Artificial Analogy: an introduction

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1. Examples of analogies
2. Cognitive models, reasoning, AI
3. Formal models of the analogical proportion
   - The analogical proportion
   - Analogical dissimilarity
   - Analogical proportion and Lattices
4. New applications in AI
   - Learning classification rules (Pattern Recognition)
   - Sequence generation for OCR
   - A typology of logical proportions
5. Conclusion

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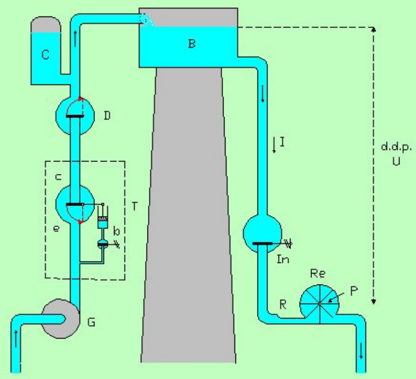
A classical analogy



	Hydraulics	Electricity
B	Tank	Battery
I	Flow	Intensity
T	Valve	Transistor
U	Height	Tension

The Tank is to the Water as the Battery is to the Current.

A numerical analogical proportion

3	6
7	

A numerical analogical proportion

3	6
7	10

$+$

A numerical analogical proportion

3	6
7	

A numerical analogical proportion

3		6
	$\times$	
7		14

A zoological analogical proportion

Calf

Cow

Foal

A zoological analogical proportion

Calf

Cow

Foal

Mare

Linguistic use of analogy

Military medecine is to medecine as military music is to music. (G. Clemenceau)

Comrade Ceaușescu is the Danube of Thought. (Unknown, c. 1970)

The accordion is the piano of the poor man. (M. Audiard)

The irish Roms are called *tinkers* or *Pavees*. (European Council).

Jessie sings soft as the Georgia rain. (K. Joshua) (comparison)

Rosy-fingered Dawn. (Homer) (metaphor)

Définition

- Analogy is the name for the identity of the relation between two objects and two other objects.
- The relations can be of several sorts :

Belonging	*The humming-bird is to birds as the shrew is to mammals.*
Difference	3 : 7 :: 6 : 10
Division	3 : 7 :: 6 : 10

- The identity can be partial.

Moby Dick is to H. Melville as
The Count of Monte-Cristo is to A. Dumas

Plan

Philosophy and linguistics

Old times

According to the Greeks, the analogy is one of the four forms of metaphor.

1. Generalizing synecdoche, the whole for a part : *To fix a flat car.*
2. Particularizing synecdoche, a part for the whole : *A herd of fifty heads.*
3. Comparison : *Your eyes like smoke and your voice like chimes.*
4. Analogy (proportion).

A way of reasoning, weaker than syllogism, but which can convince.

Uses

- Used in Linguistis and Law in classic muslim civilizations.
- Used in Linguistic and Theology in Middle Age and later in Europe.
- Important in Linguistics in the last century (not among the generativists).
- Scientific Discovery, Law, etc.

A generalization technique

For a certain number of attributes od A, B, C and D, one has

$$a_i \ : \ b_i \ :: \ c_i \ : \ d_i \quad (i = 1, n)$$

Also, a_{n+1}, b_{n+1} et c_{n+1} are known. Then d_{n+1} is computed as a solution of the equation

$$a_{n+1} \ : \ b_{n+1} \ :: \ c_{n+1} \ : \ d_{n+1}$$

Cognitive Science

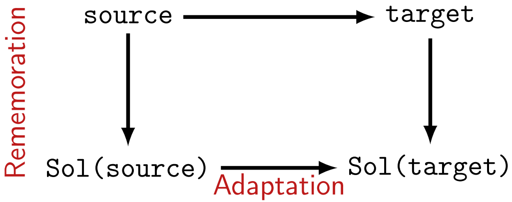
The Analogical Mind.
Edited by D. Gentner, K. Holyoak and N. Kokinov. MIT Press, 2001.

- Computable models of analogy, based on cognitive observations.
- Study of the apparition of analogy in spontaneous language : emotions, learning, reasoning, etc.

Artificial Intelligence : Case-Based Reasoning

Complete a recipe with apricot jam and apples.

Complete a recipe with apricot jam and pineapple.



Follow the book recipe.

Follow the book recipe, replacing the apples by pineapple, in the same quantity ; discard the nuts.

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Back to zoology

Calf

Cow

Foal

Mare

The four « objects » are in the same set : the farm animals.

Typography

a	c
α	γ

Stringology

aBA	αbBA
ba	

Stringology

aBA	αbBA
ba	βba

Stringology

aBA	αbBA
ba	βba

a	$\backsim$	B	A
α	b	B	A
b	$\backsim$	$\backsim$	a
β	b	$\backsim$	a

Zoology, back again

a calf *is to* **a cow** *as* **a foal** *is to* **a mare**. young $\longleftrightarrow$ adult

- An equivalent but different proportion (symmetry of *as*) :

  a foal *is to* **a mare** *as* **a calf** *is to* **a cow** young $\longleftrightarrow$ adult

- n equivalent but different proportion (permutation of the means) :

  a calf *is to* **a foal** *as* **a cow** *is to* **a mare**. bovine $\longleftrightarrow$ equine

- This proportion is not equivalent :

  a calf *is to* **a mare** *as* **a foal** *is to* **a cow**.

Why ?

	mammal	young	equine	gives milk	right profile
calf	1	1	0	0	0
foal	1	1	1	0	0
cow	1	0	0	1	1
mare	1	0	1	1	1

The column of 0 and 1 are in analogical proportion.

BackToPictures

Analogical proportion in $\mathbb{B}^d$

A_i *is to* B_i *as* C_i *is to* D_i.

$$A_i : B_i :: C_i : D_i$$

$0 : 0 :: 0 : 0$,	$0 : 0 :: 1 : 1$,	$0 : 1 :: 0 : 1$
$1 : 1 :: 1 : 1$,	$1 : 1 :: 0 : 0$,	$1 : 0 :: 1 : 0$

Look out !

$1 : 0 :: 0 : 1$ et $0 : 1 :: 1 : 0$ are not analogical proportions.

Geometric analogy in $\mathbb{R}^d$

Four vectors A, B, C et D are in an *additive proportion* when on each component one has :

$$A_i \textit{ is to } B_i \textit{ as } C_i \textit{ is to } D_i.$$

$$A_i : B_i :: C_i : D_i$$

$$\overrightarrow{A_i B_i} = \overrightarrow{C_i D_i}$$

$$\overrightarrow{OA_i} + \overrightarrow{OD_i} = \overrightarrow{OB_i} + \overrightarrow{OC_i}$$

A, B, C et D make a parallelogram.

Informal definition

Four objetcs a, b, c and d are in analogical proportion when
the differences and the similarities between a et b
are the same as those between c et d.

Let's take four subsets of $\{a, \ldots, z\}$:
$A = \{a, b, c, h\}$, $B = \{a, b, d, e, h\}$,
$C = \{f, c, h\}$ and $D = \{f, d, e, h\}$

$$A \backslash B = C \backslash D = \{c\} \quad \text{and} \quad B \backslash A = D \backslash C = \{d, e\}$$

	a	b	c	d	e	f	h
A	$\times$	$\times$	$\times$				$\times$
B	$\times$	$\times$		$\times$	$\times$		$\times$
C			$\times$			$\times$	$\times$
D				$\times$	$\times$	$\times$	$\times$

A formal definition [Lepage, 2003]

Définition (Analogical proportion)

An analogical proportion on a set $\mathbb{E}$ in a relation on $\mathbb{E}^4$ such that, four all 4-tuples A, B, C et D in relation in this order, (denoted $A : B :: C : D$) :

1. $A : B :: C : D \Leftrightarrow C : D :: A : B$
2. $A : B :: C : D \Leftrightarrow A : C :: B : D$

For each couple, one has : $A : B :: A : B$

Five other proportions are equivalent :

$\boxed{B : A :: D : C}$ $\boxed{D : B :: C : A}$ $\boxed{D : C :: B : A}$

$\boxed{B : D :: A : C}$ $\boxed{C : A :: D : B}$

Plan

Analogical dissimilarity : definition

The analogical dissimilarity (AD) is a positive real number which measures by how much a 4-tuple quadruplet « miss » the analogical proportion .

Desired properties of *AD*

Analogical coherence. $DA(u, v, w, x) = 0 \Leftrightarrow u : v :: w : x$

Symmetry of *as*. $DA(u, v, w, x) = DA(w, x, u, v)$

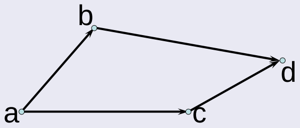
Exchange of means. $DA(u, v, w, x) = DA(u, w, v, x)$

Hexagonal inequality. $DA(u, v, z, t) \leq DA(u, v, w, x) + DA(w, x, z, t)$

Dissymetry of *is to*. Generally, $DA(u, v, w, x) \neq DA(v, u, w, x)$

Examples

Four vectors | **The binary case**



- Generally speaking :
  - $AD = 0$ is equivalent to the proportion
  - AD increases when the 4-tuple is further from a proportion
- Objects described by d attributes : $DA(a, b, c, d) = \sum_{j=1}^{d} AD(a_j, b_j, c_j, d_j)$

Examples

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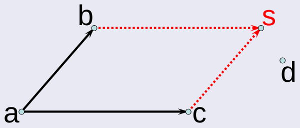

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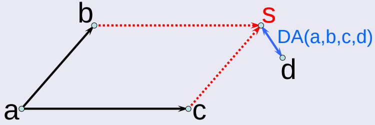

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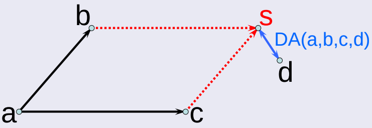

The binary case

0	0	...	0	0	...	1
0	1	...	1	1	...	1
0	0	...	1	1	...	1
0	0	...	0	1	...	1
0	1	...	2	1	...	0

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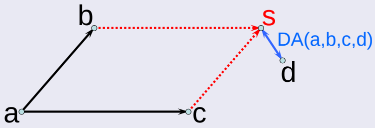

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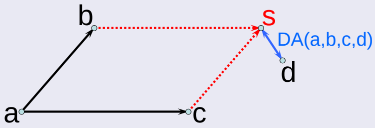

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A formal definition

N. Stroppa and F. Yvon (2005) [Stroppa, 2005], H. Prade and L. M. (2011)

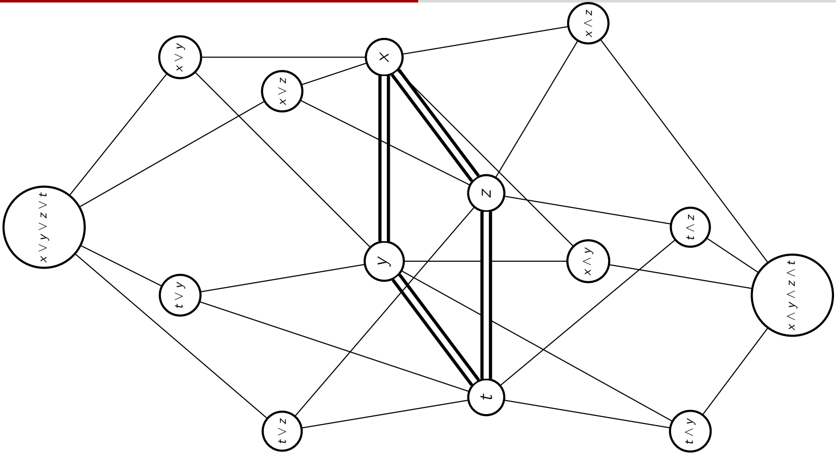
For four elements $(x, y, z, t) \in (L, \vee, \wedge)^4$, the analogical proportion denoted $(x : y :: z : t)$ holds true iff :

$$x = (x \wedge y) \vee (x \wedge z) \qquad x = (x \vee y) \wedge (x \vee z)$$

$$y = (x \wedge y) \vee (t \wedge y) \qquad y = (x \vee y) \wedge (t \vee y)$$

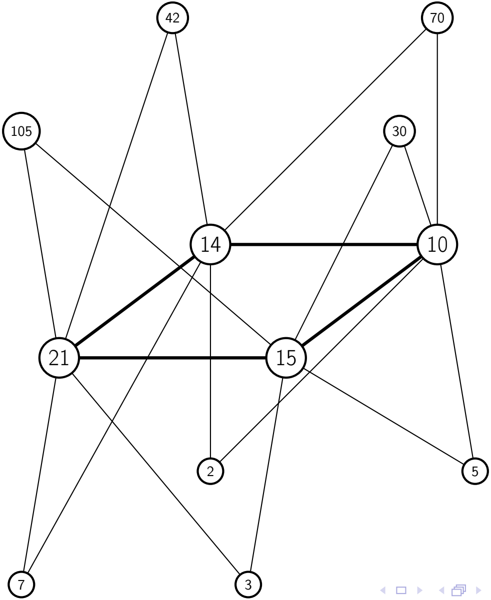
$$z = (t \wedge z) \vee (x \wedge z) \qquad z = (t \vee z) \wedge (x \vee z)$$

$$t = (t \wedge z) \vee (t \wedge y) \qquad t = (t \vee z) \wedge (t \vee y)$$

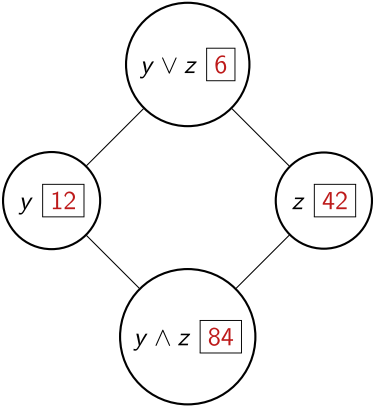


The general analogical proportion in a lattice
(either commutative or not).

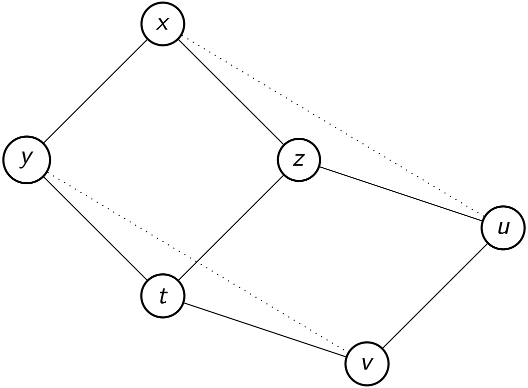
The lattice of the dividers of 120



The canonical analogical proportion in a lattice



Transitivity of the canonical proportion



Plan

Plan

Object classification in $\{0, 1\}^n$

Animals	CL	AD	MA	DM	Class
calf	0	0	0	1	*Ruminant*
bull	0	1	1	0	*Ruminant*
kitten	1	0	0	1	*Feline*
tomcat	1	1	1	0	?

CL	:	Claws
AD	:	Adult
MA	:	Male
DM	:	Drinks Milk

- 0 is to 0 as 1 is to 1 (CL)
- 0 is to 1 as 0 is to 1 (AD et MA)
- 1 is to 0 as 1 is to 0 (DM)

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calf **is to** *bull* **as** *kitten* **is to** *tomcat*

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calf **is to** *bull* **as** *kitten* **is to** *tomcat*
Ruminant **is to** *Ruminant* **as** *Feline* **is to** *Feline*

Nearest neighbor decision

Animals	HC	AD	MA	DM	class
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tomcat

- **Nearest neighbor** :
- **Analogy** : *tomcat* $\Rightarrow$ *Feline*

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tomcat $\dashrightarrow$ *bull*

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- **Nearest neighbor** : *tomcat* --→ *bull* ⇓ *Ruminant*
- **Analogy** : *tomcat* ⇒ *Feline*

Nearest neighbor decision

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- **Nearest neighbor** :

$$\begin{array}{ccc} \textit{tomcat} & \dashrightarrow & \textit{bull} \\ & & \Downarrow \\ \textit{Ruminant} & \Leftarrow & \textit{Ruminant} \end{array}$$

- **Analogy** : *tomcat* $\Rightarrow$ *Feline*

Nearest neighbor decision

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- **Nearest neighbor** :

$$\begin{array}{ccc} tomcat & \dashrightarrow & bull \\ & & \Downarrow \\ Ruminant & \Leftarrow & Ruminant \end{array}$$

- **Analogy** : *tomcat* $\Rightarrow$ *Feline*

Classification rule : the k less dissimilar triples

$\mathcal{S} = \{(c_i, h(c_i)) \mid 1 \leq i \leq m\}, x$

1. Compute AD(triple,x), triple $\in \mathcal{S}^3$.
2. Keep the triples in AP on the classes.
3. Order the triples by increasing AD.
4. Find k' from k.
5. Keep the k' triples.
6. Deduce the class of x by a vote.

Example

$o_1\, o_2\, o_3$	$h(o_1)$	$h(o_2)$	$h(o_3)$		AD
$a\ b\ c$	ω_0	ω_0	ω_1	ω_1	
$a\ b\ d$	ω_0	ω_0	ω_1	ω_1	
$a\ b\ e$	ω_0	ω_0	ω_2	ω_2	
$a\ c\ d$	ω_0	ω_1	ω_1	$\perp$	
$a\ c\ e$	ω_0	ω_1	ω_2	$\perp$	
$a\ d\ e$	ω_0	ω_1	ω_2	$\perp$	
$b\ a\ c$	ω_0	ω_0	ω_1	ω_1	
$b\ a\ d$	ω_0	ω_0	ω_1	ω_1	
$b\ a\ e$	ω_0	ω_0	ω_2	ω_2	
$b\ c\ d$	ω_0	ω_1	ω_1	$\perp$	
$b\ c\ e$	ω_0	ω_1	ω_2	$\perp$	
$b\ d\ e$	ω_0	ω_1	ω_2	$\perp$	
$c\ a\ b$	ω_1	ω_0	ω_0	$\perp$	
$c\ a\ d$	ω_1	ω_0	ω_1	ω_0	
$\vdots\ \vdots\ \vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	

Classification rule : the k less dissimilar triples

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Example

$o_1 o_2 o_3$	$h(o_1) h(o_2) h(o_3)$		AD
$a\ b\ c$	$\omega_0\ \omega_0\ \omega_1$	ω_1	6
$a\ b\ d$	$\omega_0\ \omega_0\ \omega_1$	ω_1	1
$a\ b\ e$	$\omega_0\ \omega_0\ \omega_2$	ω_2	3
$a\ c\ d$	$\omega_0\ \omega_1\ \omega_1$	$\perp$	3
$a\ c\ e$	$\omega_0\ \omega_1\ \omega_2$	$\perp$	2
$a\ d\ e$	$\omega_0\ \omega_1\ \omega_2$	$\perp$	5
$b\ a\ c$	$\omega_0\ \omega_0\ \omega_1$	ω_1	4
$b\ a\ d$	$\omega_0\ \omega_0\ \omega_1$	ω_1	0
$b\ a\ e$	$\omega_0\ \omega_0\ \omega_2$	ω_2	3
$b\ c\ d$	$\omega_0\ \omega_1\ \omega_1$	$\perp$	3
$b\ c\ e$	$\omega_0\ \omega_1\ \omega_2$	$\perp$	8
$b\ d\ e$	$\omega_0\ \omega_1\ \omega_2$	$\perp$	1
$c\ a\ b$	$\omega_1\ \omega_0\ \omega_0$	$\perp$	11
$c\ a\ d$	$\omega_1\ \omega_0\ \omega_1$	ω_0	7
$\vdots\ \vdots\ \vdots$	$\vdots\ \vdots\ \vdots$	$\vdots$	$\vdots$

Classification rule : the k less dissimilar triples

$\mathcal{S} = \{(c_i, h(c_i)) \mid 1 \leq i \leq m\}, x$

1. Compute AD(triple,x), triple $\in \mathcal{S}^3$.
2. Keep the triples in AP on the classes.
3. Order the triples by increasing AD.
4. Find k' from k.
5. Keep the k' triples.
6. Deduce the class of x by a vote.

Example

o_1	o_2	o_3	$h(o_1)$	$h(o_2)$	$h(o_3)$		AD
a	b	c	ω_0	ω_0	ω_1	ω_1	6
a	b	d	ω_0	ω_0	ω_1	ω_1	1
a	b	e	ω_0	ω_0	ω_2	ω_2	3
a	c	d	ω_0	ω_1	ω_1	$\perp$	3
a	c	e	ω_0	ω_1	ω_2	$\perp$	2
a	d	e	ω_0	ω_1	ω_2	$\perp$	5
b	a	c	ω_0	ω_0	ω_1	ω_1	4
b	a	d	ω_0	ω_0	ω_1	ω_1	0
b	a	e	ω_0	ω_0	ω_2	ω_2	3
b	c	d	ω_0	ω_1	ω_1	$\perp$	3
b	c	e	ω_0	ω_1	ω_2	$\perp$	8
b	d	e	ω_0	ω_1	ω_2	$\perp$	1
c	a	b	ω_1	ω_0	ω_0	$\perp$	11
c	a	d	ω_1	ω_0	ω_1	ω_0	7
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Classification rule : the k less dissimilar triples

$\mathcal{S} = \{(c_i, h(c_i)) \mid 1 \leq i \leq m\}, x$

1. Compute AD(triple,x), triple $\in \mathcal{S}^3$.
2. Keep the triples in AP on the classes.
3. Order the triples by increasing AD.
4. Find k' from k.
5. Keep the k' triples.
6. Deduce the class of x by a vote.

Example

o_1	o_2	o_3	$h(o_1)$	$h(o_2)$	$h(o_3)$		AD
a	b	c	ω_0	ω_0	ω_1	ω_1	6
a	b	d	ω_0	ω_0	ω_1	ω_1	1
a	b	e	ω_0	ω_0	ω_2	ω_2	3
b	a	c	ω_0	ω_0	ω_1	ω_1	4
b	a	d	ω_0	ω_0	ω_1	ω_1	0
b	a	e	ω_0	ω_0	ω_2	ω_2	3
c	a	d	ω_1	ω_0	ω_1	ω_0	7
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Classification rule : the k less dissimilar triples

$\mathcal{S} = \{(c_i, h(c_i)) \mid 1 \leq i \leq m\}, x$

1. Compute AD(triple,x), triple $\in \mathcal{S}^3$.
2. Keep the triples in AP on the classes.
3. Order the triples by increasing AD.
4. Find k' from k.
5. Keep the k' triples.
6. Deduce the class of x by a vote.

Example

$o_1 o_2 o_3$	$h(o_1)h(o_2)h(o_3)$				AD
$a\ b\ c$	ω_0	ω_0	ω_1	ω_1	6
$a\ b\ d$	ω_0	ω_0	ω_1	ω_1	1
$a\ b\ e$	ω_0	ω_0	ω_2	ω_2	3
$b\ a\ c$	ω_0	ω_0	ω_1	ω_1	4
$b\ a\ d$	ω_0	ω_0	ω_1	ω_1	0
$b\ a\ e$	ω_0	ω_0	ω_2	ω_2	3
$c\ a\ d$	ω_1	ω_0	ω_1	ω_0	7
$\vdots\ \vdots\ \vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Classification rule : the k less dissimilar triples

$\mathcal{S} = \{(c_i, h(c_i)) \mid 1 \leq i \leq m\}, x$

1. Compute AD(triple,x), triple $\in \mathcal{S}^3$.
2. Keep the triples in AP on the classes.
3. Order the triples by increasing AD.
4. Find k' from k.
5. Keep the k' triples.
6. Deduce the class of x by a vote.

Example

$o_1\, o_2\, o_3$	$h(o_1)$	$h(o_2)$	$h(o_3)$	$h(x)$	DA
b a d	ω_0	ω_0	ω_1	ω_1	0
c d e	ω_1	ω_1	ω_2	ω_2	1
a b d	ω_0	ω_0	ω_1	ω_1	1
d c e	ω_1	ω_1	ω_2	ω_2	2
d b c	ω_1	ω_0	ω_1	ω_0	2
a b e	ω_0	ω_0	ω_2	ω_2	3
b a e	ω_0	ω_0	ω_2	ω_2	3
b a c	ω_0	ω_0	ω_1	ω_1	4
$\vdots$ $\vdots$ $\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Classification rule : the k less dissimilar triples

$\mathcal{S} = \{(c_i, h(c_i)) \mid 1 \leq i \leq m\}, x$

1. Compute AD(triple,x), triple $\in \mathcal{S}^3$.
2. Keep the triples in AP on the classes.
3. Order the triples by increasing AD.
4. Find k' from k.
5. Keep the k' triples.
6. Deduce the class of x by a vote.

Example

o_1	o_2	o_3	$h(o_1)$	$h(o_2)$	$h(o_3)$	$h(x)$	DA
b	a	d	ω_0	ω_0	ω_1	ω_1	0
c	d	e	ω_1	ω_1	ω_2	ω_2	1
a	b	d	ω_0	ω_0	ω_1	ω_1	1
d	c	e	ω_1	ω_1	ω_2	ω_2	2
d	b	c	ω_1	ω_0	ω_1	ω_0	2
a	b	e	ω_0	ω_0	ω_2	ω_2	3
b	a	e	ω_0	ω_0	ω_2	ω_2	3
b	a	c	ω_0	ω_0	ω_1	ω_1	4
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

k	1	2	3	4	5	6	7
k'	1	3	3	5	5	7	7
$h(x)$							

Classification rule : the k less dissimilar triples

$\mathcal{S} = \{(c_i, h(c_i)) \mid 1 \leq i \leq m\}, x$

1. Compute AD(triple,x), triple $\in \mathcal{S}^3$.
2. Keep the triples in AP on the classes.
3. Order the triples by increasing AD.
4. Find k' from k.
5. Keep the k' triples.
6. Deduce the class of x by a vote.

Example

$o_1\, o_2\, o_3$	$h(o_1)$	$h(o_2)$	$h(o_3)$	$h(x)$	DA
$b\ a\ d$	ω_0	ω_0	ω_1	ω_1	0
$c\ d\ e$	ω_1	ω_1	ω_2	ω_2	1
$a\ b\ d$	ω_0	ω_0	ω_1	ω_1	1
$d\ c\ e$	ω_1	ω_1	ω_2	ω_2	2
$d\ b\ c$	ω_1	ω_0	ω_1	ω_0	2
$a\ b\ e$	ω_0	ω_0	ω_2	ω_2	3
$b\ a\ e$	ω_0	ω_0	ω_2	ω_2	3
$b\ a\ c$	ω_0	ω_0	ω_1	ω_1	4
$\vdots\ \vdots\ \vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

k	1	2	3	4	5	6	7
k'	1	3	3	5	5	7	7
$h(x)$	ω_1	ω_1	ω_1	?	?	ω_2	ω_2

Two operational enhancements

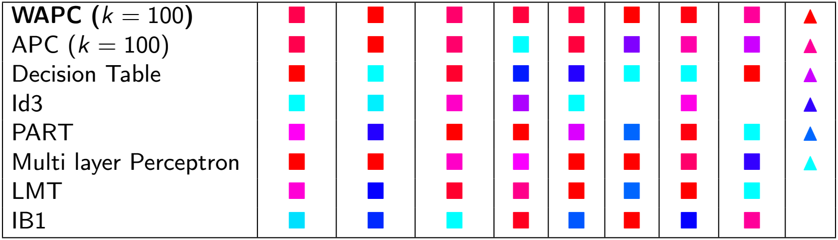
- Fast computing of the less dissemblant triple
    - Using the triangular inequality (cf. fast NN methods).
- Weighting the features
    - Learning the weights from the learning sample.

Results

Methods	*MO.1*	*MO.2*	*MO.3*	*SP.*	*B.S*	*Br.*	*H.R*	*Mu.*
nb nominal attributes	7	7	7	22	4	9	4	22
nb binary attributes	15	15	15	22	4	9	4	22
nb instances train	124	169	122	80	187	35	66	81
nb instances test	432	432	432	172	438	664	66	8043
nb classes	2	2	2	2	3	2	4	2
WAPC ($k = 100$)	**98%**	**100%**	**96%**	**79%**	**86%**	**96%**	**82%**	**98%**
APC ($k = 100$)	98%	100%	96%	58%	86%	91%	74%	97%
Decision Table	100%	64%	97%	65%	67%	86%	42%	99%
Id3	78%	65%	94%	71%	54%	*mv*	71%	*mv*
PART	93%	78%	98%	81%	76%	88%	82%	94%
Multi layer Perceptron	100%	100%	94%	73%	89%	96%	77%	96%
LMT	94%	76%	97%	77%	89%	88%	83%	94%
IB1	79%	74%	83%	80%	62%	96%	56%	98%

Results

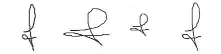
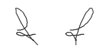
Methods	*MO.1*	*MO.2*	*MO.3*	*SP.*	*B.S*	*Br.*	*H.R*	*Mu.*	
nb nominal attributes	7	7	7	22	4	9	4	22	
nb binary attributes	15	15	15	22	4	9	4	22	
nb instances train	124	169	122	80	187	35	66	81	
nb instances test	432	432	432	172	438	664	66	8043	
nb classes	2	2	2	2	3	2	4	2	



Plan

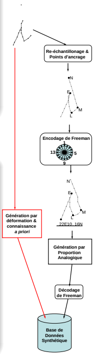
Knowledge-based generation

Distortions

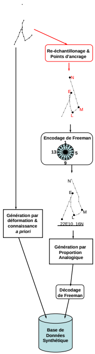
1. Scale :  ⇒ 
2. Verticality :  ⇒ 

On-line distortion :

1. Velocity :  &  ⇒  & 
2. Curvature :  &  ⇒  & 

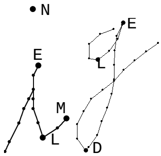


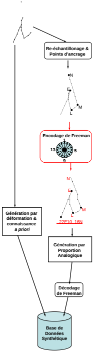
Analogy-based generation



Analogy-based generation

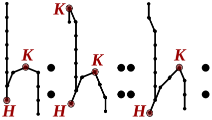
Anchor points

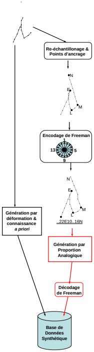
Symbol	Description	Examples
C / D	extrema max / min along y in a loop	
E / H	max angular point/ min	
K / L	extrema max extremum/ min en y	
M / N	pen up / down	



Analogy-based generation

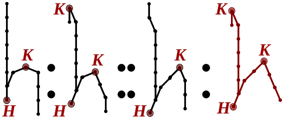
$$
\begin{array}{rcl}
h_1 & = & 9 \backsim 9 \backsim 9\,9\,9\,9\,9 \backsim H\ 1\ 2 \backsim 4\ K\ 6\ 9\ 9\ 9 \\
h_2 & = & 1\ K \backsim 8\ 9\,9\,9\,9\,9\,10\ H \backsim 2\ 2\ 4\ K \backsim 8\ 8\ 9 \\
h_3 & = & \backsim \backsim 9\ 8\ 9\,9\,9\,9\,9\,10\ H\ 2\ 2\ 3\ 3\ K\ 8\ 9\ 9 \backsim \\
x & = & 1\ K \backsim 8\ 9\,9\,9\,9\,9\,10\ H\ 2\ 2\ 3\ 3\ K\ 8\ 8\ 8 \backsim
\end{array}
$$

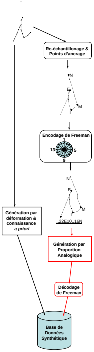




Analogy-based generation

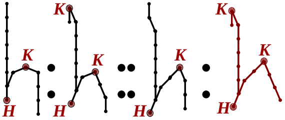
$$
\begin{array}{rcllllllllllllllllll}
h_1 & = & 9 & \backsim & 9 & \backsim & 9 & 9 & 9 & 9 & 9 & \backsim & H & 1 & 2 & \backsim & 4 & K & 6 & 9 & 9 & 9 \\
h_2 & = & 1 & K & \backsim & 8 & 9 & 9 & 9 & 9 & 9 & 10 & H & \backsim & 2 & 2 & 4 & K & \backsim & 8 & 8 & 9 \\
h_3 & = & \backsim & \backsim & 9 & 8 & 9 & 9 & 9 & 9 & 9 & 10 & H & 2 & 2 & 3 & 3 & K & 8 & 9 & 9 & \backsim \\
\color{red}{x} & \color{red}{=} & \color{red}{1} & \color{red}{K} & \color{red}{\backsim} & \color{red}{8} & \color{red}{9} & \color{red}{9} & \color{red}{9} & \color{red}{9} & \color{red}{9} & \color{red}{10} & \color{red}{H} & \color{red}{2} & \color{red}{2} & \color{red}{3} & \color{red}{3} & \color{red}{K} & \color{red}{8} & \color{red}{8} & \color{red}{8} & \color{red}{\backsim}
\end{array}
$$

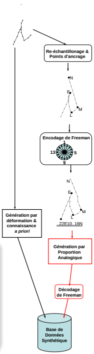




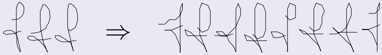
Analogy-based generation

$$
\begin{array}{rcl}
h_1 & = & 9 \backsim 9 \backsim 9\ 9\ 9\ 9\ 9 \backsim H\ 1\ 2 \backsim 4\ K\ 6\ 9\ 9\ 9 \\
h_2 & = & 1\ K \backsim 8\ 9\ 9\ 9\ 9\ 9\ 10\ H \backsim 2\ 2\ 4\ K \backsim 8\ 8\ 9 \\
h_3 & = & \backsim \backsim 9\ 8\ 9\ 9\ 9\ 9\ 9\ 10\ H\ 2\ 2\ 3\ 3\ K\ 8\ 9\ 9 \backsim \\
x & = & 1\ K \backsim 8\ 9\ 9\ 9\ 9\ 9\ 10\ H\ 2\ 2\ 3\ 3\ K\ 8\ 8\ 8 \backsim
\end{array}
$$





Examples of characters generated by analogy



Plan

Analogical proportion in propositional logic

- Analogical proportion between boolean variables

$$a : b :: c : d \quad \Leftrightarrow \quad (a \vee \overline{b} \equiv c \vee \overline{d}) \wedge (\overline{a} \vee b \equiv \overline{c} \vee d)$$

- Truth table

0	0	0	0	1	1	1	1
0	0	1	1	1	1	0	0
0	1	0	1	1	0	1	0

- The equation $a : b :: c : x$ has a solution iff

$$((a \equiv b) \vee (c \equiv d)) = 1$$

The solution is unique and has the value $a \equiv (b \equiv c)$.

Logical proportions

- Building Blocks :
  - $S_1 = (a \wedge b)$, $S_2 = (\neg a \wedge b)$, $(D_1 = a \wedge \neg b)$ and $D_2 = (\neg a \wedge \neg b)$.
  - $S'_1 = (c \wedge d)$, $S'_2 = (\neg c \wedge d)$, $D'_1 = (c \wedge \neg d)$ and $D'_2 = \neg c \wedge \neg d)$
- $(X \equiv X') \wedge (Y \equiv Y')$ with
  - $X', Y' \in \{S_1, S_2, D_1, D_2\}$ and
  - $X, Y \in \{S'_1, S'_2, D'_1, D'_2\}$
- 120 different proportions, such as :
  - $((a \wedge b) \equiv (c \wedge d)) \wedge ((\neg a \wedge \neg b) \equiv (\neg c \wedge \neg d))$ (analogical)
  - $((a \wedge b) \equiv (c \wedge d)) \wedge ((a \wedge \neg b) \equiv (\neg c \wedge d))$

About the 120 logical proportions

- Each truth table has exactly six rows with truth value 1.
- The set of logical proportions can be organised according to the syntactic type and the properties.
- In particular, 4 proportions are said « homogenuous ».
  - analogy : a is to b as c is to d.
    What a et b have in common, c et d have also in common, and inversely.
  - inverse analogy : a is to b as d is to c.
  - paralogy : a is to c as d is to à b.
  - inverse paralogy : What a et b have in common, c et d have not in common, and inversely.

Multivalued analogical inference

Analogical inference

$$\frac{\forall i \in [1,n], \quad a_i \ : \ b_i \ :: \ c_i \ : \ d_i \quad \text{true}}{a_{n+1} \ : \ b_{n+1} \ :: \ c_{n+1} \ : \ d_{n+1} \quad \text{true}}$$

Analogical proportion between boolean variables

$$a \ : \ b \ :: \ c \ : \ d \quad \Leftrightarrow \quad ((a \Rightarrow b) \equiv (c \Rightarrow d)) \wedge ((b \Rightarrow a) \equiv (c \Rightarrow d))$$

A good multivalued extension

- $x \wedge y = \min(x, y)$
- $x \Rightarrow y = min(1, 1 - x + y)$
- $x \equiv y = 1 - |x - y|$

Why ?

- $(a \ : \ b \ :: \ c \ : \ d) = 1 \quad \text{iff} \quad |a - b| = |c - d|$
- $a \leq b \Leftrightarrow c \leq d$

Multivalued analogical inference

Solving an extrapolative equation

In the multivaluate case, there exists x such as $a : b :: c : x$ is true up to degree 1
iff$x = c + b - a \in [0, 1]$.

Tri-valued case

19 correct casses among the 81.
For example :

$$\frac{1}{2} \ : \ 0 \ :: \ 1 \ : \ \frac{1}{2}$$

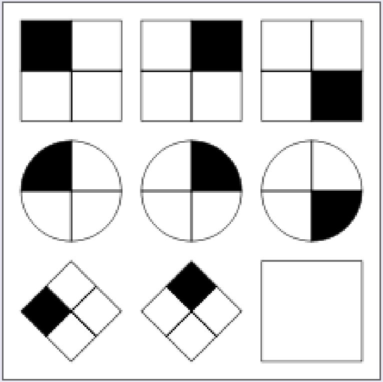
Solving an extrapolative equation

All cases are correct.

$$a \ : \ x \ :: \ x \ : \ b \quad \Leftrightarrow \quad x = \frac{a+b}{2}$$

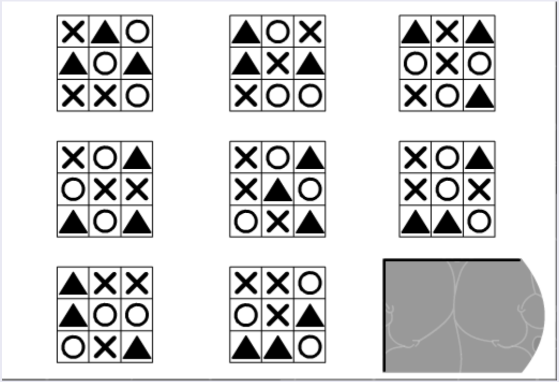
Resolving complex analogical equations (Correa, Prade et Richard)

The Raven test



Test yourself

A more difficult Raven test



Plan

Some tracks than can be followed

- Analogy and Lattices.
- Logical Proportions.
- Algorithmics for learning and decision.
- Explanation of the efficiency.
- (etc.)

Thank you
for your attention.

Thank you
for your attention.



Thank you for your attention.

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