Cautious analogical-proportion based reasoning using qualitative conceptual relations

Steven Schockaert[1] and Henri Prade[2]

[1] Cardiff University, Cardiff, UK, s.schockaert@cs.cardiff.ac.uk
[2] Université Paul Sabatier, IRIT, Toulouse, France, prade@irit.fr

Similarity based reasoning

$$\textit{chianti} \rightarrow \textit{low-tannins} \wedge \textit{medium-body}$$

barbera is similar to chianti

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barbera has an amount of tannins which is approximately low

barbera has a body which is approximately medium

How similar should chianti and barbera be for this inference to be plausible?

How is this similarity measured?

How similar to "low" can we assume the amount of tannins to be?

Interpolative reasoning

$$chianti \rightarrow low\text{-}tannins \wedge medium\text{-}body$$

$$merlot \rightarrow (low\text{-}tannins \vee mid\text{-}tannins) \wedge medium\text{-}body$$

barbera is conceptually between chianti and merlot

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barbera has an amount of tannins which is between the amounts in chianti and merlot

barbera has a body which is between the body of chianti and merlot

B is conceptually between A and C if B has all the (relevant) properties that A and C have in common

When is this inference pattern plausible?

Interpolative reasoning

$$villa(x) \wedge suburbs(x) \rightarrow luxurious(x)$$
$$apartment(x) \wedge suburbs(x) \rightarrow basic(x)$$
$$apartment(x) \wedge centre(x) \rightarrow very\text{-}comfortable(x)$$

What about apartments in the outskirts?

What about a villa in the centre?

Interpolative reasoning

$$villa(x) \wedge suburbs(x) \rightarrow luxurious(x)$$
$$apartment(x) \wedge suburbs(x) \rightarrow basic(x)$$
$$apartment(x) \wedge centre(x) \rightarrow very\text{-}comfortable(x)$$

$$apartment(x) \wedge outskirts(x) \rightarrow \quad ???$$

intermediate
conditions

Interpolative reasoning

$$
\begin{array}{r@{\;}l}
\mathit{villa}(x) \wedge \mathit{suburbs}(x) & \rightarrow \mathit{luxurious}(x) \\
\mathit{apartment}(x) \wedge \mathit{suburbs}(x) & \rightarrow \mathit{basic}(x) \\
\mathit{apartment}(x) \wedge \mathit{centre}(x) & \rightarrow \mathit{very\text{-}comfortable}(x) \\
\hline
\mathit{apartment}(x) \wedge \mathit{outskirts}(x) & \rightarrow \mathit{basic}(x) \\
& \quad \vee\, \mathit{comfortable}(x) \\
& \quad \vee\, \mathit{very\text{-}comfortable}(x)
\end{array}
$$

intermediate conditions $\Rightarrow$ intermediate conclusions

Extrapolative reasoning

$$\frac{\begin{array}{l} villa(x) \wedge suburbs(x) \rightarrow luxurious(x) \\ apartment(x) \wedge suburbs(x) \rightarrow basic(x) \\ apartment(x) \wedge centre(x) \rightarrow very\text{-}comfortable(x) \end{array}}{villa(x) \wedge centre(x) \rightarrow \quad ???}$$

analogous
changes

Extrapolative reasoning

$$\begin{array}{rcl}
\mathit{villa}(x) \wedge \mathit{suburbs}(x) & \rightarrow & \mathit{luxurious}(x) \\
\mathit{apartment}(x) \wedge \mathit{suburbs}(x) & \rightarrow & \mathit{basic}(x) \\
\mathit{apartment}(x) \wedge \mathit{centre}(x) & \rightarrow & \mathit{very\text{-}comfortable}(x) \\
\hline
\mathit{villa}(x) \wedge \mathit{centre}(x) & \rightarrow & \mathit{luxurious}(x) \vee \mathit{exclusive}(x)
\end{array}$$

analogous changes $\Rightarrow$ analogous changes

Outline

1. Introduction

2. Betweenness and analogous change

3. Formalising interpolative/ extrapolative inference

Conceptual spaces

Represent meaning of natural language terms in a geometric space, called a conceptual space (Gärdenfors 2000)

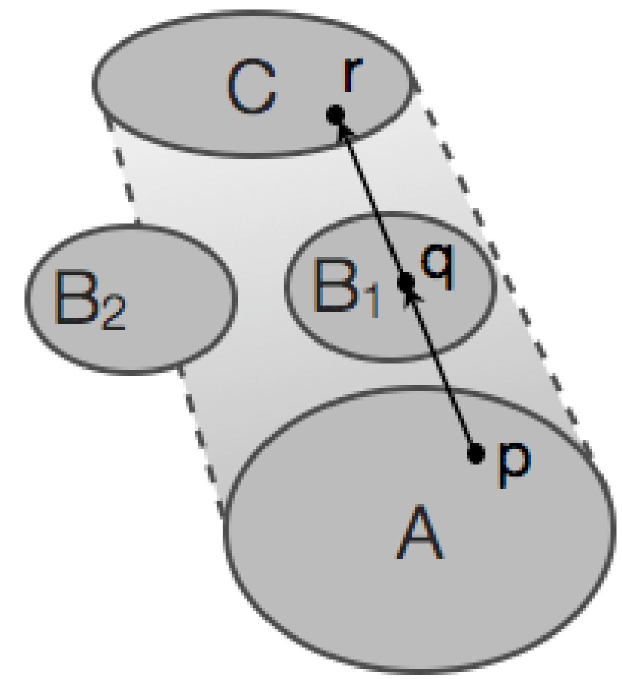
- Dimensions correspond to cognitively primitive qualities
- Natural properties or concepts can be represented as convex regions

Using conceptual spaces, we can give an operational definition for the semantic relations we need for interpolation/extrapolation:

- Conceptual betweenness corresponds to geometric betweenness in conceptual spaces
- Analogical change corresponds to parallel directions in conceptual spaces

Betweenness

Atomic propositions *a* and *c* correspond to regions A and *C* in some conceptual space



We introduce a binary modality ⋈ to refer to the convex hull of A and C at the syntactic level:

$$b \longrightarrow a \bowtie c$$

$$\Updownarrow$$

$$\forall q \in B \, . \, \exists p \in A, r \in C, \lambda \in [0,1] \, . \, \overrightarrow{pq} = \lambda \cdot \overrightarrow{pr}$$

Betweenness

Barbera is intermediate between Chianti and Merlot

$$barbera \longrightarrow chianti \bowtie merlot$$

Semi-detached houses are intermediate between row houses and detached houses

$$semi\text{-}detached \longrightarrow row\text{-}house \bowtie detached$$

The wines which are intermediate between medium-bodied wines with low-tannins and medium-bodied wines with either low or mid tannins are exactly those medium-bodied wines with low or mid tannins

$$(lt \wedge mb) \bowtie ((lt \vee mt) \wedge mb) \equiv (lt \vee mt) \wedge mb$$

Betweenness

Wine	Recommended Food
Cabernet Franc	Camembert Cheese, Cassoulet
Cabernet Franc	Pineapple and Ham Pizza, Rabbit
Cabernet Sauvignon	Blue Cheese, Hamburgers, Lamb Chops, Lamb Shanks, Lamb Stew, Macaroni and Cheese
Cabernet Sauvignon	Muenster Cheese, Roast Beef, Roast Chicken, Steak, Steak au Poivre, Swiss Cheese, Venison
Cabernet Sauvignon (Cal.)	Hare Stew
Cava	Bouillabaisse
Chablis	Fish Stew (tomato-based), Flounder, Raw Oysters
Chablis	Red Snapper, Sole
Champagne	Caviar, Raw Oysters, Risotto with Vegetables
Chardonnay	Bel Paese Cheese, Blanquette de Veau
Chardonnay	Breaded Veal Cutlets, Caprese Salad
Chardonnay	Chefs Salad, Cobb Salad, Gouda Cheese,
Chardonnay	Halibut, Macaroni and Cheese, Mussels
Chardonnay	Pasta and Shellfish, Pizza with White Sauce
Chardonnay	Pork Chops, Provolone Cheese
Chardonnay	Roast Chicken, Salmon (Grilled)
Chardonnay	Seafood with cream sauce
Chardonnay	Smoked Cheese, Swordfish, Tuna
Chateauneuf-du-Pape	Barbequed Beef Ribs, Leg of Lamb,Roast Duck
Chenin Blanc	Barbequed Chicken, Blackened Fish
Chenin Blanc	Chicken Salad, Fried Chicken
Chianti	Beef Bourguignonne, Chicken Tetrazzini

http://www.theworldwidewine.com/Wine_and_Food/This_wine_which_foods/c-wines-paired-foods.php

Betweenness

For non-atomic formulas, we can take advantage of the following inference rules, which are all compatible with the geometric view of betweenness

Interaction with ∨

$$\frac{\alpha \longrightarrow \alpha_1 \bowtie \alpha_2 \qquad \beta \longrightarrow \beta_1 \bowtie \beta_2}{(\alpha \vee \beta) \longrightarrow (\alpha_1 \vee \beta_1) \bowtie (\alpha_2 \vee \beta_2)}$$

$$\frac{\alpha_1 \bowtie \alpha_2 \longrightarrow \alpha}{(\alpha_1 \vee \beta) \bowtie (\alpha_2 \vee \beta) \longrightarrow (\alpha \vee \beta)}$$

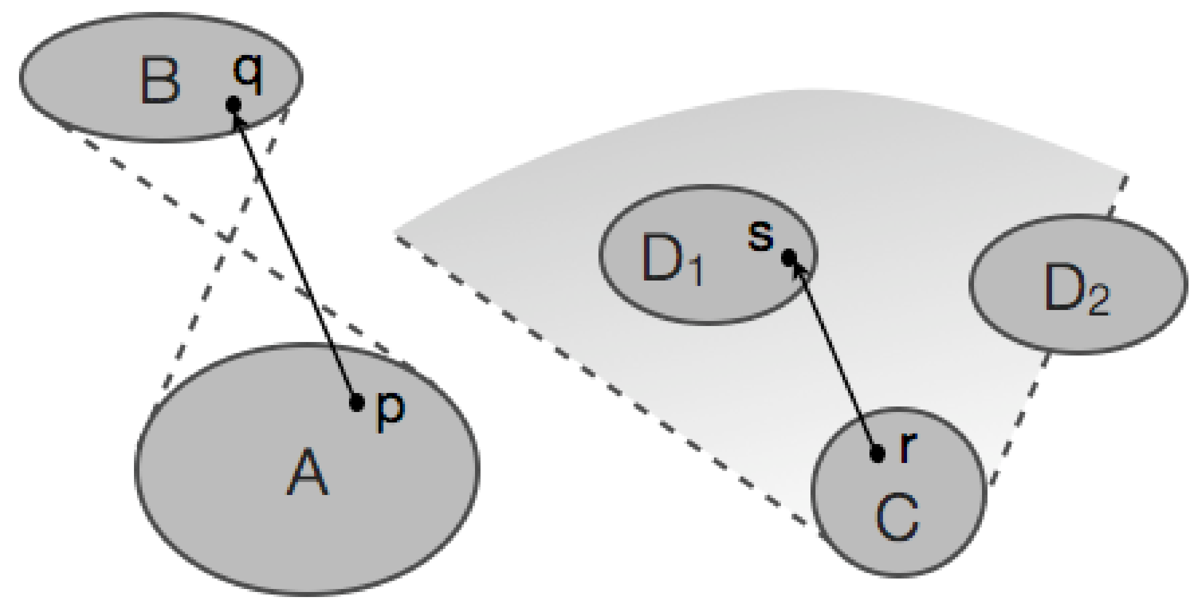
Interaction with ∧

$$\frac{\alpha \longrightarrow \alpha_1 \bowtie \alpha_2 \qquad \alpha,\ \alpha_1 \text{ and } \alpha_2 \text{ are "logically independent" from } \beta}{(\alpha \wedge \beta) \longrightarrow (\alpha_1 \wedge \beta) \bowtie (\alpha_2 \wedge \beta)}$$

$$\frac{\alpha_1 \bowtie \alpha_2 \longrightarrow \alpha \qquad \beta_1 \bowtie \beta_2 \longrightarrow \beta}{(\alpha_1 \wedge \beta_1) \bowtie (\alpha_2 \wedge \beta_2) \longrightarrow (\alpha \wedge \beta)}$$

Analogous change

We introduce a ternary modality to refer to the conical extension of C in the directions defined by A and C



$$d \longrightarrow c \rhd \langle a, b \rangle$$

$$\Updownarrow$$

$$\forall s \in D \,.\, \exists p \in A, q \in B, r \in C, \lambda \geq 0 \,.\, \overrightarrow{rs} = \lambda \cdot \overrightarrow{pq}$$

Analogous change

Progressive metal differs from heavy metal like progressive rock differs from hard rock

$$\textit{prog-metal} \longrightarrow \textit{heavy-metal} \rhd \langle \textit{hard-rock}, \textit{prog-rock} \rangle$$

The only comfort levels which differ from comfortable like comfortable differs from luxurious are basic and comfortable

$$\textit{comf} \rhd \langle \textit{lux}, \textit{comf} \rangle \longrightarrow (\textit{bas} \vee \textit{comf})$$

The house sizes which differ from medium like large differs from very small are exactly medium, large and very large

$$\textit{medium} \rhd \langle \textit{very-small}, \textit{large} \rangle \equiv \textit{medium} \vee \textit{large} \vee \textit{very-large}$$

Analogous change

Interaction with $\vee$

$$\frac{\begin{array}{l}\alpha \longrightarrow \alpha_3 \rhd \langle \alpha_1, \alpha_2 \rangle \\ \beta \longrightarrow \beta_3 \rhd \langle \beta_1, \beta_2 \rangle\end{array}}{(\alpha \vee \beta) \longrightarrow (\alpha_3 \vee \beta_3) \rhd \langle (\alpha_1 \vee \beta_1), (\alpha_2 \vee \beta_2) \rangle}$$

$$\frac{\begin{array}{l}\alpha_3 \rhd \langle \alpha_1, \alpha_2 \rangle \longrightarrow \alpha \\ \beta_3 \rhd \langle \beta_1, \beta_2 \rangle \longrightarrow \beta\end{array}}{(\alpha_3 \vee \beta_3) \rhd \langle (\alpha_1 \vee \beta_1), (\alpha_2 \vee \beta_2) \rangle \longrightarrow (\alpha \vee \beta)}$$

Interaction with $\wedge$

$$\frac{\begin{array}{l}\alpha \longrightarrow \alpha_3 \rhd \langle \alpha_1, \alpha_2 \rangle \\ \alpha,\ \alpha_1 \text{ and } \alpha_2 \text{ are ``logically independent'' from } \beta \text{ and } \gamma\end{array}}{(\alpha \wedge \gamma) \longrightarrow (\alpha_3 \wedge \gamma) \rhd \langle (\alpha_1 \wedge \beta), (\alpha_2 \wedge \beta) \rangle}$$

$$\frac{\begin{array}{l}\alpha_3 \rhd \langle \alpha_1, \alpha_2 \rangle \longrightarrow \alpha \\ \beta_3 \rhd \langle \beta_1, \beta_2 \rangle \longrightarrow \beta\end{array}}{(\alpha_3 \wedge \beta_3) \rhd \langle (\alpha_1 \wedge \beta_1), (\alpha_2 \wedge \beta_2) \rangle \longrightarrow (\alpha \wedge \beta)}$$

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1. Introduction

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Interpolation and extrapolation principle

Interpolation

$$\frac{\begin{array}{l}\alpha_1 \longrightarrow \beta_1 \\ \alpha_2 \longrightarrow \beta_2\end{array}}{\alpha_1 \bowtie \alpha_2 \longrightarrow \beta_1 \bowtie \beta_2}$$

Extrapolation

$$\frac{\begin{array}{l}\alpha_1 \longrightarrow \beta_1 \\ \alpha_2 \longrightarrow \beta_2 \\ \alpha_3 \longrightarrow \beta_3\end{array}}{\alpha_3 \rhd \langle \alpha_1, \alpha_2 \rangle \longrightarrow \beta_3 \rhd \langle \beta_1, \beta_2 \rangle}$$

Categorization rules between conceptual spaces

high-tannins ∧ full-body → opaque

Represented in a conceptual space with quality dimensions related to taste, colour, transparency, etc.

Represented in a conceptual space with quality dimensions related to transparency

Categorization rules between conceptual spaces

high-tannins $\wedge$ full-body $\rightarrow$ opaque

Represented in a conceptual space with quality dimensions related to taste, colour, transparency, etc.

Represented in a conceptual space with quality dimensions related to transparency

projecting, rescaling, ... : affine/linear mapping

Categorization rules

Categorization rules

high-tannins $\wedge$ full-body $\rightarrow$ opaque

large $\wedge$ detached $\rightarrow$ comfortable $\vee$ luxurious

$\Updownarrow$

Phenomenological rules

morning $\rightarrow$ heavy-traffic

autumn $\wedge$ UK $\rightarrow$ rainy

Handling inconsistencies

$$\begin{aligned} \textit{morning} &\longrightarrow \textit{heavy-traffic} \\ \textit{mid-day} &\longrightarrow \textit{moderate-traffic} \\ \textit{evening} &\longrightarrow \textit{heavy-traffic} \end{aligned}$$

$$\begin{aligned} \textit{morning} &\longrightarrow \textit{heavy-traffic} \\ \textit{mid-day} &\longrightarrow \textit{moderate-traffic} \end{aligned}$$

$+$

$$\begin{aligned} \textit{morning} &\longrightarrow \textit{heavy-traffic} \\ \textit{mid-day} &\longrightarrow \textit{moderate-traffic} \end{aligned}$$

Handling inconsistencies

$$bar \longrightarrow serves\text{-}wine$$
$$coffeehouse \longrightarrow \neg serves\text{-}wine$$
$$restaurant \longrightarrow serves\text{-}wine$$

$$coffeehouse \longrightarrow bar \bowtie restaurant$$

$$coffeehouse \longrightarrow serves\text{-}wine$$

Interpolating default rules

$$\begin{aligned} \mathit{bird} &\mathrel{|\!\sim} \mathit{flies} \\ \mathit{penguin} &\mathrel{|\!\sim} \mathit{bird} \\ \mathit{penguin} &\mathrel{|\!\sim} \neg \mathit{flies} \\ \mathit{aptenodytes} &\mathrel{|\!\sim} \mathit{penguin} \\ \mathit{eudyptula} &\mathrel{|\!\sim} \mathit{penguin} \end{aligned}$$

$$\mathit{pygoscelis} \longrightarrow \mathit{aptenodytes} \bowtie \mathit{eudyptula}$$

$$\mathit{pygoscelis} \mathrel{|\!\sim} \neg \mathit{flies}$$

Conclusions

We proposed a form of interpolative reasoning as a qualitative version of similarity based reasoning

We proposed a form of extrapolative reasoning as a cautious version of analogical-proportion based reasoning, taking only the “direction of change” into account and not the amount of change

We rely on a geometrical semantics based on Gärdenfors’ conceptual spaces to

- Justify when/why interpolation and extrapolation provide sound conclusions
- Induce information about betweenness and analogous change from data
- Justify how to handle inconsistencies or deal with default rules