Belief revision-based case-based reasoning

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Remark on the difference between the paper and the talk

The paper gathers several results on revision-based CBR and is rather technical

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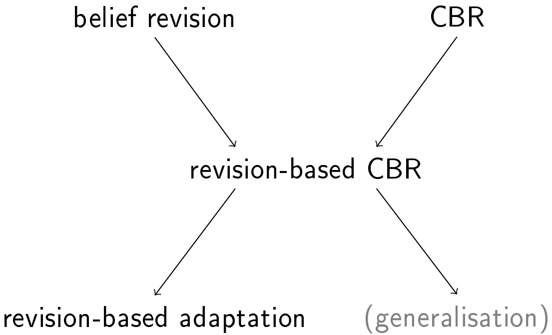
The talk will make some simplifying assumptions and will try to be more intuitive

Outline of the talk

preliminaries



Preliminaries

Formalism

- Generalisation from propositional logic with n variables:
  - $\mathcal{U} = \{\texttt{false}, \texttt{true}\}^n$: set of the interpretations
  - A formula φ represents a subset $\texttt{Mod}(\varphi)$ of $\mathcal{U}$

Formalism

- Generalisation from propositional logic with n variables:
  - $\mathcal{U} = \{\mathtt{false}, \mathtt{true}\}^n$: set of the interpretations
  - A formula φ represents a subset $\mathtt{Mod}(\varphi)$ of $\mathcal{U}$
- Generalisation:
  - $\mathcal{U}$: a given set ("the universe of instances/interpretations")
  - $\mathcal{L}$: the representation language
  - Semantics: $\varphi \in \mathcal{L} \mapsto \mathtt{Mod}(\varphi) \in 2^{\mathcal{U}}$
  - With $A = \mathtt{Mod}(\varphi)$ and $B = \mathtt{Mod}(\chi)$
    - $\varphi \models \chi$ if $A \subseteq B$
    - φ is satisfiable if $A \neq \emptyset$

Distance

- $d : \mathcal{U} \times \mathcal{U} \to [0; +\infty]$ is a distance if

$$d(x, y) = 0 \qquad \text{iff} \qquad x = y$$

(no other condition required in this talk)

Belief Revision

Belief revision (introduction)

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  - $\psi \mapsto \psi'$
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- Different ways to model *minimal modification* hence different revision operators
- In the following, $\dot{+}$ is defined on $\mathcal{L} \times \mathcal{L}$ or on $2^{\mathcal{U}} \times 2^{\mathcal{U}}$:

$$\mathtt{Mod}(\psi \dot{+} \mu) = \mathtt{Mod}(\psi) \dot{+} \mathtt{Mod}(\mu)$$

Belief revision (postulates)

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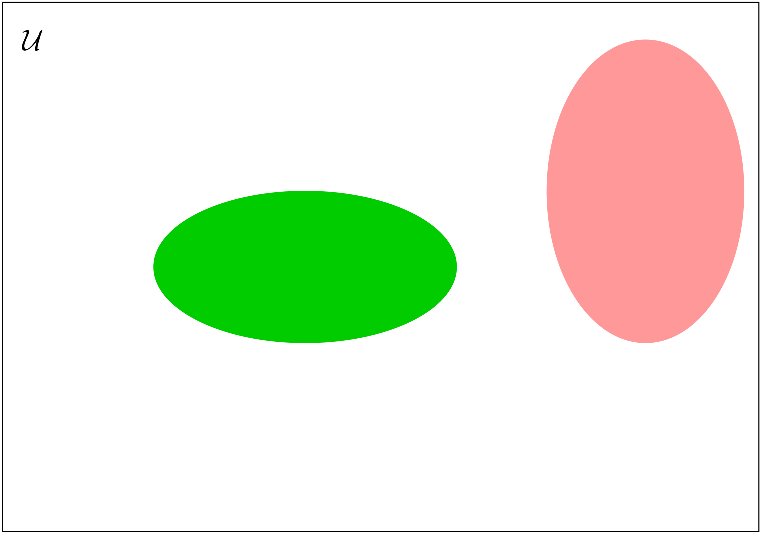
  reformulation in propositional logic
- Easy to generalise in our formalism (see further)

Belief revision (distance-based revision operators)

With d, a distance on $\mathcal{U}$, $A \subseteq \mathcal{U}$, $B \subseteq \mathcal{U}$:

$$A \dotplus^d B = \{\mathtt{b} \in B \mid d(A, \mathtt{b}) = d(A, B)\}$$



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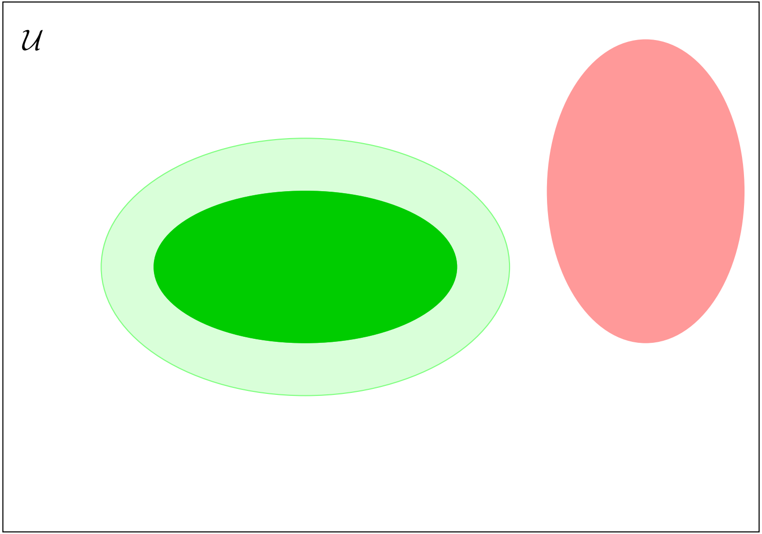

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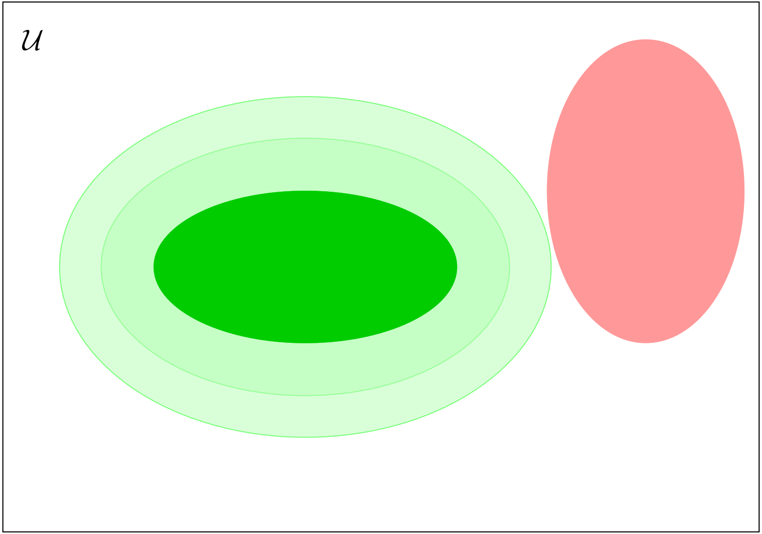

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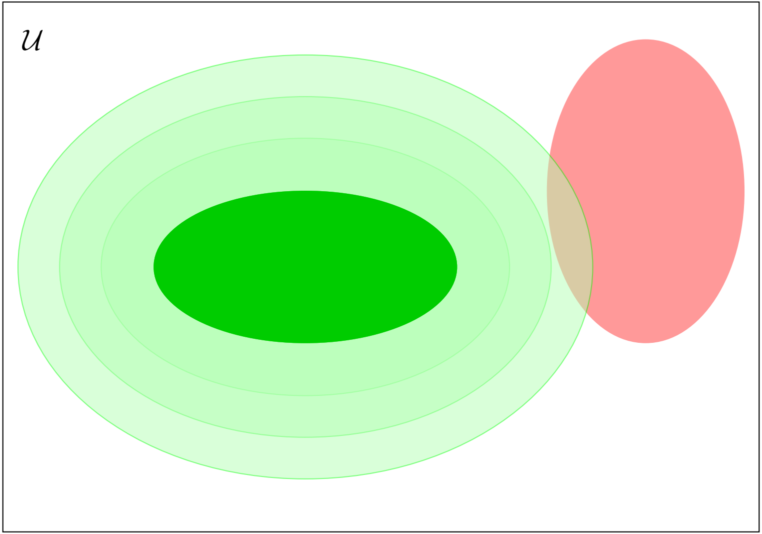

Case-Based Reasoning

CBR: problems, solutions and cases

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- Thus, $\texttt{C} \in 2^{\mathcal{U}}$ where $\mathcal{U} = \mathcal{U}_{\texttt{pb}} \times \mathcal{U}_{\texttt{sol}}$.

CBR (input/output)

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- The domain knowledge $\mathtt{DK} \subseteq \mathcal{U}$
  a necessary condition for a case instance $\mathsf{a} = (x, y) \in \mathcal{U}$ to be licit

$$\text{if } \mathsf{a} \notin \mathtt{DK} \text{ then } \mathsf{a} \text{ is not licit}$$

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- Other knowledge containers (similarity, $\texttt{AK}$)

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Output

$\mathtt{sol}(\mathtt{tgt}) \in 2^{\mathcal{U}_{\mathtt{sol}}}$ such that the hypothesis "$\mathtt{sol}(\mathtt{tgt})$ solves $\mathtt{tgt}$" is likely (...). (often $\mathtt{sol}(\mathtt{tgt}) = \{y^t\}$)

CBR (retrieval and adaptation)

tgt

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$$\texttt{srce} \xleftarrow{\textit{retrieval}} \texttt{tgt}$$

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$$\begin{tikzcd}
\texttt{srce} \arrow[d] & & \texttt{tgt} \arrow[ll, "\textit{retrieval}"'] \\
\texttt{sol(srce)} & &
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CBR (example) http://taaable.fr

Fruit pie recipe without apple?

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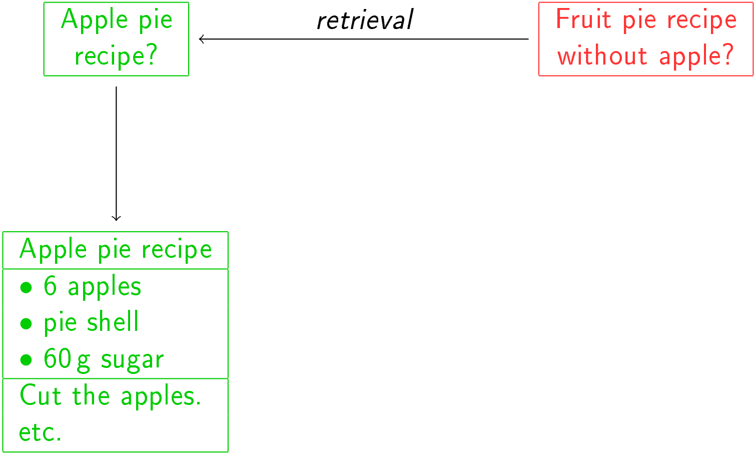

CBR (example) http://taaable.fr

Apple pie recipe?

retrieval

Fruit pie recipe without apple?

Apple pie recipe

- 6 apples
- pie shell
- 60 g sugar

Cut the apples.
etc.

adaptation

$\sigma(\texttt{sol}(\texttt{srce}))$ with

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4. σ: adaptation of the preparation

Revision-Based CBR

Revision-based adaptation (intuition)

- Adaptation: modification of $\texttt{srce} \times \texttt{sol(srce)}$
  so that it is consistent with $\texttt{tgt} \times \mathcal{U}_{\texttt{sol}}$

$$//$$

- Revision: modification of ψ
  so that it is consistent with μ

Revision-based adaptation (definition)

$$\mathtt{tgt} \times \mathtt{sol}(\mathtt{tgt}) = (\qquad \mathtt{srce} \times \mathtt{sol}(\mathtt{srce})) \qquad (\qquad \mathtt{tgt} \times \mathcal{U}_{\mathtt{sol}})$$

Revision-based adaptation (definition)

$$\mathtt{tgt} \times \mathtt{sol(tgt)} = (\mathtt{DK} \cap \mathtt{srce} \times \mathtt{sol(srce)}) \qquad (\mathtt{DK} \cap \mathtt{tgt} \times \mathcal{U}_{\mathtt{sol}})$$

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$$\mathtt{tgt} \times \mathtt{sol}(\mathtt{tgt}) = (\mathtt{DK} \cap \mathtt{srce} \times \mathtt{sol}(\mathtt{srce})) \dotplus \quad (\mathtt{DK} \cap \mathtt{tgt} \times \mathcal{U}_{\mathtt{sol}})$$

Revision-based adaptation (definition)

$$\mathtt{tgt} \times \mathtt{sol}(\mathtt{tgt}) = (\mathtt{DK} \cap \mathtt{srce} \times \mathtt{sol}(\mathtt{srce})) \dot{+}_{\mathtt{AK}} (\mathtt{DK} \cap \mathtt{tgt} \times \mathcal{U}_{\mathtt{sol}})$$

Revision-based adaptation (properties)

	revision postulates (reformulated)	$\dot{+}$-adaptation properties
$\dot{+}\,1$	$A \dot{+} B \subseteq B$	The result of the adaptation is consistent with DK.
$\dot{+}\,2$	if $A \cap B \neq \emptyset$ then $A \dot{+} B = A \cap B$	If the target case is consistent with the source case then this latter is deductively reused for solving the target case.
$\dot{+}\,3$	if $B \neq \emptyset$ then $A \dot{+} B \neq \emptyset$	Unless the target case is DK-inconsistent, the adaptation gives a consistent result.
$\dot{+}\,4$ $\dot{+}\,5$	if $(A \dot{+} B) \cap C \neq \emptyset$ then $A \dot{+} (B \cap C) = (A \dot{+} B) \cap C$	Adaptation by *minimal* modification (according to $\dot{+}$).
$\dot{+}\,6$	The result of the revision is representable in $\mathcal{L}$.	The result of the adaptation is representable in $\mathcal{L}$.

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- $\mathtt{DK} = \left\{\begin{matrix} \mathtt{apple} \lor \mathtt{pear} \Leftrightarrow \mathtt{pome\_fruit}, & \\ \mathtt{pome\_fruit} \Rightarrow \mathtt{fruit}, & \text{etc.} \end{matrix}\right\}$

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- Use of adaptation knowledge given by adaptation rules

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- $\dot{+} = \dot{+}^{d_{\mathtt{AK}}}$ with $d_{\mathtt{AK}}$ defined by
  - Version 1:

$$d_{\mathtt{AK}}(\mathcal{I}, \mathcal{J}) = \text{ length of the shortest } \mathtt{AK}\text{-path from } \mathcal{I} \text{ to } \mathcal{J}$$

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  - Version 2:
$$d_{\mathtt{AK}}(\mathcal{I}, \mathcal{J}) = \begin{array}{l} \text{length of the shortest path from } \mathcal{I} \text{ to } \mathcal{J} \\ \text{using adaptation rules and flips of variables} \end{array}$$
(when $\mathtt{AK} = \emptyset$, $d_{\mathtt{AK}} = d_H$)

Example 2: in prop. logic, using adaptation rules

- Same $\mathtt{tgt}$, $\mathtt{srce}$, $\mathtt{sol}(\mathtt{srce})$ and $\mathtt{DK}$
- Use of adaptation knowledge given by adaptation rules
$$\mathtt{AK} = \left\{\begin{matrix} \mathtt{salad} \land \mathtt{vinegar} \rightsquigarrow \mathtt{salad} \land \mathtt{lemon} \land \mathtt{salt}, \\ \mathtt{apple} \rightsquigarrow \mathtt{pear} \land \mathtt{almond\_powder}, \qquad \text{etc.} \end{matrix}\right\}$$
- $\dot{+} = \dot{+}^{d_{\mathtt{AK}}}$ with $d_{\mathtt{AK}}$ defined by
  - Version 1:
  $$d_{\mathtt{AK}}(\mathcal{I}, \mathcal{J}) = \quad \text{length of the shortest } \mathtt{AK}\text{-path from } \mathcal{I} \text{ to } \mathcal{J}$$
  Limitation: $d_{\mathtt{AK}}(\mathcal{I}, \mathcal{J}) = +\infty$ if no $\mathtt{AK}$-path from $\mathcal{I}$ to $\mathcal{J}$ exist
  - Version 2:
  $$d_{\mathtt{AK}}(\mathcal{I}, \mathcal{J}) = \quad \begin{array}{l} \text{length of the shortest path from } \mathcal{I} \text{ to } \mathcal{J} \\ \text{using adaptation rules and flips of variables} \end{array}$$
  (when $\mathtt{AK} = \emptyset$, $d_{\mathtt{AK}} = d_H$)

$$\mathtt{tgt} \land \mathtt{sol}(\mathtt{tgt}) = \mathtt{pie} \land \boxed{\mathtt{pear} \land \mathtt{almond\_powder}} \land \mathtt{pie\_shell} \land \mathtt{sugar}$$

Example 3: adaptation of quantities (1/2)

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- $\mathcal{L}$: conjunction of linear constraints
  (on integers and real numbers)

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- $\mathcal{L}$: conjunction of linear constraints (on integers and real numbers)
- $\mathtt{tgt} = (\mathtt{nb}_{\mathtt{apple}} = 0)$
- $$\mathtt{srce} \wedge \mathtt{sol}(\mathtt{srce}) = \begin{array}{l} (\mathtt{nb}_{\mathtt{pie\_shell}} = 1) \wedge (\mathtt{nb}_{\mathtt{apple}} = 6) \wedge \\ (\mathtt{mass}_{\mathtt{sugar}} = 60) \wedge (\mathtt{nb}_{\mathtt{pear}} = 0) \wedge \\ (\mathtt{nb}_{\mathtt{tomato}} = 0) \wedge \ldots \end{array}$$

Example 3: adaptation of quantities (1/2)

- $\mathcal{L}$: conjunction of linear constraints (on integers and real numbers)
- $\texttt{tgt} = (\texttt{nb}_{\texttt{apple}} = 0)$
- $$\texttt{srce} \wedge \texttt{sol}(\texttt{srce}) = \begin{array}{l} (\texttt{nb}_{\texttt{pie\_shell}} = 1) \wedge (\texttt{nb}_{\texttt{apple}} = 6) \wedge \\ (\texttt{mass}_{\texttt{sugar}} = 60) \wedge (\texttt{nb}_{\texttt{pear}} = 0) \wedge \\ (\texttt{nb}_{\texttt{tomato}} = 0) \wedge \ldots \end{array}$$
- $$\texttt{DK} = \begin{array}{l} (\texttt{mass}_{\texttt{apple}} = 242 \times \texttt{nb}_{\texttt{apple}}) \wedge \\ (\texttt{mass}_{\texttt{pear}} = 166 \times \texttt{nb}_{\texttt{pear}}) \wedge \ldots \\ (\texttt{mass}_{\texttt{fruit}} = \texttt{mass}_{\texttt{apple}} + \texttt{mass}_{\texttt{pear}} + \ldots) \wedge \\ (\texttt{mass}_{\texttt{carbohydrates}} = \texttt{mass}_{\texttt{sugar}} + 33 \times \texttt{nb}_{\texttt{apple}} \\ \qquad\qquad + 26 \times \texttt{nb}_{\texttt{pear}} + \ldots) \wedge \ldots \end{array}$$

Example 3: adaptation of quantities (1/2)

- $\mathcal{L}$: conjunction of linear constraints (on integers and real numbers)
- $\mathtt{tgt} = (\mathtt{nb}_{\mathtt{apple}} = 0)$
- $\mathtt{srce} \wedge \mathtt{sol}(\mathtt{srce}) = \begin{array}{l} (\mathtt{nb}_{\mathtt{pie\_shell}} = 1) \wedge (\mathtt{nb}_{\mathtt{apple}} = 6) \wedge \\ (\mathtt{mass}_{\mathtt{sugar}} = 60) \wedge (\mathtt{nb}_{\mathtt{pear}} = 0) \wedge \\ (\mathtt{nb}_{\mathtt{tomato}} = 0) \wedge \ldots \end{array}$
- $\mathtt{DK} = \begin{array}{l} (\mathtt{mass}_{\mathtt{apple}} = 242 \times \mathtt{nb}_{\mathtt{apple}}) \wedge \\ (\mathtt{mass}_{\mathtt{pear}} = 166 \times \mathtt{nb}_{\mathtt{pear}}) \wedge \ldots \\ (\mathtt{mass}_{\mathtt{fruit}} = \mathtt{mass}_{\mathtt{apple}} + \mathtt{mass}_{\mathtt{pear}} + \ldots) \wedge \\ (\mathtt{mass}_{\mathtt{carbohydrates}} = \mathtt{mass}_{\mathtt{sugar}} + 33 \times \mathtt{nb}_{\mathtt{apple}} \\ \qquad\qquad + 26 \times \mathtt{nb}_{\mathtt{pear}} + \ldots) \wedge \ldots \end{array}$
- $x = (x_1, x_2, \ldots, x_n) \in \mathcal{U}$, $x_k \in \mathbb{R}_+$ or $x_k \in \mathbb{N}$

Example 3: adaptation of quantities (1/2)

- $\mathcal{L}$: conjunction of linear constraints (on integers and real numbers)
- $\mathtt{tgt} = (\mathtt{nb}_{\mathtt{apple}} = 0)$
- $\mathtt{srce} \wedge \mathtt{sol(srce)} = \begin{array}{l} (\mathtt{nb}_{\mathtt{pie\_shell}} = 1) \wedge (\mathtt{nb}_{\mathtt{apple}} = 6) \wedge \\ (\mathtt{mass}_{\mathtt{sugar}} = 60) \wedge (\mathtt{nb}_{\mathtt{pear}} = 0) \wedge \\ (\mathtt{nb}_{\mathtt{tomato}} = 0) \wedge \ldots \end{array}$
- $\mathrm{DK} = \begin{array}{l} (\mathtt{mass}_{\mathtt{apple}} = 242 \times \mathtt{nb}_{\mathtt{apple}}) \wedge \\ (\mathtt{mass}_{\mathtt{pear}} = 166 \times \mathtt{nb}_{\mathtt{pear}}) \wedge \ldots \\ (\mathtt{mass}_{\mathtt{fruit}} = \mathtt{mass}_{\mathtt{apple}} + \mathtt{mass}_{\mathtt{pear}} + \ldots) \wedge \\ (\mathtt{mass}_{\mathtt{carbohydrates}} = \mathtt{mass}_{\mathtt{sugar}} + 33 \times \mathtt{nb}_{\mathtt{apple}} \\ \qquad\qquad\qquad + 26 \times \mathtt{nb}_{\mathtt{pear}} + \ldots) \wedge \ldots \end{array}$
- $x = (x_1, x_2, \ldots, x_n) \in \mathcal{U}$, $x_k \in \mathbb{R}_+$ or $x_k \in \mathbb{N}$
- $\dot{+} = \dot{+}^d$ with $d(x, y) = \sum_k w_k \cdot |y_k - x_k|$

Example 3: adaptation of quantities (2/2)

- For some sets of weights:

$$\begin{aligned}\texttt{tgt} \land \texttt{sol}(\texttt{tgt}) = \texttt{DK} \land (\texttt{nb}_{\texttt{apple}} = 0) \\ \land \boxed{(\texttt{nb}_{\texttt{pear}} = 9) \land (\texttt{mass}_{\texttt{sugar}} = 24)}\end{aligned}$$

- Algorithm: transformation of the problem into a linear programming problem

Example 4: adaptation of preparations

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- Representation of recipes using a temporal qualitative algebra

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- Representation of recipes using a temporal qualitative algebra
- Using a revision operator on qualitative constraint networks

Conclusion and Future Work

Conclusion: Belief revision offers a useful framework (or a viewpoint) for specifying adaptation in CBR

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Future work:

- What part of the adaptation processes does it cover?

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- What part of the adaptation processes does it cover?
- What revision operator choosing? (adaptation knowledge acquisition issue)

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Conclusion Belief revision offers a useful framework (or a viewpoint) for specifying adaptation in CBR

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- What part of the adaptation processes does it cover?
- What revision operator choosing?
  (adaptation knowledge acquisition issue)

There is other work not mentioned there
(but mentioned in the paper)