Locally identifying colorings of graphs

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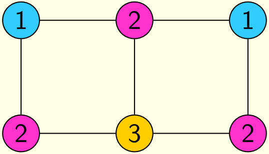
Identification with colors ?

Identifying coloring of a graph G:

- $c : V \to \mathbb{N}$
- $c(N[x]) \neq c(N[y])$ for any vertices $x \neq y$
- $\chi_{id}(G)$: minimum number of colors needed to identify G



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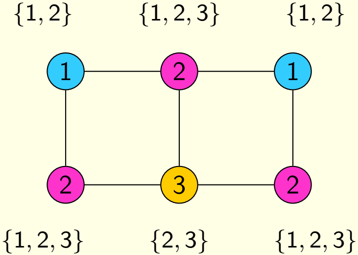

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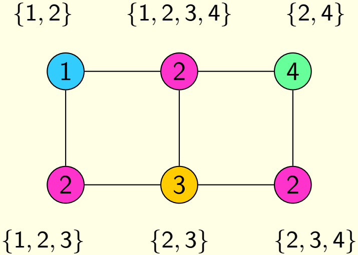

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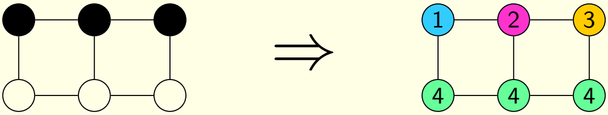
Few remarks:

- only exists for twin-free graphs (like id-codes)
- $\chi_{id}(G) \leq \gamma^{ID}(G) + 1$



Global to local colorings

Identifying coloring of a graph $G = (V, E)$:

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Locally identifying coloring (lid-coloring) of a graph $G = (V, E)$:

- $c : V \to \mathbb{N}$, $c(x) \neq c(y)$ for $xy \in E$;
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- $\chi_{lid}(G)$: min. number of colors needed to locally identify G.

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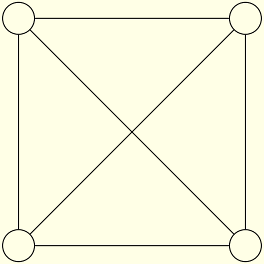
Why?

- Always exists.
- Refinment of classic colorings: $\chi(G) \leq \chi_{lid}(G)$

An example

Def: $\forall xy \in E$, $c(x) \neq c(y)$ and $c(N[x]) \neq c(N[y])$



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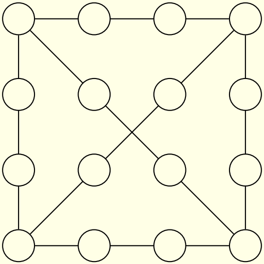

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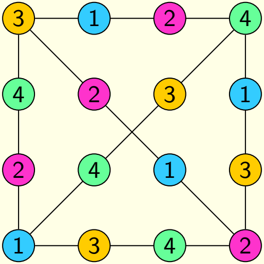

$$\chi_{lid}(G) \leq 4$$

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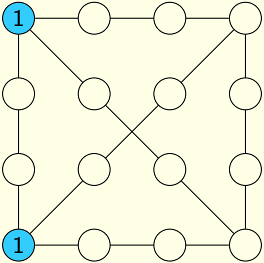

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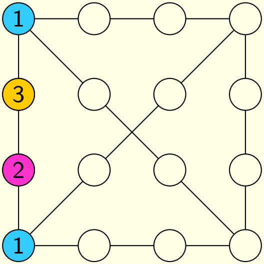

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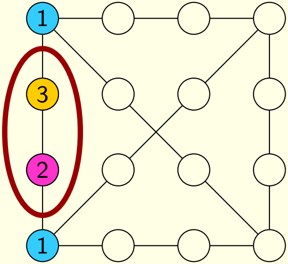

$\chi_{lid}(G) = 4$ but $\chi(G) = 3$

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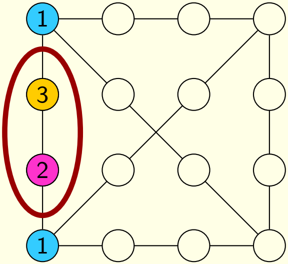

$$\chi_{lid}(G) = 4 \text{ but } \chi(G) = 3$$

For each k, there exists graph G_k s.t $\chi(G_k) = 3$ and $\chi_{lid}(G_k) = k$

No upper bound with χ !

Upper bound on a graph with n vertices ?

Classic colorings: $\chi(G) = n \Leftrightarrow G = K_n$

Lid-colorings: for which graphs $\chi_{lid}(G) = n$?

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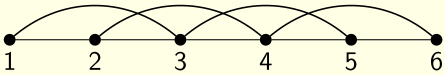
- K_n
- P_{2k}^{k-1}:



Extremal graph for identifying codes !

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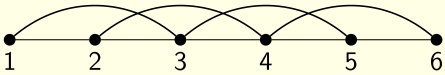


Extremal graph for identifying codes !

- ... ?

Open question

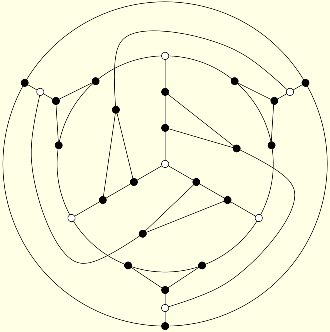
Caracterize graphs G such that $\chi_{lid}(G) = n$.

Maximum degree

Classic colorings: $\chi(G) \leq \Delta + 1$, tight

Lid-colorings:

- $\chi_{lid}(G) \leq \chi(G^3) \leq \Delta^3 - \Delta^2 + \Delta + 1$
- Graphs with $\chi_{lid}(G) \geq \Delta^2 - \Delta + 1$



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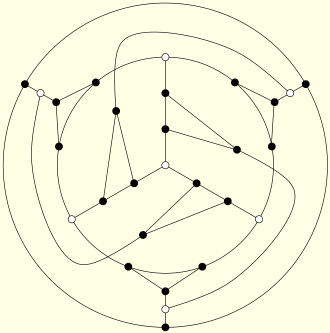

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Theorem (Foucaud,Honkala,Laihonen,P.,Perarnau, 2011^+)

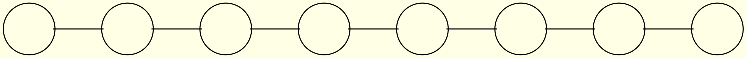
For any graph G with $\Delta \geq 3$: $\chi_{lid}(G) \leq 2\Delta^2 - 3\Delta + 3$

Open question

Do we always have $\chi_{lid}(G) \leq \Delta^2 + O(\Delta)$?

Bipartite graphs: the paths

With 4 colors :



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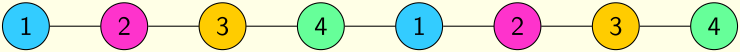

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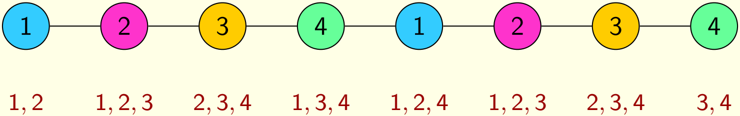

So:

$$\chi_{lid}(P_k) \leq 4$$

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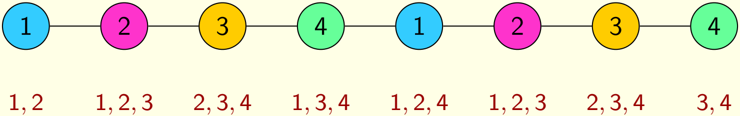

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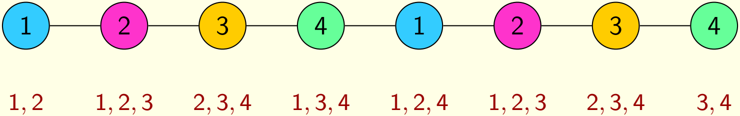

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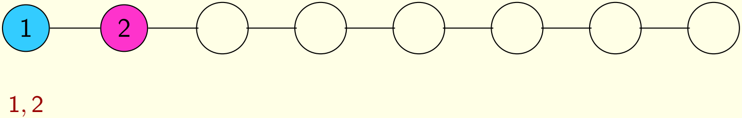

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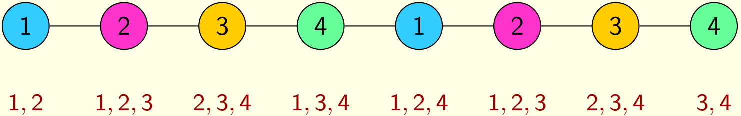

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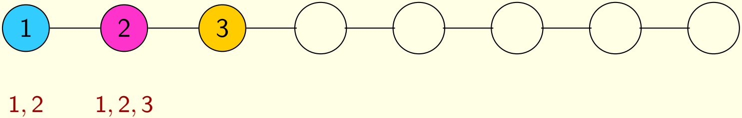

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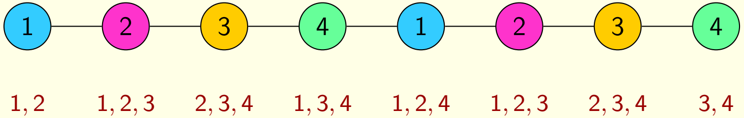

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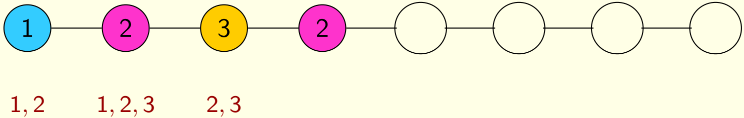

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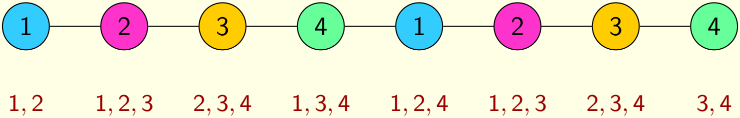

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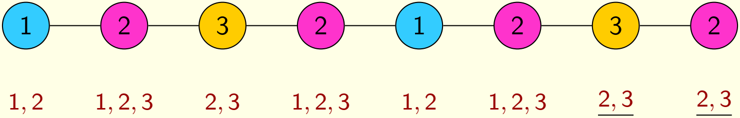
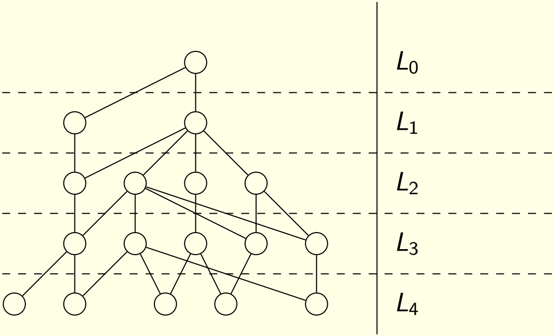


$\chi_{lid}(P_k) = 3 \Leftrightarrow k$ is odd ... χ_{lid} is not heriditary !

Bipartite graphs



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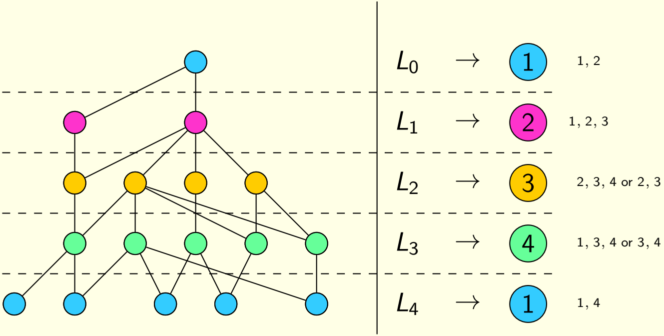
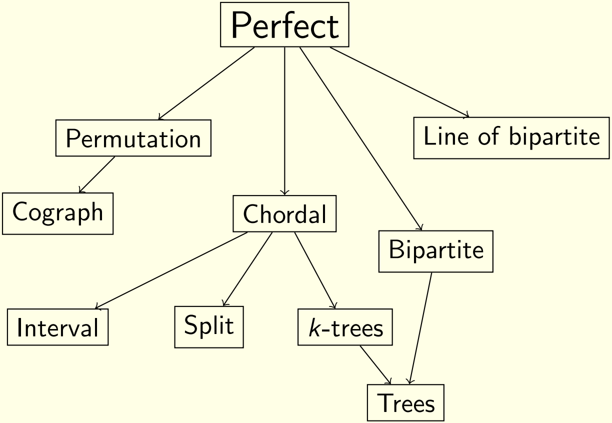


- $3 \leq \chi_{lid}(B) \leq 4$
- To decide between 3 and 4 is NP-complete (reduction from 2-coloring of hypergraph)
- Polynomial for trees, grids and hypercubes ($\chi_{lid} = 3$), regular bipartite graphs...

Perfect Graphs



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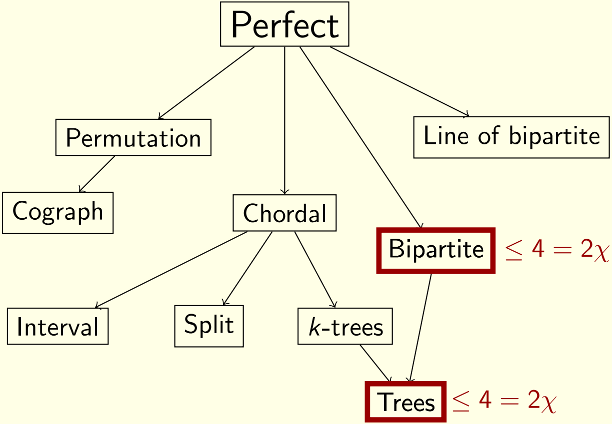

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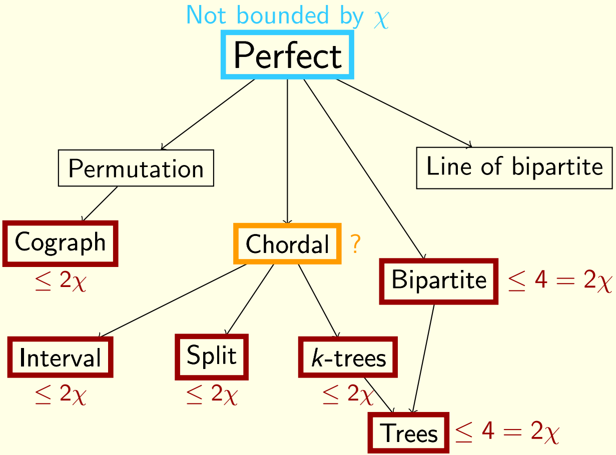


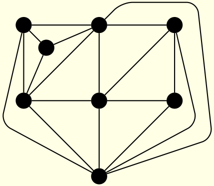
Open question

Do we have $\chi_{lid}(G) \leq 2\chi(G)$ for a chordal graph G?

Planar Graphs

Planar graphs:

- Worse example : 8 colors,
- Really large (1000 ?) bound by Gonzcales and Pinlou



P_8^3

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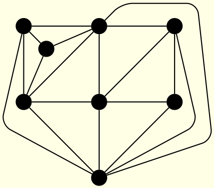

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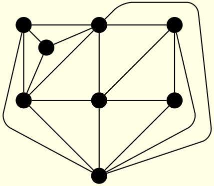

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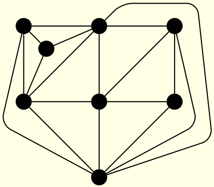
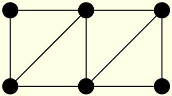


P_8^3

Outerplanar graphs:

- General bound: 20 colors,
- Max outerplanar graphs: ≤ 6 colors,
- Without triangles: ≤ 8 colors,
- Examples with at most 6 colors



P_6^2

Open question

Do we have $\chi_{lid}(G) \leq 8$ for planar graphs and $\chi_{lid}(G) \leq 6$ for outer-planar graphs ?

A remark

- For some subclasses of perfect graphs : $\chi_{lid}(G) \leq 2\omega(G) = 2\chi(G)$
- For planar graphs, worse example : $\chi_{lid}(G) \leq 8 = 2\chi(G)$
- For outerplanar graphs, worse example : $\chi_{lid}(G) \leq 6 = 2\chi(G)$
- ...

Open question

For which graphs do we have $\chi_{lid}(G) \leq 2\chi(G)$?

Another remark

- $\chi_{lid}(G) = 2 \Leftrightarrow G = K_2$
- $\chi_{lid}(G) = 3 \Rightarrow G = K_3$ or G is bipartite
- $\chi_{lid}(G) = 3$ and $\chi(G) = 3 \Leftrightarrow G = K_3$

Open question

Caracterize graphs G such that $\chi_{lid}(G) = \chi(G)$. Are they only the complete graphs ?

Conclusion

Lot of open questions:

- Graphs with $\chi_{lid} = n$?
- Graphs with $\chi_{lid} = \chi$?
- Do we have $\chi_{lid}(G) \leq \Delta^2 + O(\Delta)$?
- Do we have $\chi_{lid}(G) \leq 2\chi(G)$ for chordal graphs? for planar graphs? for which graphs?
- Find a good bound for planar graphs.
- Find a "nice" application of lid-colorings

Thanks !