Codes identifiants tolérants utilisant des boules Euclidiennes.

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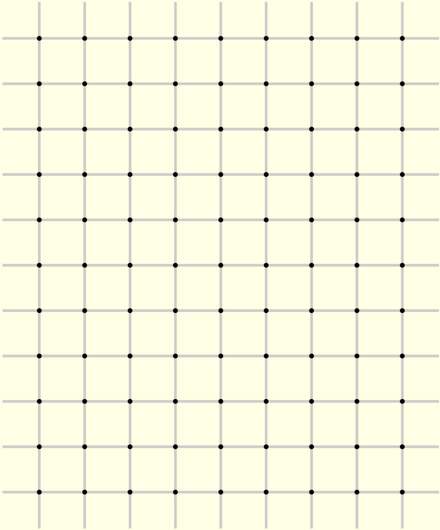
En collaboration avec Ville Junnila et Tero Laihonen

Journées Graphes et Algorithmes 2011

ANR IDEA

Cadre

- Sommets (capteurs) : $\mathbb{Z}^2$



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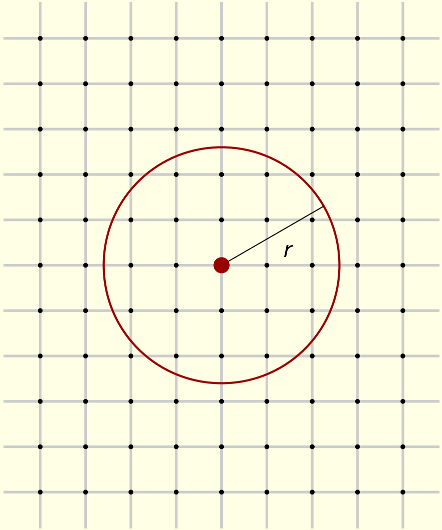
- Sommets (capteurs) : $\mathbb{Z}^2$
- Capable de détecter à distance Euclidienne r



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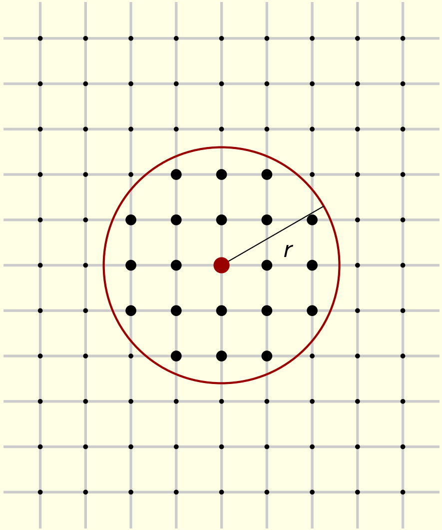
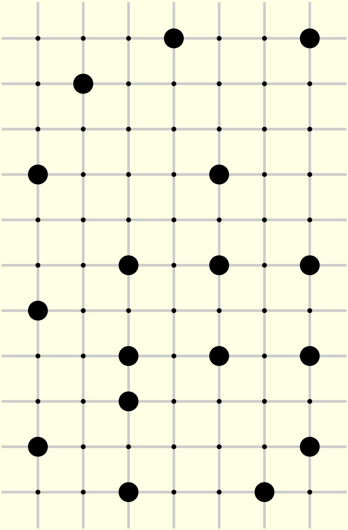
$\rightarrow$ Grille Euclidienne



Codes identifiants dans la grille euclidienne

Code identifiant : ensemble $C \subseteq \mathbb{Z}^2$ tel que :



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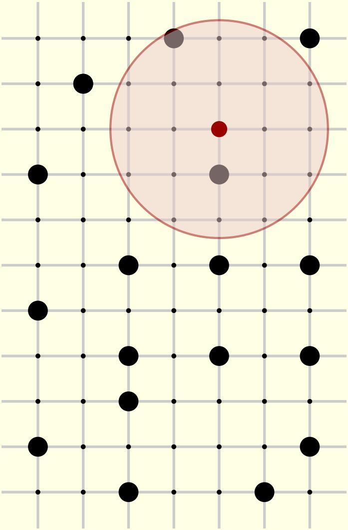

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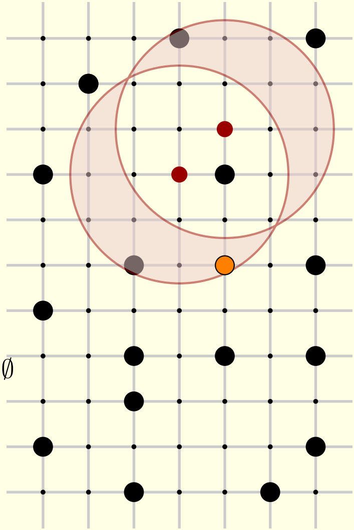
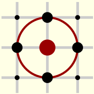
- C soit r-séparant : pour tous $u, v \in \mathbb{Z}^2$,

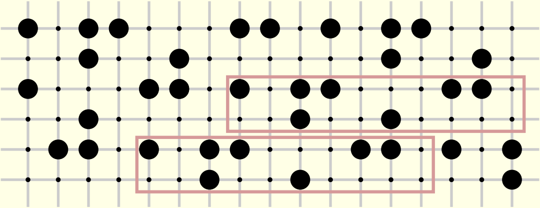
$$C \cap ((B_r(u) \setminus B_r(v)) \cup (B_r(v) \setminus B_r(u))) \neq \emptyset$$

But : minimiser la densité $D(r)$ de C



L'exemple de la grille carrée

$r = 1 \rightarrow$ grille carrée classique 



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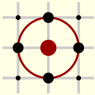
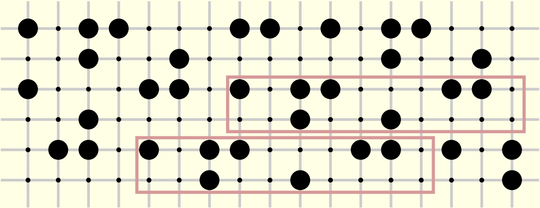
$r = 1 \rightarrow$ grille carrée classique 



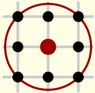
Proposition Cohen *et al*, 1999 et Ben-Haim et Litsyn, 2005

La densité optimale d'un code identifiant dans la grille carrée est : $D(1) = \frac{7}{20}$

D'autres résultats pour des petites valeurs de r

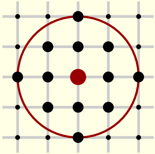
$r = \sqrt{2}$, Grille du roi :

$$D(\sqrt{2}) = \frac{2}{9}$$



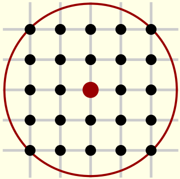
$r = 2$, Grille carrée de rayon 2 :

$$\frac{6}{35} \leq D(2) \leq \frac{5}{29}$$



$r = 2\sqrt{2}$ Grille du roi de rayon 2:

$$D(2\sqrt{2}) = \frac{1}{8}$$



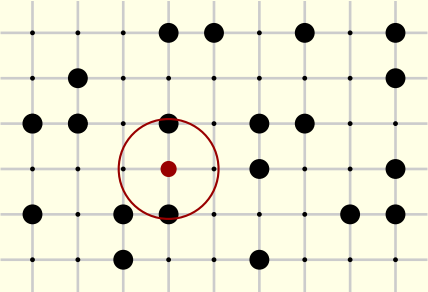
Pour un rayon quelconque

Proposition Junnila et Laihonen, 2011

Borne générale :

$$\frac{1}{3.22r+4} \leq D(r) \leq \frac{1}{2\lfloor r \rfloor}$$

Les codes optimaux ne sont pas robustes



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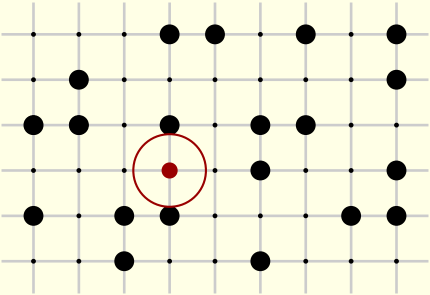

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Codes identifiants tolérants

Les boules varient entre les boules de rayons r et $r + \Delta$.

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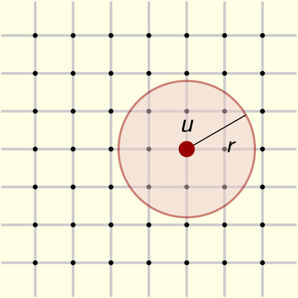

$B_r(u)$

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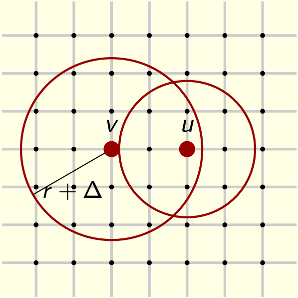

$B_r(u)$ $B_{r+\Delta}(v)$

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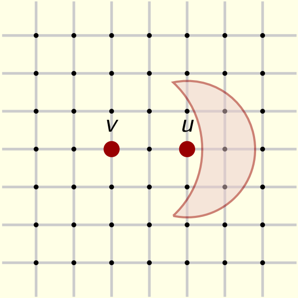

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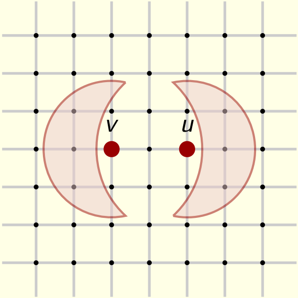
$D(r, \Delta)$: densité minimale



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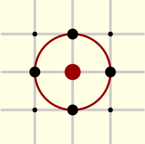
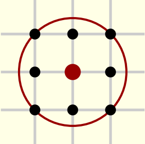
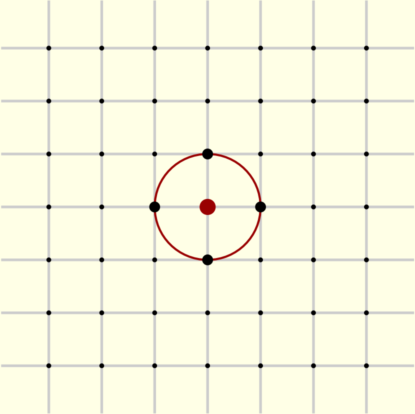
Premier exemple : $(r, \Delta) = (1, \sqrt{2} - 1)$



$r = 1$



$r + \Delta = \sqrt{2}$



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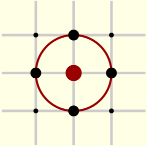

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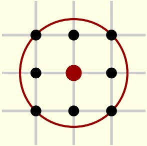

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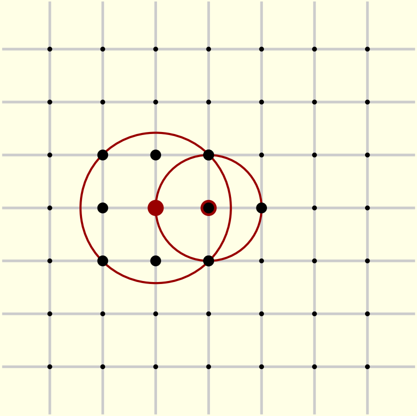

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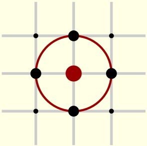

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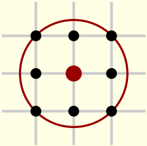

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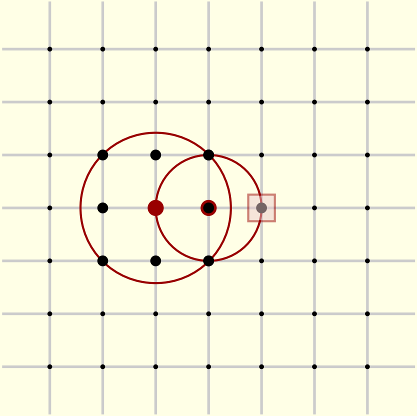

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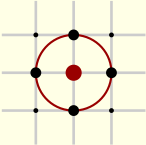

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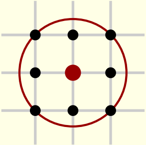

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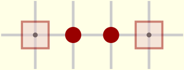
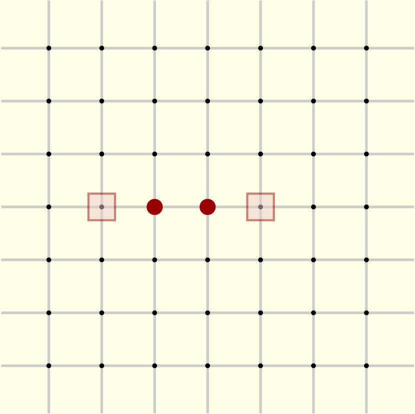
Motif horizontal $\mathcal{S}_{r,\Delta}(\bullet\ \bullet)$:



$\Rightarrow D(r, \Delta) \geq 1/2$



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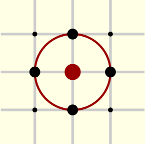

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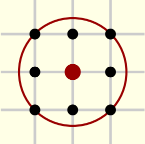

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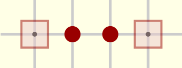

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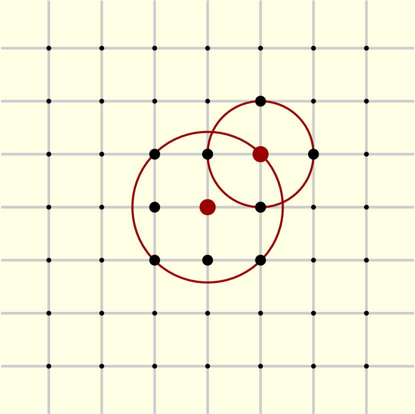

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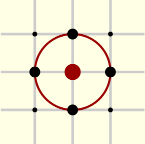

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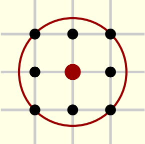

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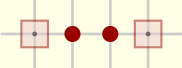

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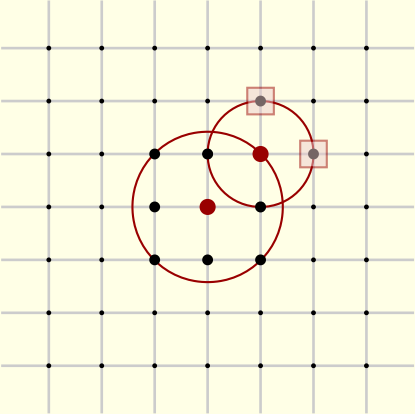

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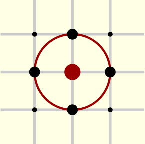

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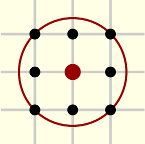

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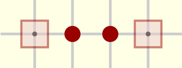

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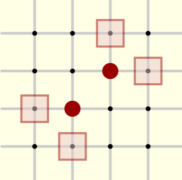

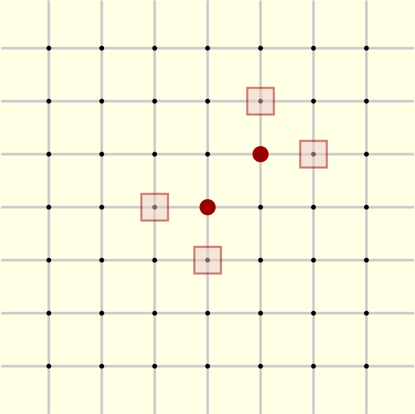

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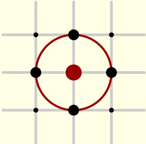

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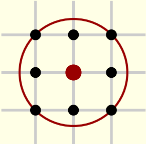

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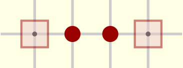

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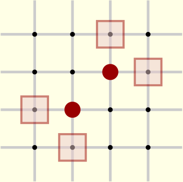

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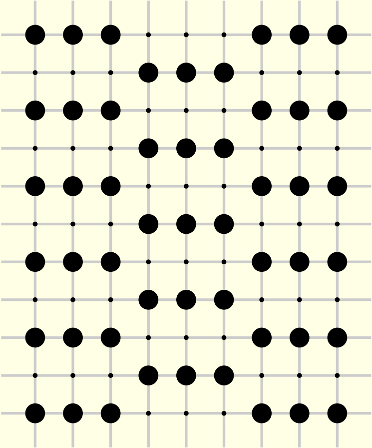
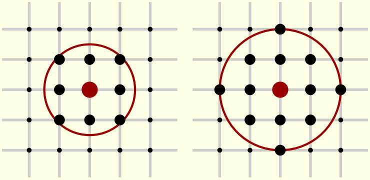


Au suivant ! $(r, \Delta) = (\sqrt{2}, 2 - \sqrt{2})$



$r = \sqrt{2}$ $\qquad$ $r + \Delta = 2$

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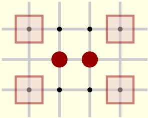

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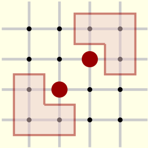

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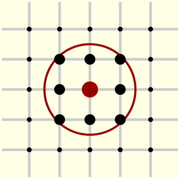

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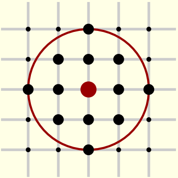

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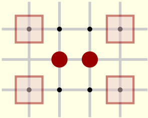

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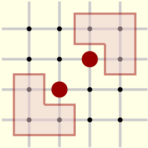
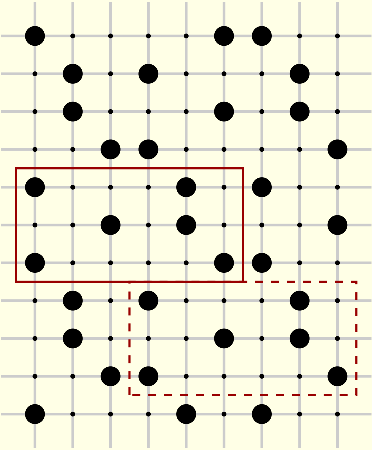
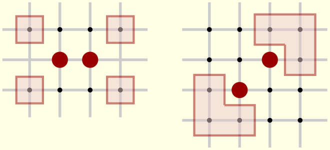


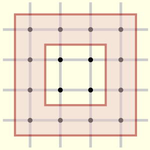
$\frac{1}{4} \leq D(r, \Delta) \leq \frac{1}{3}$:



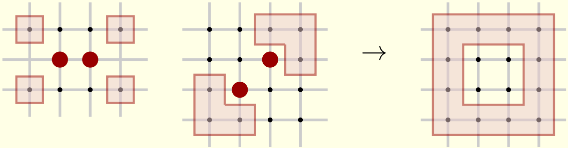
Amélioration de la borne inférieure



$\rightarrow$

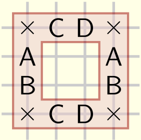


Amélioration de la borne inférieure

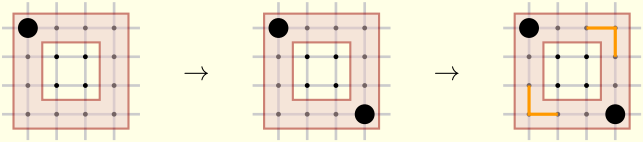


Au moins 3 sommets du code dans la fenêtre:

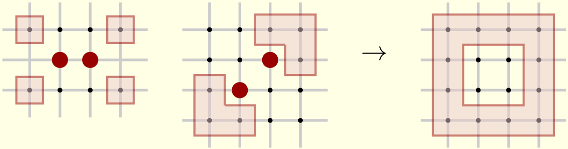
- Si aucun sommet dans les coins, au moins 4 sommets du code:



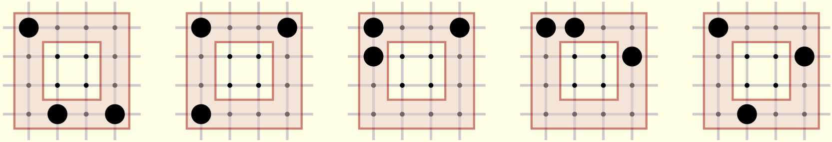
- si un sommet dans un coin :



Amélioration de la borne inférieure



Au moins 3 sommets du code et seulement 5 cas possibles:



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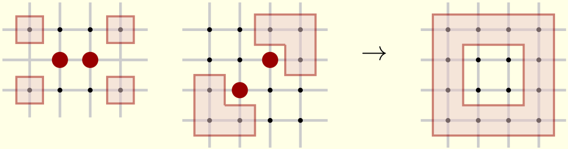

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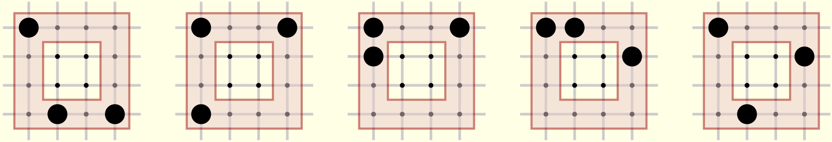
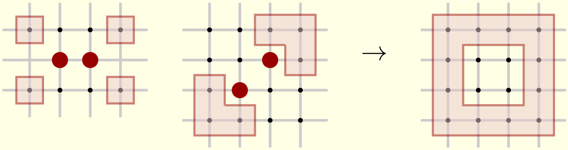


Chaque fenêtre a une "fenêtre voisine" avec au moins 4 sommets du code.
Chaque fenêtre avec 4 codes ou plus donne 1/5 aux fenêtres voisines avec 3 codes.

$$D(\sqrt{2}, 2-\sqrt{2}) \geq \frac{(3+\frac{1}{5})}{12} = \frac{4}{15} \simeq 0.26$$

Amélioration de la borne inférieure



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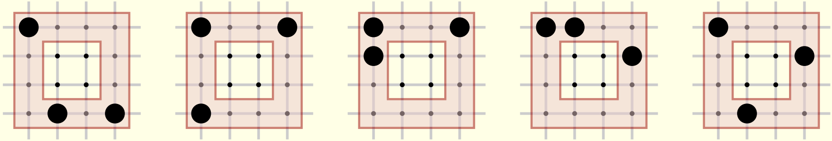

Avec d'autres règles :

$$D(\sqrt{2}, 2-\sqrt{2}) \geq \frac{16}{57} \simeq 0.28$$

Résultats pour les petites valeurs

$r \backslash r+\Delta$	1	$\sqrt{2}$	2	$\sqrt{5}$	$\sqrt{8}$	3	$\sqrt{10}$
1	1	$\frac{1}{2}^{b}$	X	X	X	X	X
$\sqrt{2}$	–	$\frac{2}{9}^{a}$	$[\frac{16}{57}^{c}, \frac{1}{3}]$	X	X	X	X
2	–	–	$[\frac{6}{35}, \frac{5}{29}]^{a}$	$\frac{1}{2}^{b}$	$\frac{1}{2}^{b}$	X	X
$\sqrt{5}$	–	–	–	$\frac{3}{8}^{a}$	$[\frac{17}{100}^{c}, \frac{2}{9}]$	$[\frac{1}{4}^{b}, \frac{1}{3}]$	X
$\sqrt{8}$	–	–	–	–	$\frac{1}{8}^{a}$	$[\frac{1}{7}^{c}, \frac{4}{21}]$	$[\frac{1}{4}^{b}, \frac{3}{8}]$
3	–	–	–	–	–	$[\frac{1}{13.66}^{a}, \frac{1}{6}]^{a}$	$[\frac{1}{6}^{b}, \frac{13}{48}]$

X: Pas de (r, Δ)-code identifiant

a: Anciens résultats

b: Borne inférieure triviale avec le motif horizontal

c: Borne inférieure relevée avec du déchargement

Quelques remarques sur Δ

Pour garantir l'existence : $\mathcal{S}_{r,\Delta}(\bullet\ \bullet) \neq \emptyset$

$$\Delta_m(r) = \sup\{\Delta | \mathcal{S}_{r,\Delta}(\bullet\ \bullet) \neq \emptyset\}$$

On a $\Delta_m(r) \leq 1$ et $\Delta_m(r) = 1$ ssi $r \in \mathbf{N}$, et :

$$\Delta_m(r) \geq 1 - O(\frac{1}{\sqrt{r}})$$

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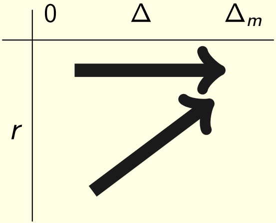
Monotonicité de $D(r, \Delta)$:



Grandes valeurs de r

Borne inférieure : $D(r,\Delta) \geq \frac{1}{|\mathcal{S}_{r,\Delta}(\bullet\ \bullet)|} \rightarrow$ majorer $|\mathcal{S}_{r,\Delta}(\bullet\ \bullet)|$
Pour les grandes valeurs de Δ : $D(r,\Delta) \geq O(\frac{1}{\sqrt{r}})$

Pour les autres valeurs : $D(r,\Delta) \geq D(r,0) \geq \frac{1}{3.22r+4}$

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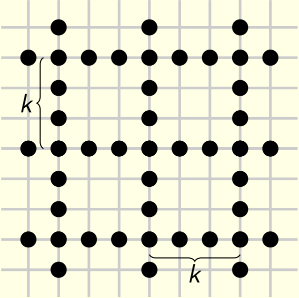
Borne supérieure :

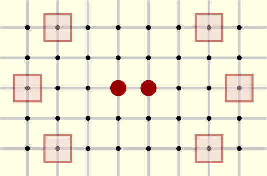


$$\begin{aligned} D(r,\Delta) &\leq \frac{2}{k} \\ D(r,\Delta) &\leq \frac{4}{r(2-\Delta-\sqrt{2-\Delta^2})+C} \end{aligned}$$

On peut faire mieux quand on connait les motifs

Exemple : $(r, \Delta) = (3, \sqrt{10} - 3)$:

Motif horizontal:



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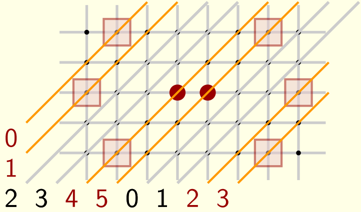

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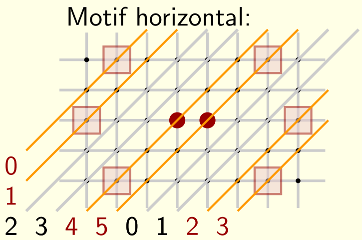

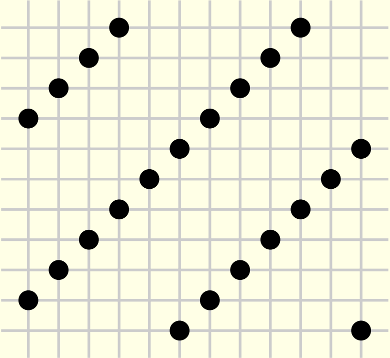

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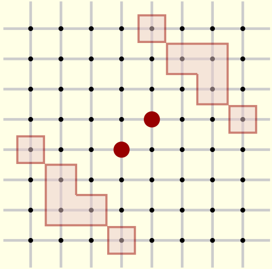
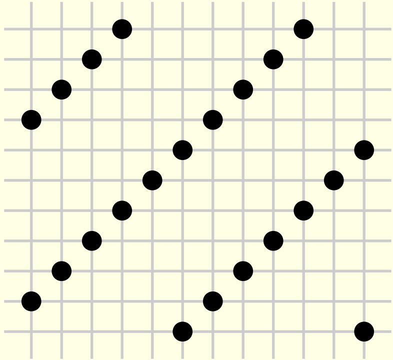
Motif diagonal:





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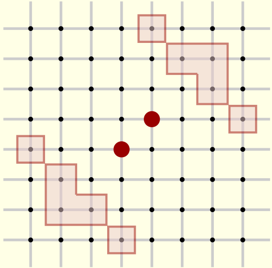

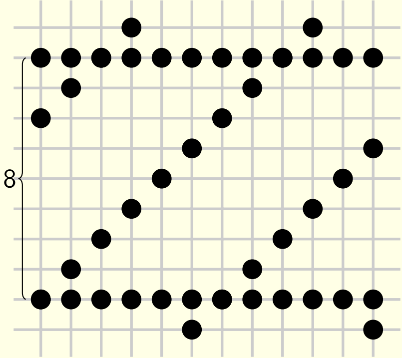

Code de densité $\frac{1}{6} + \frac{1}{8} - \frac{1}{48} = \frac{13}{48}$ (on peut faire $\frac{1}{4}$)

Et encore mieux

Parfois, on peut enlever les lignes horizontales :

- $r = \sqrt{101}$, $r + \Delta = 11$, densité optimale $\frac{1}{4}$
- $r = \sqrt{257}$, $r + \Delta = \sqrt{281}$, densité optimale $\frac{1}{8}$

Pourquoi ?

$\rightarrow$ Exactement d sommets dans $\mathcal{S}_{r,\Delta}(\bullet\ \bullet)$ et toutes les diagonales modulo d.

$\rightarrow$ d diagonales consécutives dans $\mathcal{S}_{r,\Delta}(\bullet\ ^{\bullet})$

$\Rightarrow$ construction de familles infinies pour lesquelles les codes sont optimaux

Exemple : $r = 2k^2 + 1, r + \Delta = \sqrt{r^2 + 2r - 3}$ *avec* $k \not\equiv 0[3]$, *k impair : pour r assez grand, code optimal de densité* $\frac{1}{6}$

C'est fini

Merci !