Identifying colorings of graphs *

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We are interested in colorings of a graph in which each vertex is identified from the others by the sets of colors appearing in its neighborhood. Burris and Schelp introduced in [1] edge colorings of graphs where each vertex is distinguished from the others by the set of colors of its incident edges. This type of coloring was later extended to require only adjacent vertices to be distinguished [3], which was in turn extended to distinguishing total colorings [2] (vertices and edges are both colored). Here, we are interested in a version where only vertices are colored.

For any vertex u of a graph G, let $N[u]$ denote its closed neighborhood: $N[u] = \{v \mid uv \in E(G)\} \cup \{u\}$. A vertex-coloring c is a *locally-identifying coloring* (*lid-coloring* for short) of the graph G, if for each edge uv of G, $c(u) \neq c(v)$ (i.e. c is a proper coloring) and $N[u] \neq N[v] \Rightarrow c(N[u]) \neq c(N[v])$ (i.e. if two adjacent vertices u, v have distinct closed neighorhood, then the sets of colors appearing on $N[u]$ and $N[v]$ are distinct). We define the *lid-chromatic number* of G, denoted by $\chi_{lid}(G)$, as the minimum number of colors required by a lid-coloring of G.

We study the parameter χ_{lid} for different families of graphs, such as bipartite graphs and chordal graphs. Every bipartite graph G satisfies $\chi_{lid}(G) \leq 4$, but deciding whether a subcubic bipartite graph G with large girth satisfies $\chi_{lid}(G) = 3$ is an NP-complete problem. However, it can be checked in polynomial time whether a tree or a subcubic planar graph G satisfies $\chi_{lid}(G) = 3$. We also remark that the Cartesian product of two bipartite graphs (and thus hypergraphs and grids in any dimension) has lid-chromatic number 3.

In general, there is no connection between the chromatic number χ and the lid-chromatic number of a graph. However we conjecture that every chordal graph G satisfies $\chi_{lid}(G) \leq 2\chi(G)$. We prove that it holds for k-trees and show that split graphs satisfy $\chi_{lid}(G) \leq 2\chi(G) - 1$. We also provide examples showing that both bounds are sharp.

We also study the connection between $\chi_{lid}(G)$ and the maximum degree Δ of G. We prove that $\chi_{lid}(G) \leq \Delta^3 - \Delta^2 + \Delta + 1$ by observing that a proper coloring of G^3 is a lid-coloring of G. On the other hand, we construct graphs with maximum degree Δ that require at least $\Delta^2 - \Delta + 1$ colors in any lid-coloring.

References

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