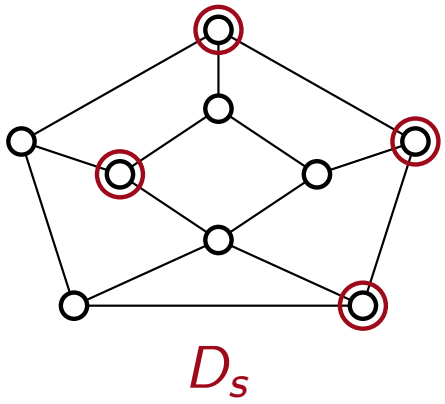

Reconfiguration of dominating sets in the TAR model

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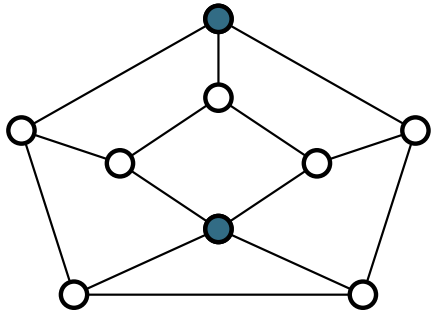
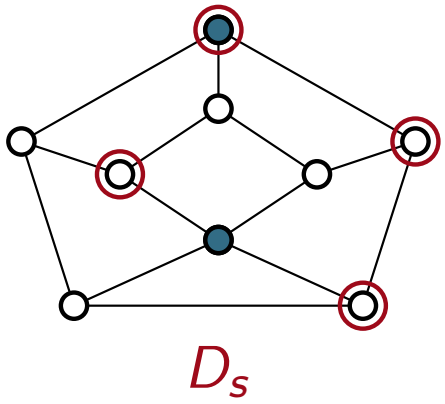


D_t

Reconfiguration of dominating sets in the TAR model

Can we transform D_s into D_t with a sequence of vertex additions/removals keeping G dominated ?



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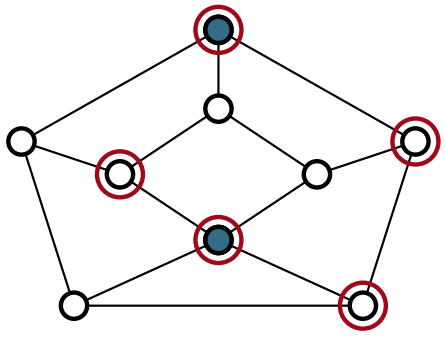
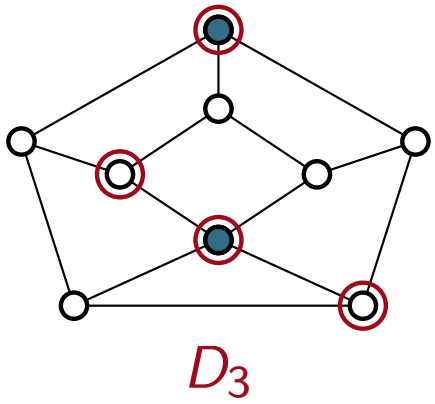


D_2

Reconfiguration of dominating sets in the TAR model

Can we transform D_s into D_t with
a sequence of vertex additions/removals
keeping G dominated ?



Reconfiguration of dominating sets in the TAR model

Can we transform D_s into D_t with
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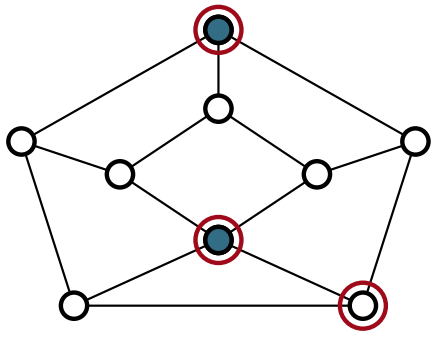
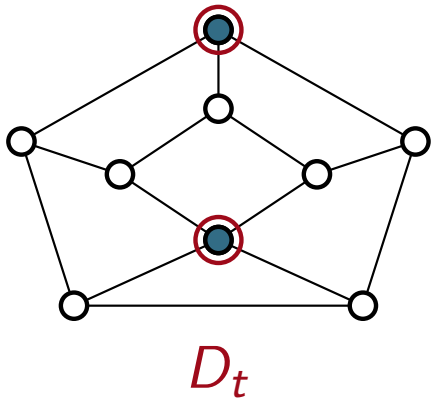


D_4

Reconfiguration of dominating sets in the TAR model

Can we transform D_s into D_t with
a sequence of vertex additions/removals
keeping G dominated ?



Reconfiguration of dominating sets in the TAR model

Can we transform D_s into D_t with
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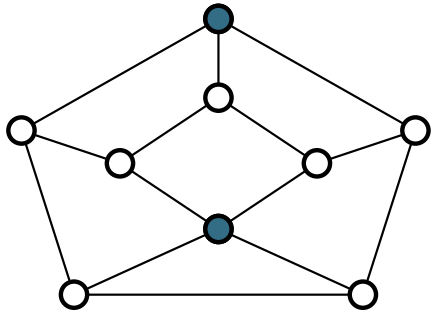
keeping G dominated ?



$\mathcal{R}$: the vertices are the dominating sets, two dominating sets are adjacent if they differ by a vertex addition/deletion

Reconfiguration of dominating sets in the TAR model

Can we transform D_s into D_t with
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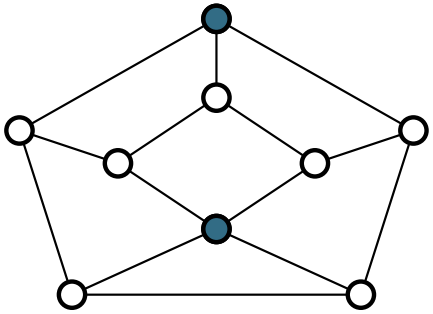

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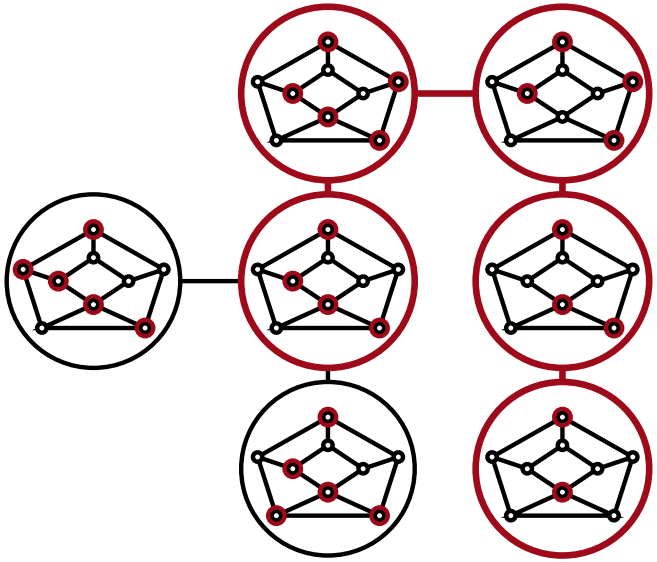

Reconfiguration of dominating sets in the TAR model

Can we transform D_s into D_t with
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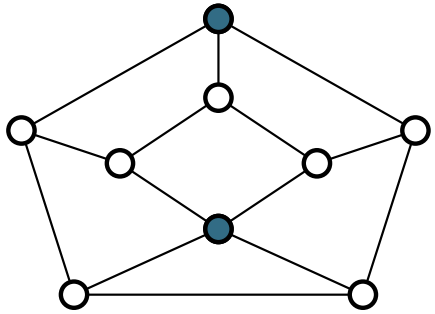

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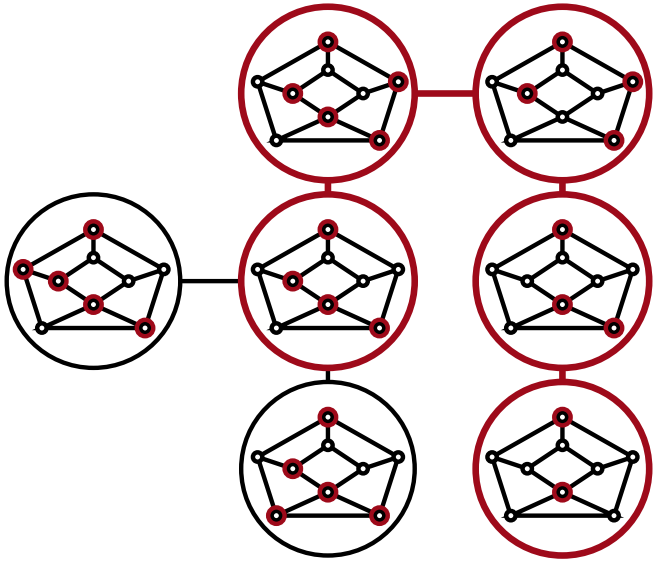

Is $\mathcal{R}$ connected ? What is its diameter ?

Threshold

Threshold

If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:

Threshold

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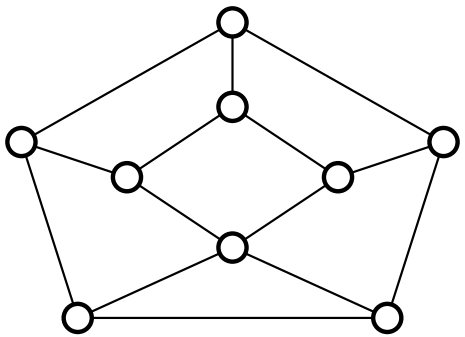

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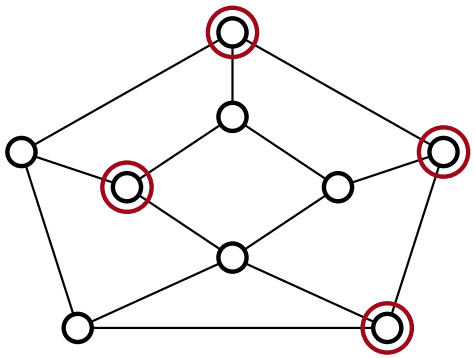
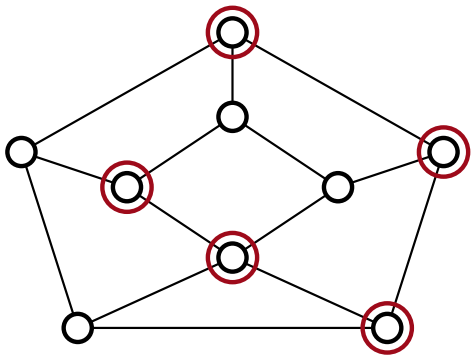


D_s

Threshold

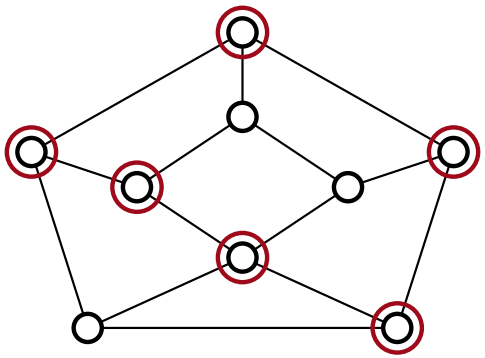
If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



D_2

Threshold

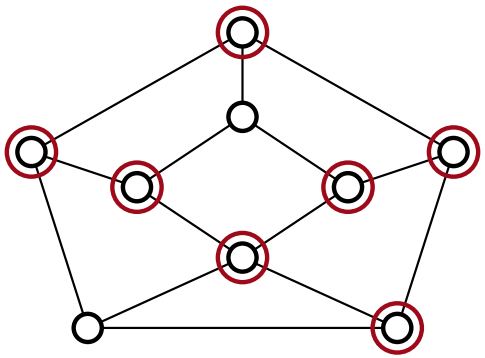
If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



D_3

Threshold

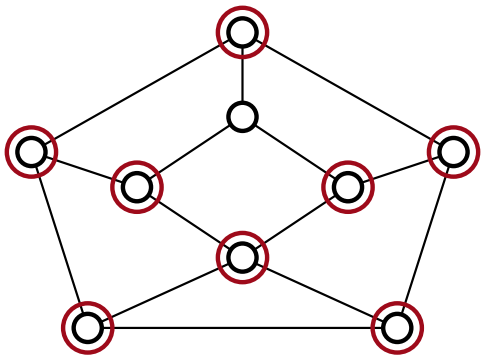
If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



D_4

Threshold

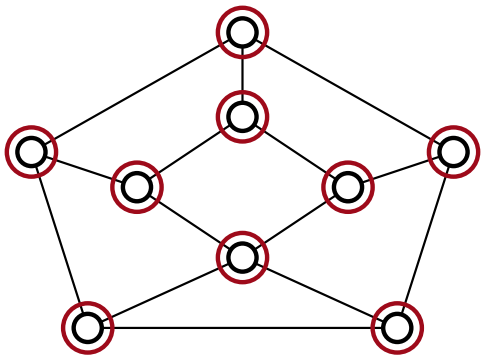
If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



D_5

Threshold

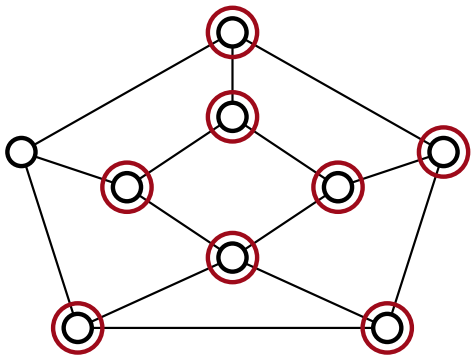
If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



D_6

Threshold

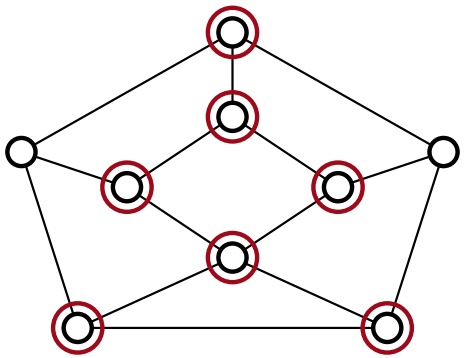
If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



D_7

Threshold

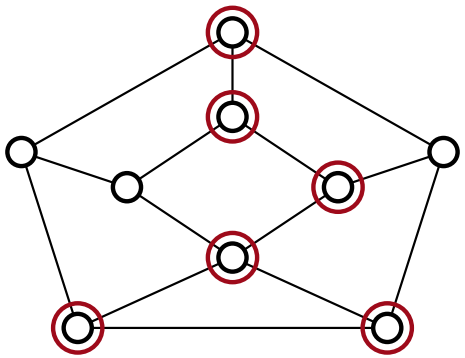
If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



D_8

Threshold

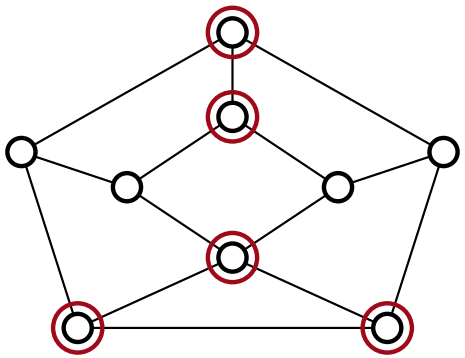
If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



D_9

Threshold

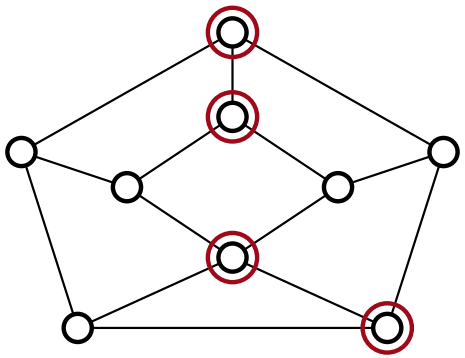
If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



D_{10}

Threshold

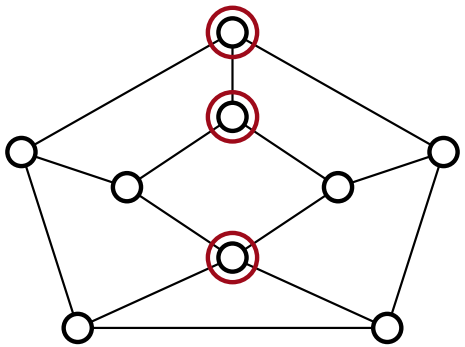
If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



D_{11}

Threshold

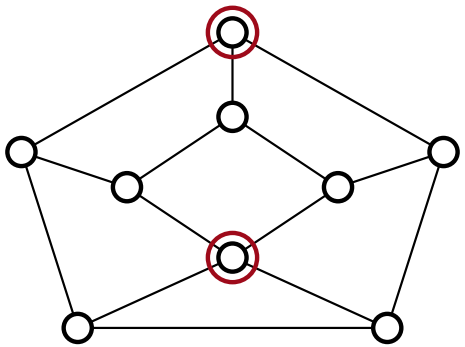
If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



D_{12}

Threshold

If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



D_t

Threshold

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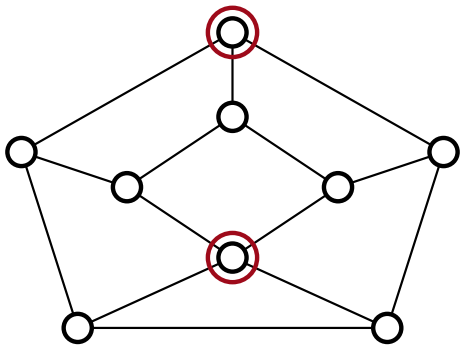

D_t

Threshold: maximum size k of the dominating sets in $\mathcal{R}_k(G)$

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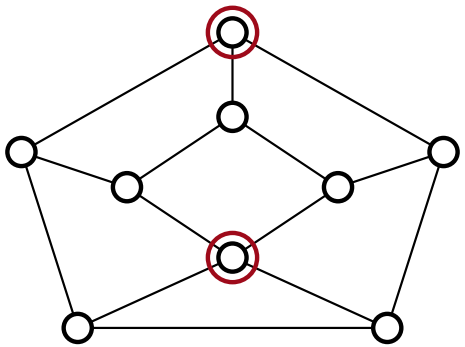

D_t

Threshold: maximum size k of the dominating sets in $\mathcal{R}_k(G)$

Remark: $\mathcal{R}_k(G)$ connected $\not\Rightarrow \mathcal{R}_{k+1}(G)$ connected:

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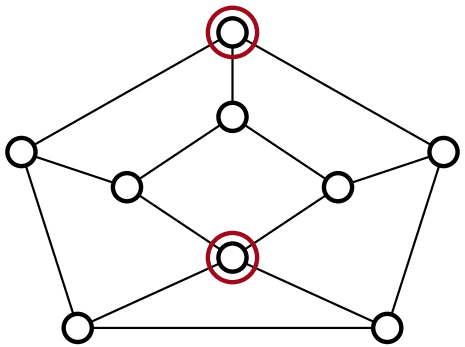

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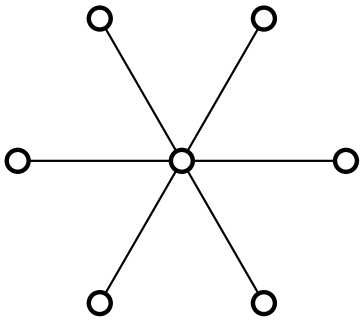

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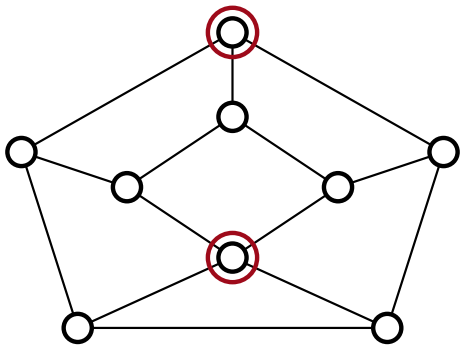

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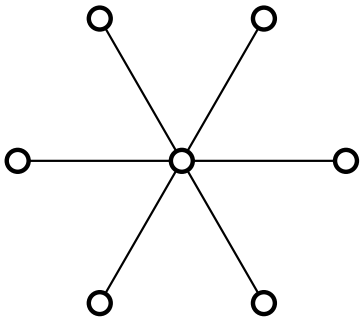
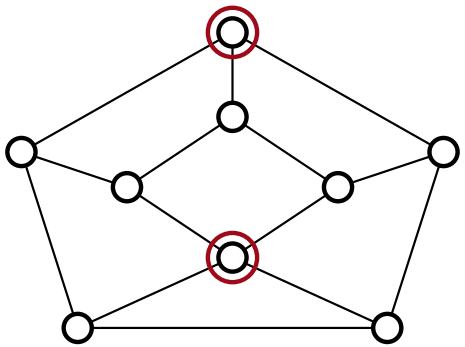


$k = n-2 : \mathcal{R}_k(S_n)$ is connected

Threshold

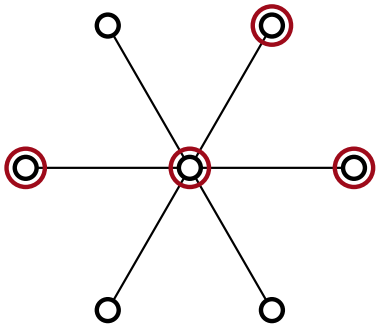
If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



D_t

Threshold: maximum size k of the dominating sets in $\mathcal{R}_k(G)$

Remark: $\mathcal{R}_k(G)$ connected $\not\Rightarrow \mathcal{R}_{k+1}(G)$ connected:

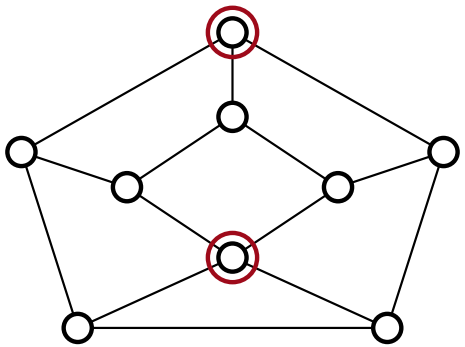


D_s

$k = n-2 : \mathcal{R}_k(S_n)$ is connected

Threshold

If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



D_t

Threshold: maximum size k of the dominating sets in $\mathcal{R}_k(G)$

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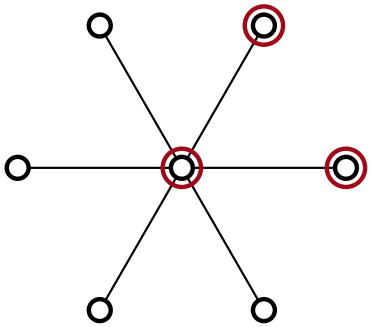

D_2

$k = n-2 : \mathcal{R}_k(S_n)$ is connected

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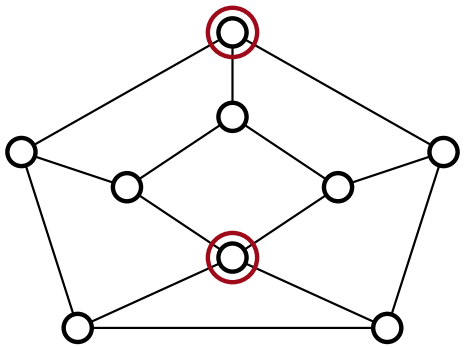
If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



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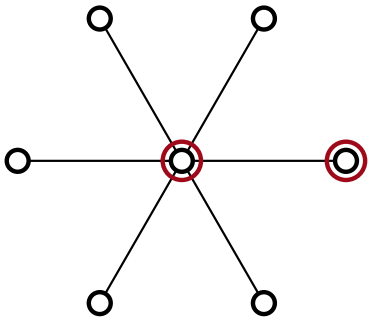
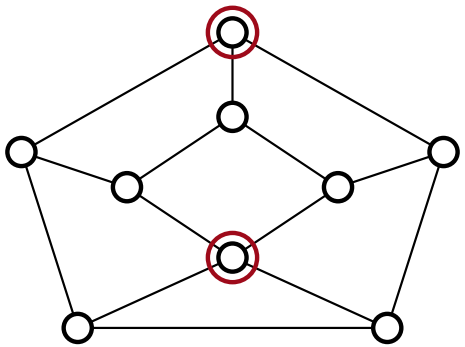


D_3

$k = n-2 : \mathcal{R}_k(S_n)$ is connected

Threshold

If $\mathcal{R}(G)$ contains all the dominating sets, it is connected:



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Threshold: maximum size k of the dominating sets in $\mathcal{R}_k(G)$

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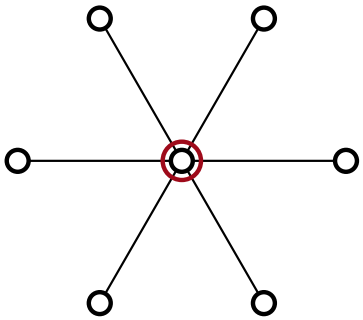

D_4

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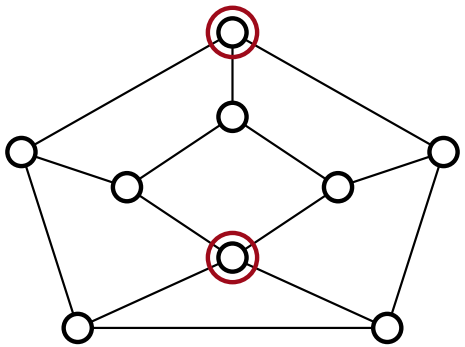

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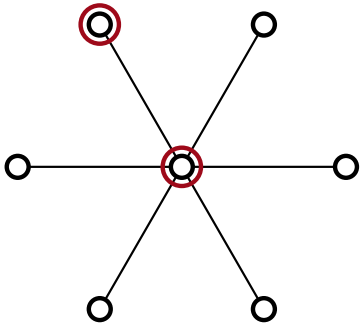

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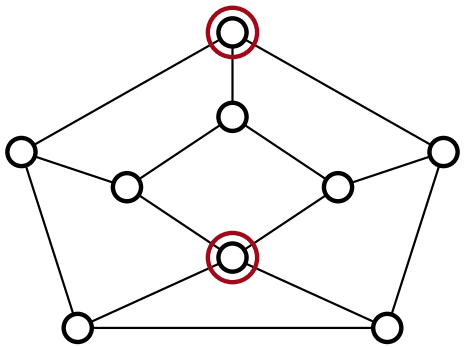

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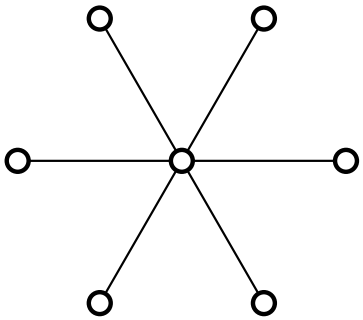

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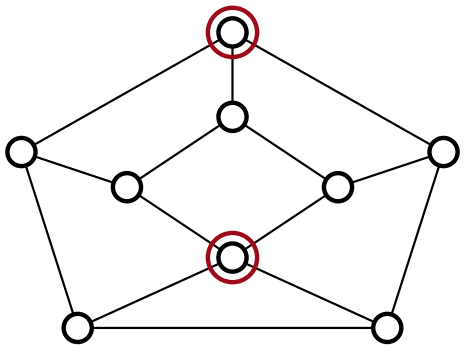

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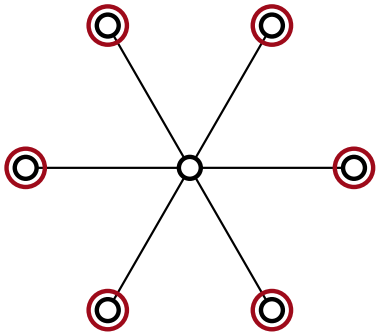

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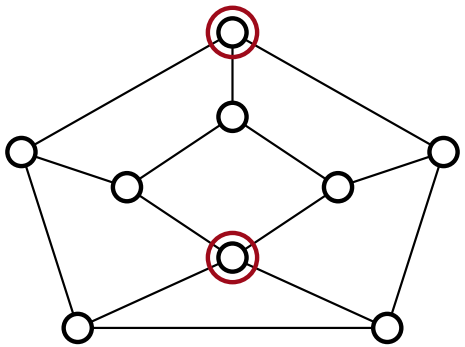

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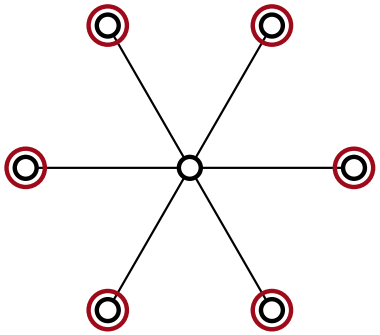

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What is the smallest d_0 s.t. $\mathcal{R}_k(G)$ is connected for any $k \geq d_0$?

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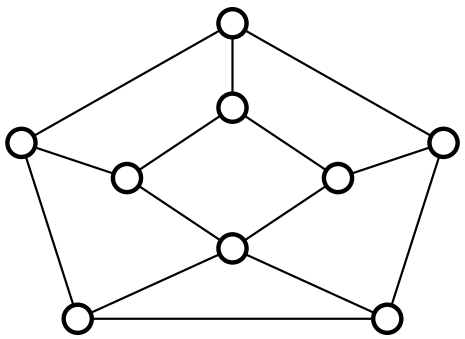

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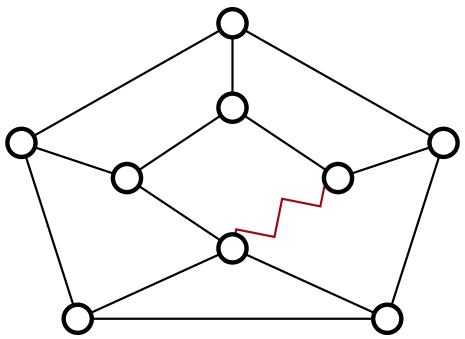

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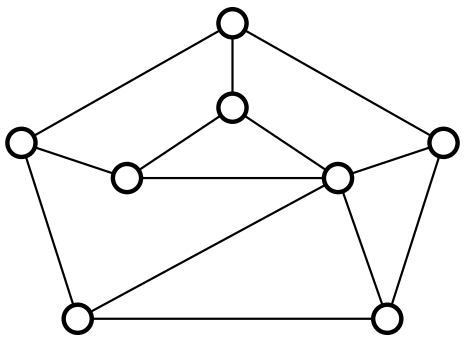

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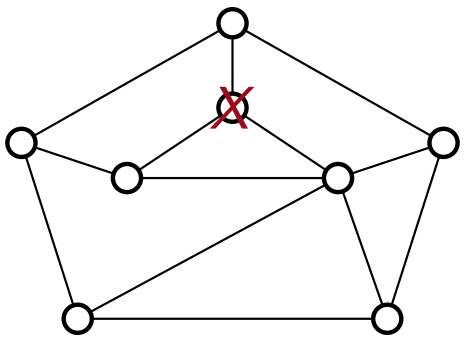

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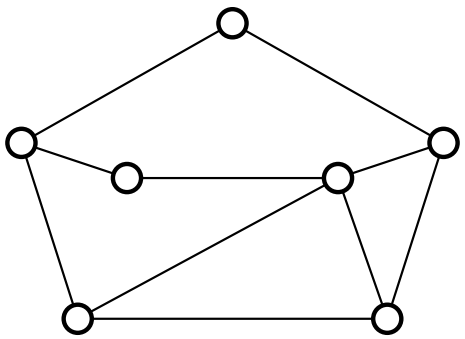

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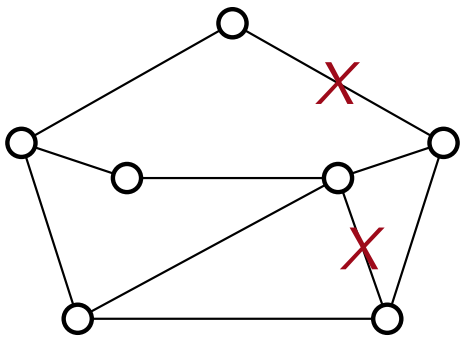

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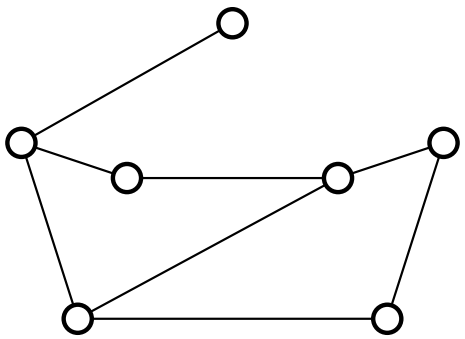

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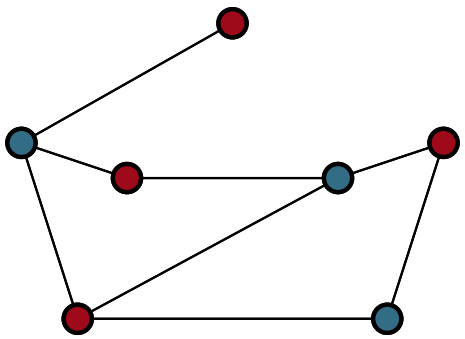

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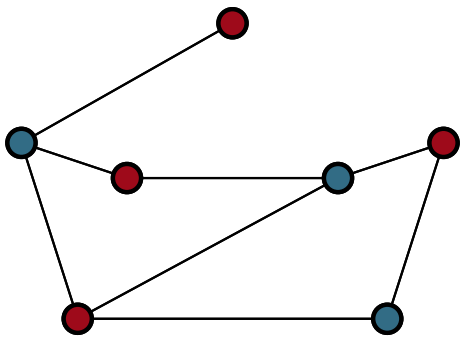

7 vertices, 8 edges $\rightarrow d > \frac{16}{7}$

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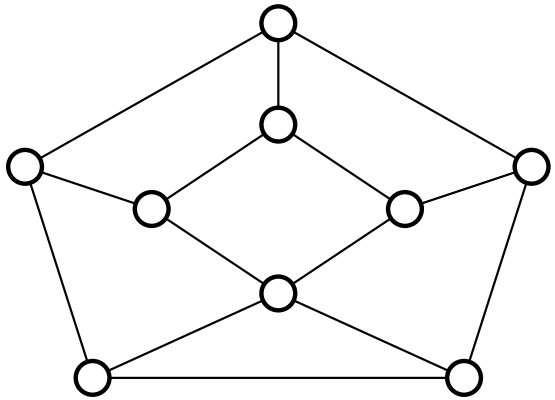

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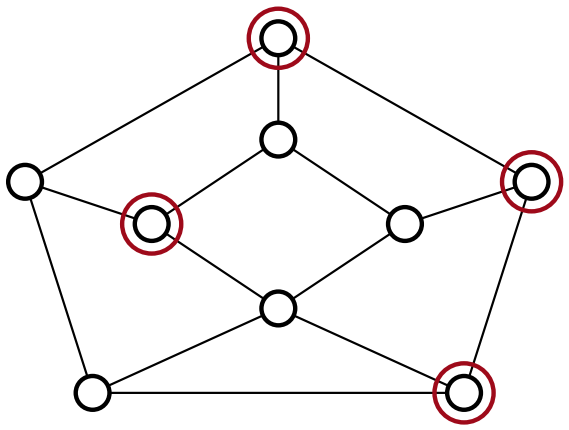

D_s

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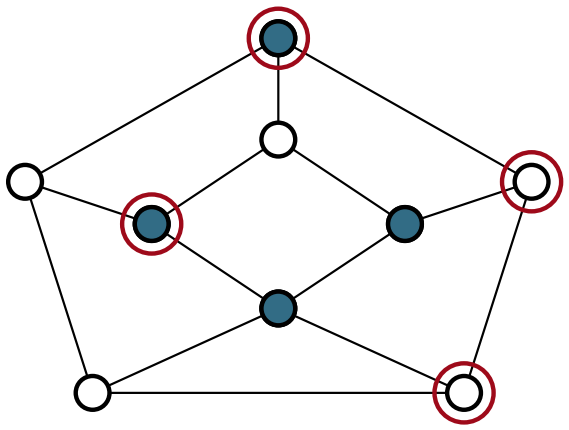

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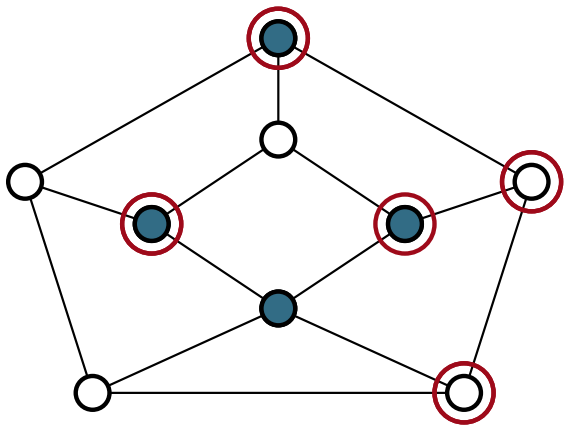

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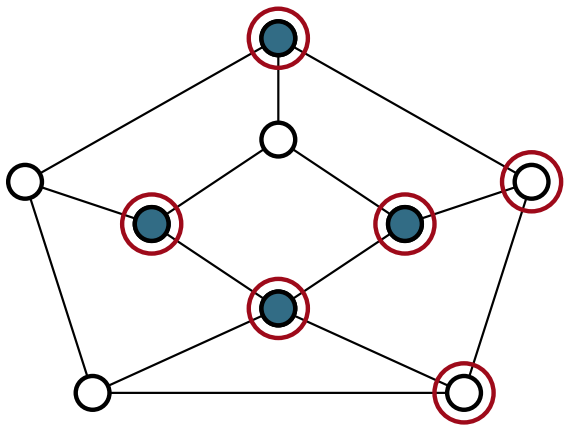

D_3

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Minor: obtained by contracting, deleting edges, and deleting isolated vertices

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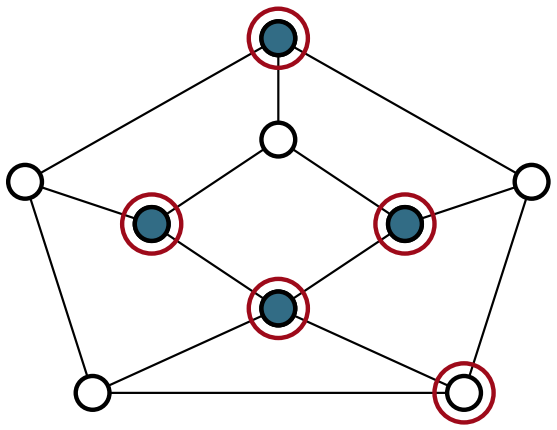

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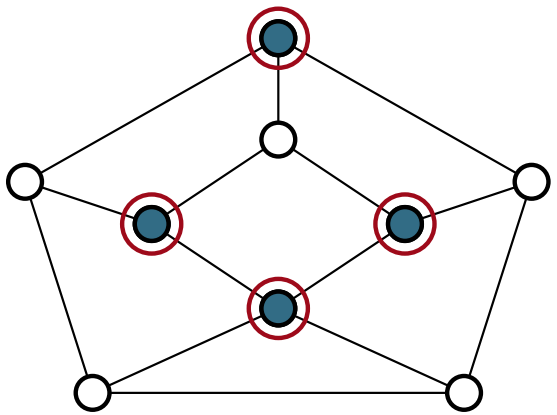

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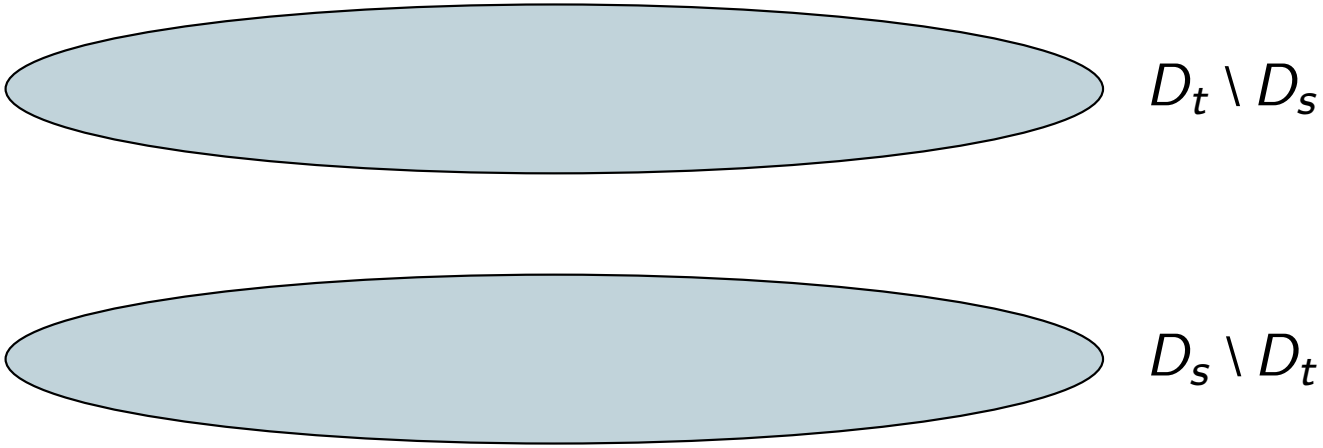

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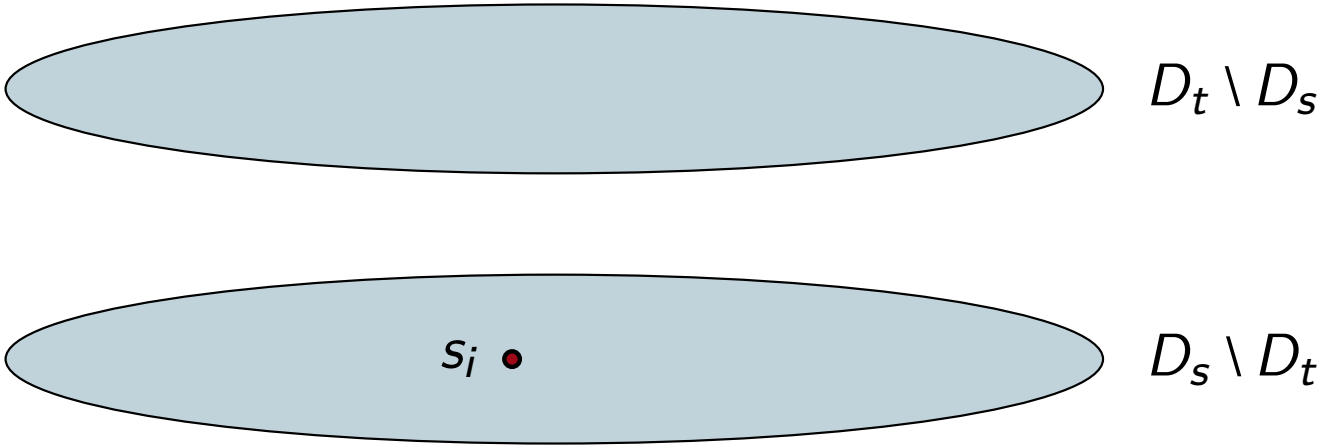

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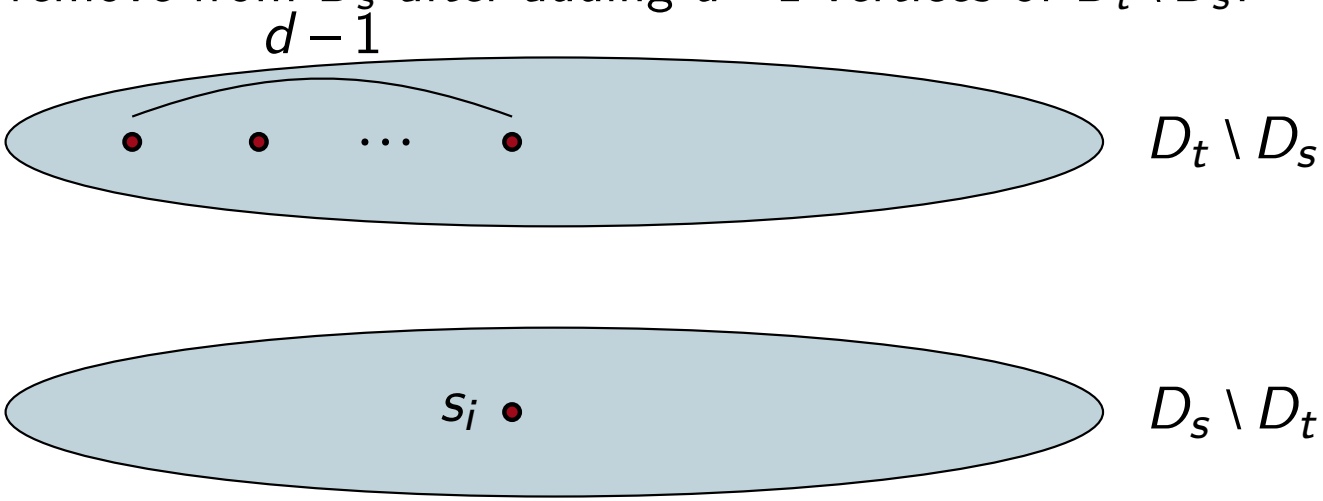

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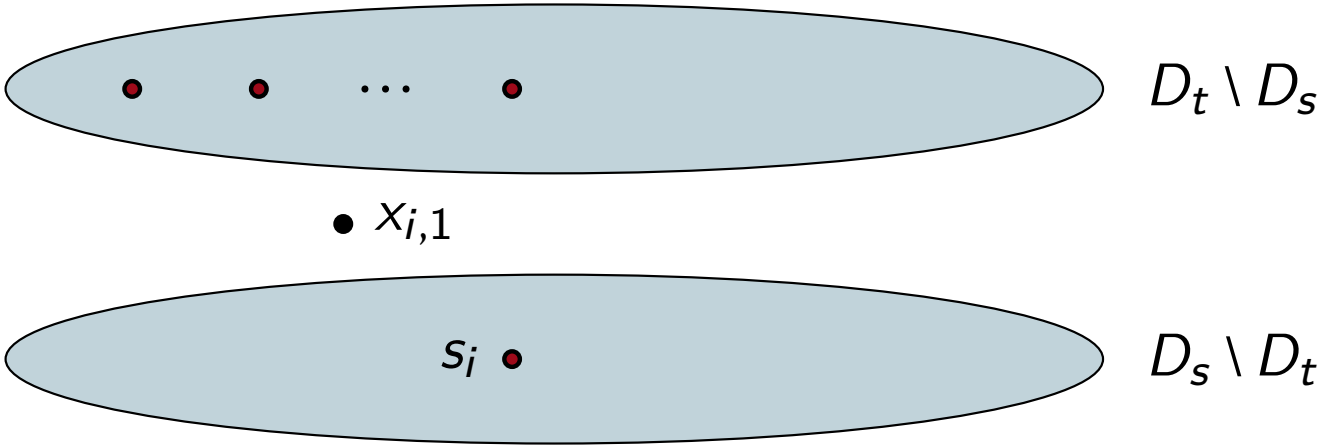

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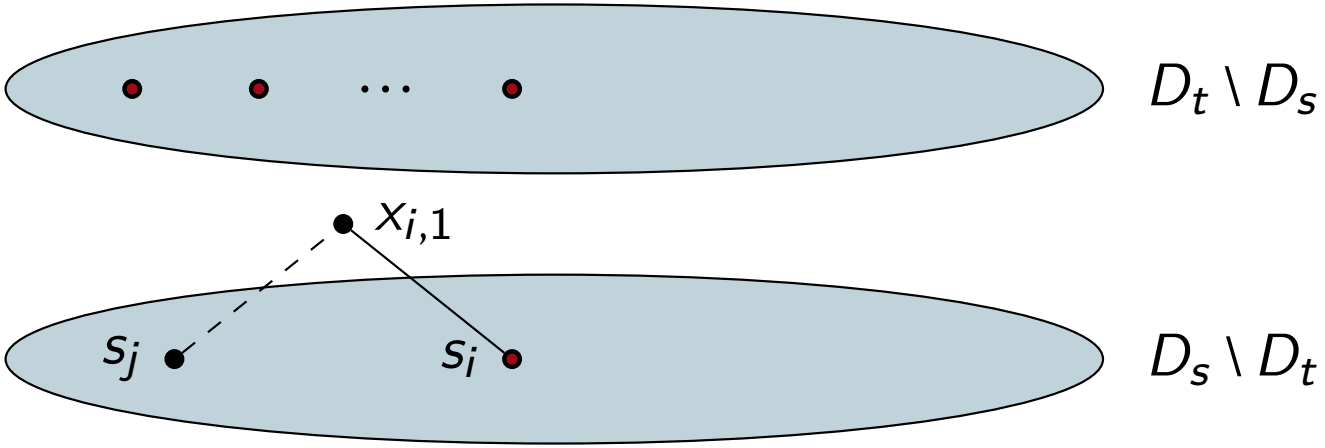

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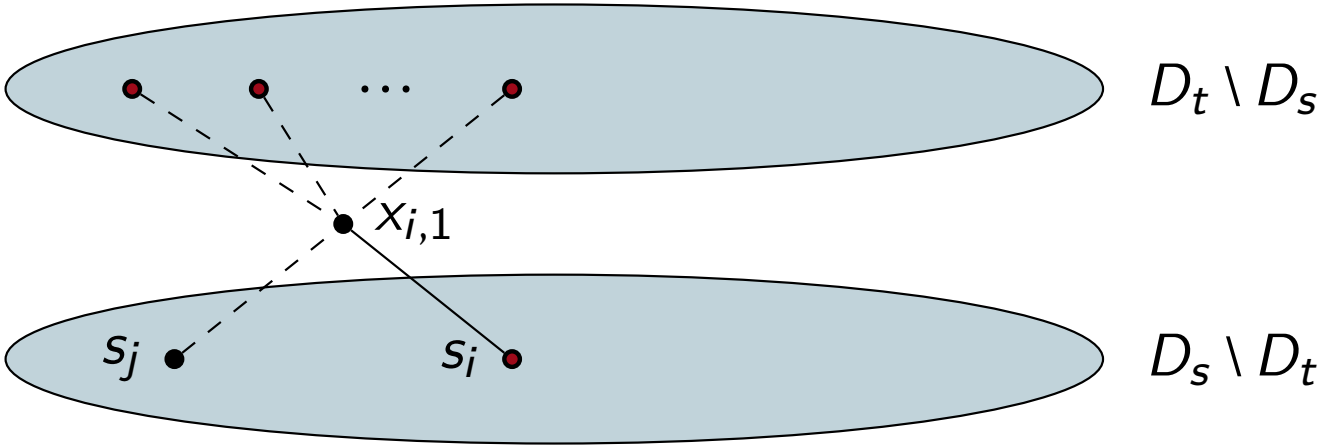

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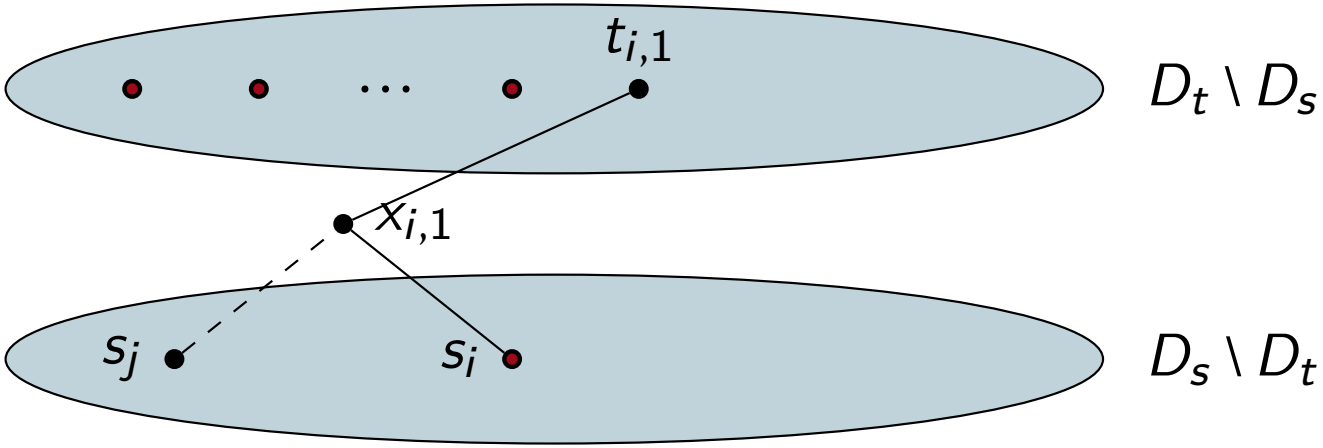

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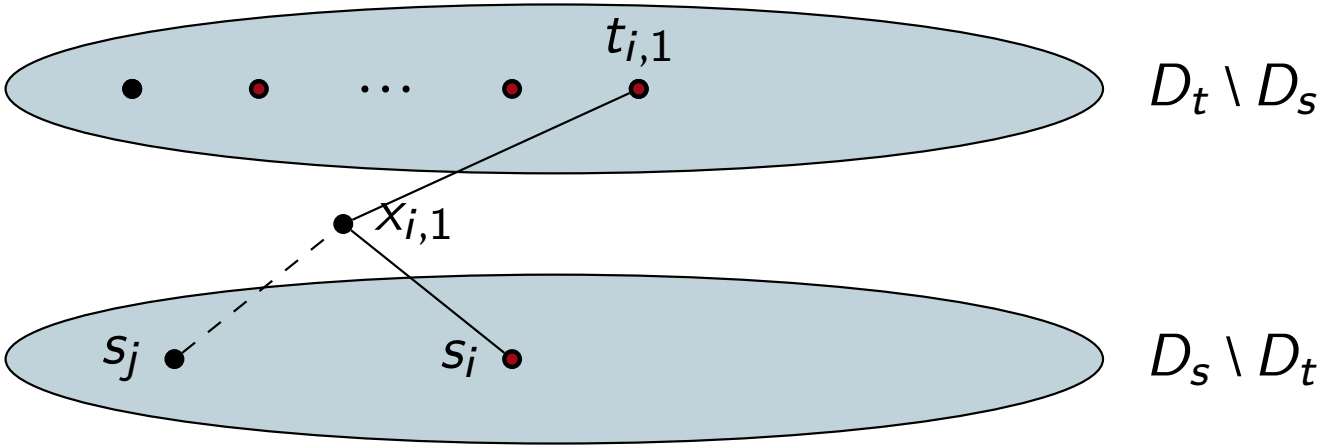

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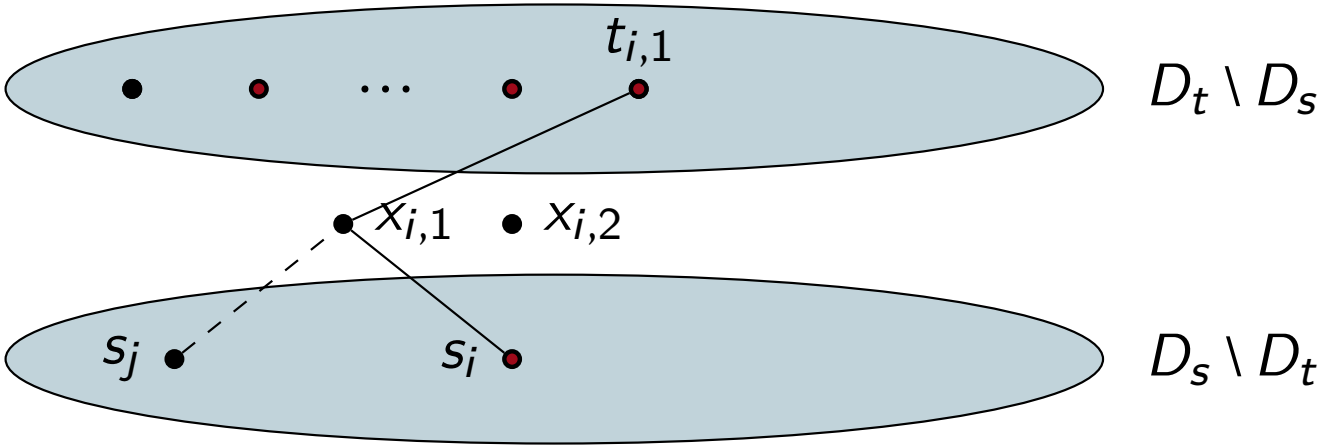

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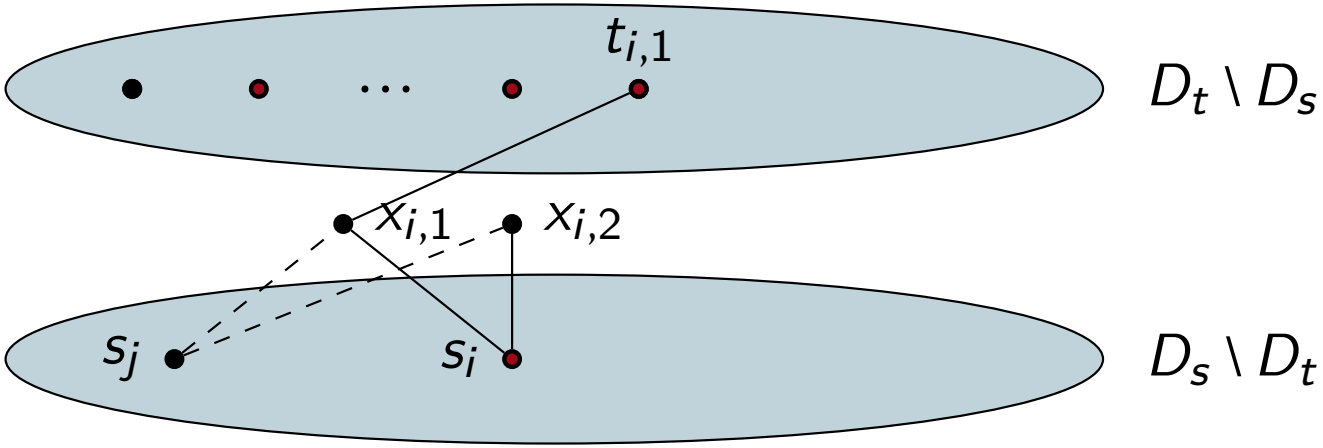

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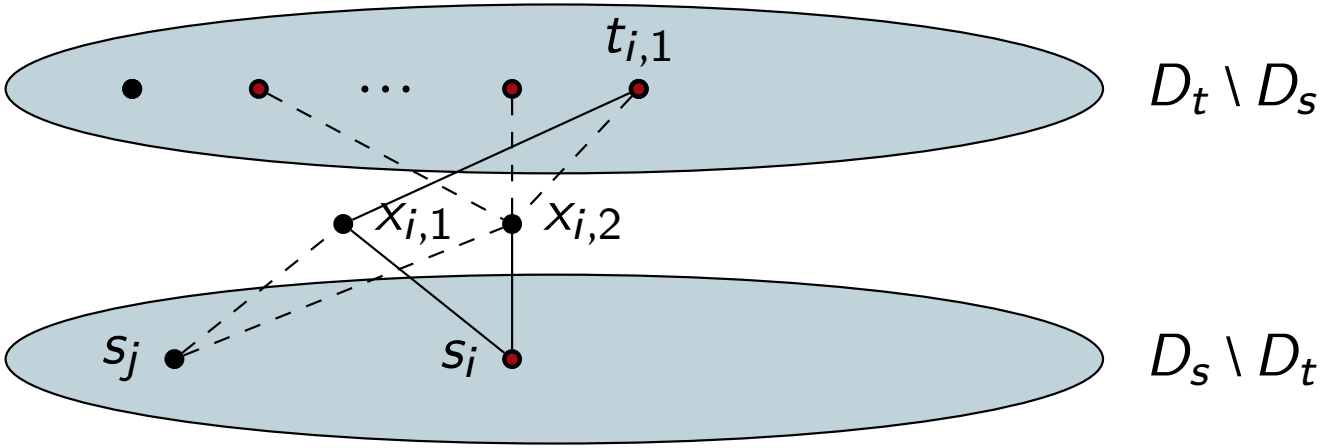

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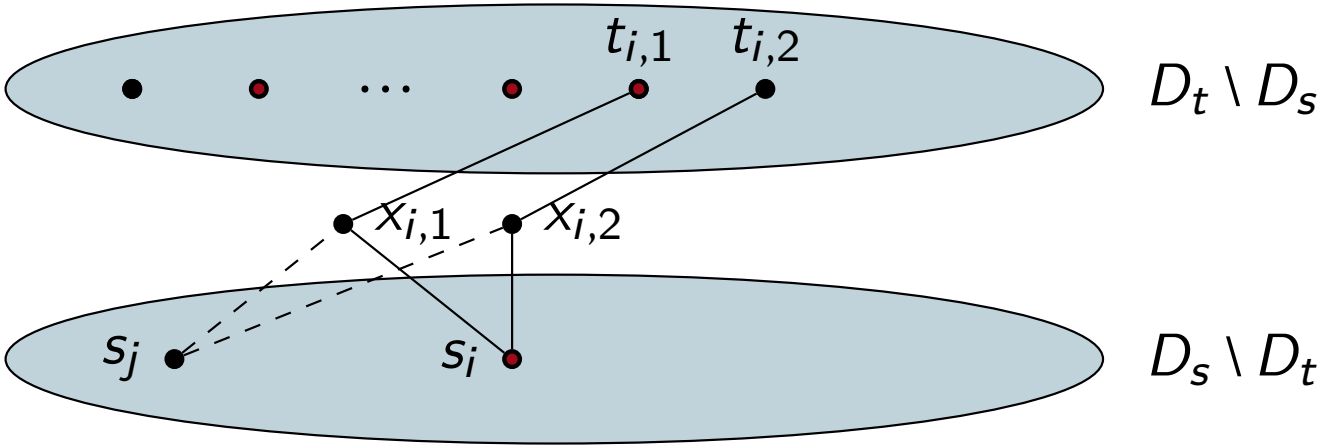

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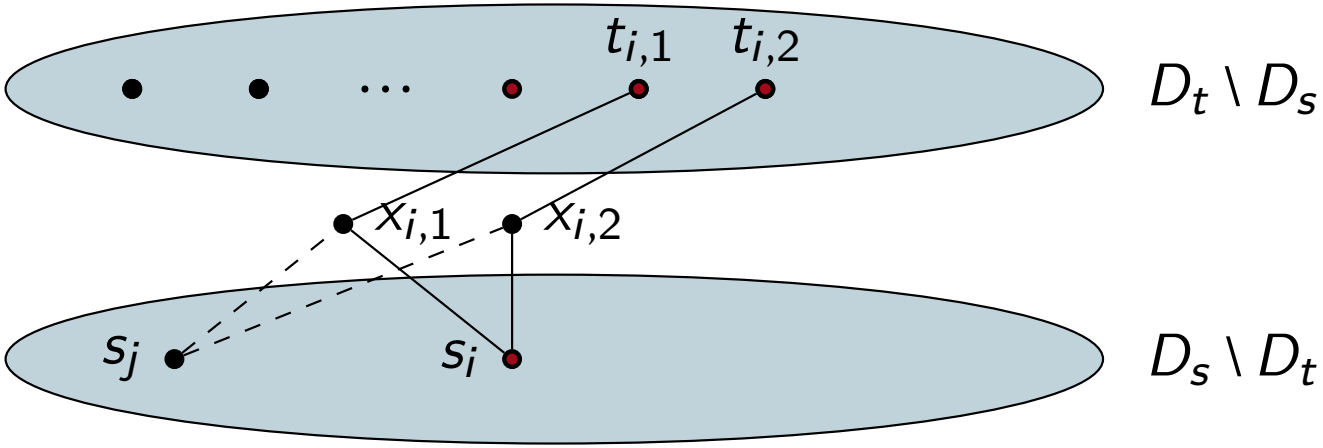

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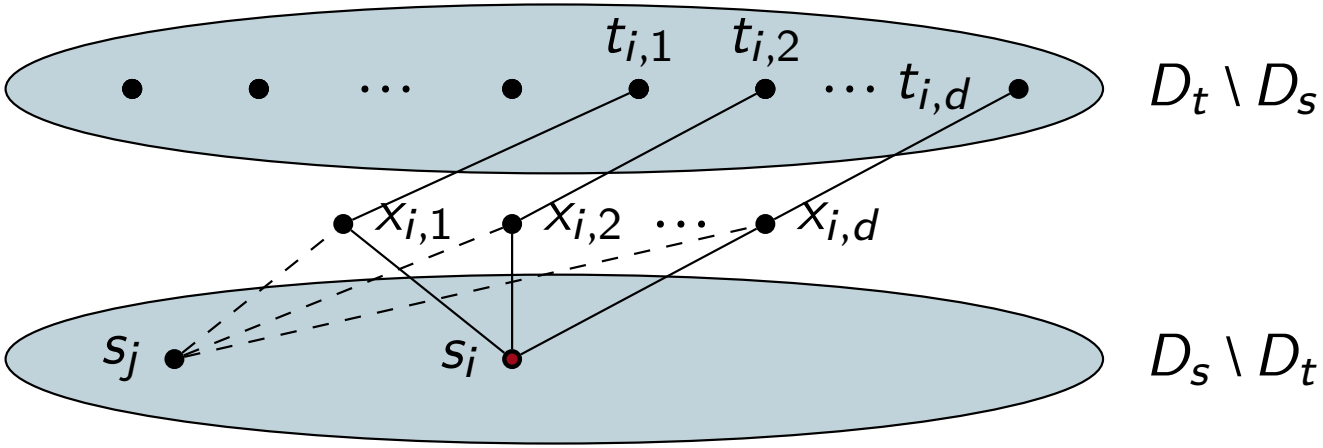

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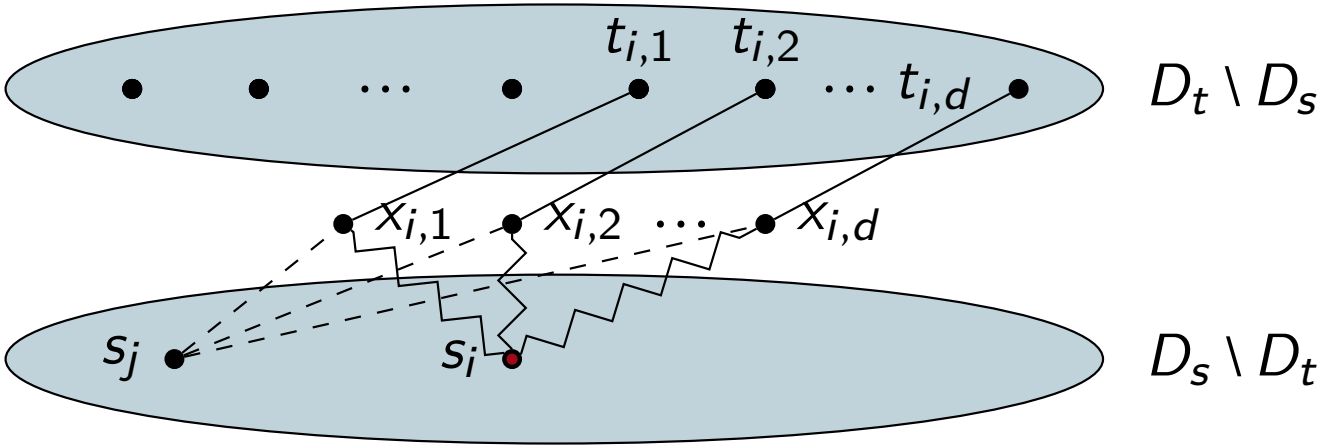

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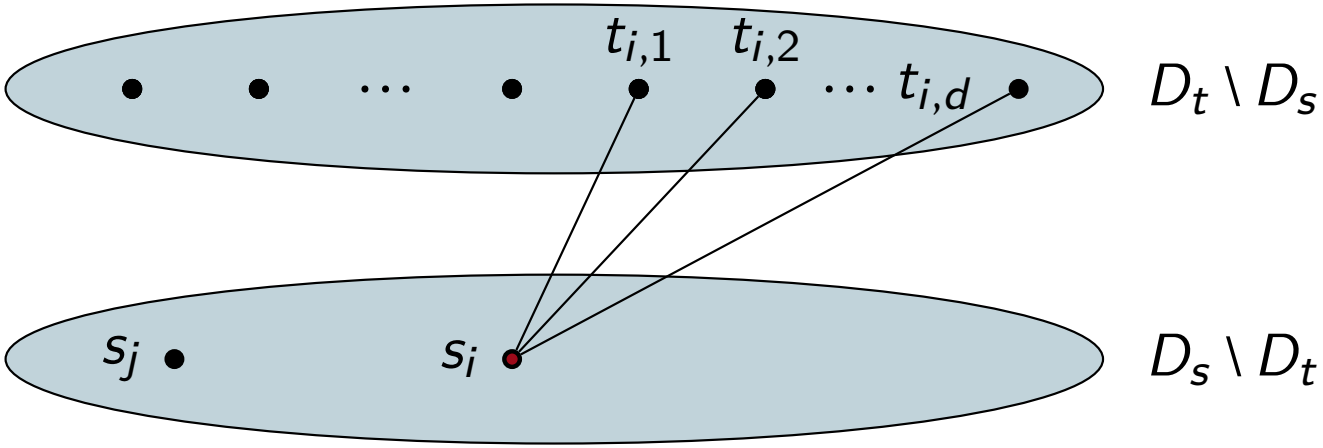

K_ℓ-minor free graphs and planar graphs

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Theorem: For G K_ℓ-minor free, there exists C s.t. if $k \geq \Gamma(G) + C\ell(\log\log\ell)^{18}$ then $\mathcal{R}_k(G)$ is connected and has linear diameter

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Proof: Bipartite planar have at most $2n-4$ edges $\rightarrow$ 4-minor sparse

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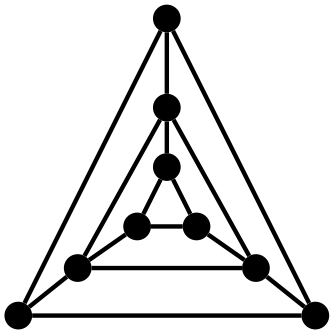

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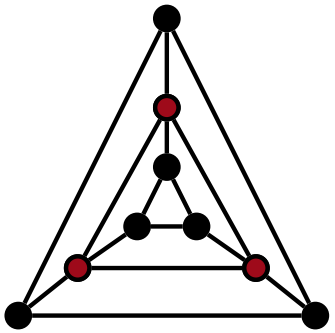

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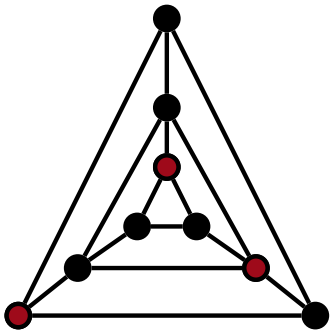

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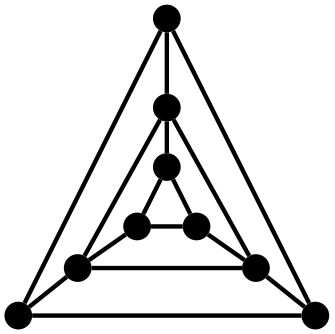

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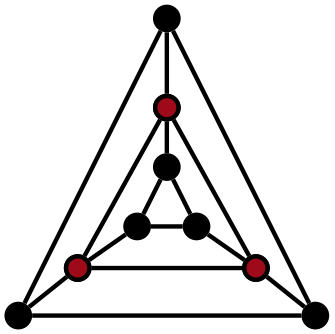

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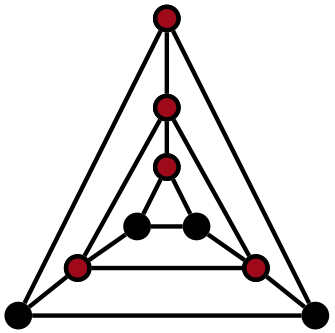

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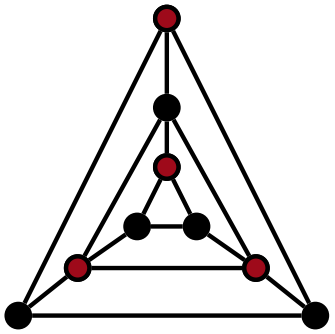

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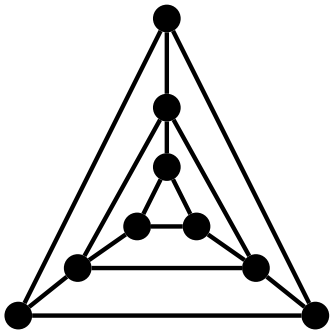

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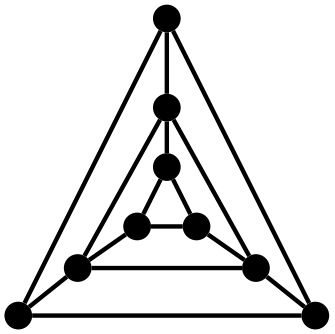

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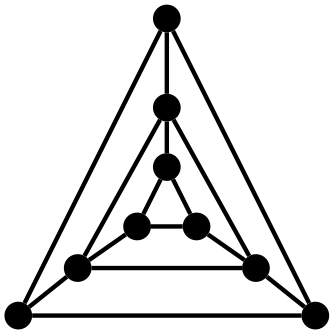

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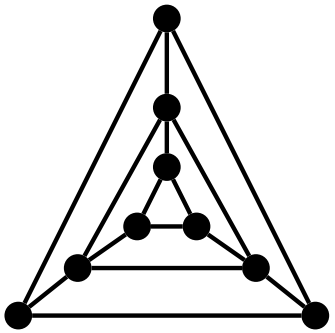

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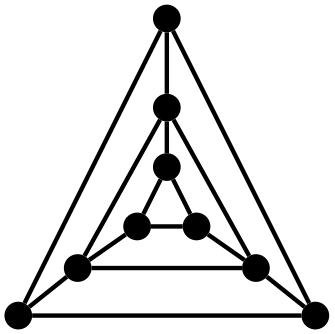

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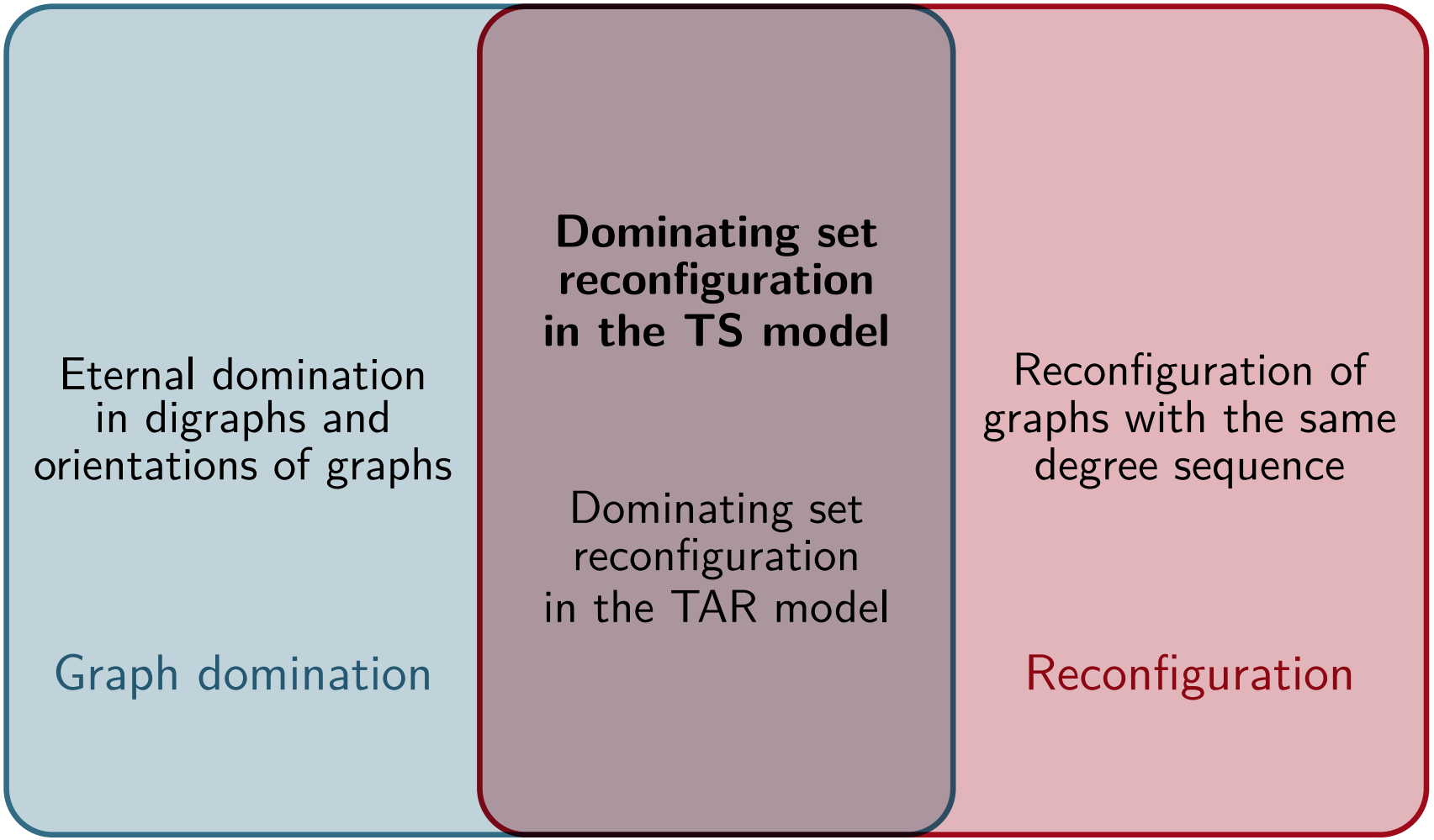
- Find a better upper bound depending on the pathwidth and the bandwidth

Outline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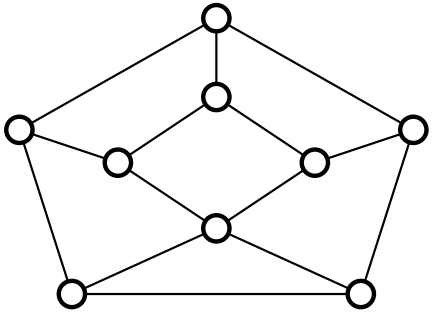

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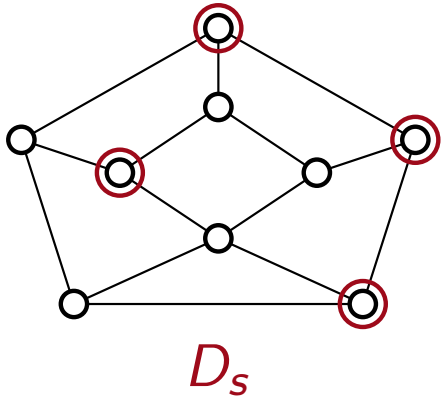

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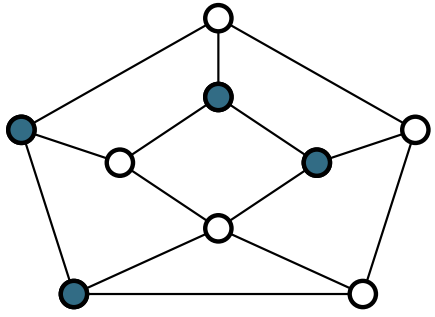

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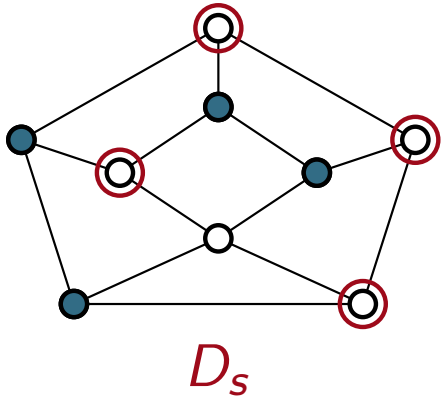

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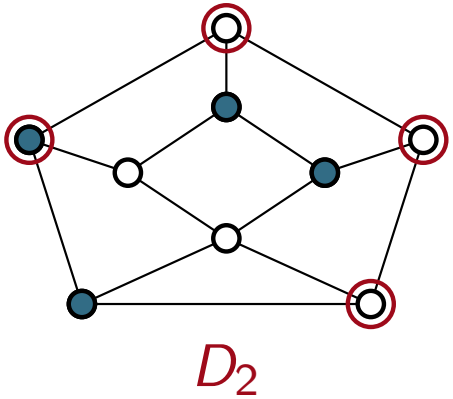

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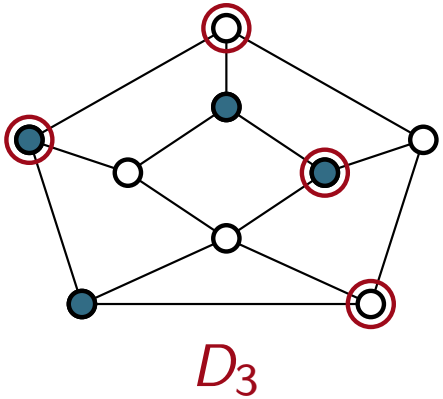

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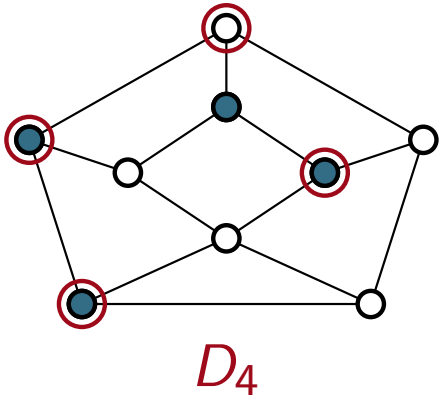

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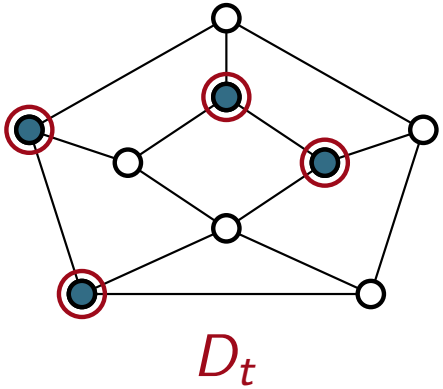

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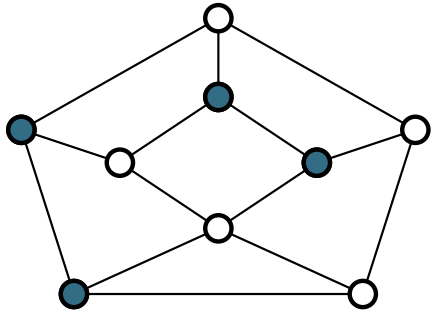

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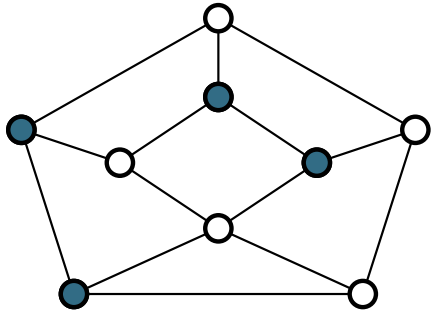

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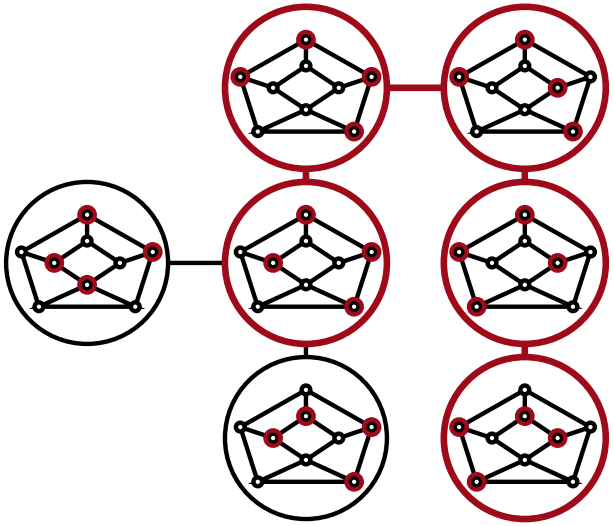

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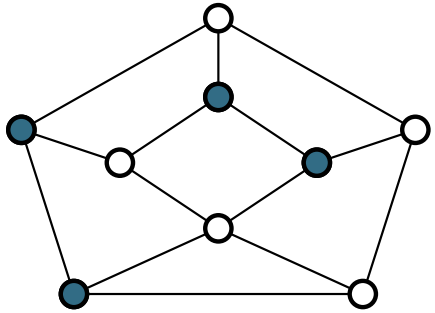

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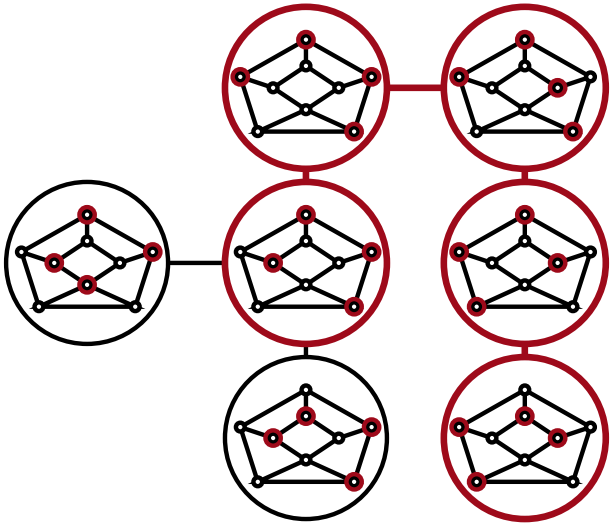

Given D_s and D_t, what is the complexity of deciding if there is a path from D_s to D_t ?

Results

Results

TS-REACHABILITY

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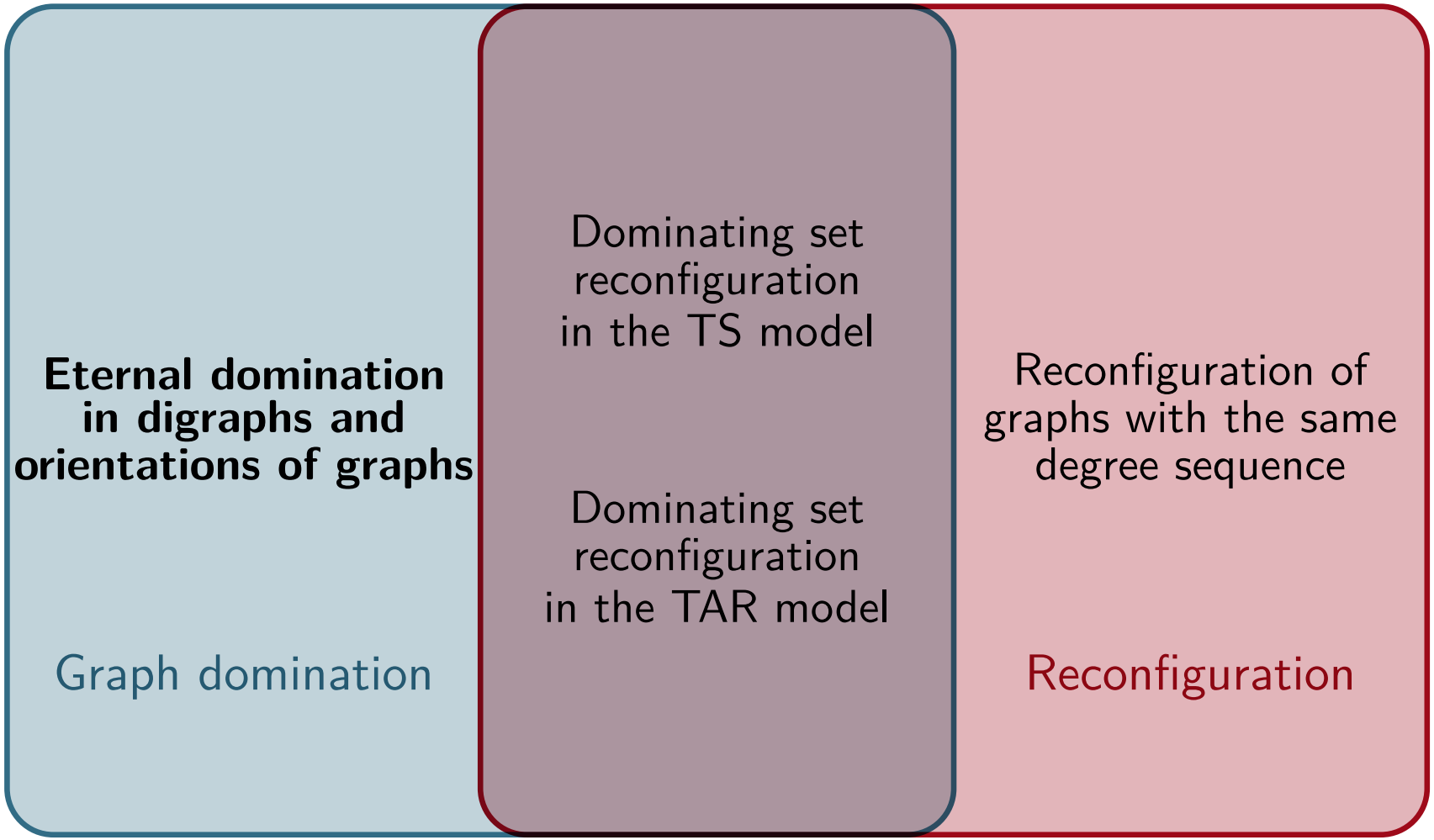
Outline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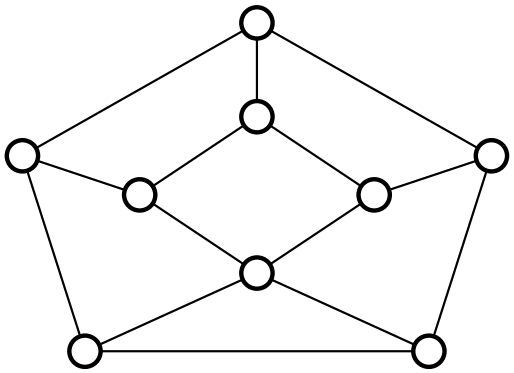

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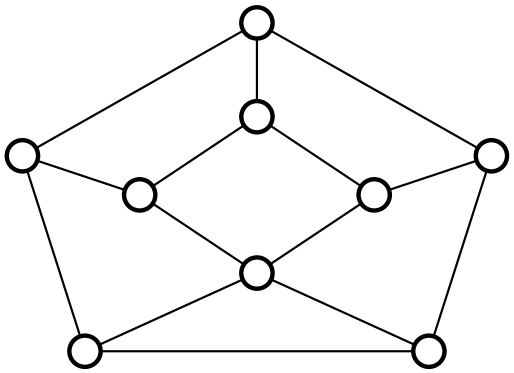

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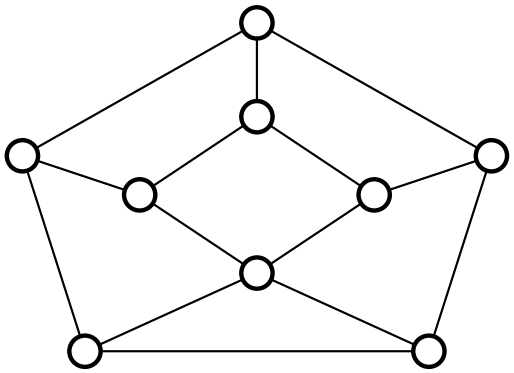

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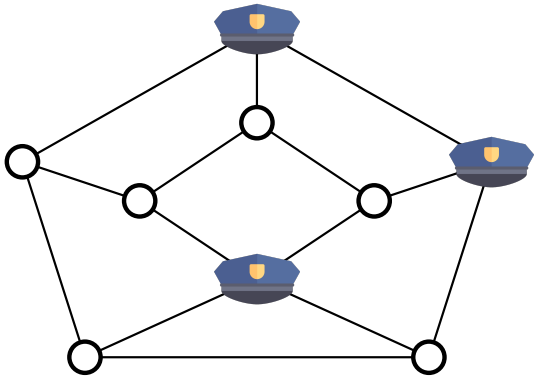

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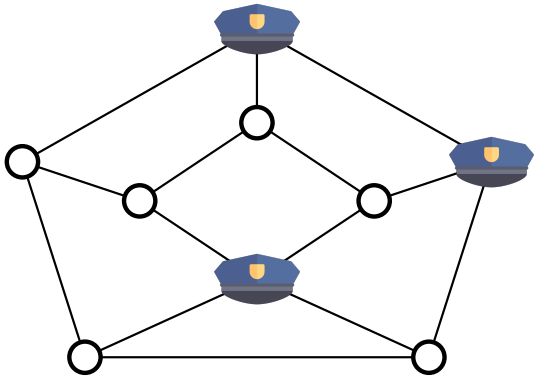

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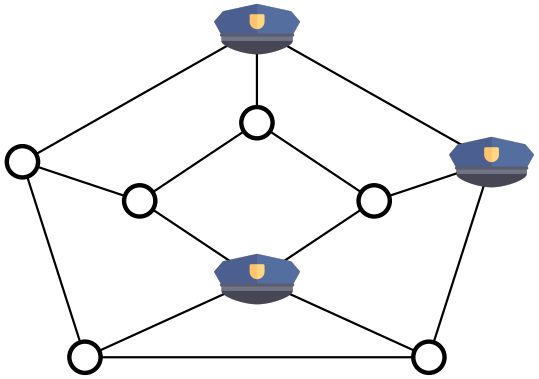

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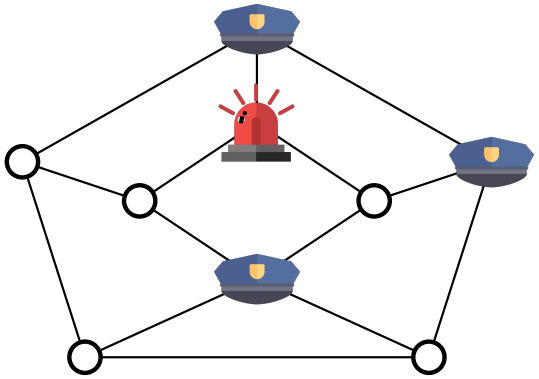

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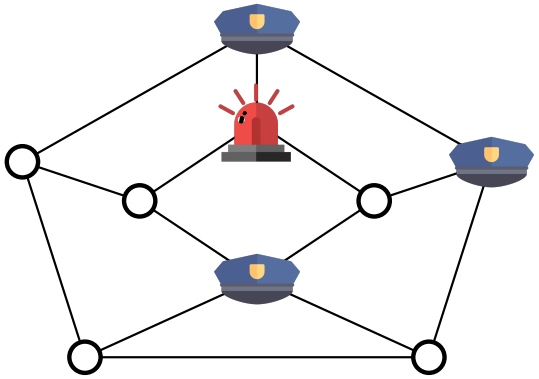

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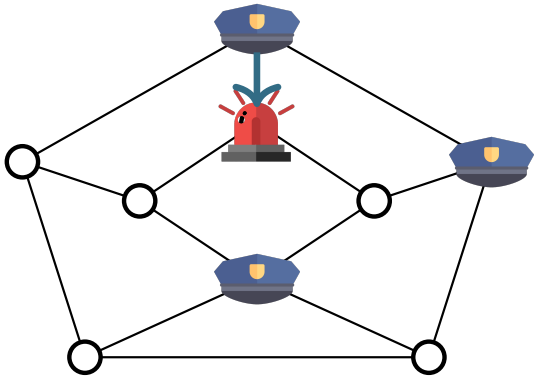

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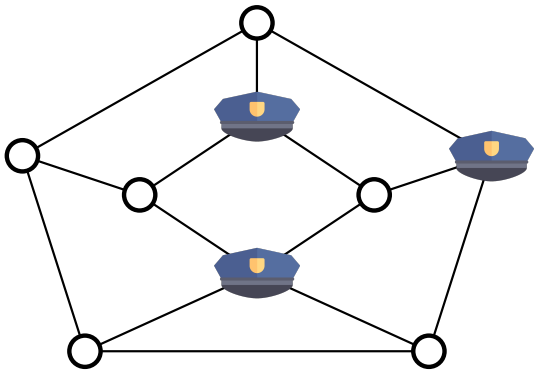

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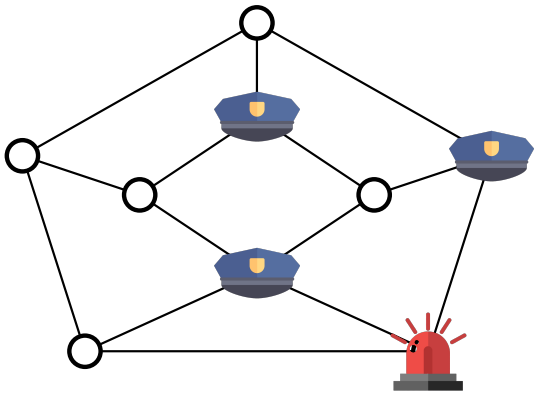

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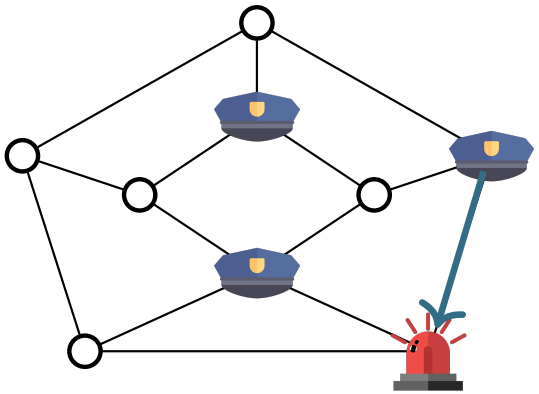

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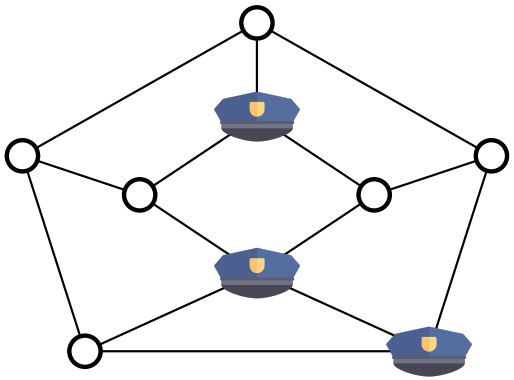

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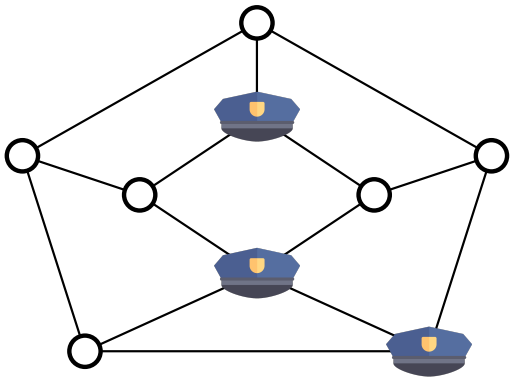
- D wins if he/she can defend against any infinite sequence of attacks



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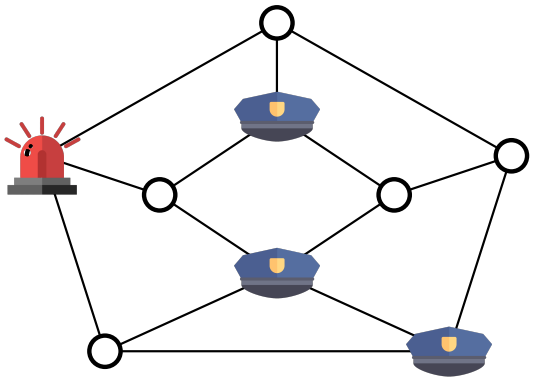

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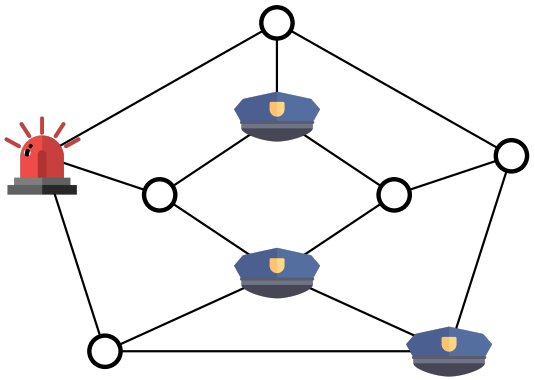

$\gamma^{\infty}(G)$: min number of guards necessary for the defender to win

Results

Results

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Results

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- State of the art:
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  - $\alpha(G) \leq \gamma^\infty(G) \leq \binom{\alpha(G)+1}{2}$ (Klostermeyer & MacGillivray '07)

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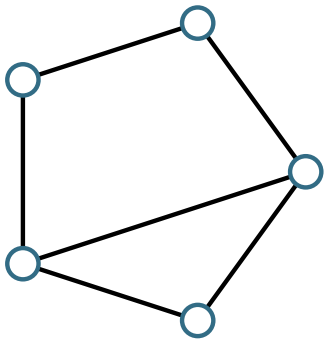

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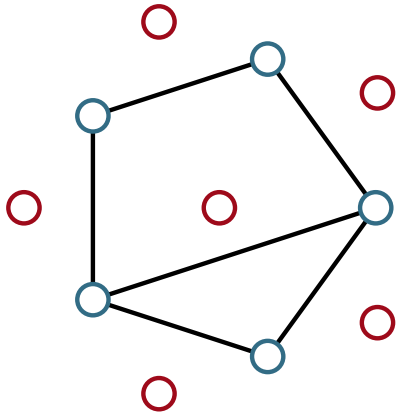

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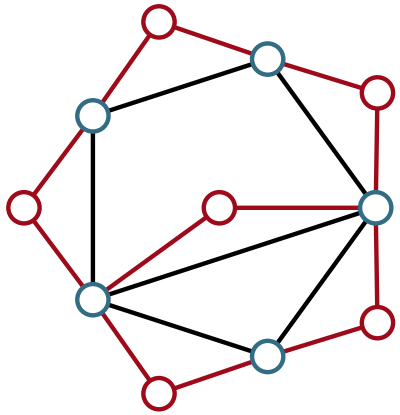

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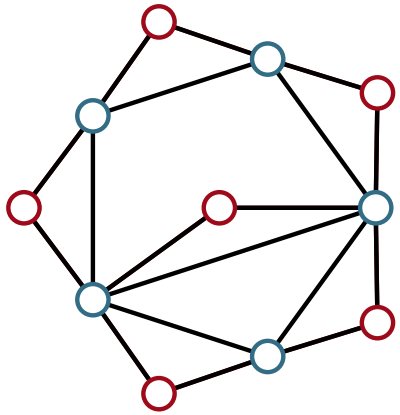

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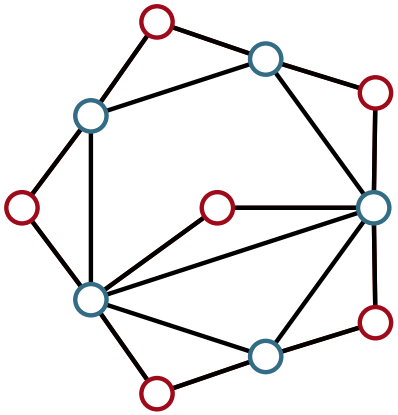

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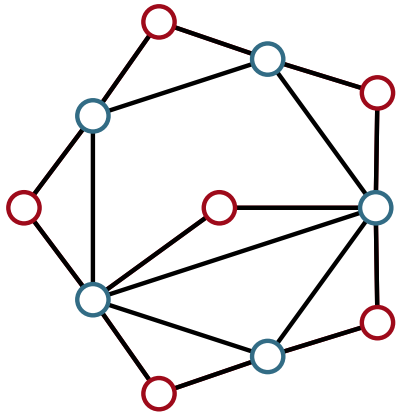

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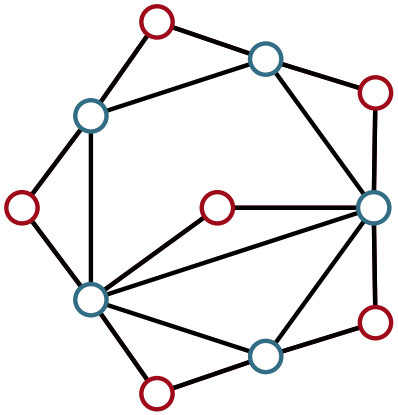

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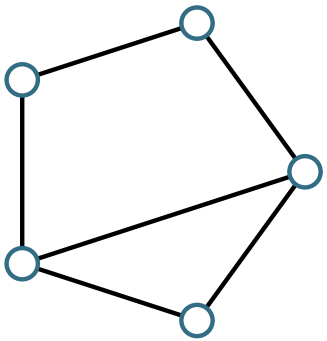

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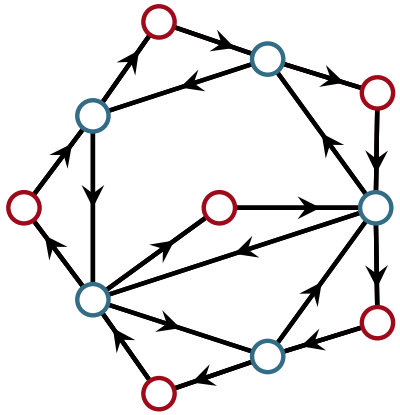

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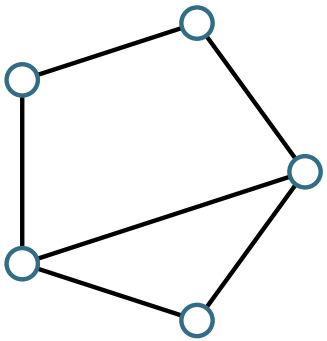

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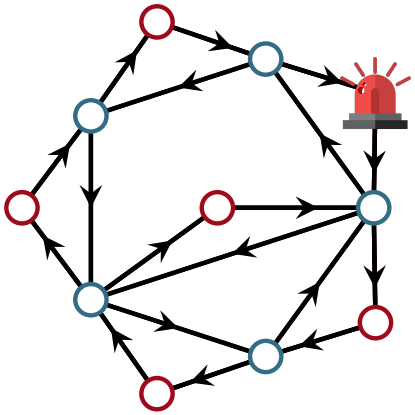

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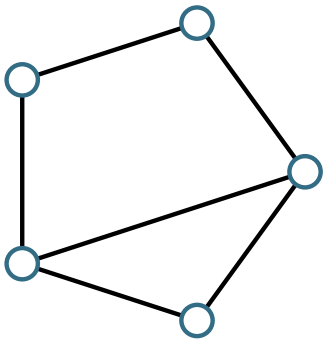

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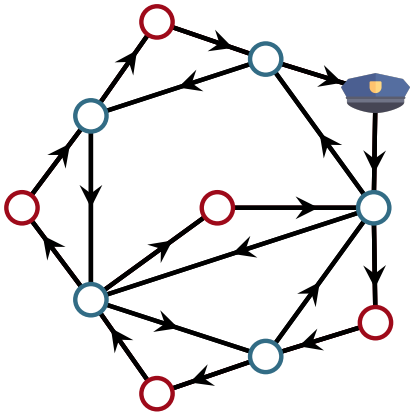

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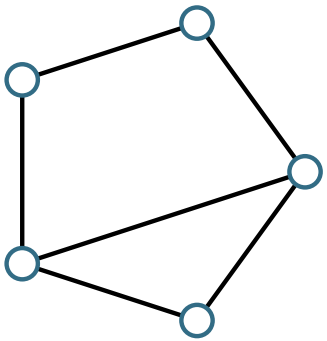

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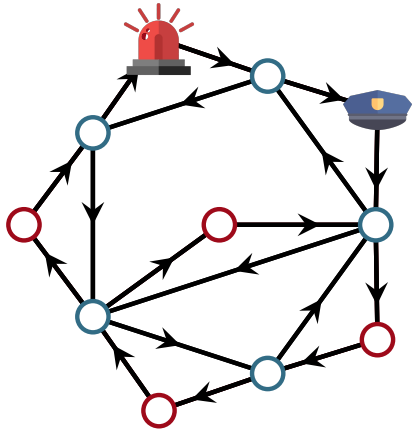

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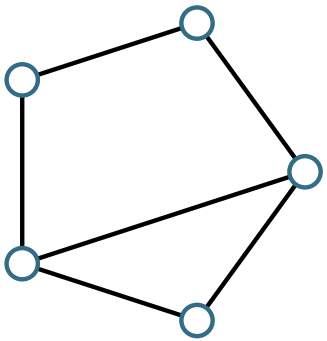

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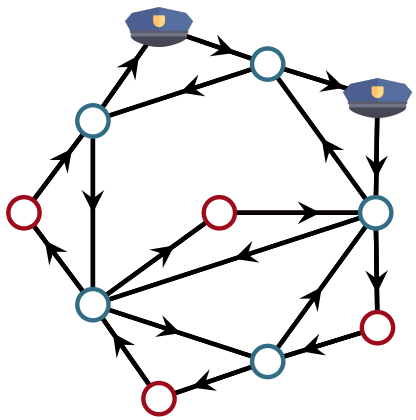

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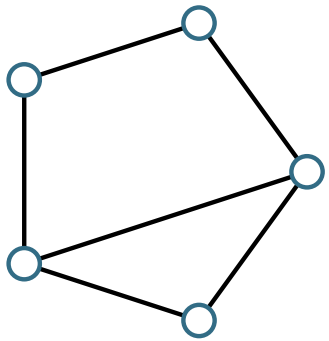

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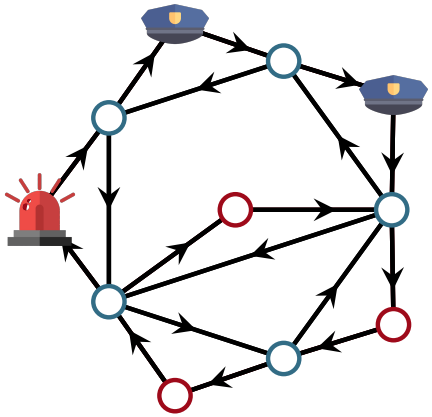

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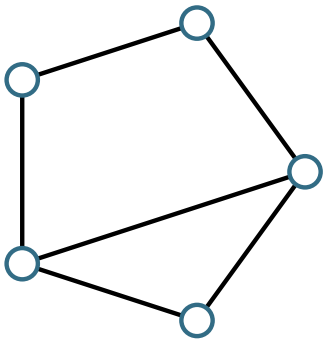

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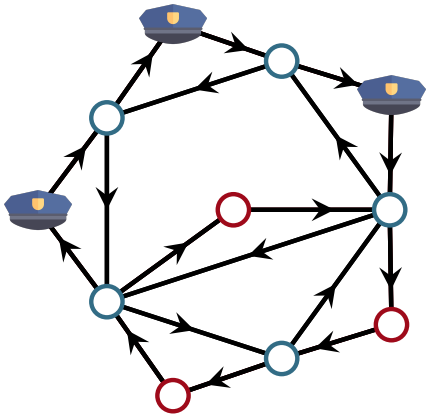

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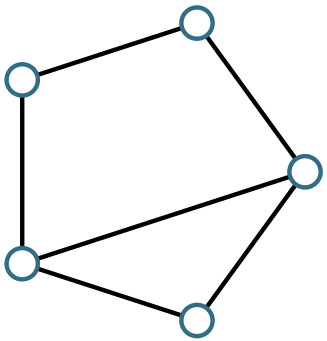

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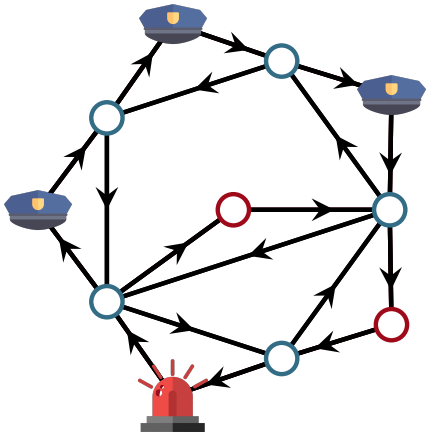

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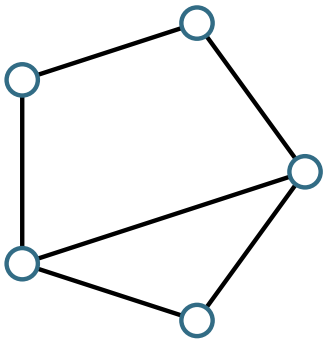

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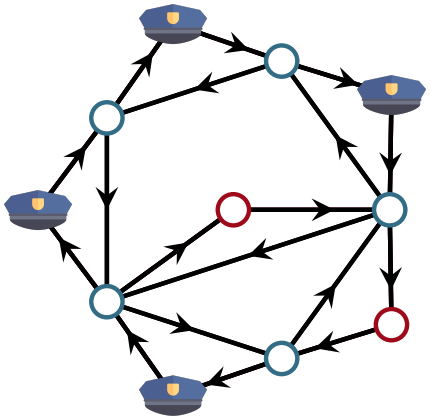

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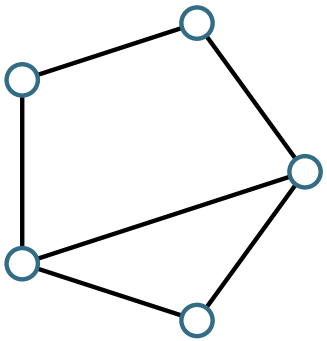

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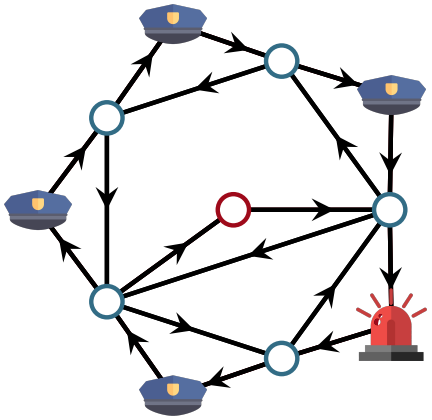

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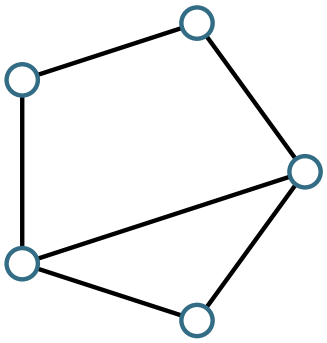

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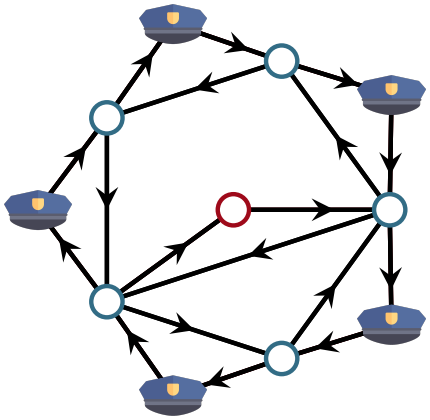

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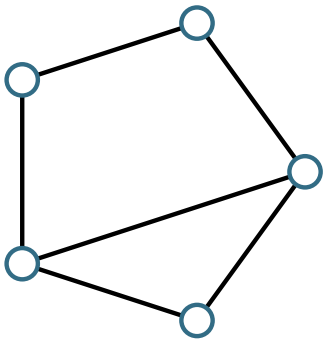

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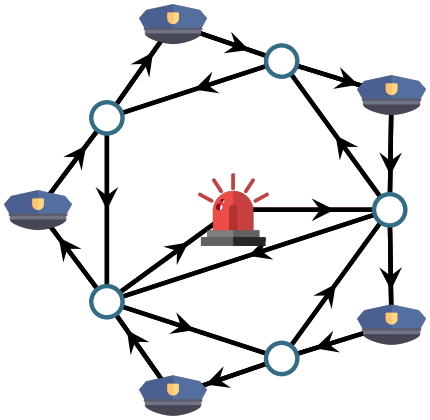

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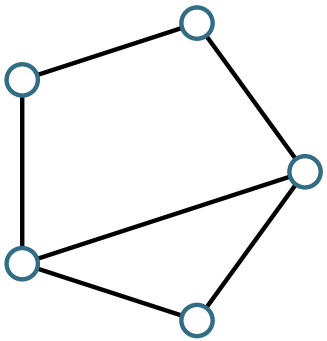

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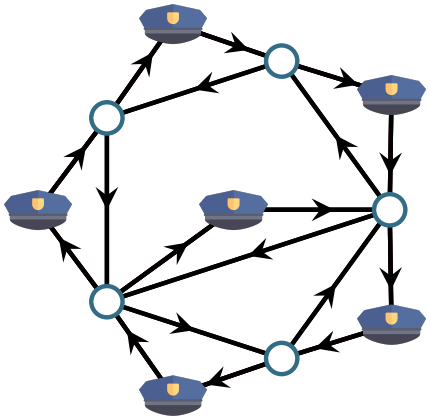

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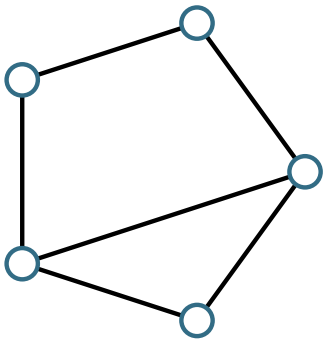

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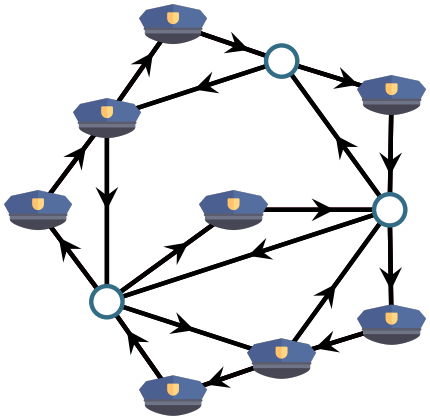

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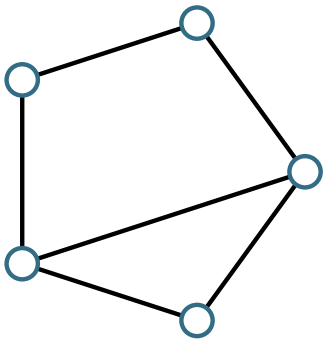

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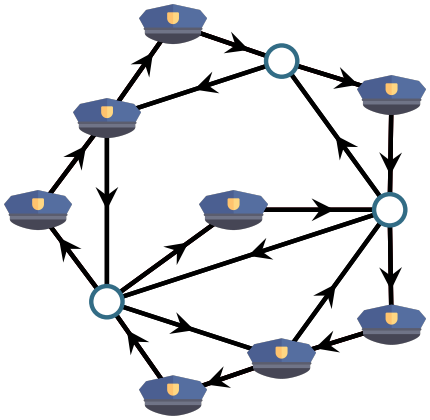

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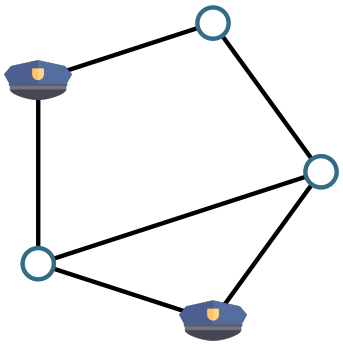

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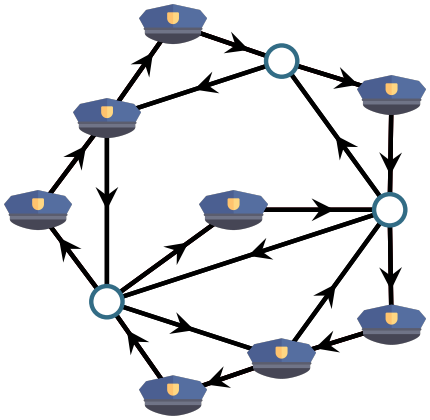

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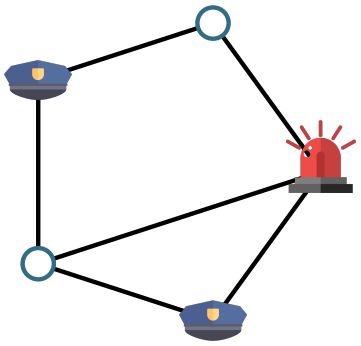

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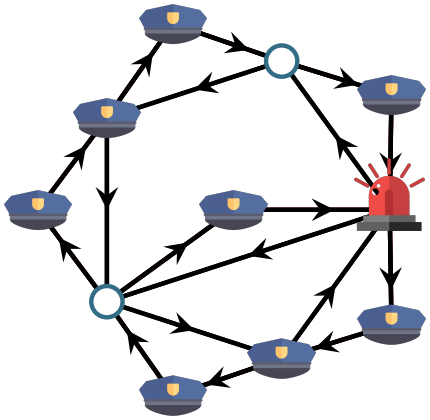

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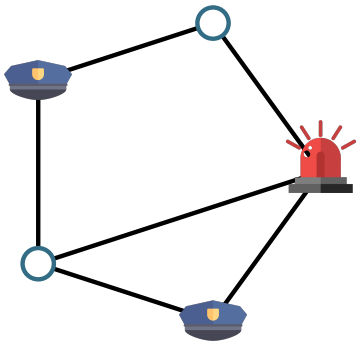

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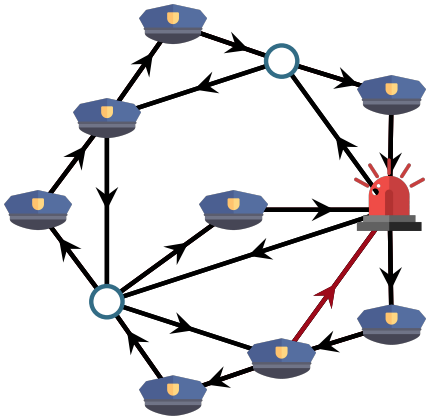

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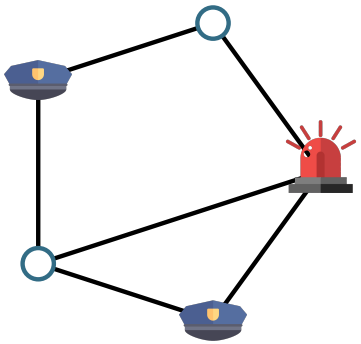

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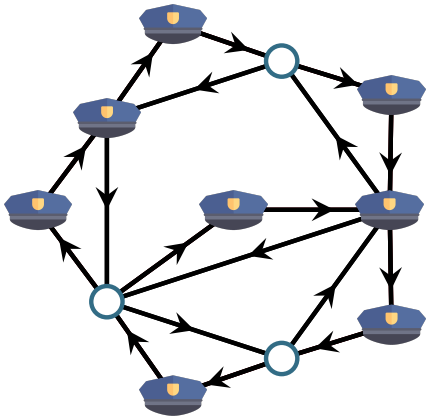

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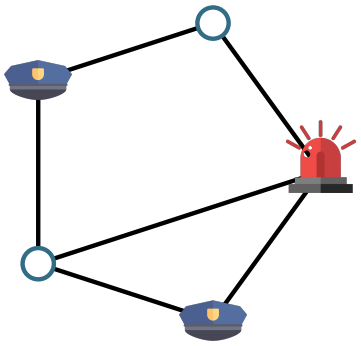

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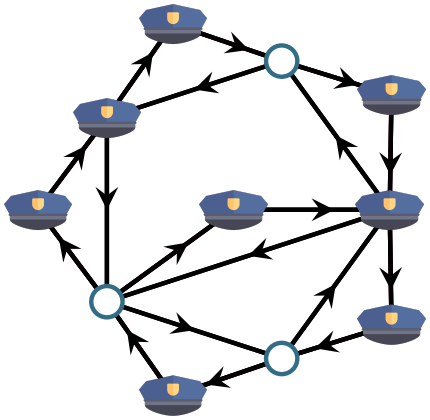

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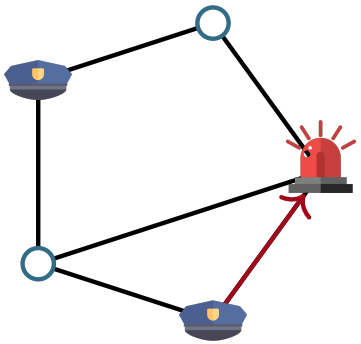

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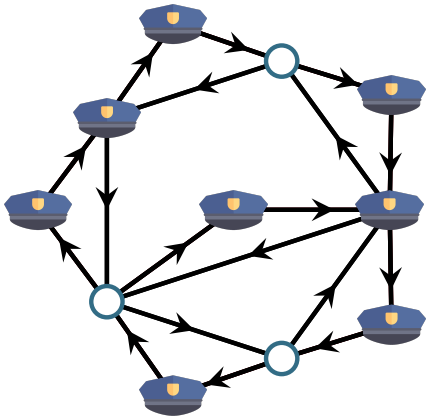

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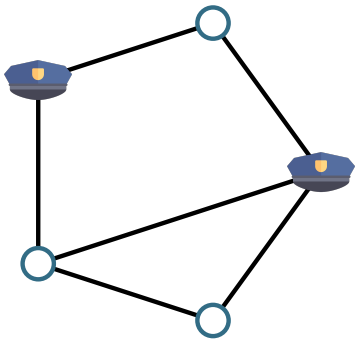

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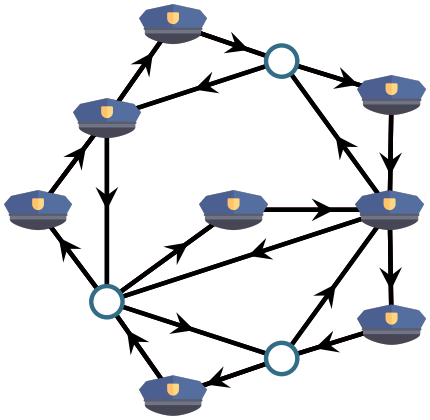

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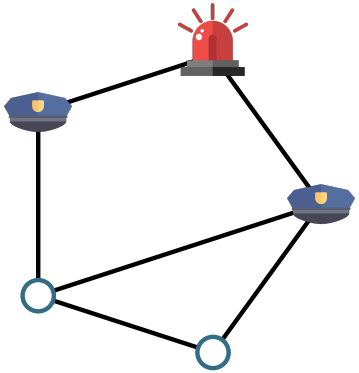

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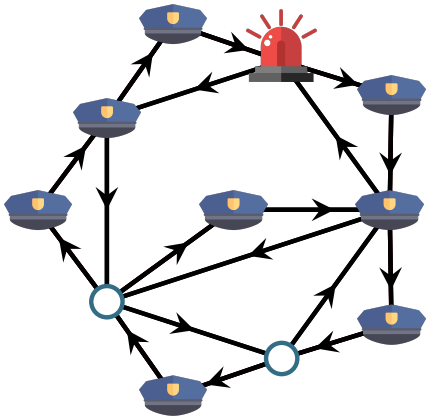

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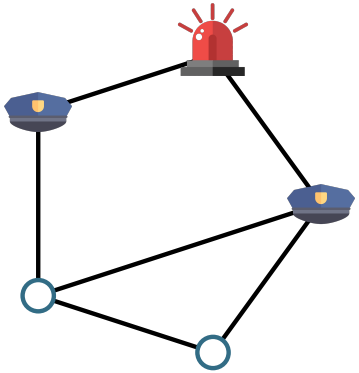

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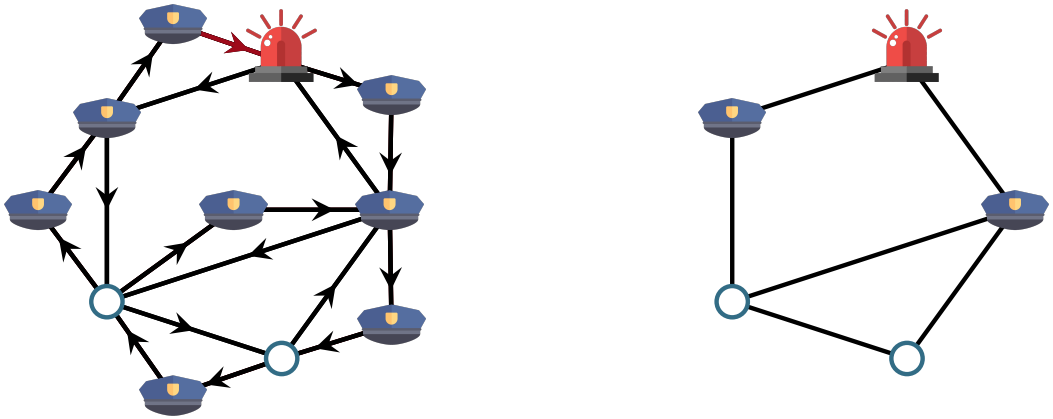

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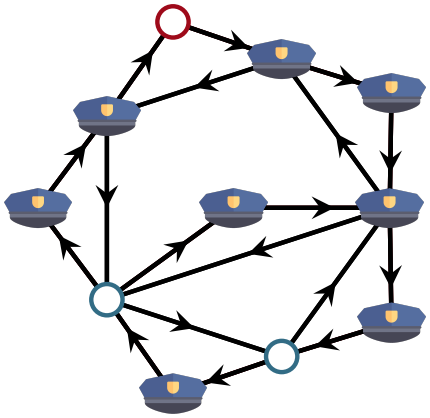

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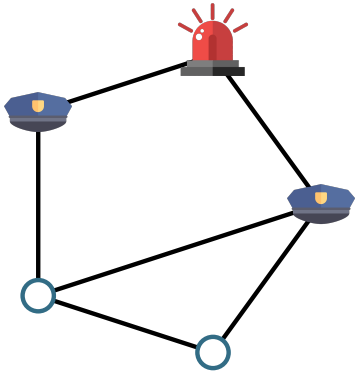

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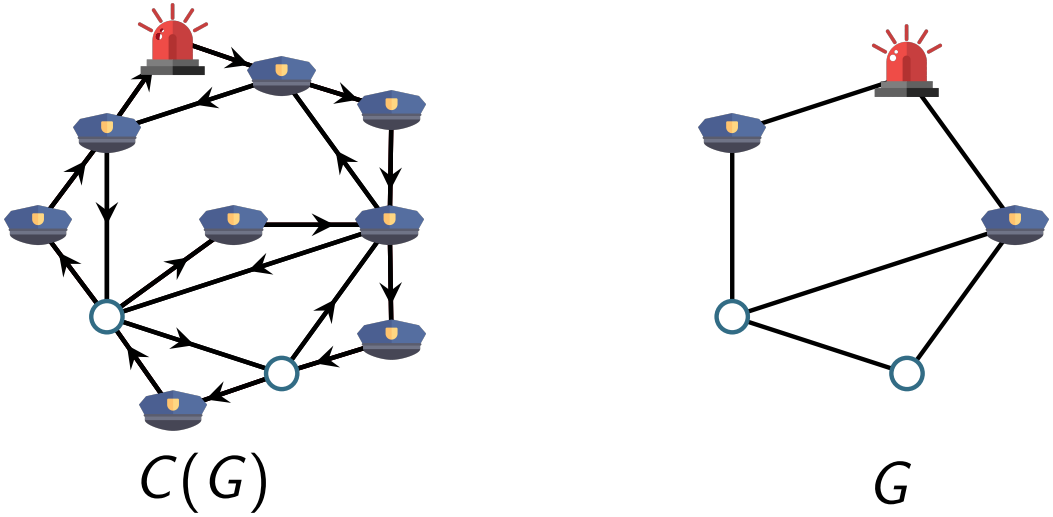

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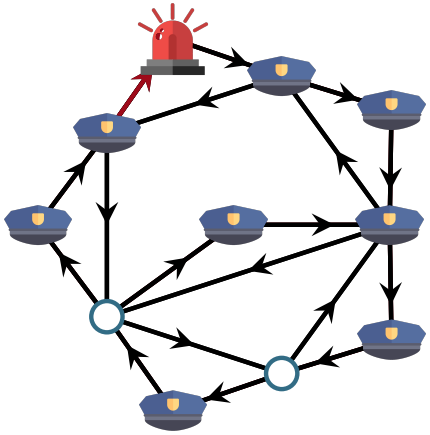

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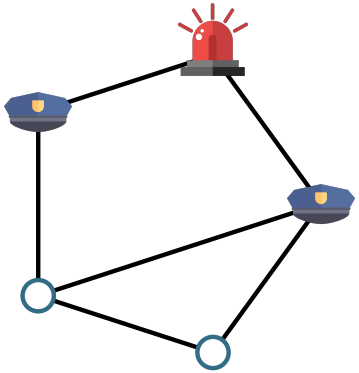

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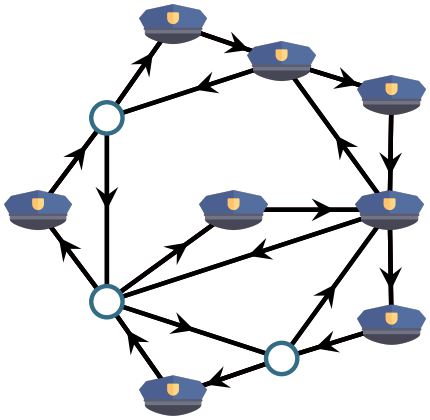

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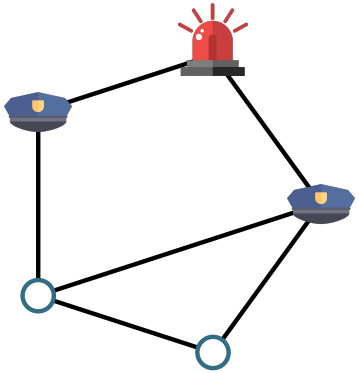

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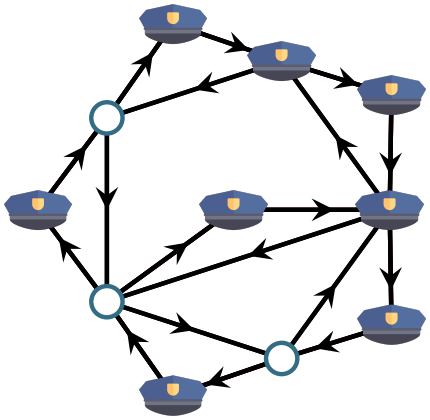

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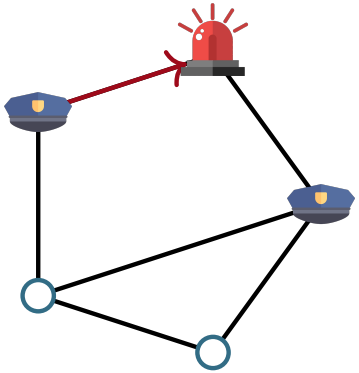

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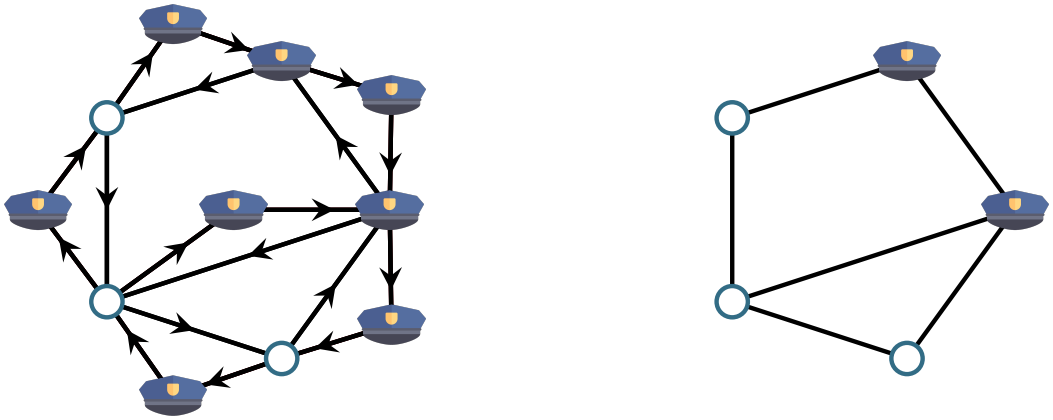

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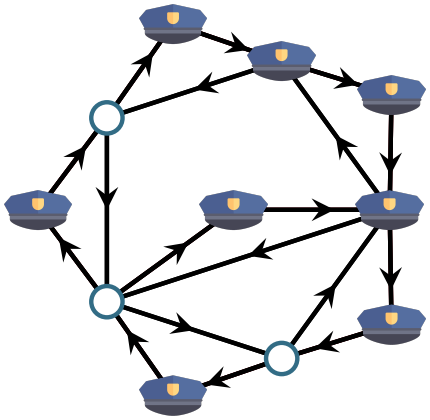

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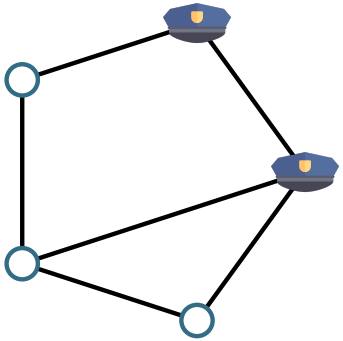

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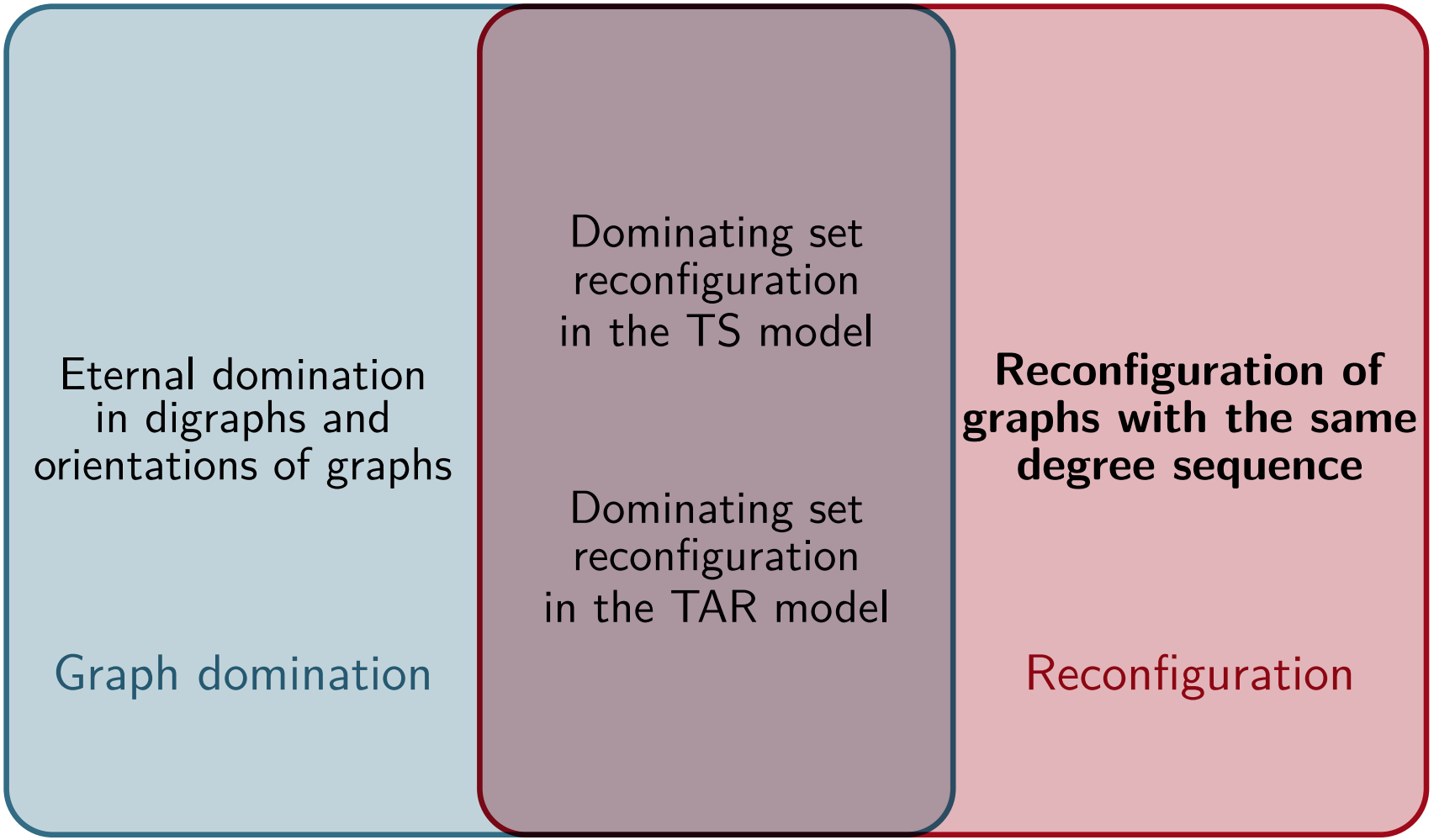
Conjecture: Deciding if $\overrightarrow{\gamma^\infty}(G) \leq k$ is a PSPACE-complete problem

Outline


Eternal domination in digraphs and orientations of graphs
Graph domination
Dominating set reconfiguration in the TS model
Dominating set reconfiguration in the TAR model
Reconfiguration of graphs with the same degree sequence
Reconfiguration

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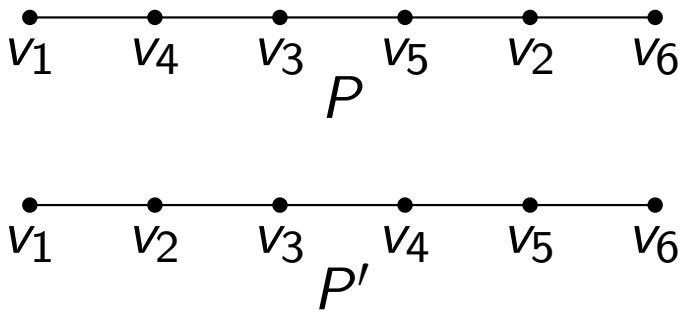

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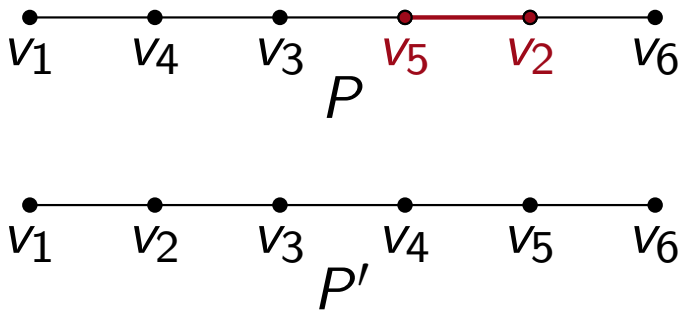

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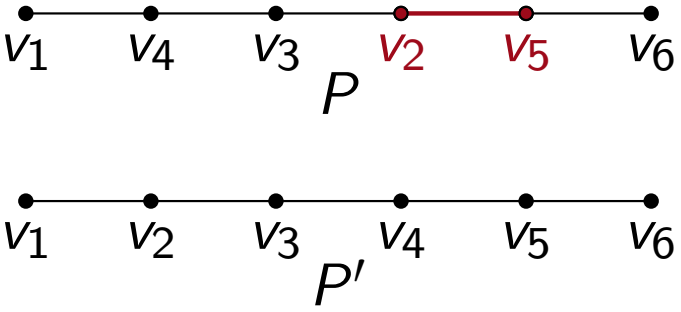

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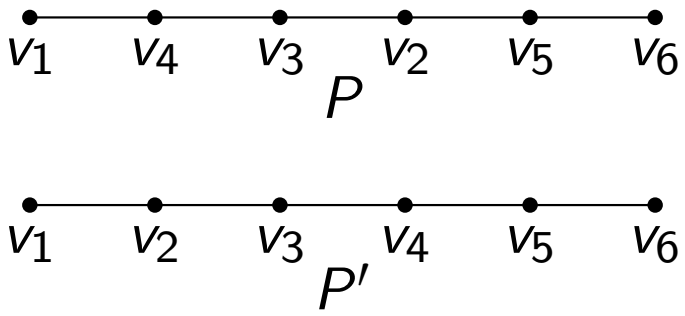

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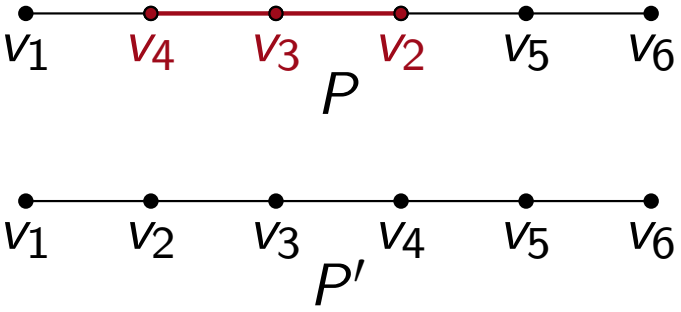

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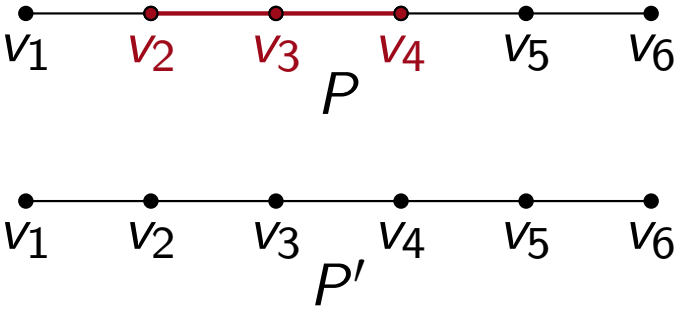

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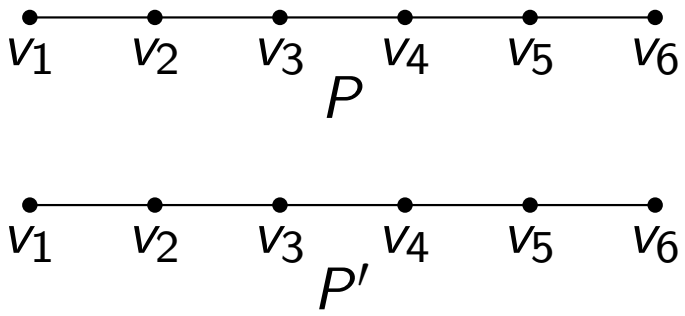

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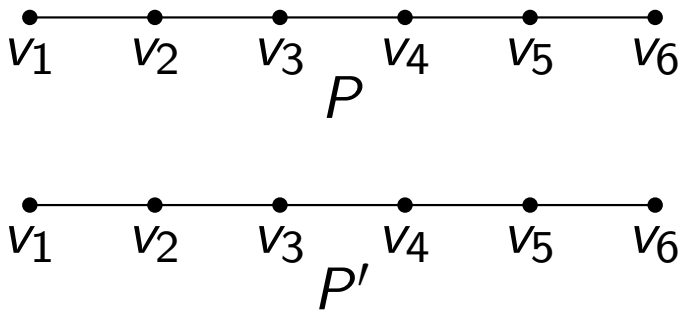

Application to the computation of genetic distance between species

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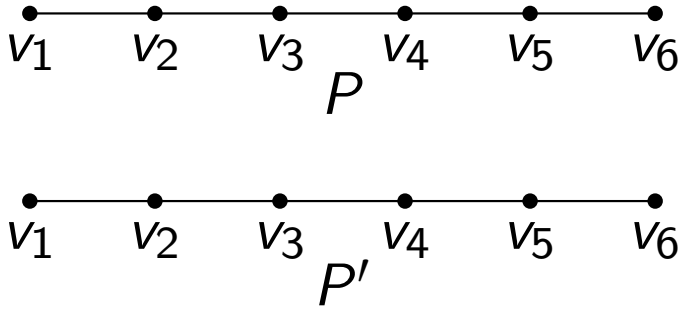
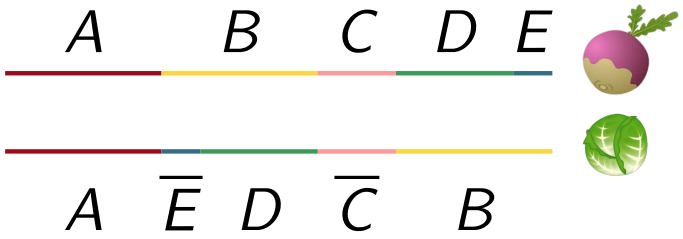


Application to the computation of genetic distance between species



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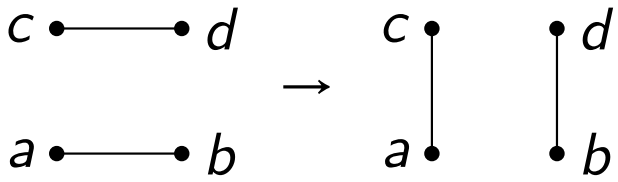

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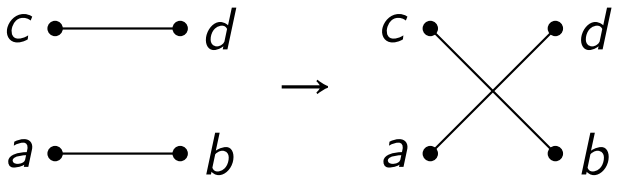

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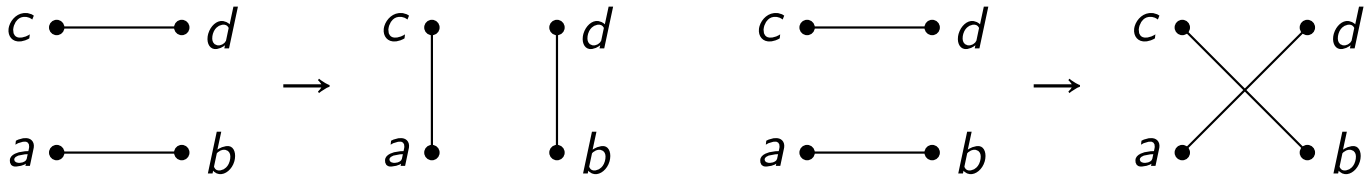

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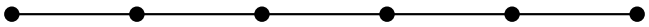

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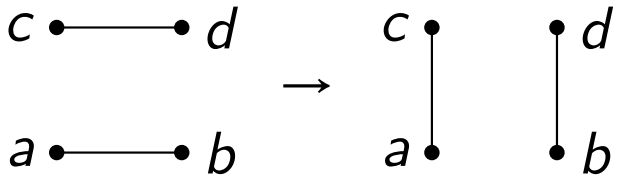

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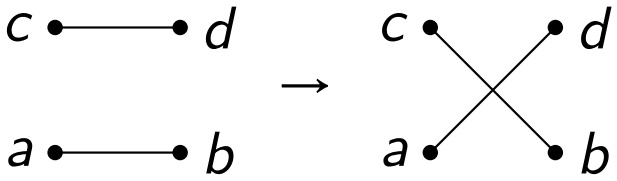

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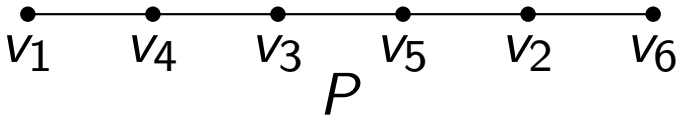

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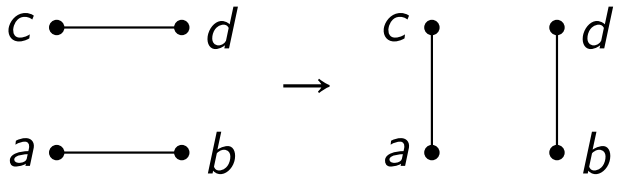

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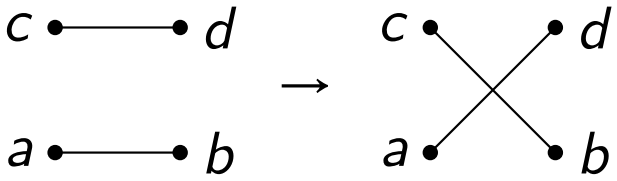

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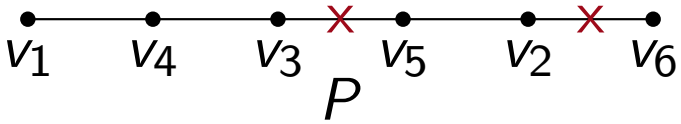

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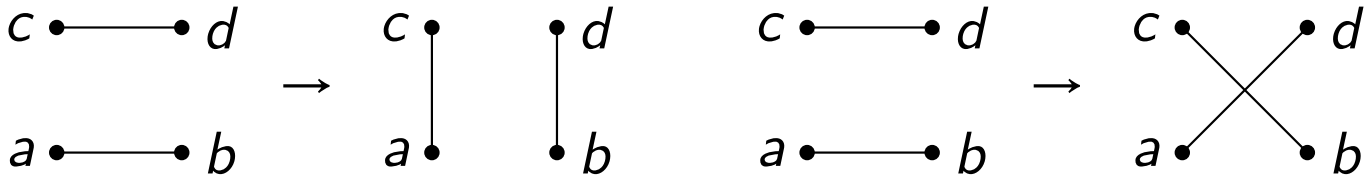

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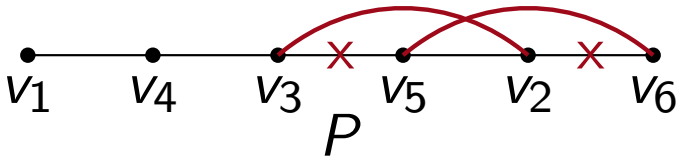

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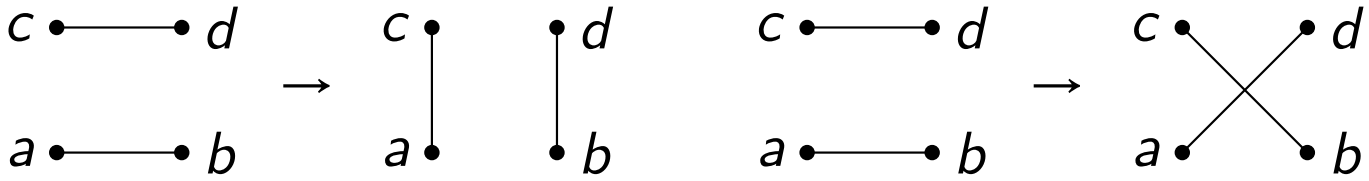

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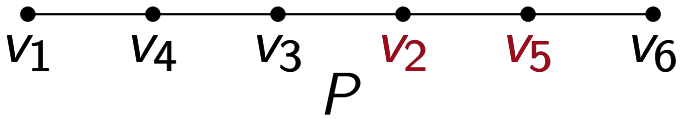

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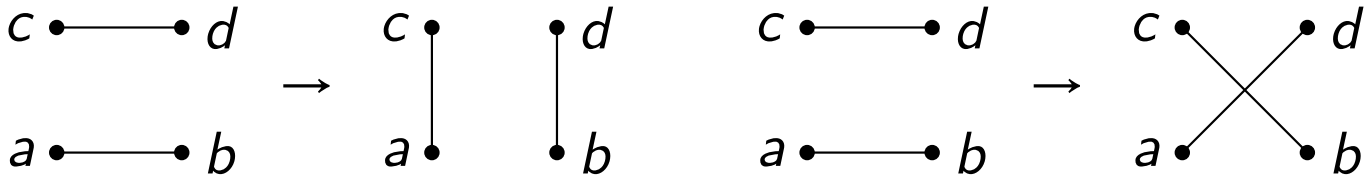

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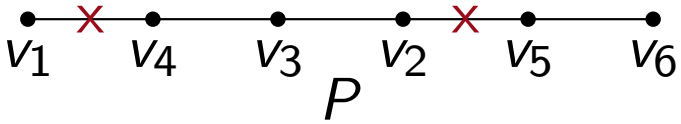

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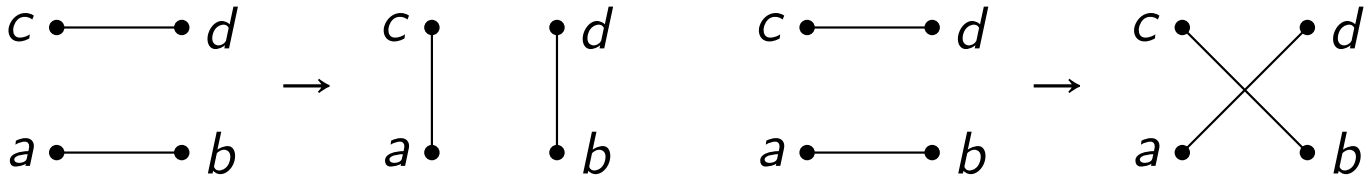

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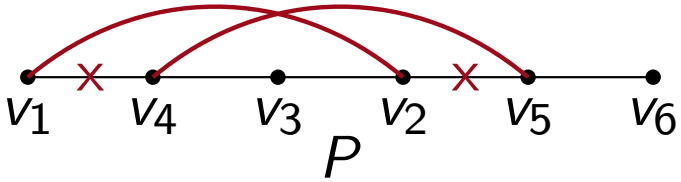

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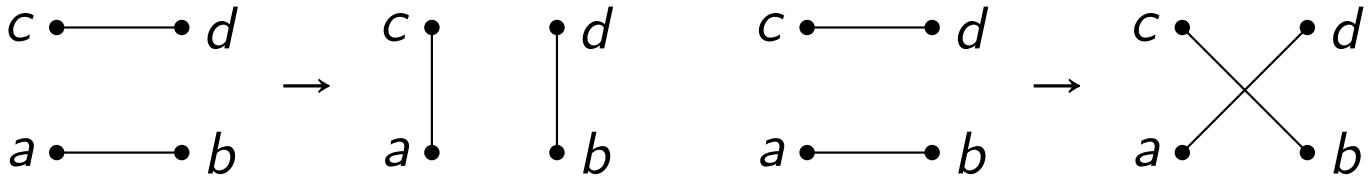

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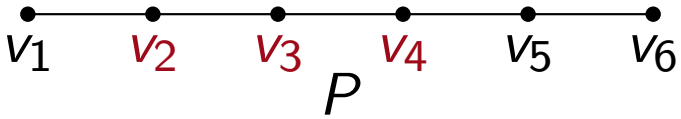

Shortest Connected Graph Transformation

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SHORTEST CONNECTED GRAPH TRANSFORMATION (SCGT)

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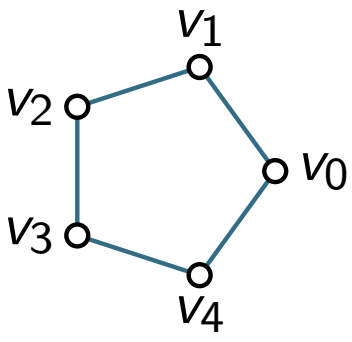

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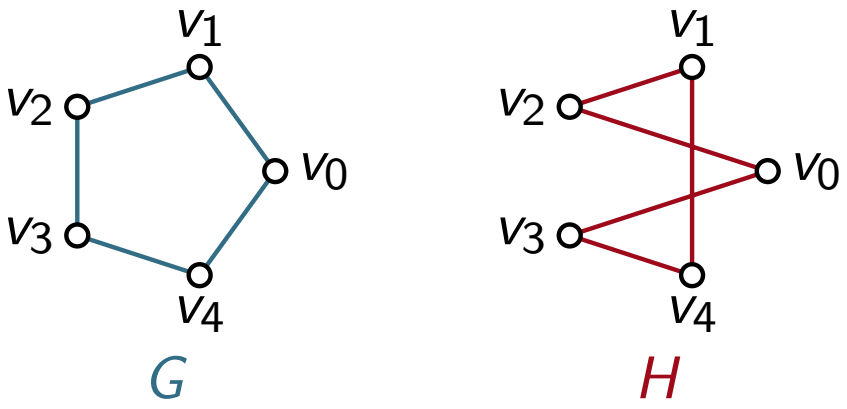

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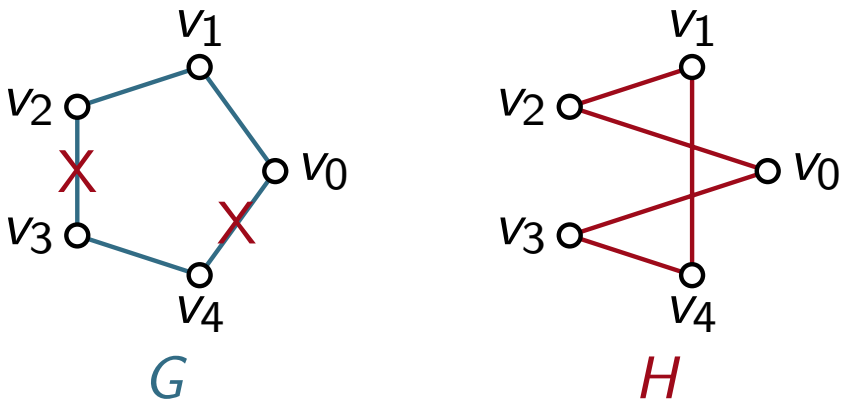

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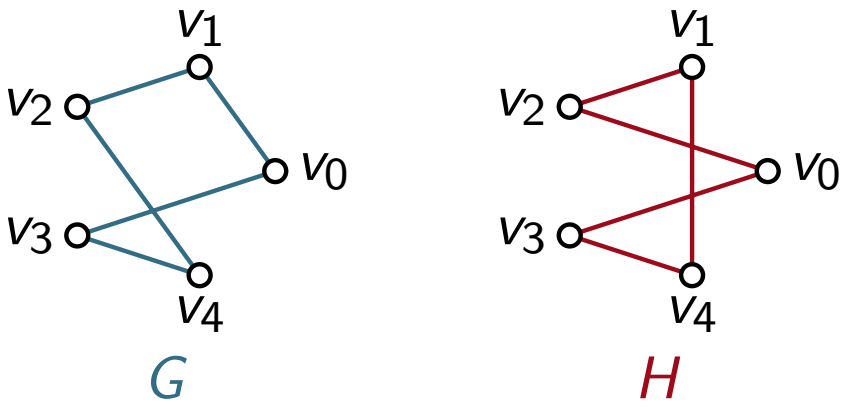

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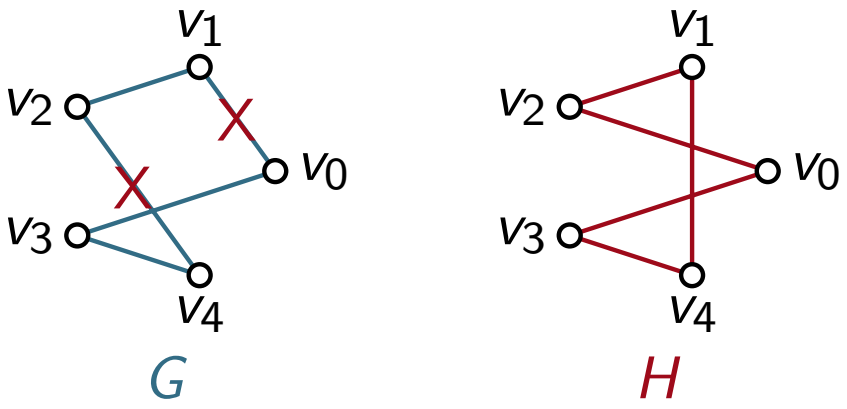

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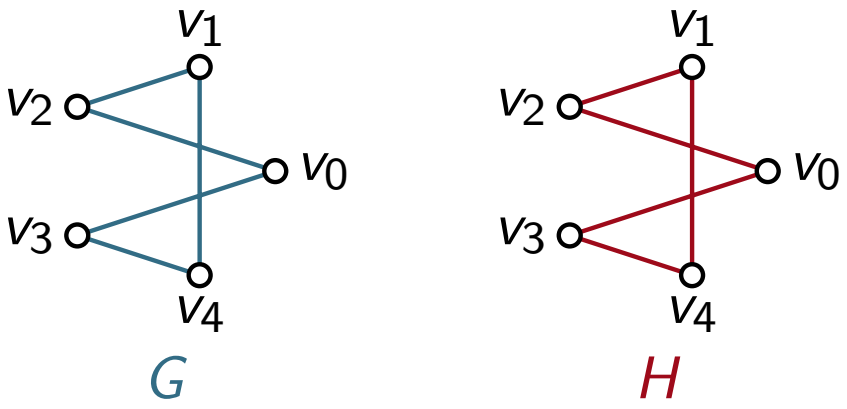

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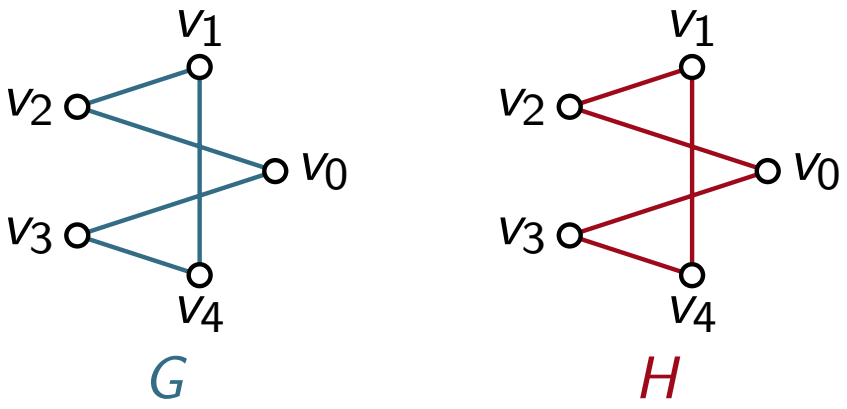

Application to mass spectrometry

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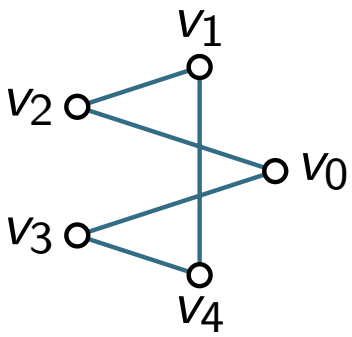

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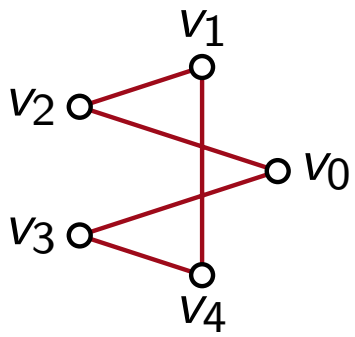

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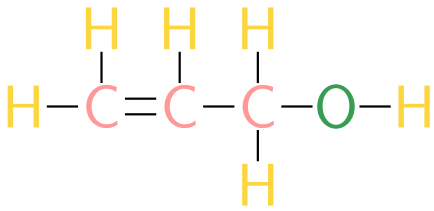
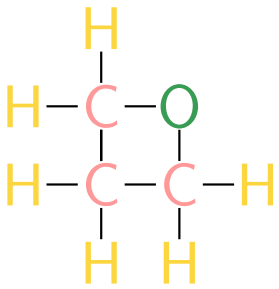


allyl alcohol C_3H_6O



oxetane C_3H_6O

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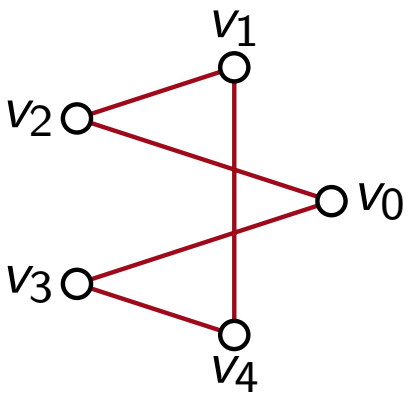

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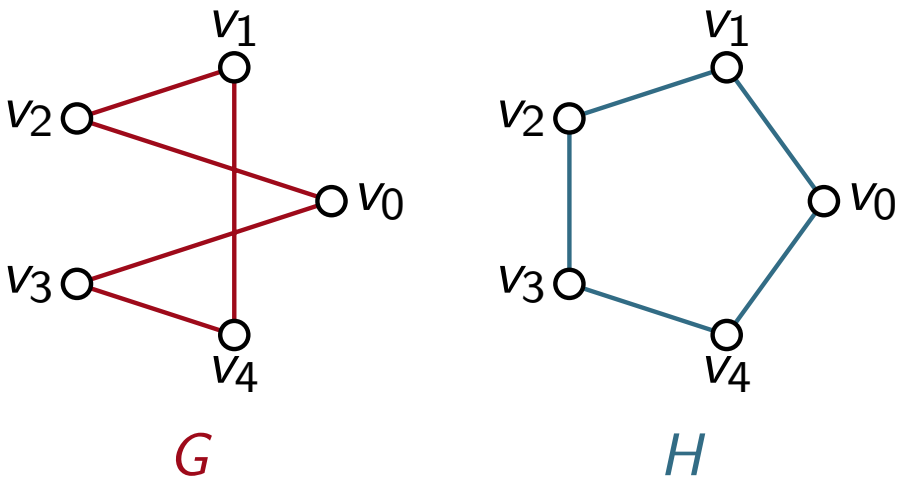

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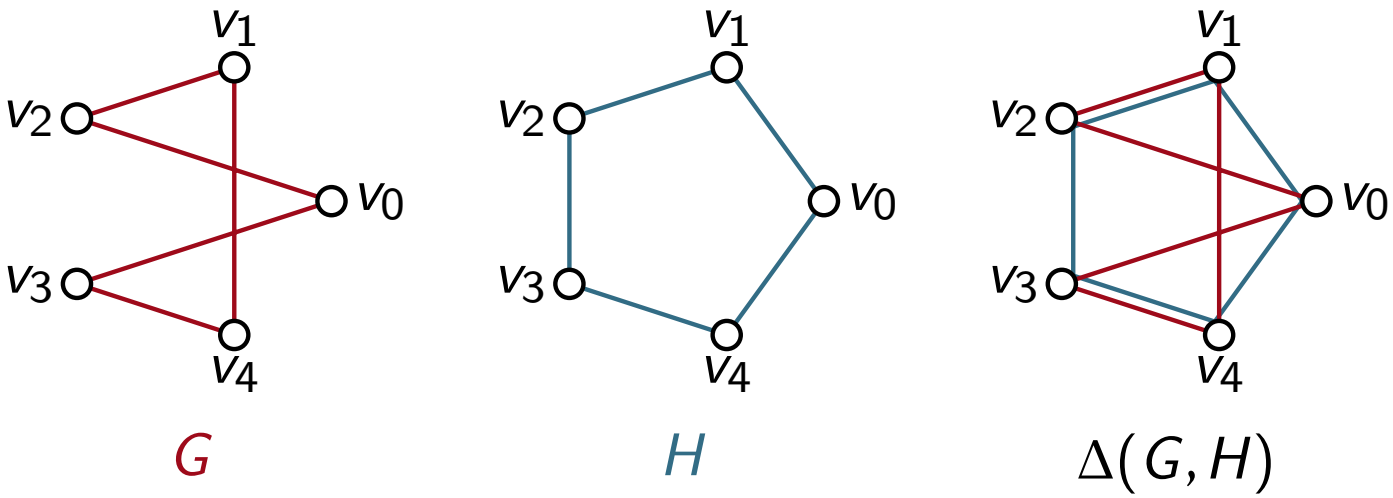

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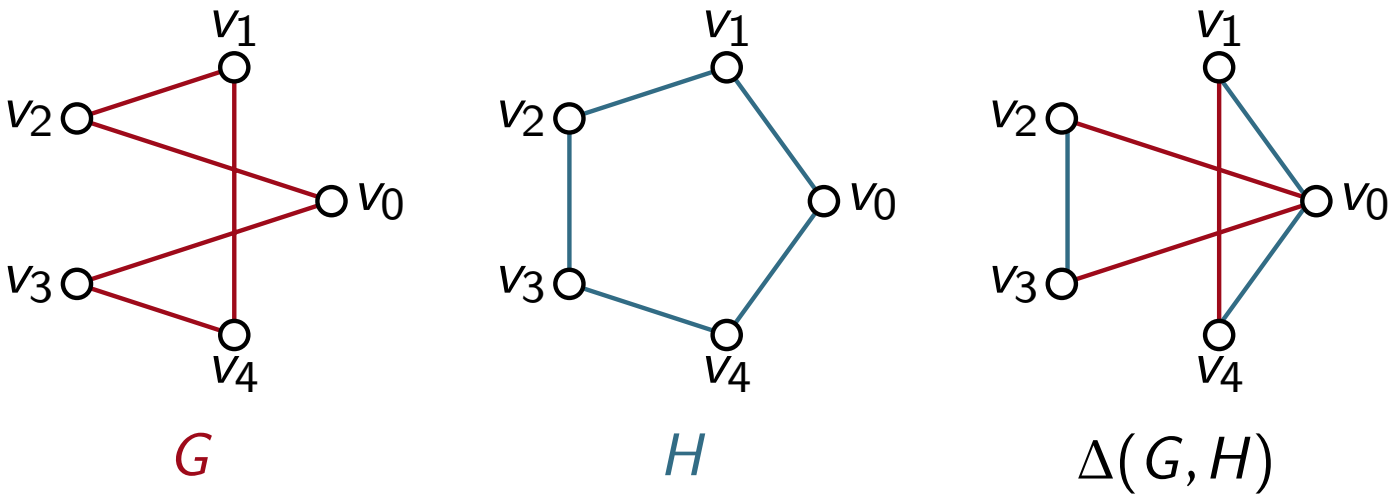

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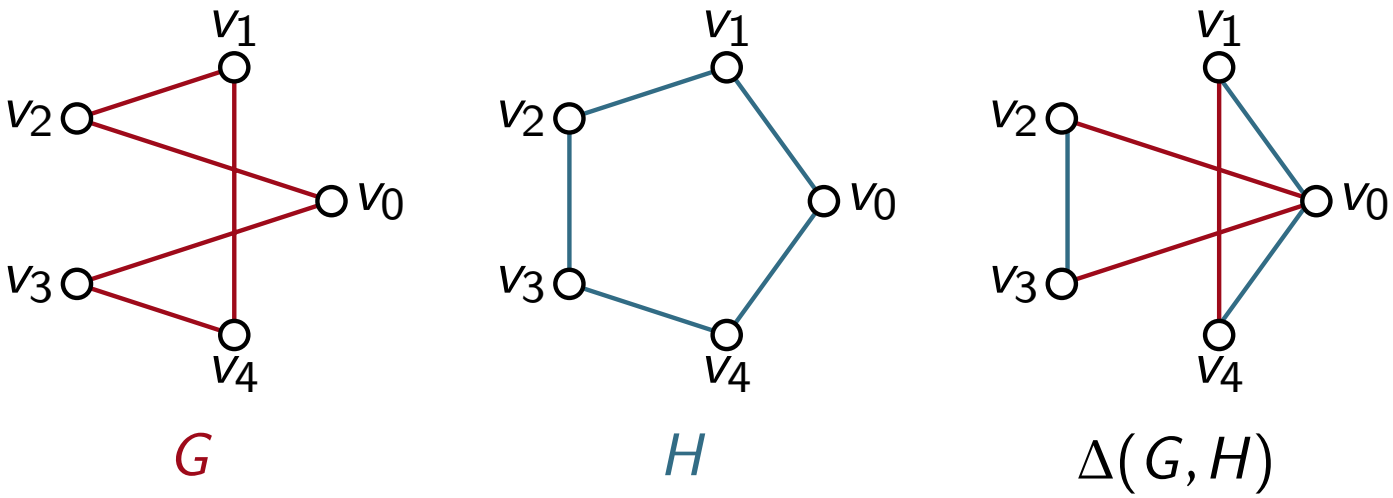

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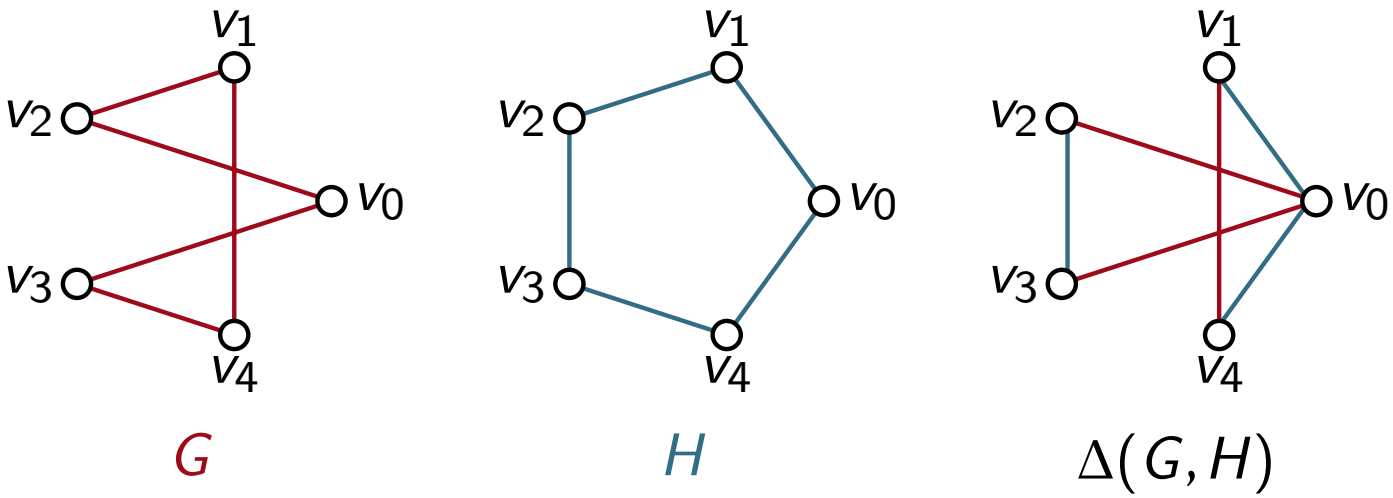

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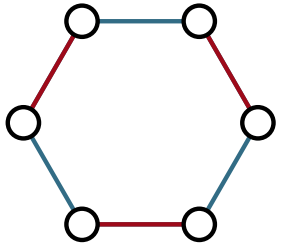

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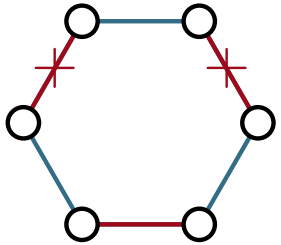

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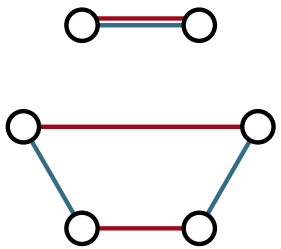

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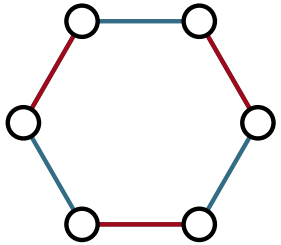

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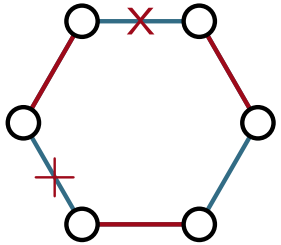

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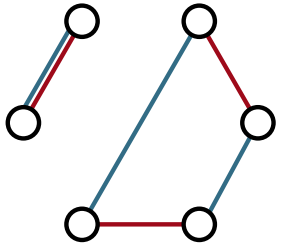

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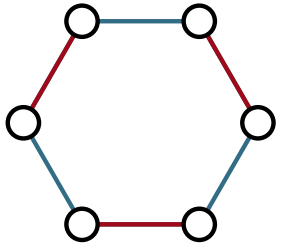

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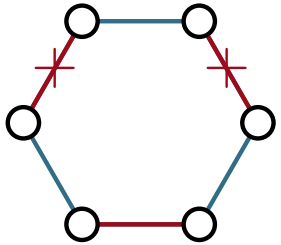

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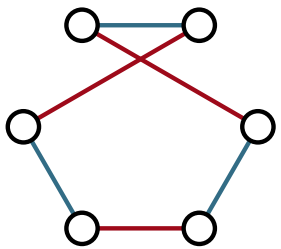

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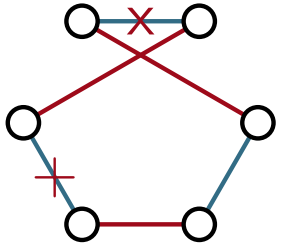

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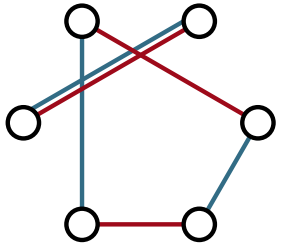

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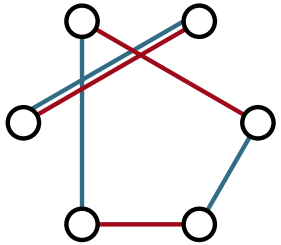

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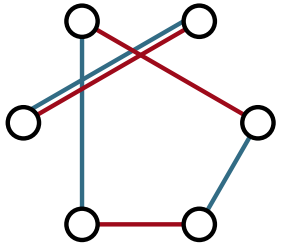

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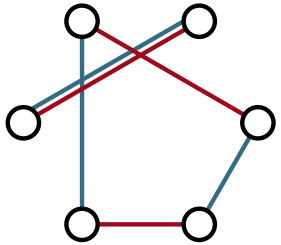

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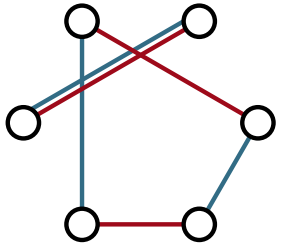

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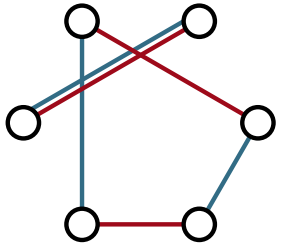

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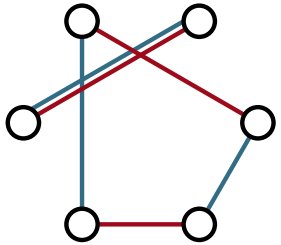

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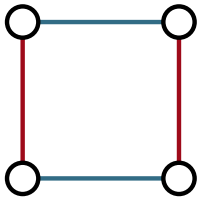

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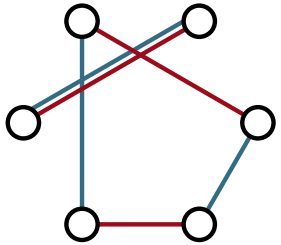

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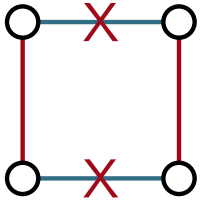

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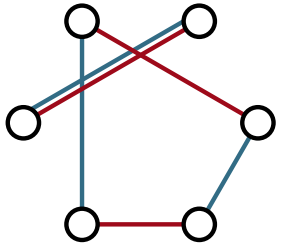

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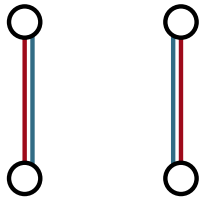

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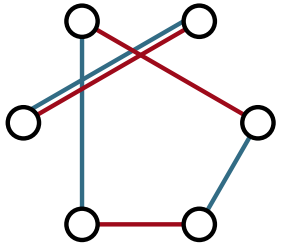

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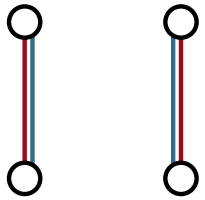

  - Improvement: Approximate the number of alternating C_4

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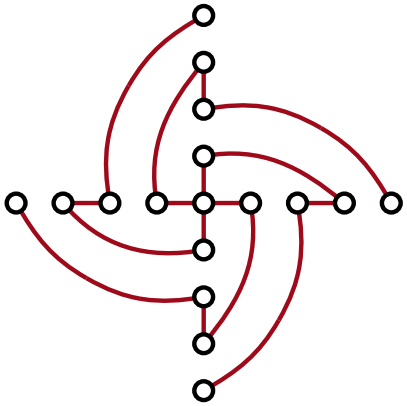
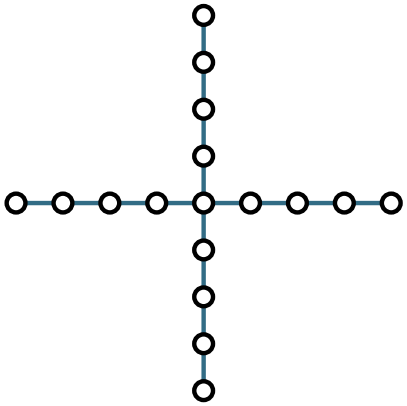
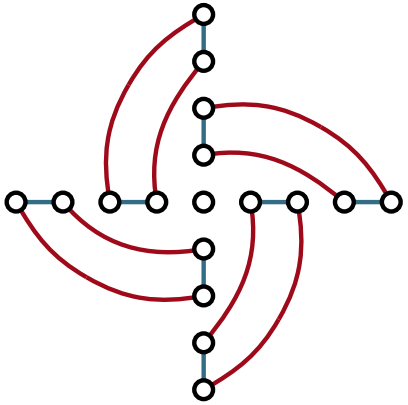


G_4



H_4



$\Delta(G_4, H_4)$

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- Find a better lower bound to improve our approximation ratio

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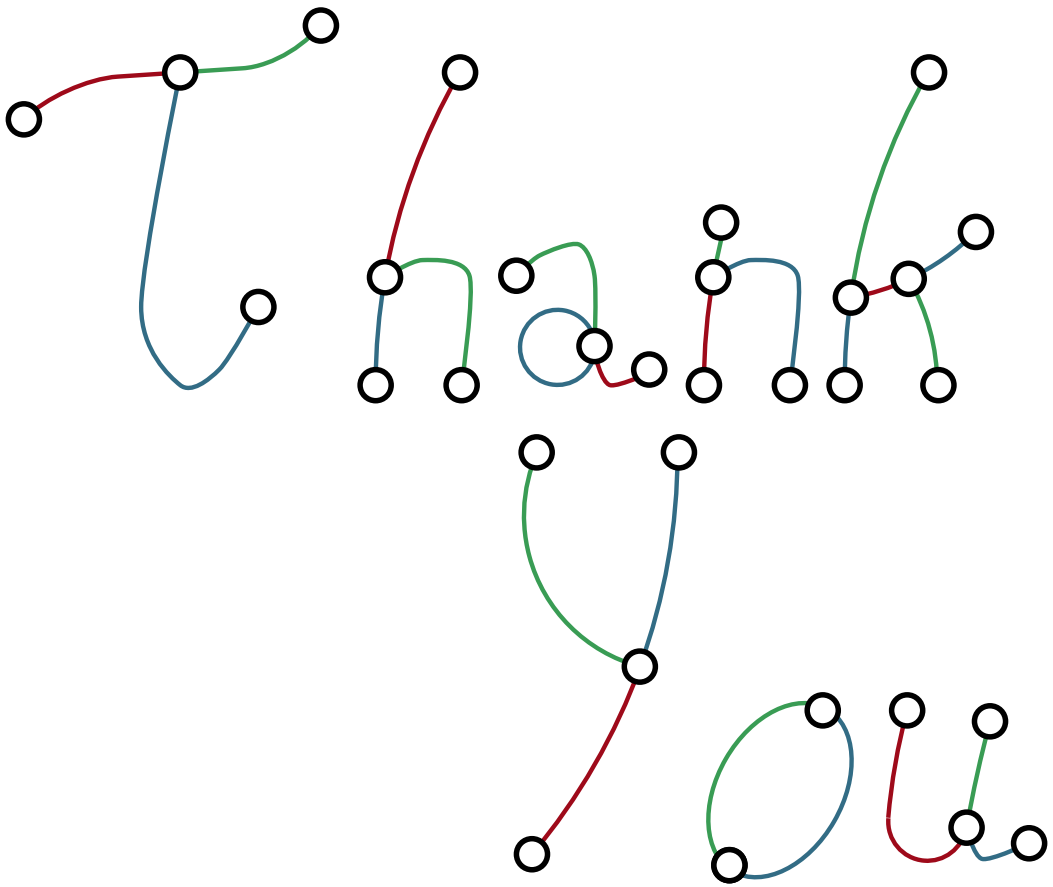
Thank you