Algorithmique pour l'analyse et la modélisation en géométrie discrète

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5 décembre 2007

Short Bio and Educational Activities

Education

- ♢ Since December 2003 Chargé de Recherche CNRS, Laboratoire LIRIS, UMR 5205.
- ♢ Sept. 2003 - Dec . 2003 Attaché Temporaire d'enseignement et de Recherche à l'Institut National des Sciences Appliquées de Lyon (INSA), laboratoire de rattachement LIRIS UMR 5205.
- ♢ March 2003 - June 2003 Post Doctorat, Laboratoire LIS, Université Joseph Fourier, Grenoble
- ♢ Sept. 2000 - Dec. 2002 Doctorat d'université, Spécialité Informatique, Université Lumière Lyon 2
- ♢ Sept. 1997 - Sept. 2000 Magistère Informatique et Modélisation, Ecole Normale Supérieure de Lyon, Université Claude Bernard Lyon 1
- ♢ Sept. 1995 - Sept. 1997 DEUG MIAS, Université Claude Bernard Lyon 1

Lectures

	Filière	Matière	Type	Vol.
07–08	Master 1ere année, UCBL	Responsable UE “TER”	-	-
06–07	Master 2ième année, UCBL	Cours de spécialité recherche	TD	6h
05–06	Master 2ième année, UCBL 3ième année école d'ingénieurs IEG, Grenoble	Cours de spécialité recherche Cours de spécialité recherche	CM CM	10h 8h
04–05	Master 2ième année, UCBL 3ième année école d'ingénieurs IEG, Grenoble	Cours de spécialité recherche Cours de spécialité recherche	CM CM	10h 8h
03–03	ATER (demi-poste) département informatique, INSA, Lyon	Programmation C++	TP	80h
00–03	Moniteur à l'Institut de la Communication, Lyon2	-	TD	192h

Scientific Responsibilities

International

- Chair of the Technical Committee 18 “Discrete Geometry”, IAPR (06-08)
- Co-General Chair of the 14th International Conference on Discrete Geometry for Computer Imagery
- Reviewer for several international journals and conferences

National

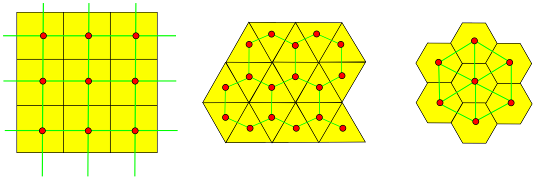
- Organizer and co-organizer of several meetings (JIG 2006, Working Group “Géométrie Discrète”)
- Chair of the ACI “Jeunes Chercheurs” GeoDiGit
- Member of the ANR GeoDiB

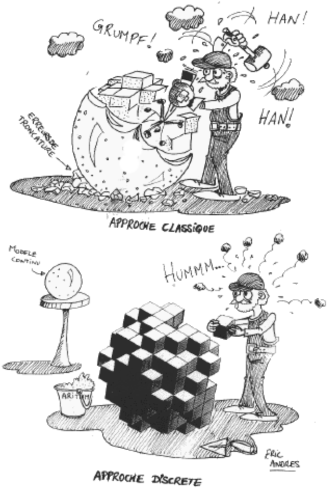
Local

- Member of the *Conseil Scientifique*, Université Claude Bernard Lyon 1
- Member of the *CSES 26-27-61*, Université Lumière Lyon 2

Discrete Geometry in one slide

Analysis of geometrical problems on objects defined on regular grids





[Eric Andres 95]

Motivations

Pragmatic approach *(data driven)*

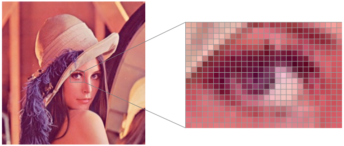
- Data produced by acquisition devices which consider an underlying grid (CDD, Scanner, Scanner+T, . . .)
- Modeling of numerical problems on grids/integer numbers
- [BERNOUILLI, ROSENFELD, . . .]

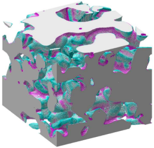
⇒ *Arithmetization*

Constructive approach *(model driven)*

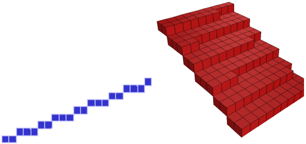
- Construction from scratch of a geometry based on integer numbers
- *e.g.* Theory based on the Non-Standard Analysis
- [HARTONG, REEB, REVEILLES, . . .]

⇒ *Modeling*



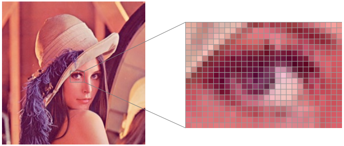


[Météo-France/ESRF]



⇒ Complete Geometrical Paradigm (objects, axioms, ALGORITHMS,. . .) well-adapted to image analysis

Motivations



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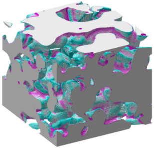

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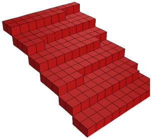

⇒ Complete Geometrical Paradigm (objects, axioms, ALGORITHMS,. . .) well-adapted to image analysis

Discrete Geometry Model

[Continuous world]

↓

Discrete Analytic Models

Grid definitions, digitization schemes, fundamental algebraic and arithmetic facts, . . .

Fundamental Objects and Properties

Points, straight lines, planes, simplexes, circles, intersection, parallelism, . . .

↻

Objects Analysis/Modeling

Feature extraction, distance transformation, . . .

Loss-less Model Conversion

Reversible reconstruction with linear structures, volumic encoding with unions of balls, . . .

↓

[Polygons, Meshes, Poly-ball structures]

ALGORITHMIC POINT OF VIEW

Discrete Geometry Model

[Continuous world]

↓

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[Polygons, Meshes, Poly-ball structures]

ALGORITHMIC POINT OF VIEW

Contribution Overview

Discrete Analytic Models

Supercover & Interval Arithmetic, Irregular isothetic grid formulation

Fundamental Objects and Properties

Computational analysis of fundamental object recognition algorithms

Objects Analysis/Modeling

Separable techniques for the EDT and REDT problems

Loss-less Model Conversion

MA extraction algorithms and NP-completeness of the Minimal MA, Reversible reconstruction on irregular grids, Reversible surface reconstruction

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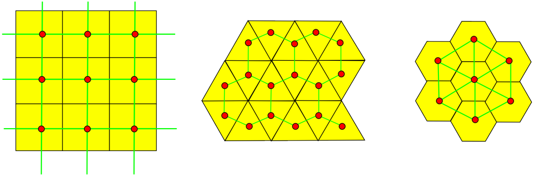
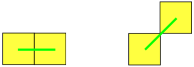
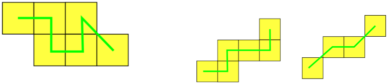
Regular Grids and Basic Topological Principles



- Mapping on $\mathbb{Z}^2$
- Easy access to neighbors

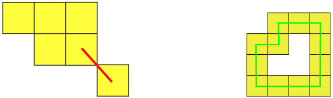


- $(1)-$adjacency
- $(0)-$adjacency



- $(r)-$path
- $(r)-$curve

$r \in \{0, 1\}$

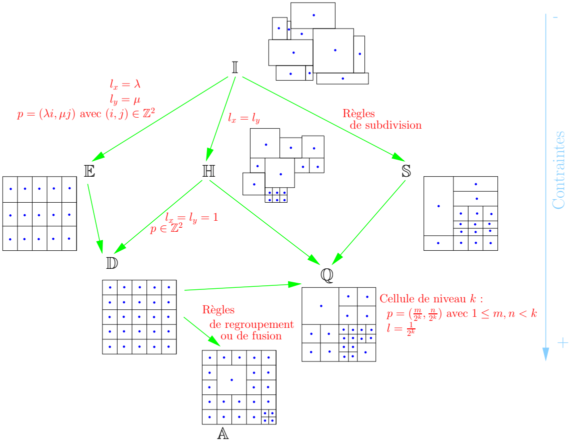


- one $(0)-$object but two (1)-objects
- (1)-curve but only a $(0)-$path

The Irregular Isothetic Model

Main idea

Relax the constraints on the size and on the center position of isothetic cells



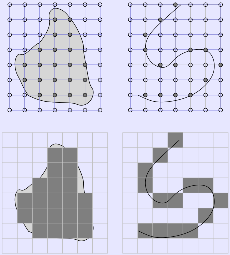
- Subdivision grids
- Adaptive grids
- QuadTree decomposition based grids
- Elongated grids
- ...

ACI GeoDigit

- Objective: Design uniform definitions (*e.g.* Straight lines) and algorithms
- Antoine Vacavant, PhD (defence: Dec. 2008)

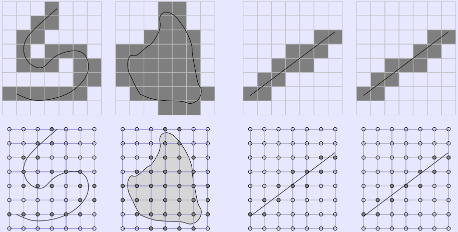
Classical Digitization Schemes and Analytic Models

Grid-point based approaches



GAUSS digitization, GIQ, BBQ,...

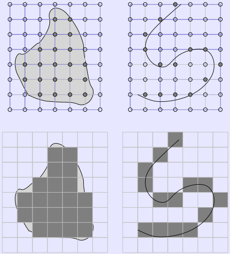
Cell based approaches



Supercover, Standard model,...

Classical Digitization Schemes and Analytic Models

Grid-point based approaches



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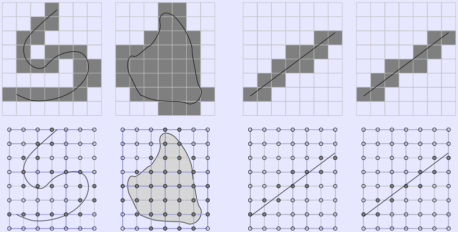
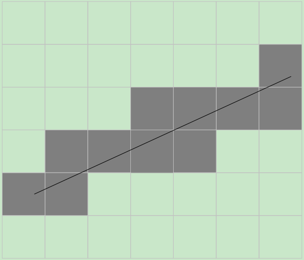


{ Supercover, Standard model,...} ⊂ Analytic Models

Discrete Analytic Models: Supercover Model



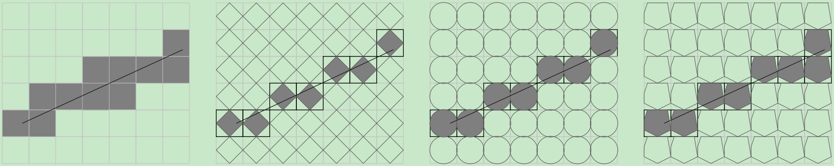
Def.

$$\mathbb{S}(F) = \{X \in \mathbb{Z}^d \mid B(X) \cap F \neq \emptyset\}$$
$$= \{X \in \mathbb{Z}^d \mid d_\infty(X,F) \leq \frac{1}{2}\}$$
$$= (F \oplus M) \cap \mathbb{Z}^d$$

Properties

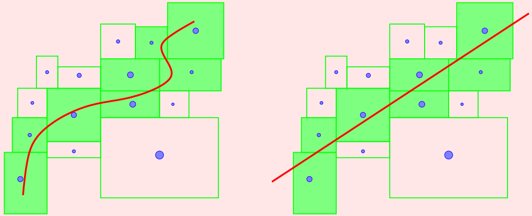
$$\mathbb{S}(F \cup G) = \mathbb{S}(F) \cup \mathbb{S}(G)\,,$$
$$\mathbb{S}(F) = \bigcup_{p \in F} \mathbb{S}(p)\,,$$
$$\mathbb{S}(F \cap G) \subseteq \mathbb{S}(F) \cap \mathbb{S}(G)\,,$$
$$\text{if } F \subseteq G \quad \text{then} \quad \mathbb{S}(F) \subseteq \mathbb{S}(G)\,.$$

Discrete Analytic Models - Examples



Supercover Closed Naive Model Pythagorician

Supercover Generalization to $\mathbb{I}-$grids



[DGCI 2005, Computer & Graphics 2005]

Interval Arithmetic

Elements = Intervals

$x \sim X = [\overline{x}, \underline{x}]$ such that $\overline{x} = \uparrow x \uparrow$ and $\underline{x} = \downarrow x \downarrow$ are representable numbers, *e.g.* $\mathbb{I}_{\mathbb{Z}+\frac{1}{2}}$

Examples and arithmetic operations on $\mathbb{I}_{\mathbb{Z}+\frac{1}{2}}$

$$3.144546 \longrightarrow [2+\frac{1}{2}, 3+\frac{1}{2}]$$

$$[2+\frac{1}{2}, 3+\frac{1}{2}] \oplus [0+\frac{1}{2}, 4+\frac{1}{2}] = [2+\frac{1}{2}, 8+\frac{1}{2}]$$

$$\ldots$$

Fundamental principle = Inclusion

Let $f : \mathbb{R} \longrightarrow \mathbb{R}$, and $\Box f : \mathbb{I} \longrightarrow \mathbb{I}$ the interval function (extension) associated to f. For all $x \in X$, $\Box f$ should be such that :

$$f(x) = y \quad \Rightarrow \quad y \in \Box f(X)$$

Interval Arithmetic and Supercover Model

Interval Arithmetic & Supercover: same ideas

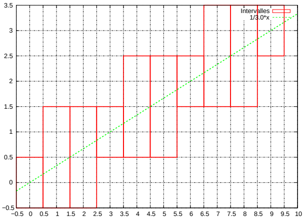
- Uncertainty on the underlying Euclidean object position
- We propagate the uncertainty through the construction/computation

Main result on $\mathbb{I}_{\mathbb{Z}+\frac{1}{2}}$

$$\mathbb{S}(f) \subseteq \bigcup_{k\in\mathbb{Z}} [K \times \Box f(K)] \quad \text{strict equality if } \Box f \text{ is optimal}$$

$f(x) = \frac{1}{3}x$

$f(x) = 10\sin(x)\exp^{-x^2/50}$



Conclusion

Contributions

- Supercover:
  - Cell/curve intersection $B(X) \cap F \neq \emptyset$
  - Metric definition $d_\infty(X, F) \leq \frac{1}{2}$
  - Morphological definition $(F \oplus M) \cap \mathbb{Z}^d$
  - Algebraic definition
- Interval Arithmetic based interpretation of the Supercover Model (or conversely ;))
- Applications:
  - Certified digitization of complex functions (*e.g.* implicit functions with the help of an Interval Arithmetic Solver)
  - Novel approach to model image transformations (*certified* transformations)

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Separable techniques for the EDT and REDT problems

Loss-less Model Conversion

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Definitions

Pragmatic approach

Digitization of the Euclidean object

Constructive approach

Model Driven definition

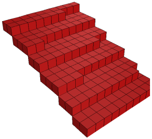
Example

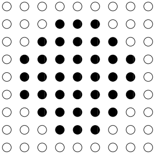
DSS = the result of the BRESENHAM's drawing algorithm

Example

DSS = set of grid point solution a discrete resolution of $y' = a$

Objects are usually identical
but the representation choice matter when you derive properties





Recognition Problem

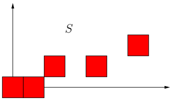
Statement

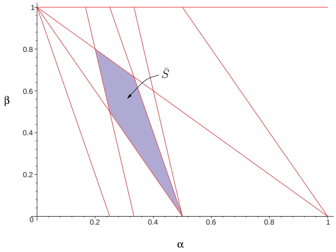
Given a set of grid-point S, is S a piece of <*yourfavoriteobjecthere*> ?

Answer Types

- Binary answer: Yes/No
- A valid parametrization of <*yourfavoriteobjecthere*> (if applicable)
- The set of parameters of all valid <*yourfavoriteobjecthere*> $\Rightarrow$ Preimage

e.g.: DSS





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Goals

Exploit discrete object properties to design fast recognition algorithms

What kind of Properties ?

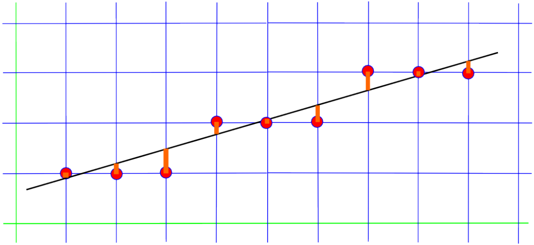
Related to discrete mathematics

Arithmetic, Number Theory, Theory of words, Patterns, Lattice Polytopes

e.g. DSS

Euclidean Straight line with rational slope ($r \in [0, 1]$)

- $\Rightarrow$ Finite possible intercepts with vertical lines
- $\Rightarrow$ The sequence of intercepts is periodic
- $\Rightarrow$ Periodic patterns in the DSS and arithmetical properties of DSS parameters
- $\Rightarrow$ Efficient algorithms



Toolbox: Computational Geometry, Linear Programming, Arithmetic, ...

Arithmetic

Euclid's division algorithm

$O(\log(min(|a|, |b|)))$

Computation Geometry

Convex hull and Integer Convex Hull

Linear Programming and Integer LP

Solve a linear system:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2 \\ \ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m \end{cases} \Rightarrow \begin{array}{c} \max cx \\ Ax \leq b \\ x \in \{\mathbb{R}^n, \mathbb{Z}^n\} \end{array}$$

LP: $O(m)$ (n fixed)
ILP: NP-hard

Focus: Integer Convex Hull / Lattice Polytopes

Consider an object included in a N^d window or with volume $(Vol\ P)$ (P not empty))

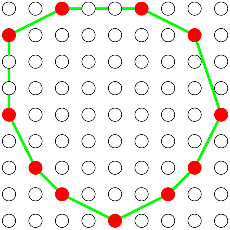
in 2-D

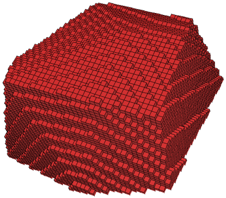
$$f_1(N) = \frac{12}{(4\pi^2)^{1/3}} N^{2/3} + O(N^{1/3} log(N))$$

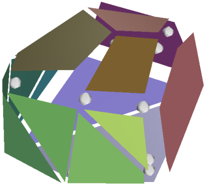
General Formula

f_k denotes the number of $k-$facets of $CH(P)$

$$f_k \leq c_n (\text{Vol } P)^{\frac{n-1}{n+1}}$$







Important trick to design tight computation costs
[DAM 2005, IWCIA 2006]

Example of the approach

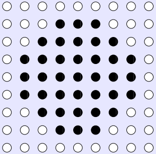
Problem

Digital Disk Recognition

Solution in C&G

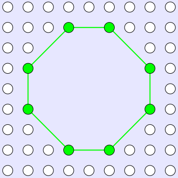
Separability of two sets S and T by an circle $\Rightarrow O(|S| + |T|)$

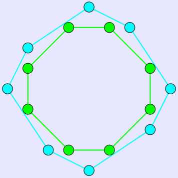
Brute-force approach



$\Rightarrow O(n^2)$

Arithmetic approach





Integer Convex Hull + BEZOUT's points

$\Rightarrow O(n^{2/3} \log n)$

[DAM 2004,IWCIA 2001]

Digital Plane and Hyperplane Recognition

Main problem

Asymptotic computational analysis $\not\Leftrightarrow$ Experimental Analysis

Contributions

Use lattice polytope properties to obtain tight asymptotic bounds

- Preimage based recognition algorithms
- Convex hull width based recognition algorithms
- ...

Contribution Overview

Discrete Analytic Models

Supercover & Interval Arithmetic, Irregular isothetic grid formulation

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Problem Description

Objectives

- Extract metric information from discrete objects
- Many applications:
  - Shape description
  - Motion planning
  - Image synthesis
  - ...
  - First step of the Medial Axis Extraction

Distance

- Euclidean: vector displacement, square of the EDT $\Rightarrow$ isotropic but complex algorithms
- Chamfer: mask based approximation of the Euclidean metric $\Rightarrow$ fast algorithms but anisotropic

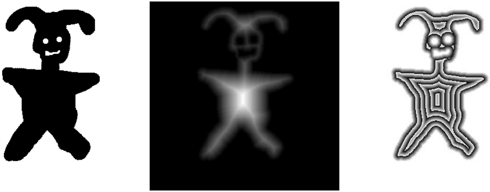
Contribution

Fast algorithms for the error-free Euclidean metric

Definitions

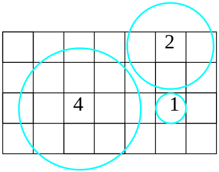
Distance Transformation Definition

Label each object grid point with the distance to the closest background point



Reverse Distance Transformation Definition

Given a set of discs, reconstruct the shape



0	0	0	0	1	2	1
0	2	3	2	0	1	0
0	3	4	3	0	1	0
0	2	3	2	0	0	0

Uniform problem and Uniform Algorithmic Tool

E²DT

$$s(q) = \min_{p \in \bar{X}} \{d^2_{euc}(p,q)\}$$

$$s(q) = \min_{p(x,y) \in \bar{X}} \{(x-i)^2 + (y-j)^2\}$$

$$g(i,j) = \min_x \{(i-x)^2\}$$

$$h(i,j) = \min_y \{g(i,y) + (j-y)^2\}$$

REDT

Disc MA: $\{(x_m, y_m, r_{(x_m, y_m)}\}$:

$$X = \{(i,j) \mid \exists m,\ (i-x_m)^2 + (j-y_m)^2 < r_{x_m y_m}\}$$

$$X = \{(i,j) \mid \max_{(x_m, y_m) \in MA} \{r_{x_m y_m} - (i-x_m)^2 - (j-y_m)^2\} > 0\}$$

$$g(i,j) = \max_x \{f(x,j) - (x-i)^2\}$$

$$h(i,j) = \max_y \{g(i,y) - (j-y)^2\}$$

Uniform problem and Uniform Algorithmic Tool

E^2DT

$$s(q) = \min_{p \in \bar{X}} \{d^2_{euc}(p,q)\}$$

$$s(q) = \min_{p(x,y) \in \bar{X}} \{(x-i)^2 + (y-j)^2\}$$

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REDT

Disc MA: $\{(x_m, y_m, r_{(x_m,y_m)}\}$:

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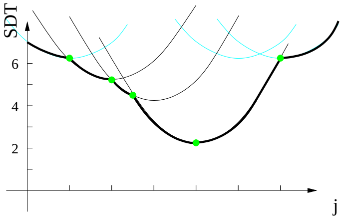
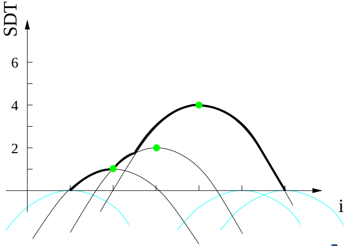




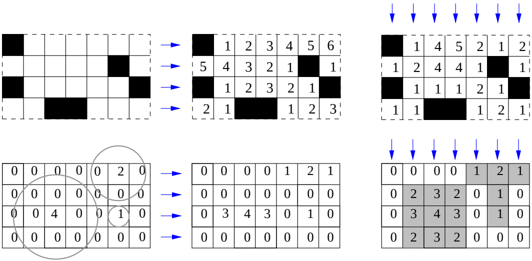
Uniform problem and Uniform Algorithmic Tool (bis)

To sum up:

- Separable 1D processes (dimension by dimension)
  $\Rightarrow$ Straightforward generalization to higher dimensions
- On each 1D slice: Upper/Lower Envelope Computation
  $\Rightarrow$ $O(n)$ (stack based technique)

$\Rightarrow$ Optimal in time algorithms $O(n^d)$

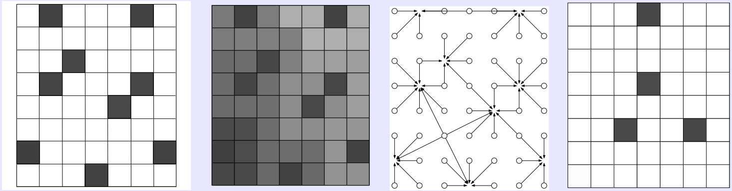
[DGCI 2002, IEEE PAMI 2007]

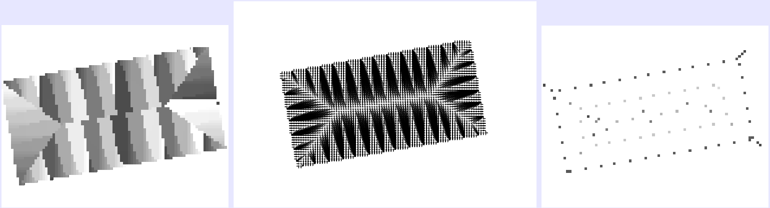


Variation on the Theme (1): The Discrete Voronoi Diagram

Contribution

Separable algorithm to compute the complete Discrete Voronoi mapping $O(n \cdot c)$





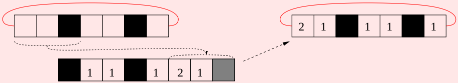
Variation on the Theme (2): Toric Spaces

Toric domain

Contribution

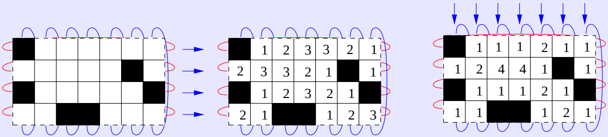
Take benefits from the separability:

- Process each column/row independently
- For each row, extract a break point b and we use $i + b \ modulo \ N$ indices



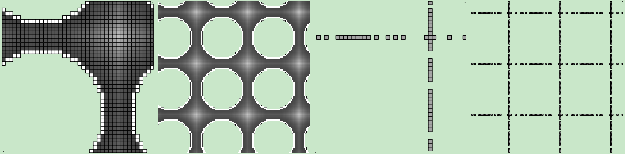
Variation on the Theme (2): Toric Spaces

Overall process for the EDT

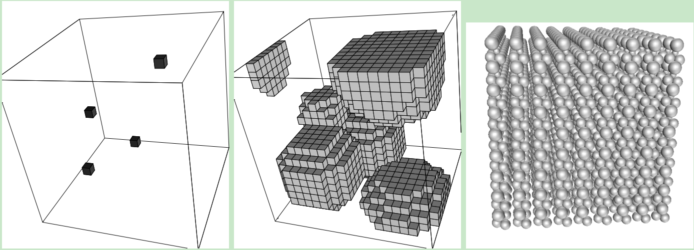


Variation on the Theme (2): Toric Spaces

2-D EDT example (+MA extraction)



3-D REDT example



Conclusion

Optimal separable techniques for the:

- Euclidean Distance Transformation
- Reverse Euclidean Distance Transformation
- Complete Discrete Voronoi Diagram mapping
- (Coming next) Discrete Euclidean Medial Axis Extraction

on:

- $d-$dimensional Images
- $d-$dimensional Toric Images

Contribution Overview

Discrete Analytic Models

Supercover & Interval Arithmetic, Irregular isothetic grid formulation

Fundamental Objects and Properties

Computational analysis of fundamental object recognition algorithms

Objects Analysis/Modeling

Separable techniques for the EDT and REDT problems

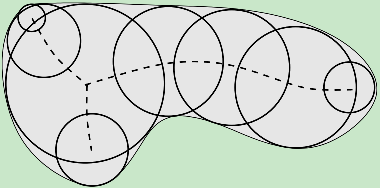
Loss-less Model Conversion

MA extraction algorithms and NP-completeness of the Minimal MA, Reversible reconstruction on irregular grids, Reversible surface reconstruction

Medial Axis

Definition

- Maximal ball: an open ball $B \subseteq X$ is *maximal* in X if for all included open balls B':
$$B \subseteq B' \subseteq X \implies B = B'.$$
- Medial Axis: denoted $AM(X)$, set of maximal ball centers in X



Many applications

- Shape description/matching
- Image synthesis
- ...

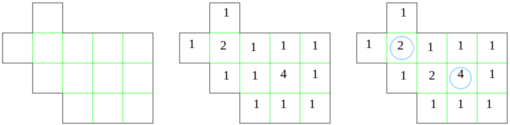
Discrete Medial Axis

Reformulation

Discrete domain, discrete metric and discrete balls

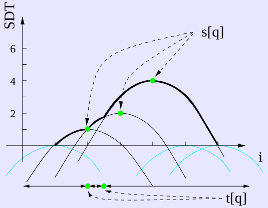
Extracted from the E^2DT

$E^2DT \longrightarrow$ disks included in the shape with maximal radii



Separability rocks...

1-D process

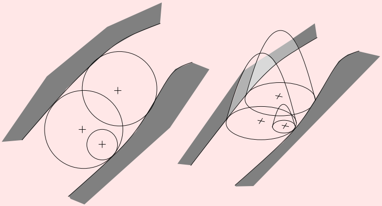


F	0	0	1	2	4	0	0
G	0	0	1	3	4	3	0
REDT	0	0	1	1	1	1	0

Idea

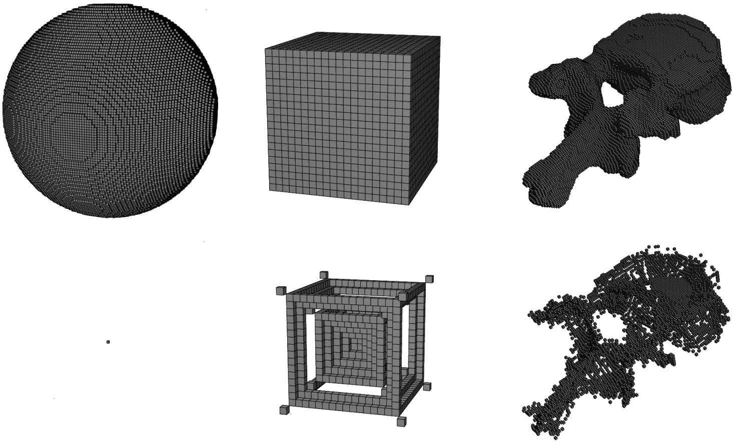
- Start with all disks obtained during the EDT
- Mark upper envelope parabola apex and propagate labeling through dimensions

Maximal disks ⇔ Upper envelope of elliptic paraboloids



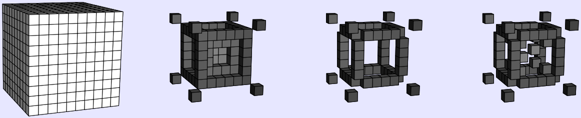
$$h_B(x, y) = r^2 - (x - i)^2 - (y - j)^2$$

Examples...



DMA Minimality

Statement of the problem



→ subsets of the MA may describe the same object

Two problems

1. Find the Minimum Discrete Medial Axis (min. in number of disks)
2. Find a subset of the DMA with less that k balls that covers the entire object ($k-$MIN)

Contribution

Problem 2 is NP-complete and thus Problem 1 is NP-hard

DMA Minimality

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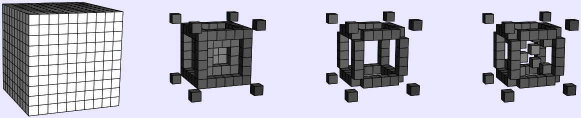

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Contribution

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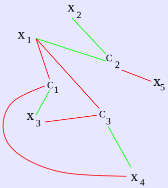
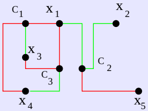
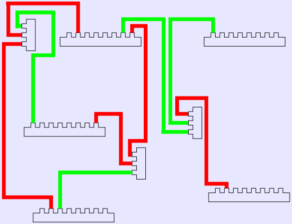
NP-completeness proof: PLANAR-4-3-SAT reduction

Overview

$\phi = (x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_5) \wedge (x_1 \vee x_3 \vee \neg x_4)$







Instance of PLANAR-4-3-SAT $\Rightarrow$ Discrete Orthogonal Embedding $\Rightarrow$ Discrete Object X

Key-points

- Polynomial reduction of ALL PLANAR-4-3-SAT instances into a subset of $k-$MIN instances
- An algorithm able to solve $k-$MIN should also solve PLANAR-4-3-SAT

Conclusion

Main result

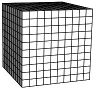
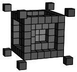
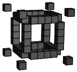
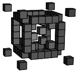
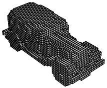
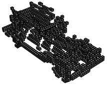
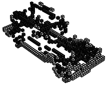
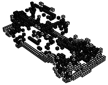
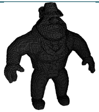
The Minimum Discrete Medial Axis is NP-hard

Future Works and Open Questions

- Complete the proof for other metrics (Chamfer,. . .)
- Is the problem still NP-complete on hole-free discrete objects ?

What's next?

Next Step: Approximation heuristics with bounds (if possible)

Object	$\mathcal{F} = \text{AMD}(X)$	$\hat{\mathcal{F}}$ BORGEFORS ET AL.	$\hat{\mathcal{F}}$ Greedy (with bound !)
	 104	 56 (-46%) [<0.01s]	 66 (-36%) [< 0.01s]
	 1292	 795 (-38%) [0.1s]	 820 (-36%) [0.19s]
	 17238	 6177 (-64%) [48.53s]	 6553 (-62%) [57.79s]

Reversible Reconstruction with linear structures

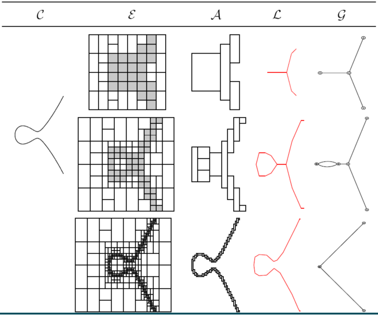
Objective

Reconstruct a reversible polygonal (resp. polyhedral) object from a discrete object

- Intensive use of digital straight lines and digital plane recognition algorithms

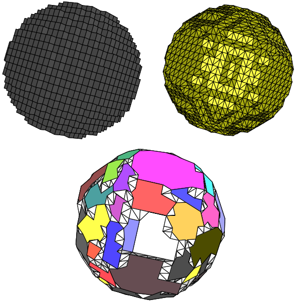
2-D

- Topological and Geometrical dynamic reconstruction in the Irregular Isothetic Model
- Application to Interval Arithmetic solver reconstruction



3-D

- Topologically correct polyhedral reconstruction based on MC simplification



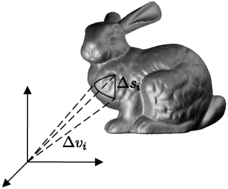
2D/3D Shape Description

Context: SEMANTIC-3D

- Objective: 3D-Objects indexing and retrieval from 2D and 3D queries
- JULIEN RICARD (PhD), defence 12/2005. Advised in coordination with ATTILA BASKURT (Pr - LIRIS)

Main Contributions

- Generalization of the MPEG-7 Angular Radial Transform
- Fourier based 2D/3D matching
- Analytic computation of volumic integrals on a 3D mesh (geometrical moments, Fourier Transform,. . .)



[PRL-2005, ICPR, ICIP]

Conclusion

Algorithmic solution to discrete shape analysis and modeling

From many fields:

- Arithmetic / Number theory
- Computational Geometry
- Complexity Analysis

Topics

- Model Definitions
- Fundamental Object Recognition
- EDT, REDT, RDMA, ...
- Reversible reconstruction

Two-way Cooperation

- C&G Algorithms to solve Discrete Geometry problems
- Use Discrete Geometry specificities to tune classical C&G algorithms

Collaborations

[EXTERNAL] [INTERNAL] [PHD STUDENTS] [MASTER STUDENTS]

Discrete Analytic Models

[L. TOUGNE] [A. VACAVANT] [L. ZERARGA]

Fundamental Objects and Properties

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[Y. GÉRARD] [J.-P. REVEILLÈS]
[L. TOUGNE] [I. SIVIGNON] [A. VACAVANT]

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[A. VACAVANT] [C. CHEVARIER]
[L. JOSPIN]

Loss-less Model Conversion

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RNRT SEMANTIC-3D

co-encadrement J. RICARD

ACI GeoDiGit

2005 - 2007 (Responsable)

ANR GeoDiB

- 2006-2010 (Membre)
- LORIA, LaBRI, LAIC, LIRIS

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Future Works

Discrete Analytic Models

- Theoretical framework: Non-Standard Analysis, Constructive maths, . . .
- Continue the interaction analysis between IA and the discrete model

[SIC-Poitiers, LMA-La Rochelle]

Fundamental Objects and Properties

- Uncertainty and noise integration in definitions and algorithms

[ANR GeoDiB (2006-2010)]

Objects Analysis/Modeling

- GPU based Discrete Object Volumic Analysis [Internal]

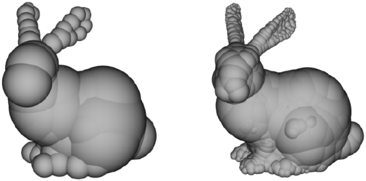
Loss-less Model Conversion

- Parallel discrete contours reconstruction with combinatorial maps [G. DAMIAND]
- THE DIGITAL SPHERE TREE

The Digital Sphere Tree

Objectives: Hierarchical/Multiresolution Discrete Medial Axis

- Multiresolution of the underlying grid
- Multiresolution and hierarchical sphere structures
  ⇒ Sphere-Tree
  - Trade-Off: Structure compactness & approximation
  - Many developments and applications in geometric processing

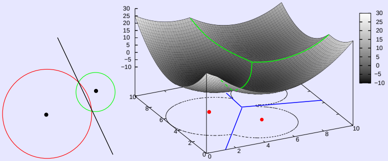


[G. Bradshaw and C. O'Sullivan]

Intermediate Steps

- First sphere structure (*flat* graph) could be extracted from the Discrete Power Diagram
- Better comprehension of interactions between Power diagram and Discrete Medial Axis
- . . .

⇝ Power (Laguerre) diagram



⇒ Interactions C&G, Discrete Geometry

Convex hulls

Objects	Computational cost	References
Points 2D	$O(m \log m)$	GRAHAM
Points 2D	$O(mh)$	JARVIS
Points 2D	$O(m \log h)$	CHAN
Points 3D	$O(m \log h)$	CHAN
Points dD	$O(m \log m + m^{\lfloor n/2 \rfloor})$	CHAZELLE
Chaîne simple 2D	$O(m)$ incrémental $O(1)$	MELKMAN
Chaîne simple 2D	$O(m)$ dynamique $O(1)$	BUZER
Courbe discrète	$O(m)$	VOSS
Objet discret convexe X	$O(h \log \delta(X))$	HAR-PELED

Back

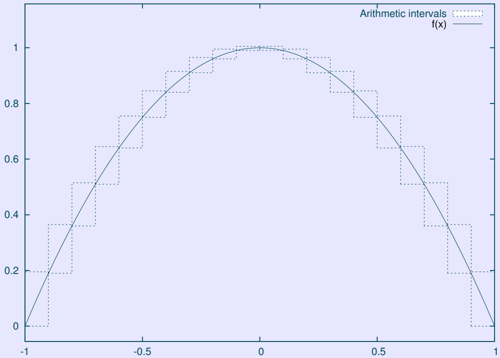
IA Operators

Arithmetic operators

$$X \oplus Y = [\downarrow \underline{x} + \underline{y} \downarrow, \uparrow \overline{x} + \overline{y} \uparrow]$$
$$X \ominus Y = [\downarrow \underline{x} - \overline{y} \downarrow, \uparrow \overline{x} - \underline{y} \uparrow]$$
$$X \odot Y = [\min(\downarrow \underline{x} \cdot \underline{y} \downarrow, \downarrow \underline{x} \cdot \overline{y} \downarrow, \downarrow \overline{x} \cdot \underline{y} \downarrow, \downarrow \overline{x} \cdot \overline{y} \downarrow), \max(\uparrow \underline{x} \cdot \underline{y} \uparrow, \uparrow \underline{x} \cdot \overline{y} \uparrow, \uparrow \overline{x} \cdot \underline{y} \uparrow, \uparrow \overline{x} \cdot \overline{y} \uparrow)]$$
$$\ldots$$

Interval Arithmetic Analysis Example

Example on $\mathbb{I}_{\mathbb{F}}$

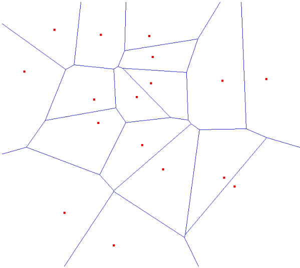


- $f(x) = (x+1)(x-1)$ on $[-1, 1]$
- $\Box f(X) = (X \oplus [1,1]) \odot (X \ominus [1,1])$

Optimality of an extension $\Box f$

$$\forall X \in \mathbb{I}, \nexists Y \subset \Box f(X) \text{ such that } \forall x \in X, f(x) \in Y$$

Variation on the Theme (1): The Discrete Voronoi Diagram



Voronoi Diagrams

Given a set of sites $\mathcal{S} = \{s_i\}$ in $\mathbb{R}^2$, the Voronoi diagram is a decomposition of the plane into cells $\mathcal{C} = \{c_i\}$ (one cell $c_i \subset \mathbb{R}^2$ per site s_i) such that each point p in the (open) cell c_i, we have $d(p, s_i) < d(p, s_j)$ for $i \neq j$.

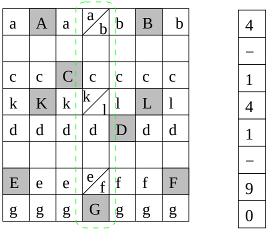
DT and Voronoi Diagrams

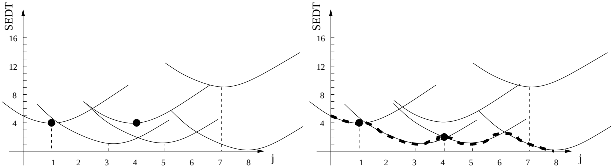
- DT $\leftarrow$ Voronoi Diagram of background grid points
- DT algorithms $\longrightarrow$ Discrete Voronoi Diagram mapping
  - Association of each grid point to its associated cell s_i
  - Difficulties to handle equidistant background points (when Voronoi diagram edges/vertices intersect $\mathbb{Z}^d$ points)

Variation on the Theme (1): The Discrete Voronoi Diagram

Ideas

Propagate parabola apex indices and detect when two parabolas intersect at an integer coordinate position





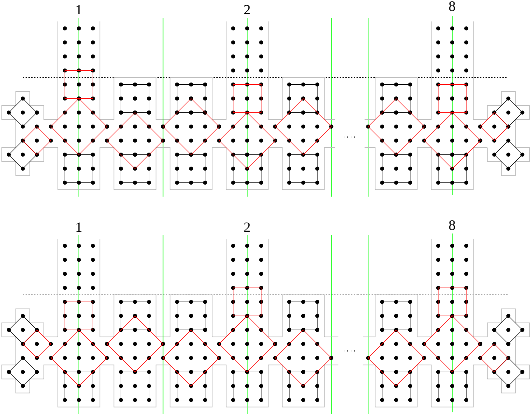
Geometrical interpretation of a PLANAR-4-3-SAT instance

Variable

- Eight slots to encode the uses of the variable in a PLANAR-4-3-SAT formula
- Two minimal decomposition with d_E balls (72 balls)
  - one protrudes out only at even slots ⇒ True
  - one protrudes out only at odd slots ⇒ False
- Constant size

Wires

Clauses



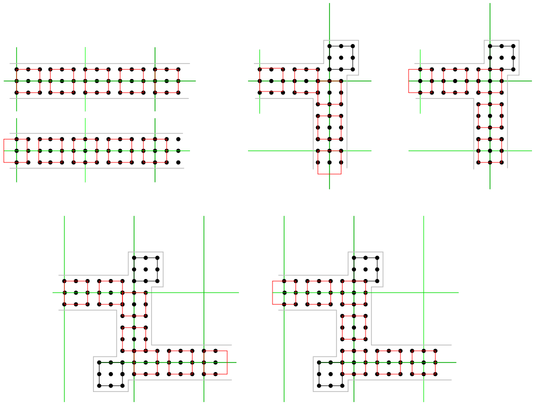
Geometrical interpretation of a PLANAR-4-3-SAT instance

Variable

Wires

- Transmission of the Truth assignment signals from variables to clauses
- Wires can be bent without changing the signal
- Extremities of the wires are placed on a 6×6 sub-grid
- Size independent to the signal value

Clauses



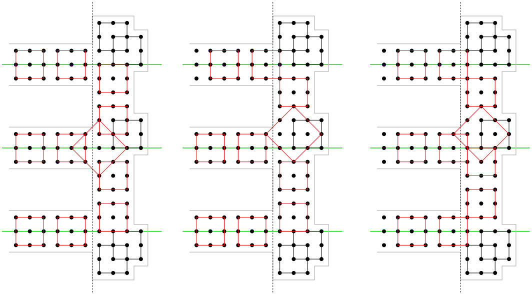
Geometrical interpretation of a PLANAR-4-3-SAT instance

Variable

Wires

Clauses

- Input: three wires
- 10 balls required to cover the shape if all signals are False and 9 otherwise
- Constant size



Geometrical interpretation of a PLANAR-4-3-SAT instance

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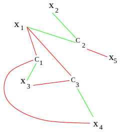

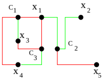

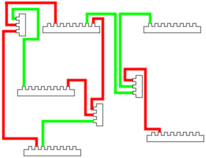

Putting all together

ϕ Planarity + Discrete Orthogonal Embedding + sub-grid alignments

$\Rightarrow$ $(1)-$connected discrete object such that there is no intersection between wires, variables and clauses

$\Rightarrow$ Object size polynomial in the ϕ size

[Details skipped. . .]

The Minimum Medial Axis problem is NP-complete

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