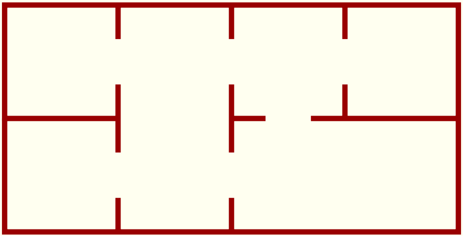
Problèmes d'identification dans les graphes

Aline Parreau

Séminaire DOLPHIN

20 septembre 2012

Fire detection in a museum?



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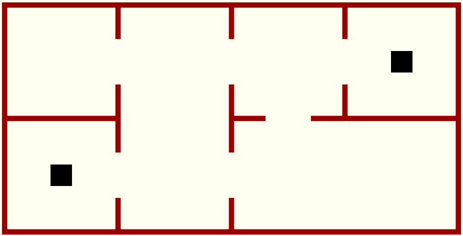

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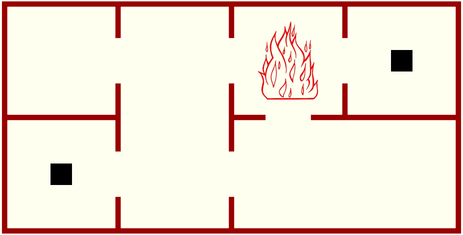

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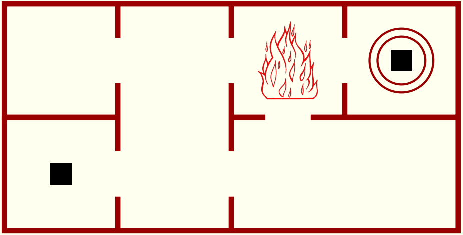

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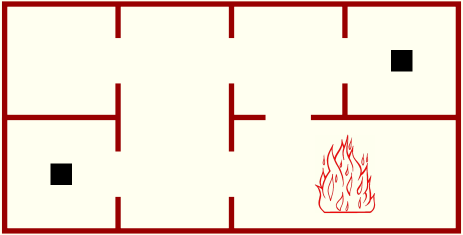

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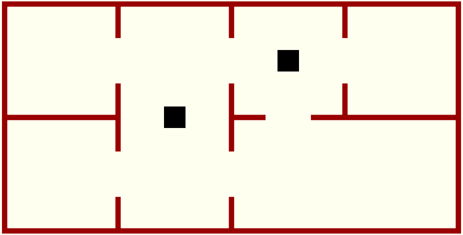

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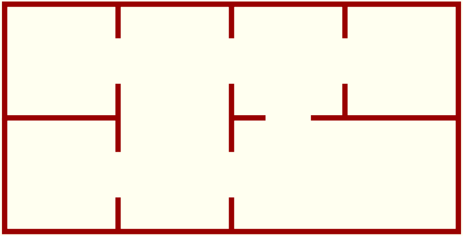
Modelization with a graph



- Vertices V: rooms
- Edges E: between two neighboring rooms

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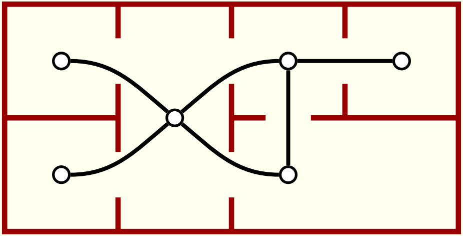

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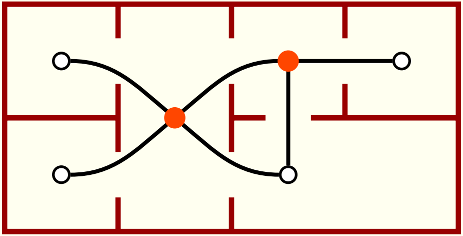

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- Set of detectors = dominating set S:

$$\forall u \in V, N[u] \cap S \neq \emptyset$$

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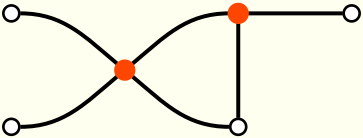

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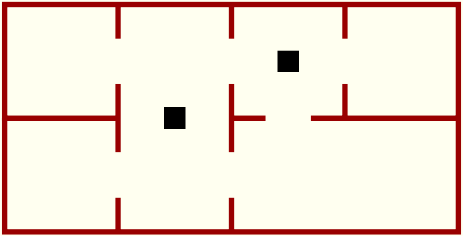

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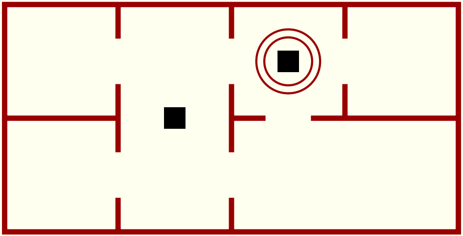

Where is the fire ?

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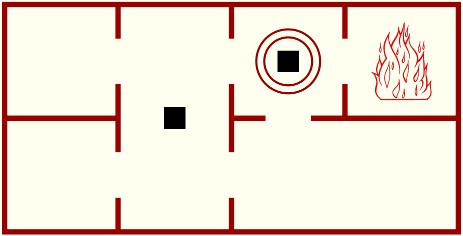

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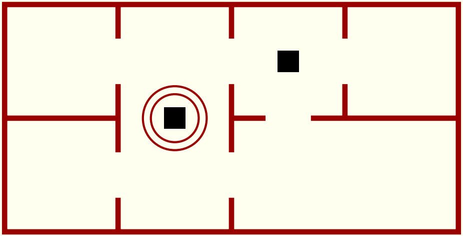

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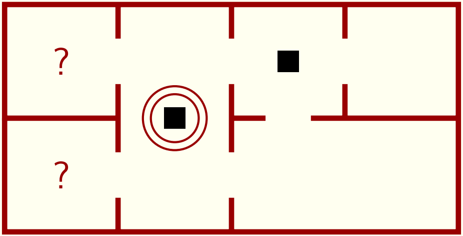
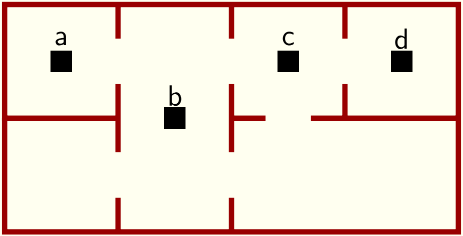


Where is the fire ?

To locate the fire, we need more detectors.

Identifying where is the fire



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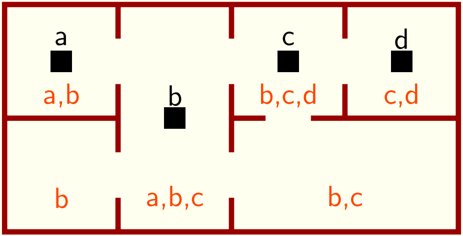
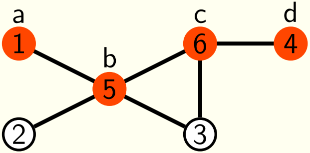


In each room, the set of detectors in the neighborhood is unique.

Modelization with a graph

Identifying code C = subset of vertices of a graph which is

- dominating : $\forall u \in V, N[u] \cap C \neq \emptyset$,
- separating : $\forall u, v \in V, N[u] \cap C \neq N[v] \cap C$.



V \ C	a	b	c	d
1	•	•	-	-
2	-	•	-	-
3	-	•	•	-
4	-	-	•	•
5	•	•	•	-
6	-	•	•	•

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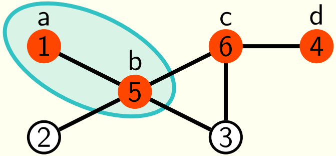

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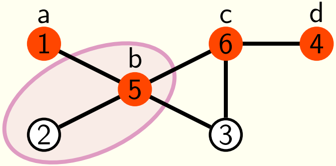

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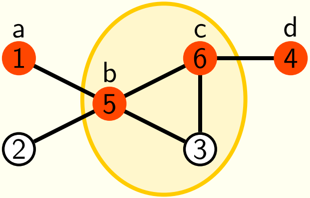

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- Main question:

  Given a graph G, what is the size $\gamma^{ID}(G)$ of minimum identifying code ?

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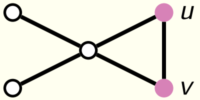
- Existence $\Leftrightarrow$ no twins in the graph:



Twins: $N[u] = N[v]$

A difficult question...

IDENTIFYING CODE : Given a twin-free graph G and an integer k, is there an identifying code of size k in G?

Proposition Charon, Hudry, Lobstein, 2001

IDENTIFYING CODE is NP-complete.

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IDENTIFYING CODE is NP-complete.

- Best polynomial approximation with logarithmic factor
- Polynomial for trees

Outline

1. Bounds and extremal graphs
2. Study in restricted classes of graphs
3. Identifying colorings
4. Some perspectives

Part I

Bounds and extremal graphs

Bounds

$|V|$: number of vertices

$$\log(|V|+1) \leq \gamma^{ID}(G) \leq |V|-1$$

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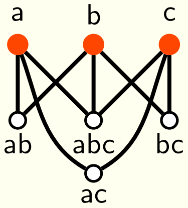
- Tight example:



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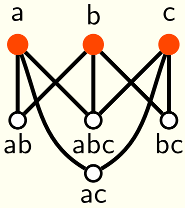

- Complete characterization by Moncel in 2006.

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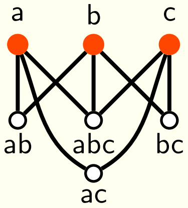

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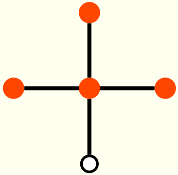
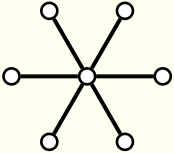
- Bertrand and Gravier, Moncel in 2001.
- Tight example:



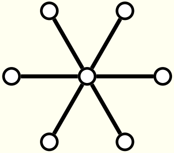
- Complete characterization?

Some tight examples and a conjecture



Stars

Some tight examples and a conjecture



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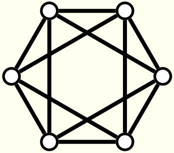

Complete graphs minus maximal matching

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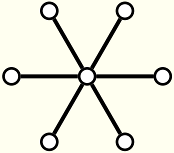

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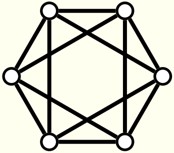


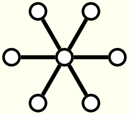
Complete graphs minus maximal matching

Conjecture Charbit, Charon, Cohen, Hudry, Lobstein, 2008

These are the only graphs with $\gamma^{ID} = |V| - 1$.

Characterization of graphs with $\gamma^{ID}(G) = |V| - 1$

(1) Star $K_{1,n}$,



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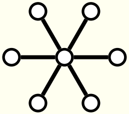

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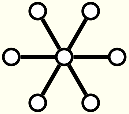

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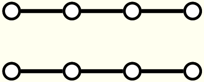

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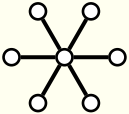

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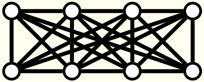

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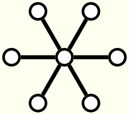

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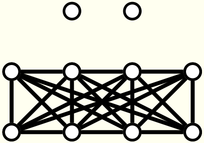

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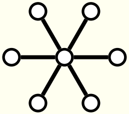

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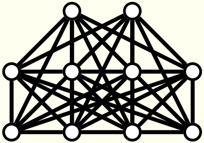

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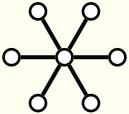

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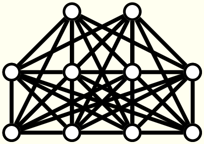

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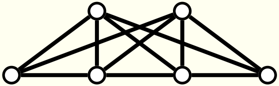

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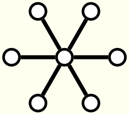

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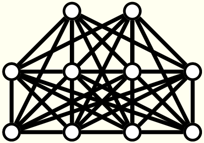

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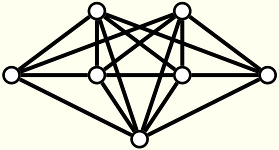

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Theorem Foucaud, Guerrini, Kovše, Naserasr, P., Valicov, 2011

Let G be a connected twin-free graph.

$$\gamma^{ID}(G) = |V| - 1 \Leftrightarrow G \text{ in } (1), (2), (3) \text{ or } (4)$$

Ideas of the proof

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$\Leftarrow$ By induction

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$\Leftarrow$ By induction

$\Rightarrow$ Let G be a minimal counter-example.

- There is $u \in V$ s.t. $G - u$ extremal.
- By minimality, $G - u$ is in (1), (2), (3) or (4).
- We can construct an identifying code of size $|V| - 2$ of G, contradiction.

Consequence

Corollary

If $\gamma^{ID}(G) = |V| - 1$, G has maximum degree $\Delta \geq |V| - 2$.

Consequence

Corollary

If $\gamma^{ID}(G) = |V| - 1$, G has maximum degree $\Delta \geq |V| - 2$.

Upper bound with the maximum degree Δ?

Conjecture Foucaud, Klasing, Kosowski, Raspaud, 2012

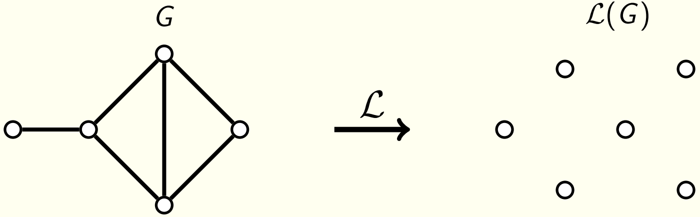
$$\gamma^{ID}(G) \leq |V| - \frac{|V|}{\Delta} + O(1).$$

Part II

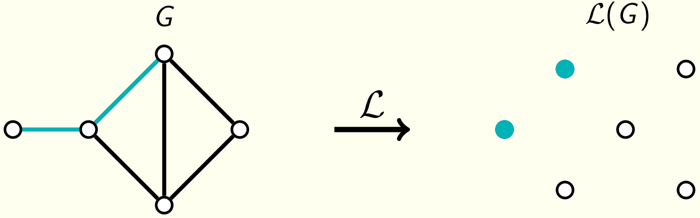
Study in a restricted class of graphs:

Line graphs

Identifying code in line graphs



Identifying code in line graphs



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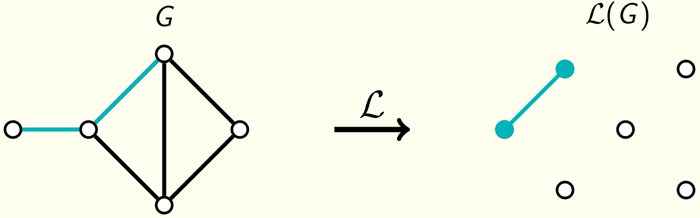

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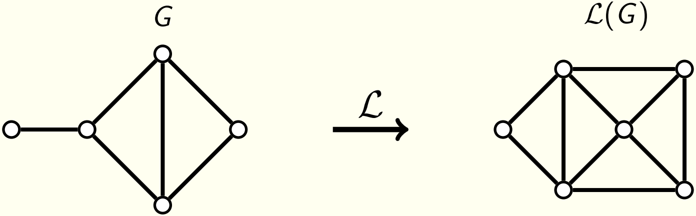

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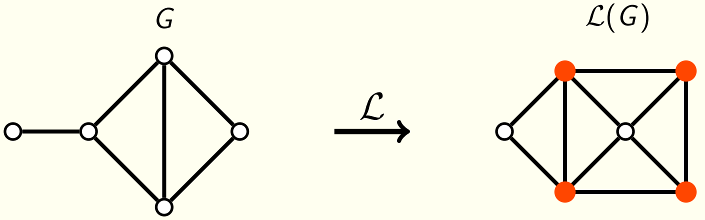
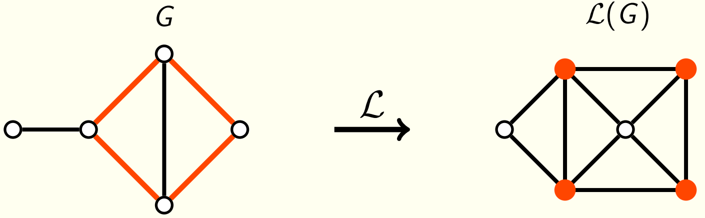


Identifying code

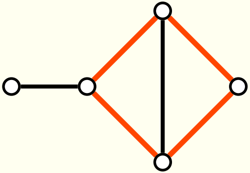
Identifying code in line graphs



Edge identifying code $\longleftrightarrow$ Identifying code

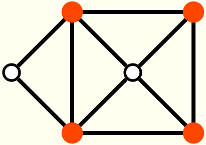
Identifying code in line graphs

G



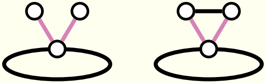
$\xrightarrow{\mathcal{L}}$

$\mathcal{L}(G)$

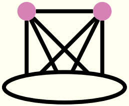


Edge identifying code $\longleftrightarrow$ Identifying code

$\gamma^{EID}(G) = \gamma^{ID}(\mathcal{L}(G))$



$\longleftrightarrow$



Pendant edges

Twins

Still difficult

EDGE-IDCODE : Given G pendant-free and k, $\gamma^{EID}(G) \leq k$?

Theorem Foucaud, Gravier, Naserasr, P., Valicov, 2012

EDGE-IDCODE is NP-complete even for planar subcubic bipartite graphs with large girth.

Reduction from PLANAR ($\leq 3, 3$)-SAT.

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Reduction from PLANAR $(\leq 3, 3)$-SAT.

Corollary

IDENTIFYING CODE is NP-complete even for perfect planar 3-colorable line graphs with maximum degree 4.

Bounds using the number of vertices

Proposition Foucaud, Gravier, Naserasr, P., Valicov, 2012

$$\frac{1}{2}|V(G)| \leq \gamma^{EID}(G) \leq 2|V(G)| - 3$$

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- Lower Bound: a code must cover $\simeq$ half of vertices.
  $\rightarrow$ Tight for hypercubes.

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- Upper Bound: a minimal code is 2-degenerate.
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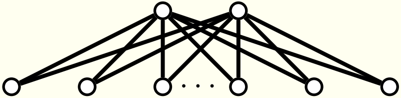
  $\rightarrow$ Infinite family with $\gamma^{EID}(G) = 2|V(G)| - 6$:



Bounds using the number of vertices

Proposition Foucaud, Gravier, Naserasr, P., Valicov, 2012

$$\frac{1}{2}|V(G)| \leq \gamma^{EID}(G) \leq 2|V(G)| - 3$$

Corollary

EDGE-IDCODE has a polynomial 4-approximation.

- Best polynomial approximation for identifying codes in $\log(|V|)$.
  (Laifenbeld, Trachtenberg, Berger-Wolf, 2006 and Gravier, Klasing, Moncel, 2008)

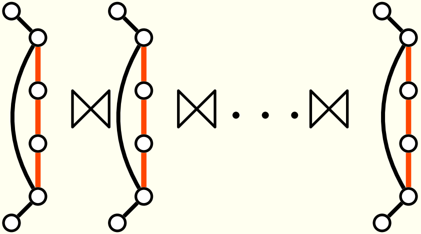
Bounds using the number of edges

Proposition Foucaud, Gravier, Naserasr, P., Valicov, 2012

$$\frac{3}{2\sqrt{2}}\sqrt{|E(G)|} \leq \gamma^{EID}(G) \leq |E(G)| - 1$$

- Upper Bound: from identifying code
- Lower Bound: using the lower bound for vertices

$\rightarrow$ Tight for:



Bounds using the number of edges

Proposition Foucaud, Gravier, Naserasr, P., Valicov, 2012

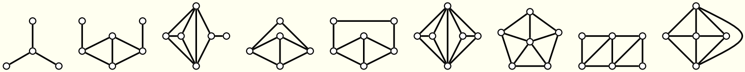
$$\frac{3}{2\sqrt{2}}\sqrt{|E(G)|} \leq \gamma^{EID}(G) \leq |E(G)| - 1$$

Corollary

If G is a line graph, $\gamma^{ID}(G) \geq \Theta(\sqrt{|V|})$

Conclusion for line graphs

- Class of graph for which $\gamma^{ID}(G) \geq \Theta(\sqrt{|V|})$ (instead of $\Theta(\log(|V|))$).
- Defined by forbidden induced subgraphs:



- Is the lower bound still true with less restrictions? For other classes defined by forbidden induced subgraphs?
  - $\rightarrow$ False for claw-free graphs.
  - $\rightarrow$ True for interval graphs.

Part III

A variation of identifying code:

Identifying colorings of graphs

Some variations

- Locating-dominating codes
- Resolving sets
- $(r, \leq \ell)$-identifying codes
- Weak and light codes
- Tolerant identifying codes
- Watching systems
- Discriminating codes
- Adaptative identifying codes
- Locating colorings
- ...

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- Resolving sets
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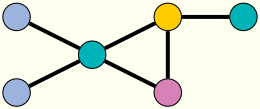
- ...

One more:

Identifying coloring

Proper coloring of graphs

$\rightarrow$ Two adjacent vertices have different colors.

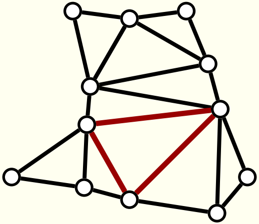


$$\chi(G) = 3$$

Chromatic number $\chi(G)$: minimum number of colors needed

Proper coloring of graphs - a lower bound

Clique number $\omega(G)$: max. number of vertices that induces a complete graph

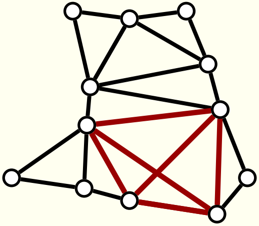


$\omega(G) = 3$

For any graph G, $\chi(G) \geq \omega(G)$

Proper coloring of graphs - a lower bound

Clique number $\omega(G)$: max. number of vertices that induces a complete graph

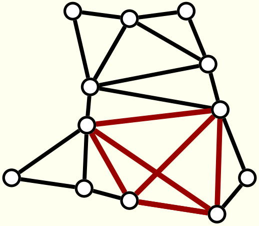


$\omega(G) = 4$

For any graph G, $\chi(G) \geq \omega(G)$

Proper coloring of graphs - a lower bound

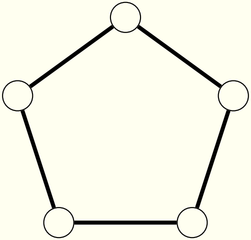
Clique number $\omega(G)$: max. number of vertices that induces a complete graph



$\omega(G) = 4$

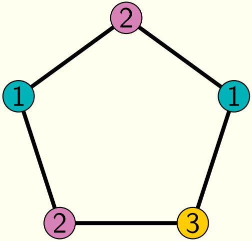
For any graph G, $\chi(G) \geq \omega(G)$

...that is not always reached



$\chi(C_5) = 3$ but $\omega(C_5) = 2$

...that is not always reached



$\chi(C_5) = 3$ but $\omega(C_5) = 2$

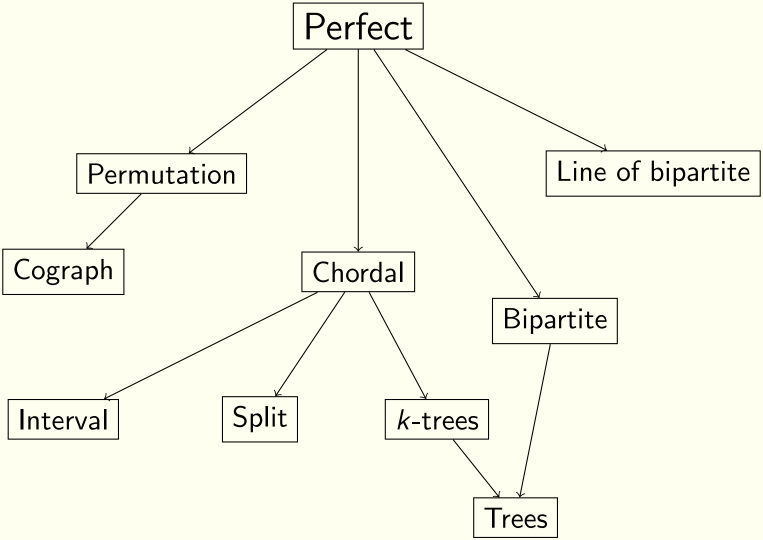
Perfect graphs

Perfect graph (1963): G is *perfect* if $\omega(H) = \chi(H)$ for any induced subgraph H of G

Theorem Strong Perfect Graph Theorem (Chudnovsky *et al.* 2002)

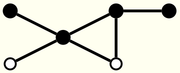
G is perfect if and only if it has no induced odd cycle or complement of odd cycle with more than 4 vertices

A part of the big family of perfect graphs

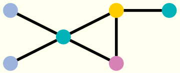


Identification with colors

Identifying codes



Proper graph colorings



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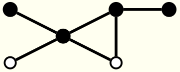

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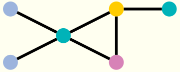
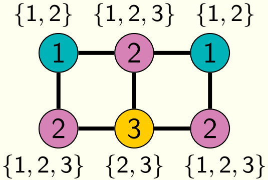


Identifying colorings

Locally identifying coloring

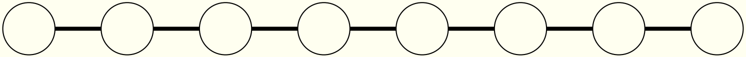
- Proper vertex coloring $c : V \to \mathbb{N}$
- local identification by the colors in the neighborhood: $c(N[x])$



$$c(N[x]) \neq c(N[y]) \text{ for } xy \in E$$

- $\chi_{lid}(G)$: min. number of colors in a lid-coloring of G.

An example: the path



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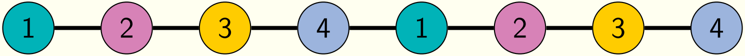

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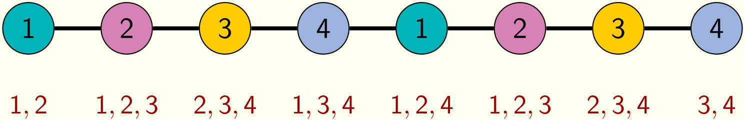

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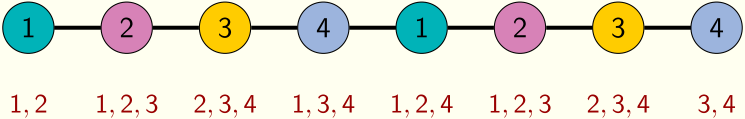
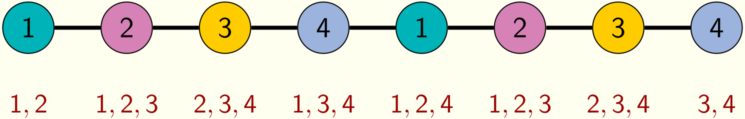


$$\chi_{lid}(P_k) \leq 4$$

An example: the path



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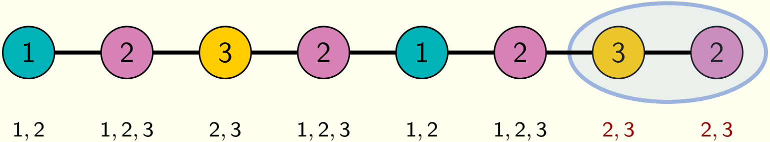
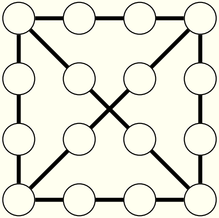
With 3 colors :



$\chi_{lid}(P_k) = 3$ iff k is odd.

Link with chromatic number

- A lid-coloring is a proper coloring: $\chi_{lid} \geq \chi$.
- No upper bound with χ.
  $\rightarrow$ complete graph K_k subdivided twice: $\chi_{lid} = k$, $\chi = 3$



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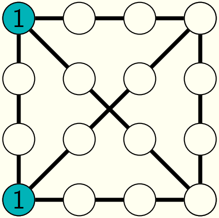

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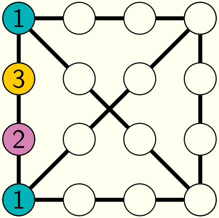

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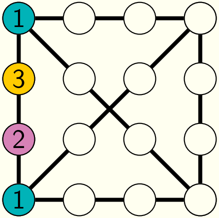
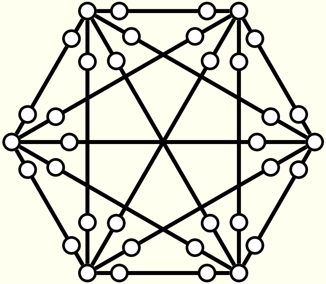
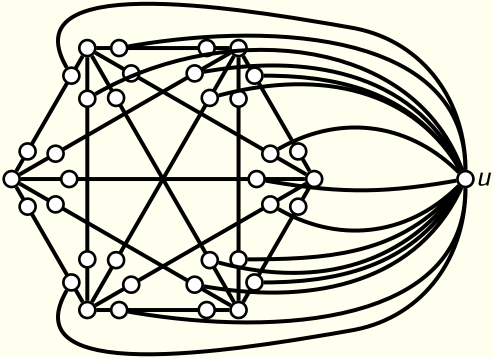


- Not monotone: $\chi_{lid}(P_5) \leq \chi_{lid}(P_4)$

χ_{lid} is not monotone at all

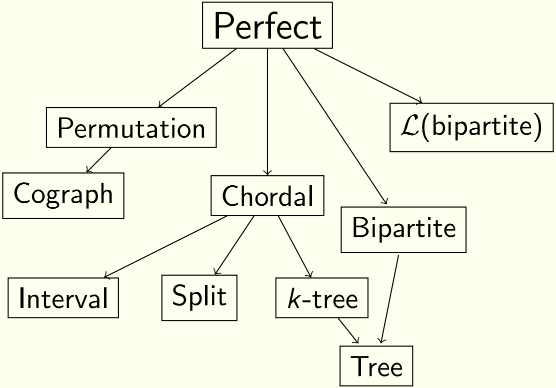


χ_{lid} is not monotone at all

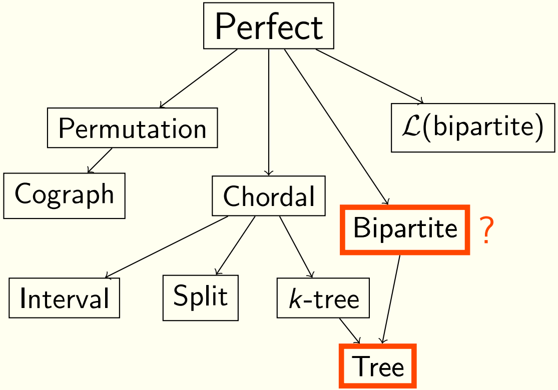


$$\chi_{lid}(G) = 5 \ll k = \chi_{lid}(G - u)$$

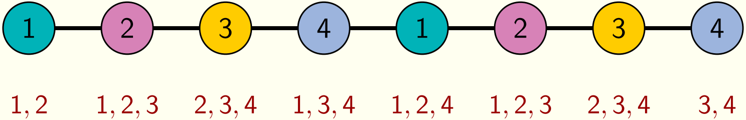
Study in perfect graphs



Study in perfect graphs

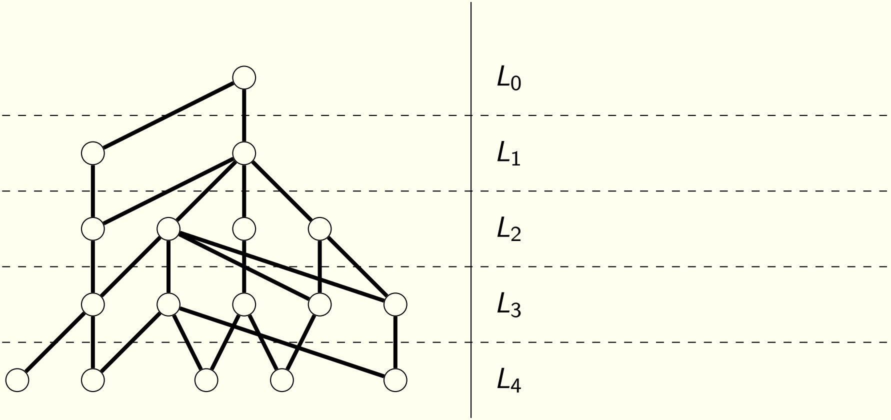


Bipartite graphs: the path



$$\chi_{lid}(P_k) \leq 4$$

Bipartite graphs are 4-lid-colorable



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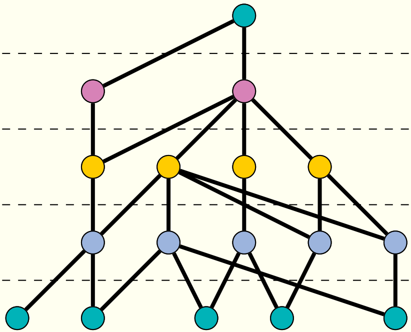
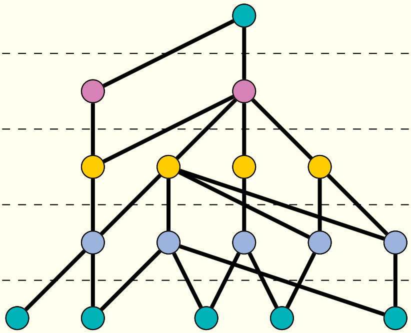


L_0	$\rightarrow$	1	$\{1,2\}$
L_1	$\rightarrow$	2	$\{1,2,3\}$
L_2	$\rightarrow$	3	$\{2,3,4\}$ or $\{2,3\}$
L_3	$\rightarrow$	4	$\{1,3,4\}$ or $\{3,4\}$
L_4	$\rightarrow$	1	$\{1,4\}$

Bipartite graphs are 4-lid-colorable



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L_3	$\rightarrow$	4	$\{1, 3, 4\}$ or $\{3, 4\}$
L_4	$\rightarrow$	1	$\{1, 4\}$

If G is bipartite, $\chi_{lid}(G) \leq 4$.

Bipartite graphs

General bounds: $3 \leq \chi_{lid}(B) \leq 4$.

Bipartite graphs

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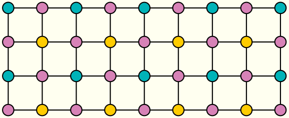
$\chi_{lid}(B) = 3$:



Bipartite graphs

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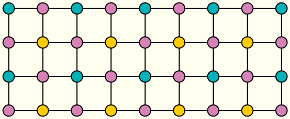

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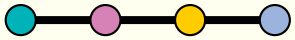

Bipartite graphs

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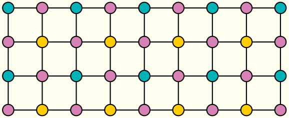
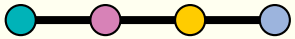


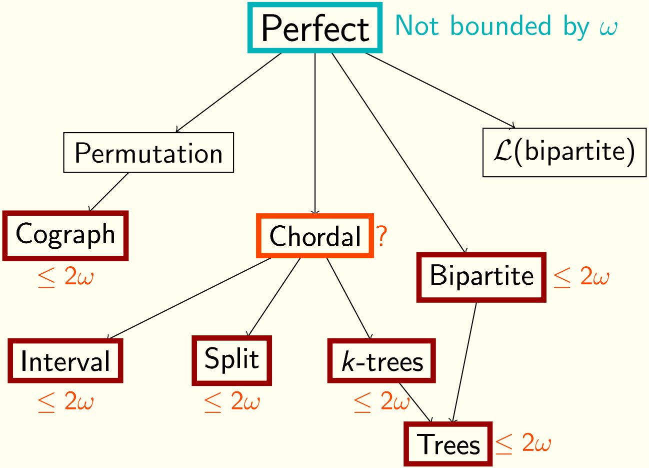
$\leftarrow ? \rightarrow$

$\chi_{lid}(B) = 4$:

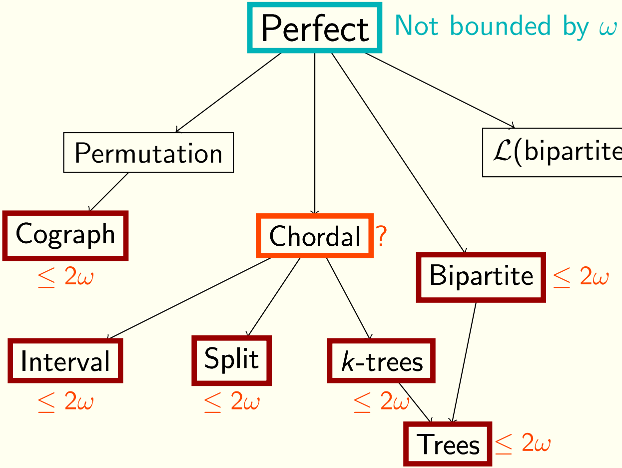


In general... 3-LID-COLORING is NP-complete in bipartite graphs

Perfect graphs - results and conjecture



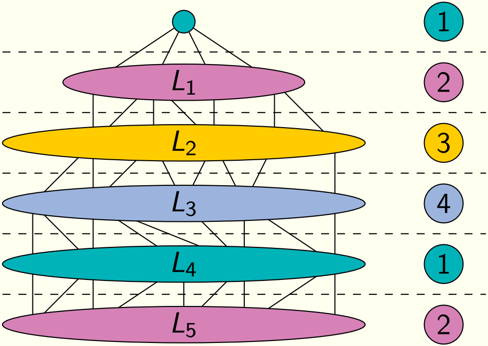
Perfect graphs - results and conjecture



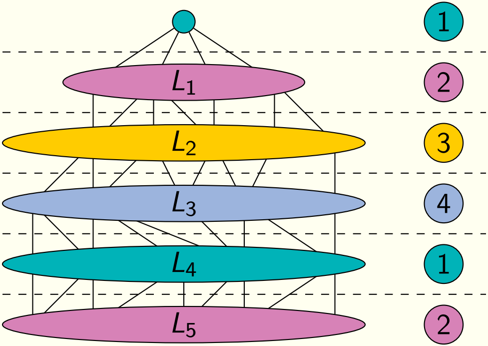
Conjecture Esperet, Gravier, Montassier, Ochem, P., 2012

Any chordal graph G has a lid-coloring with $2\omega(G)$ colors.

A good method for coloring

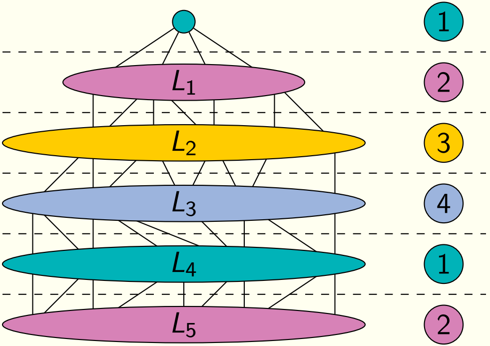


A good method for coloring



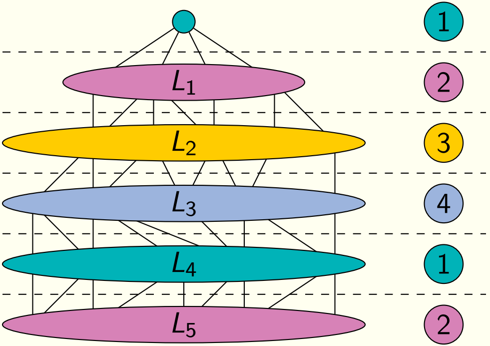
- Outerplanar graphs: L_i = union of paths, 5 colors
  $\rightarrow 4 \times 5 = 20$ colors

A good method for coloring



- Outerplanar graphs: L_i = union of paths, 5 colors
  $\rightarrow 4 \times 5 = 20$ colors
- Planar graphs: L_i = outerplanar, 20 colors and 16 more colors
  $\rightarrow 4 \times 20 \times 16 = 1280$ colors (Gonçalves, P., Pinlou, 2012)

A good method for coloring



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- Same idea for K_k-minor free graphs (Gonçalves, P., Pinlou, 2012)

Part IV

Perspectives

Open questions

- Bounds and extremal graphs
  $\rightarrow$ Conjecture $\gamma^{ID}(G) \leq n - \frac{n}{O(\Delta)}$
- Study in restricted classes of graphs
  $\rightarrow$ Other classes with $\gamma^{ID}(G) \geq \Theta(\sqrt{|V|})$?
  $\rightarrow$ Better approximation for line graphs ?
- Identifying colorings
  $\rightarrow$ Better bound for planar graphs (between 8 and 1280...)
  $\rightarrow$ Conjecture $\chi_{lid} \leq 2\omega$ for chordal graphs
- Generalization to hypergraph

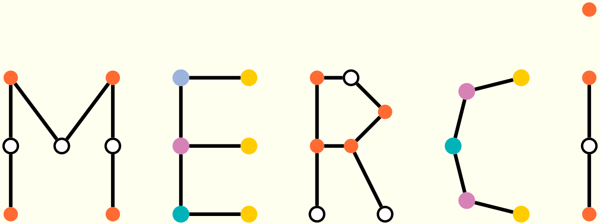
A new approach with integer linear programming?

Identifying code problem is equivalent to the following problem :

$$
\begin{array}{lllr}
\min & \sum_{u \in V} x_u & & \\
\text{s.t} & \sum_{u \in N[v]} x_u \geq 1 & \forall v \in V & \text{(domination)} \\
 & \sum_{u \in N[v] \Delta N[v']} x_u \geq 1 & \forall v \neq v' \in V^2 & \text{(separation)} \\
 & x_u \in \{0, 1\} & &
\end{array}
$$

$\rightarrow$ Subproblem of hitting set, covering set problems

$\rightarrow$ New lower bounds, approximations, polynomial algorithm?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