

Arguing by analogy – Towards a formal view

A preliminary discussion

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Motivation

- *arguing by analogy* is a current practice
- arguments based on analogies
are easy to grasp, intuitively appealing,
and may be especially convincing in public uses
- little attention has been devoted to the study of this form of argumentation, *especially at a formal level*
in spite of a rich *AI* literature on formal argumentation
- need for a logical modeling of *analogical relations*, and *analogical proportions*

Contents

- Recent results on the logical modeling of analogy
 - propositional logic modeling of **analogical proportions**
 - **Polya**'s pattern of analogical reasoning
- A *preliminary* study of analogical arguments
 - different types
 - different attacks

Identity, resemblance, and analogy (J. - B. Grize)

- **Resemblance** is strictly weaker than identity.
- **S resembles T** if they belong to the *same domain* and have *common features* (which are easily observable)
- **S is analogous to T** rather means that S and T *may belong to different domains*, and that S has the same relation with an object U as T has with another object V

E.g., “Fishes (S) breathe through their gills (U), mammals (T) breathe through their lungs (V)” (Aristotle)

Analogy

- make a parallel between 2 systems of objects, each related by similar relations, or even equations,
has been investigated for a long time, and is at the core of the *structure-mapping model*
- *case-based reasoning* relies on the comparison of 2 pairs, (Prob1, Sol1) and (Prob2, Sol2), where Prob1 and Prob2 are multiple-features descriptions of 2 problems, whose solutions Sol1, Sol2 are respectively known and *unknown*
- analogy is as much a matter of dissimilarity as a matter of similarity

Analogical proportion

“*A is to B as C is to D*”, denoted $A : B :: C : D$ where A, \dots stand for objects, or situations, described by *sets of features*

- holds if $A \cap \overline{B} = C \cap \overline{D}$ and $\overline{A} \cap B = \overline{C} \cap D$

“*A differs from B as C differs from D, and
B differs from A as D differs from C*” (Miclet, Prade)

- logical counterpart for *each binary feature* viewed as a Boolean variable, denoted $a : b :: c : d$

$$((a \rightarrow b) \equiv (c \rightarrow d)) \wedge ((b \rightarrow a) \equiv (d \rightarrow c))$$

- the proportion $a : b :: c : d$ is viewed as a *Boolean formula*

Analogical proportion: truth table and inference

a	b	c	d
0	0	0	0
1	1	1	1
0	0	1	1
1	1	0	0
0	1	0	1
1	0	1	0

- satisfies *symmetry*, *central permutation*, $a : b :: a : b$, $a : a :: b : b$ (but not $a : b :: b : a$), and $a : b :: \neg b : \neg a$
- $a : b :: c : x$ is *solvable* iff $(a \equiv b) \vee (a \equiv c)$ holds, the *unique solution* is $x = c \equiv (a \equiv b)$

$$\bullet \frac{\forall i \in [1, m], \quad a_i : b_i :: c_i : d_i}{\forall j \in [m + 1, n], \quad a_j : b_j :: c_j : d_j}$$

“Analogical jump” pattern of inference

$$P(s), P(t), Q(s)$$

$$Q(t)$$

“P(s) is to P(t) as Q(s) is to Q(t)” (they are similar changing s into t), or by central permutation that “P(s) is to Q(s) as P(t) is to Q(t)” (changing P into Q). It may be restated as

$$P(s) : P(t) :: Q(s) : Q(t)$$

$$P(s), P(t), Q(s)$$

$$Q(t)$$

which is a valid pattern of inference

Polya's pattern of analogical reasoning

a and b are analogous
 a is true

b true is more credible

“ a and b are analogous” $a \sim b$ iff $\vdash a \equiv b$ (\vdash preferential nonmonotonic consequence relation). $a \sim b$ iff $\neg a \sim \neg b$

$\vdash a : b :: c : d \quad a \sim b$

$c \sim d$

$a \sim b \quad c \sim d$

$\vdash a : b :: c : d$

An example (mentioned by Aristotle)

Iphicrates, an Athenian general, provided the following argument about his son for whom one wanted that he serves in a public position

- “if one deals with adults as tall children, are we going to deal with short adult as children?”
- it can be checked that
 $tall\ child : adult :: child : short\ adult$ holds, considering that child and adult are normally short and tall respectively
Then considering that $tall\ child \sim adult$ leads to admit that $child \sim short\ adult$

Analological argument - 1

“An *analogy* is a comparison between two objects, or systems of objects, that highlights respects in which they are thought to be similar. *Analological reasoning* is any type of thinking that relies upon an analogy. An *analological argument* is an explicit representation of analological reasoning that cites accepted similarities between two systems in support of the conclusion that some further similarity exists.” (*Bartha*)

For ex., given that “Peter is like Paul, they like good life”, and that “Paul spoilt his fortune in a few years”, one may argue that “Peter (who is presently rich) will do the same”

Analogical argument - 2

- An argument by analogy involves *at least one premise* which *refers to an analogy* and as such departs from deductive arguments
- An analogy may be a simple statement relating two objects “*a is analogous to b*” (or “*a is like b*”), or the *statement of an analogical proportion* ; one may also state that “Objects *A* and *B* are similar in having properties P_1 , ..., P_n ”, making explicit the basis of the analogy
- The different patterns of analogical inference provide a formal basis for discussing analogical arguments. Polya’s pattern provides the simplest form of argument by analogy

Example of argument involving an analogical proportion

“credit rating agencies are useful”,

since

“credit rating agency is to crisis as thermometer is to fever”

and

“thermometers are useful”

Using analogical argument in attack

An analogical argument, as any argument may be attacked, or used in attacks against other arguments (analogical or not), as in [Iphicrates example](#), where the analogical proportion is not challenged.

On the contrary, it is used to show that given this analogical proportion,

as soon as one accepts to consider a = tall child and b = adult as analogous,

one is led to accept an *absurd conclusion*,

i.e., considering c = child and s = short adult as analogous

Attacking analogical arguments - 1

by

- *disputing the relevance of the similarities*

This amounts in the “analogical jump” pattern to say that properties P and Q are unrelated. It may be done by providing a *counterexample* by pointing out an object for which property P is true, but for which property Q is false

- *disputing an alleged similarity*, or *challenging an analogical proportion* by pointing out that the 2 situations are in fact dissimilar wrt another (relevant) property

Attacking analogical arguments - 2

- In the “credit rating agency” example, the analogy becomes debatable once one states that “credit rating agencies have effect on crisis”, while “thermometers have no effect on fever”
- *pointing out undesirable consequences*. David Hume attacked the teleological argument according to which since a complex object like a watch requires an intelligent designer, a (more) complex object like the universe should also have an intelligent designer. Hume argued that since watches are often the result of the work of several people, the reasoning support polytheism also

A sequence of analogical arguments may involve analogical proportions

In a debate,

a discussant d states that situation $S2$ is like situation $S1$ and that what took place in $S1$ will happen in $S2$ as well

The opponent d' , will argue that in fact there is an (important) feature where they differ, and that what took place in $S1$ may not happen in $S2$

Then d may produce another pair of situations $S3, S4$, where the same difference can be observed without affecting the conclusion advocated by d for $S2$

Then d' may counter-argue if he knows another pair of situations $S'3, S4'$ where the same difference does lead to a different conclusion

This kind of exchange can be analyzed in terms of analogical proportions. Indeed, depending if we consider $S3 : S4 :: S1 : S2$, where the same effects have been observed for $S1, S3, S4$, or if we consider $S'3 : S'4 :: S1 : S2$ where different effects have been reported, one may conclude in opposite ways about $S2$ (using the transfer pattern of the previous section for inferring new analogical proportions)

Concluding remarks

- preliminary study
- existence of formal inference patterns provides a basis for the formal study of analogical argumentation
- variety of patterns; variety of attacks