Identifying coloring of graphs

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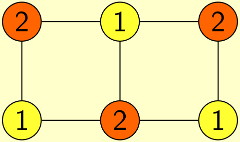
Aline Parreau

July 1st, 2010

ANR IDEA

Proper coloring

Two adjacent vertices have distinct colors .



$$B_t(u) = \{v \mid d(u, v) \leq t\}$$

For any edge uv, $c(B_0(u)) \neq c(B_0(v))$

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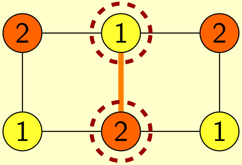

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Locally identifying coloring (lid-coloring)

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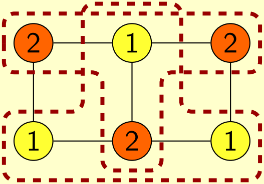

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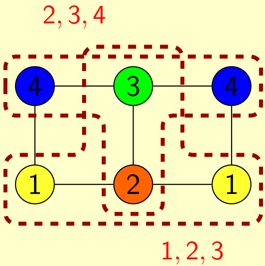

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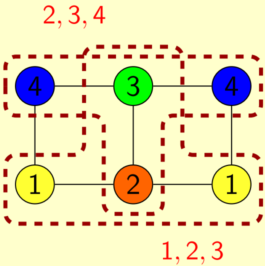

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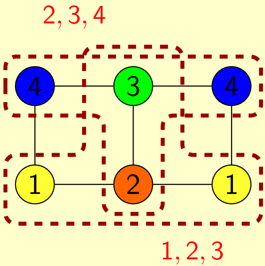

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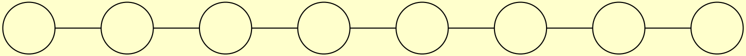
$\chi_{lid}(G)$: lid-chromatic number

An example: the path

With 4 colors :



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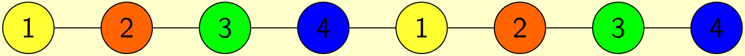

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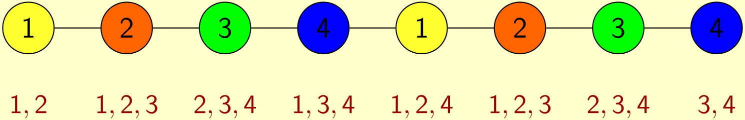

So:

$$\chi_{lid}(P_k) \leq 4$$

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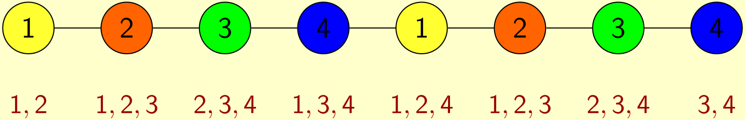

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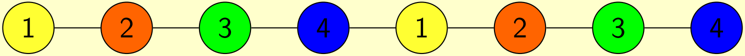

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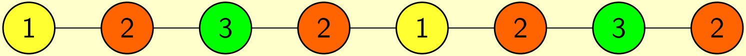

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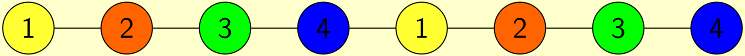

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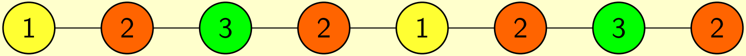

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$$\chi_{lid}(P_k) = 3 \Leftrightarrow k \text{ is odd}$$

Related works

With edge colorings:

- Vertex-distinguishing edge colorings (Observability of a graph) (Hornak et al, 95'),
- Adjacent vertex-distinguishing edge colorings (Zhang et al, 02')

With total colorings:

- Adjacent vertex-distinguishing total colorings (Zhang, 05')

Link with chromatic number

Do we need much more than $\chi(G)$ colors ?

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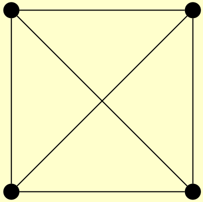

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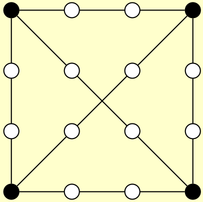

An example with $\chi(G) = 3$ and $\chi_{lid}(G) \geq k$

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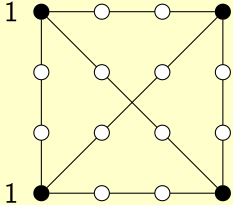

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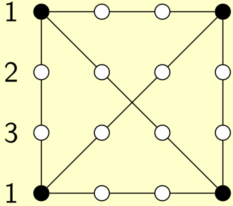

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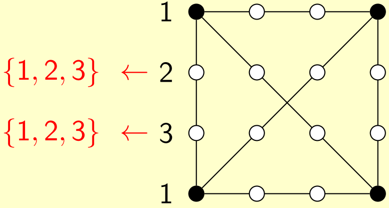

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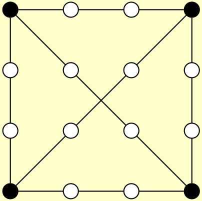

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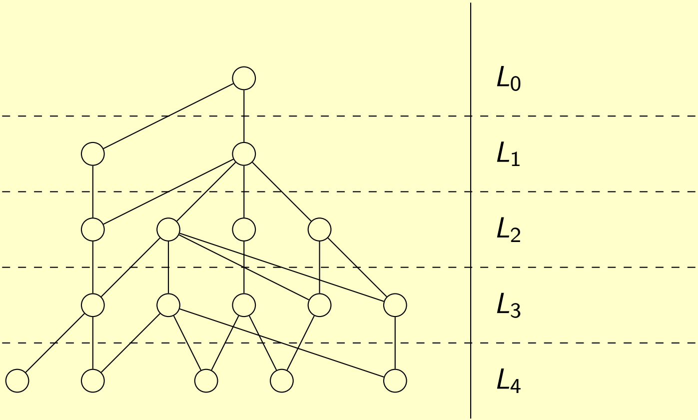
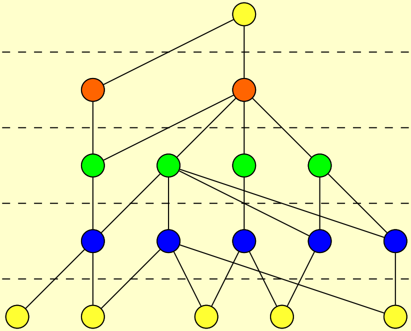
What about "good classes" for classical colorings ?

Bipartite graphs



Bipartite graphs



$L_0 \rightarrow$ 1   1, 2

$L_1 \rightarrow$ 2   1, 2, 3

$L_2 \rightarrow$ 3   2, 3, 4 or 2, 3

$L_3 \rightarrow$ 4   1, 3, 4 or 3, 4

$L_4 \rightarrow$ 1   1, 4

Bipartite graphs

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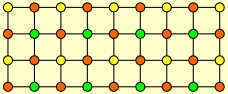

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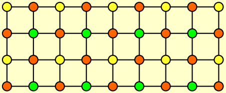

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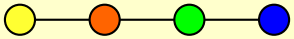

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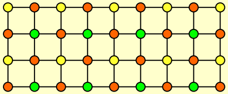

$\leftarrow ? \rightarrow$

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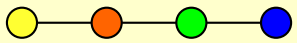
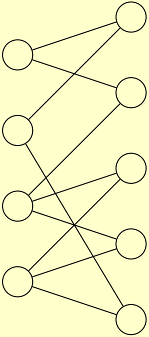


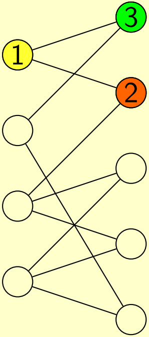
In general... 3-LID-COLORING is NP-complete in bipartite graphs

Link with 2-coloring of hypergraph



Link with 2-coloring of hypergraph

$1, 2, 3$



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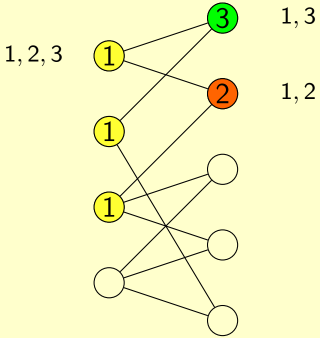

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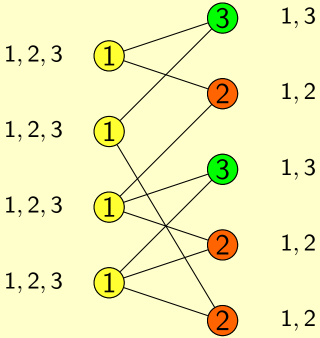

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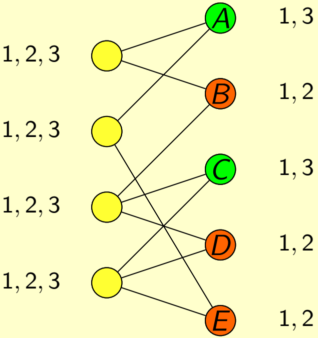

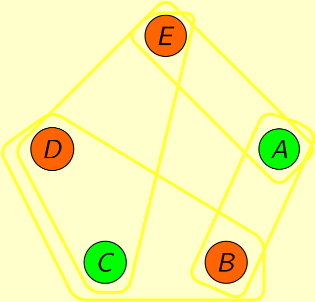

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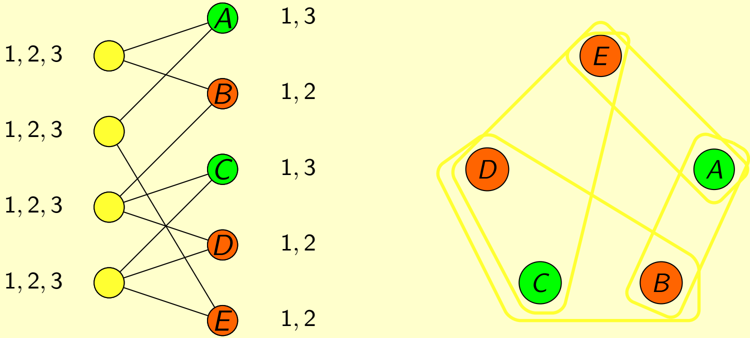
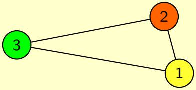


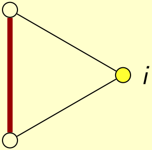
- 3-LID-COLORING in bipartite graph is NP-Complete
- Polynomial if B regular, if B is planar with maximum degree 3.

To perfect graph : *k*-trees

Lid-coloring of 2-trees with 6 colors :



- Color the triangle with colors $1, 2, 3$
- Step:



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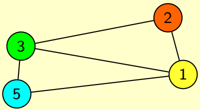

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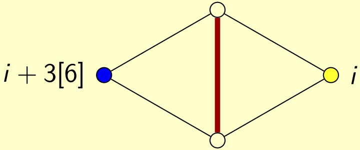

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  - proper coloring
  - no edge $(i, i+3)$

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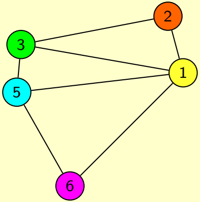

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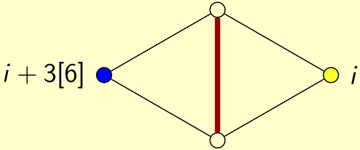

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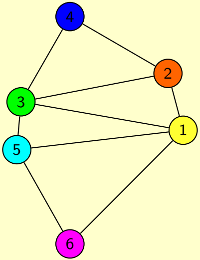

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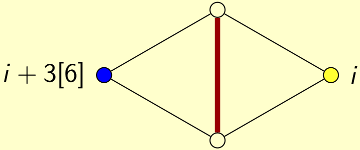

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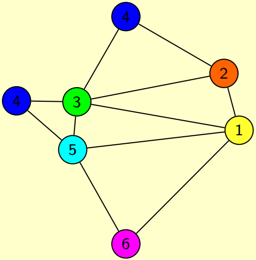

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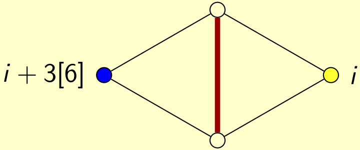

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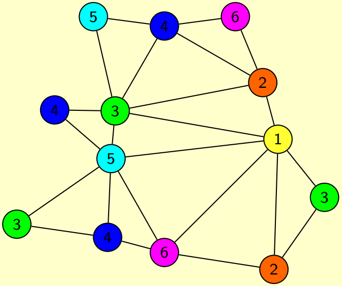

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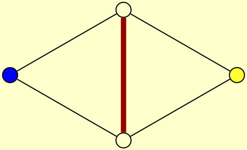

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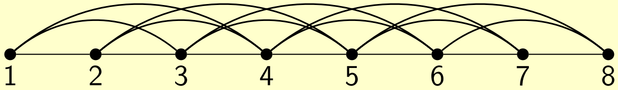
  - no edge $(i, i + 3)$

To perfect graph: k-trees

We can extend the construction to k-trees:

$\rightarrow$ A k-tree has lid-chromatic number at most $2k+2$

This bound is sharp: P^k_{2k+2}



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Question: Can we color any perfect graph G with $2\omega(G)$ colors?

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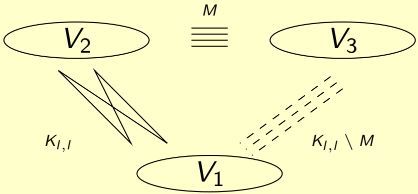
No !



V_1, V_2, V_3 stable sets of size l

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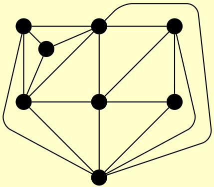

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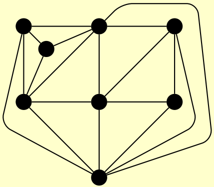
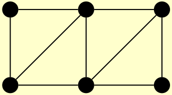


Outerplanar graphs:

- General bound: 20 colors,
- Max outerplanar graphs: 6 colors,
- Without triangles: 8 colors,



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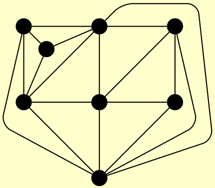
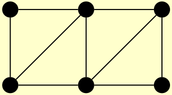
- With large girth (36): 5 colors,
- Examples with at most 8 colors

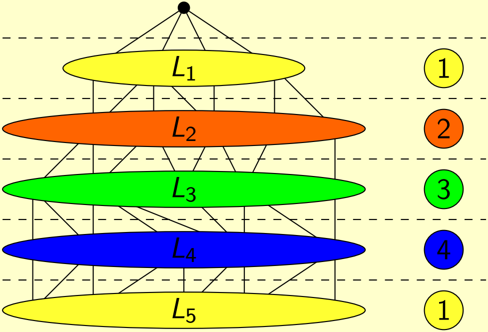


Outerplanar graphs:

- General bound: 20 colors,
- Max outerplanar graphs: 6 colors,
- Without triangles: 8 colors,
- Examples with at most 6 colors



A bound for outerplanar graphs



- a layer = union of paths,
- 5 colors in a layer,
- $4 \times 5 = 20$

Some open questions

- Is χ_{lid} bounded for planar graphs?
- For which graphs $\chi_{lid} = \chi$?
- Link with maximum degree Δ ?
- What about a global version ?

Thanks !