Possibilistic Logic

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Overview

- motivation
- possibilistic logic
- extension of possibilistic logic
- application

Part I

Introduction

from fuzzy sets to possibilistic logic

- fuzzy set F on set U defined by membership function $\mu_F : U \rightarrow [0,1]$
- mathematical tool describing model people use when reasoning about systems
- "John is tall" modelled as a fuzzy set F of with a universe U of sizes,
- $\mu_F(x)$ with $x \in U$ expresses membership of x to fuzzy set of tall sizes (x is known)
- attention: possibilistic logic $\neq$ classical fuzzy logic (multi-valued logic)
- *but*: fuzzy restriction possible truth values of a formula

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Possibility theory

- Let x be a variable in U
- possibility distribution $\pi_x : U \rightarrow [0, 1]$ attached to x
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  - $\pi_x(u) = 0$ means $x = u$ is impossible
  - $\pi_x(u) = 1$ means $x = u$ is completely allowed
  - $\pi_x(u) > \pi_x(u')$ means $x = u$ is preferred to $x = u'$
- normalization requirement: $\exists u \in U \pi_x(u) = 1$ (at least one value of x is completely allowed)
- $\pi_x < \pi'_x$ means π_x is more specific than π'_x (more informative)
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Two measures deriving from possibilty distribution π_x

- possibility measure $\Pi(A) = sup_{u \in A} \pi_x(u)$ (extent of value $a \in A$ that stands as a value for x)
- necessity measure $N(A) = 1 - \Pi(\bar{A})$ (extent all possible values of x belong to A)
- measures allowing modelling uncertainty-qualified statements: ex. $N(A) \geq \alpha$ means x is A is at least α-certain

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- later general possibilistic logic (possibility qualified statements $\Pi(\phi) \geq \alpha$)

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Syntax

Definition

A *necessity-valued formula is* $(\phi\alpha)$, *where* ϕ *classical first-order formula,* $\alpha \in (0,1]$, *thus* $N(\phi) \geq \alpha$, *only conjunctions are allowed*
A necessity-valued knowledge base*: finite set of necessity valued formulas*

Little repetition: semantics in classical logic

- $F = \{\phi_1, \ldots, \phi_n\}$ a set of formulas
- Ω be a set of interpretations,
- $M(F)$: set of all models of F ($\forall I \in M, \forall f \in F : I \models f$)
- F induces a partition of Ω into two subsets: $M(F)$ and $M(\neg F)$

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Link to necessity-valued logic

- valuations can be intermediary
- Let $\mathcal{F} = \{(\phi_1 \alpha_1), \ldots, (\phi_n \alpha_n)\}$ a set of necessity valued formulas
- $\pi_{\mathcal{F}}(\omega) = min\{1 - \alpha_i | \omega \models \neg\phi_i, i = 1, \ldots, n\}$
- $\pi_{\mathcal{F}}$ is the least specific possibility distribution (principle of minimal specifity)
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Best models

Definition

The interpretations $\omega^\star$ maximizing $\pi_{\mathcal{F}}$ is called *best* models of $\mathcal{F}$.

Most compatible with $\mathcal{F}$ among the set of all interpretations Ω.
(membership degree of fuzzy set of models of $\mathcal{F}$)

Deduction problem

Let $\mathcal{F}$ be a set of possibilistic formulas, ϕ a classical formula, then

$$\mathcal{F} \models (\phi\alpha) \leftrightarrow \pi_{\mathcal{F}} \models (\phi\alpha)$$

Knowing $\pi_{\mathcal{F}}$ sufficient for any deduction

Part II

Formal system and Deduction

Axioms schemata

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A1 $(\varphi \rightarrow (\psi \rightarrow \varphi)\ 1)$

A2 $((\varphi \rightarrow (\psi \rightarrow \xi)) \rightarrow ((\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \xi))\ 1)$

A3 $((\neg\varphi \rightarrow \neg\psi) \rightarrow ((\neg\varphi \rightarrow \psi) \rightarrow \varphi)\ 1)$

A4 $((\forall x\,(\varphi \rightarrow \psi)) \rightarrow (\varphi \rightarrow (\forall x\,\psi))\ 1)$
if x does not appear in φ and is not bound in ψ

A5 $((\forall x\,\varphi) \rightarrow \psi_{x|t}\ 1)$ if x is free for t in φ

The axioms of the Hilbert formal system for classical logic weighted by 1.

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Inference rules

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- $\dfrac{(\varphi\ \ \alpha),(\varphi\rightarrow\psi\ \ \beta)}{\vdash(\psi\ \ \min(\alpha,\beta))}$
- $\dfrac{(\varphi\ \ \alpha)}{\vdash((\forall x\ \varphi)\ \ \alpha)}$ if x is not bound in φ
- $\dfrac{(\varphi\ \ \alpha)}{\vdash(\varphi\ \ \beta)}$ if $\beta\leq\alpha$

A formal system for necessity-valued logic

The proposed system is sound and complete with respect to the inconsistency-tolerant semantics of possibilistic logic :

Theorem

$$\mathcal{F} \models (\varphi\ \alpha) \Leftrightarrow \frac{\mathcal{F}}{\vdash (\varphi\ \alpha)}$$

Two automated deduction methods

Two well-known automated deduction methods:

- Resolution (in a second...)
- The Davis and Putman semantic evaluation procedure

Resolution

Resolution rule

$\dfrac{(c_1\ \ \alpha_1),(c_2\ \ \alpha_2)}{\vdash (R(c_1,c_2)\ \ \min(\alpha_1,\alpha_2))}$ where R is any classical resolvent of c_1 and c_2

Soundness of the resolution rule

Let $\mathcal{C}$ be a set of possibilistic clauses, and $C = (c\ \ \alpha)$ a possibilistic clause obtained by a finite number of successive applications of the resolution rule to $\mathcal{C}$; then $\mathcal{C} \models C$

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Refutation by resolution

Given a set $\mathcal{F}$ of possibilistic formulas and a classical formula φ, $\mathrm{Val}(\varphi, \mathcal{F}) = \sup\{\alpha \in (0,1] | \mathcal{F} \models (\varphi\ \alpha)\}$ can be computed as follows:

Refutation by resolution

1. Put $\mathcal{F}$ into clausal form $\mathcal{C}$
2. Put φ into clausal form; let $c_1, \ldots, c_m$ the obtained clauses
3. $\mathcal{C}' \leftarrow \mathcal{C} \cup \{(\neg c_1\ 1), \ldots, (\neg c_n\ 1)\}$
4. Search for a deduction of $(\perp\ \bar{\alpha})$ by applying repeatedly the resolution rule from $\mathcal{C}'$, with $\bar{\alpha}$ maximal
5. $\mathrm{Val}(\varphi, \mathcal{F}) \leftarrow \bar{\alpha}$

Super Senior!

The knowledge base $\mathcal{F}$

Φ_1 ((Eliminated(Loretta) $\vee$ Eliminated(Georgette)) $\wedge$ ($\neg$ Eliminated(Loretta) $\vee$ Eliminated(Georgette)) 1)

Φ_2 ($\forall x \neg$ Snores(x) $\vee$ Eliminated(x) 0.5)

Φ_3 (Snores(Loretta) 1)

Φ_4 ($\forall x \neg$ LosesHerDentures(x) $\vee$ Eliminated(x) 0.6)

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Φ_6 ($\forall x \neg$ SwimsWith(x, Jean-Edouard) $\vee \neg$ Eliminated(x) 0.7)

The burning question

How certain will Loretta be eliminated?

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$\mathcal{C}$ is completely consistent: $\text{Incons}(\mathcal{C}) = 0$

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Let's search for a deduction of $(\bot\ \bar{\alpha})$ by applying repeatedly the resolution rule from $\mathcal{C} \cup \{(\neg$ Eliminated(Loretta) 1)$\}$, with $\bar{\alpha}$ maximal.

Hot news: Loretta swims with Jean-Edouard!

The new set of possibilistic clauses $\mathcal{C}'$

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$\mathcal{C}'$ is inconsistent: Incons($\mathcal{C}'$) $= 0.5$

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A few remarks

- Some implementation finds out the refutation with $\bar{\alpha}$ maximal first
- A qualitative possibilistic knowledge base is enough
- Possibilistic logic is linked to the belief revision theory (since possibilistic logic is non-monotonic)
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Part III

Generalization of possibilstic logic

We need to handle:

- **possibility-qualified** sentences

  i.e: "It is **possible** *that John comes."*

- **conditional** sentences

  i.e: "The **later** *John arrives, the more certain the meeting will not be quiet."*

- sentences involving **vague predicates**

  i.e: "If the temperature is **high enough**, *there will be only few participants."*

We had **two kinds** of formulas:

- **necessity-valued formulas**
  - denoted $(\varphi\ (\mathrm{N}\ \alpha))$
  - expressing that $\mathrm{N}(\varphi) \geq \alpha$
- **possibility-valued formulas**
  - denoted $(\varphi\ (\Pi\ \beta))$
  - expressing that $\Pi(\varphi) \geq \alpha$

Then we **extend the language** to make it handle both.
Thus, a **knowledge base** $\mathcal{F}$ (a set of formulas) can combine both forms.

$$i.e:\ \mathcal{F} = \{(p\ (\mathrm{N}\ 0.7)),\ (\neg p \vee q\ (\Pi\ 0.8))\}$$

Knowing the **relation between the two forms**, we can extend simply all the previous notions.

> *Relation:* $\Pi(\varphi) = 1 - \mathrm{N}(\neg\varphi)$
> *"The less an event is possible, the more it is sure that it will not happen."*

Thus, we can:

- express knowledge about **ignorance** from **what we know**
- express **inconcistancy** in a better way
- ...

Previously the valuation was either:

- constant,

  as in $(\varphi\ (\Pi\ w))$ *(w is constant)*

- depending on a quantifier (either $\forall$ or $\exists$),

  as in $(\exists x)(\varphi\ (\Pi\ w))$

Then, the new notion is: "qualified possibilistic logic with **variable valuations**":

i.e: $(\varphi(x)\ (\Pi\ w(x)))$

This boils down to have "**dependencies** between **distributions**".

The choice of a **unit interval** is **not compulsory**:
We just need a **partially ordered set**.
i.e:

- a finit chain of symbolic certainty level (i.e: certain, quite sure, likely, likely not, impossible)
- periods of time (i.e: this morning, this afternoon, this evening, this night)
- ...

Using back [0,1], we can **change the axiom**:

$$\Pi(\varphi \wedge \psi) = \min(\Pi(\varphi), \Pi(\psi))$$

to something else that **keeps the four extrema points**.

For instance, we can use:

- $\Pi(\varphi \wedge \psi) = \max(0, \Pi(\varphi) + \Pi(\psi) - 1)$
- $\Pi(\varphi \wedge \psi) = \Pi(\varphi)\Pi(\psi)$
- ...

All these **generalisations** allow to handle more accurately **real problems** because we can now take into account all the precise aspects of one particular real problem.

Part IV

Applications

Part V

Applications

ATMS

ATMS - Part 1/2

ATMS = Assumption-based Truth-Maintenance System

Possibilistic ATMS can answering the following questions :

1. Under what configuration of assumption is the proposition p certain to the degree α ?
2. What is the inconsistency degree of a given configuration of assumption ?
3. In a given configuration of assumptions, to what degree is each proposition certain ?

ATMS

ATMS - Part 2/2

In a possibilistic ATMS, each piece of information is represented by a propositional clause, which enables :

1. A uniform representation for all pieces of knowledge,
2. The capability of handling negated assumptions as assumptions,
3. A simple and uniform algorithm for the computation of labels and nogoods.

Usage : Smart help system, Planification, etc.

Discrete optimisation

Possibility and necessity degrees can be interpreted in the scope of constraint-based reasoning instead of linked to the partial absence of information. In this framework :

1. Tautologies are imperatives : $N(\top)(= \Pi(\top)) = 1$;
2. Contradictions are tolerated : $(\varphi(N\alpha)) \wedge (\neg\varphi(N1)) \vdash (\perp(N\alpha))$;
3. Violating one of two constraints can be allowed while preserving a level of feasibility at most equal to $1 - min(\alpha, \beta)$;
4. The possibility distribution $\pi_{\mathcal{F}}$ induced by a set of N-Valued constraints represents the fuzzy feasibility domain, subnormalization indicating that some constraints which are not fully imperative must be violated.

Negotiating agents

What is a negotiation ?

In a multi-agent system, every agent has his own set of goals, and his own knowledge, but he needs to share informations and resources with the others. Sometimes, two agents need to find a compromise.
For instance lets imagine a buyer and a seller discussing the prices of an object.

Possibilistic logic as a negotiation model

Two ways to find a compromise

- An external agent or program find the best compromise, for instance by using heuristics.
- or the two agents have a "real" discussion and negotiate.

We want a real negotiation

We want to "simulate" the real process of negotiation. That means that each agent has to convince the other one that his solution is the best for them : the elements of the knowledge and the priority of the goals are not fixed.
With possibilistic logic we can represent this unstable mental state of an agent ; degree of belief about his knowledge and priority of his goals.

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Mental state of an agent

Let's consider a negotiation between only two agents. Every agent's mental state can be defined as three possibilistics bases, containing uncertain values or priorities :

Possibilistic bases

- a static set of goals : $\mathcal{K} = \{(k_i, \alpha_i)$, i = 1,...,n$\}$,
- a knowledge base : $\mathcal{G} = \{(g_i, \beta_i)$, i = 1,...,m$\}$,
- a set of goals for the other agent : $\mathcal{GO} = \{(go_i, \delta_i)$, i = 1,...,p$\}$.

A possibility distribution is associated with each base : $\pi_{\mathcal{K}}$, $\pi_{\mathcal{G}}$, $\pi_{\mathcal{GO}}$. X is the set of all the offers that can be made by an agent. For instance, $x \in X$ it is the price of the object.

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The acts of the negotiation

During the negotiation the agents play a dialog composed by actions. Each action is chosen regarding the possibilistic model of beliefs (possibilistic knowledge) and goals.

Set of possible actions (1/2)

Basic actions :

- Offer(x) : the agent makes an offer of value $x \in X$,
- Defy(x) : the agent asks for an explanation of the offer,
- Argue(S) : the agent gives a new set of knowledge to explain his last offer,
- GiveUp() : the agent stops negotiating.

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The acts of the negotiation

Set of possible actions (2/2)

And answers :

- Accept(x) : the offer x is accepted, the negotiation ends,
- Accept(S) : the knowledge set S is accepted,
- Refuse(x) : the agent considers the offer x as unacceptable.

Every negotiation terminates. And this termination is an Accept(x) or a GiveUp() action.

An example of negociation...

What they want, what they believe...

- Peter wants a country not expensive, possibly sunny.
- Mary wants a country sunny, not very hot and not expensive
- Peter believes that Tunisia is not expensive and that Italy is expensive.
- Mary believes that Tunisia is sunny and that Italy is sunny, not hot, and should not be expensive.

An example...

Dialog between Peter and Mary

Mary : I suggest Italy. $((Offer(Italy))$

Peter : Why do you prefer Italy ? $(Defy(Italy))$

Mary : Because Italy is sunny, not very hot not very expensive.
$(Argue(\{Sunny(Italy), \neg Hot(Italy), \neg Expensive(Tunisie)\})$

Peter : No, Italy is expensive, Tunisia is not expensive.
$(Argue(\{Expensive(Italy), \neg Expensive(Tunisie)\})$

Mary : I didn't know !
$(Accept(\{Expensive(Italy), \neg Expensive(Tunisie)\})$

Peter : What do you think about Tunisia ?
$(Offer(Tunisie))$

Mary : OK ! $(Accept(Tunisie))$

Part VI

To conclude

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- Possibilistic logic is a logic of uncertainty.
- It takes into account incomplete evidences an partially inconsistent knowledge.
- Possibilistic logic is different from fuzzy or probabilistic logic.
- Possibilistic logic is adapted to human reasoning.
- It is an useful tool for artificial life.