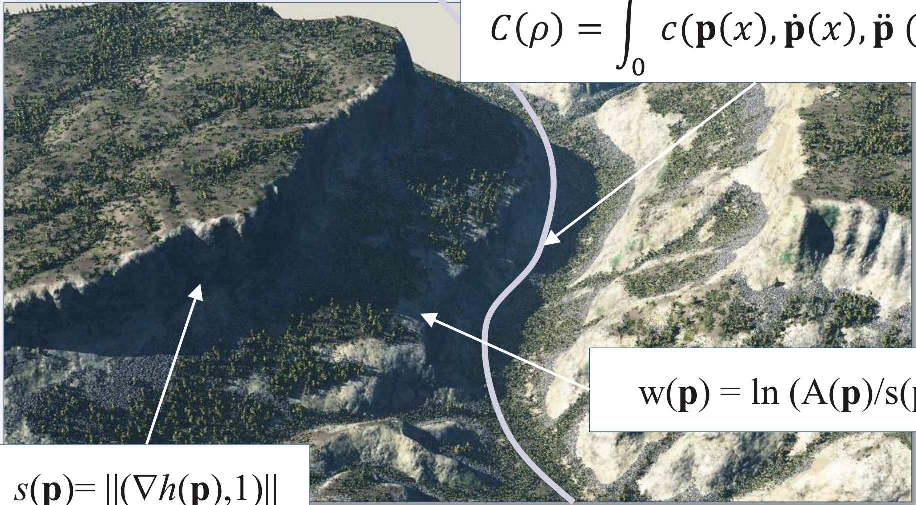
Digital World Modeling

From mathematics …



$$C(\rho) = \int_0^1 c(\mathbf{p}(x), \dot{\mathbf{p}}(x), \ddot{\mathbf{p}}(x))dx$$

$$w(\mathbf{p}) = \ln(A(\mathbf{p})/s(\mathbf{p}))$$

$$s(\mathbf{p}) = \|(\nabla h(\mathbf{p}), 1)\|$$

… to the screen

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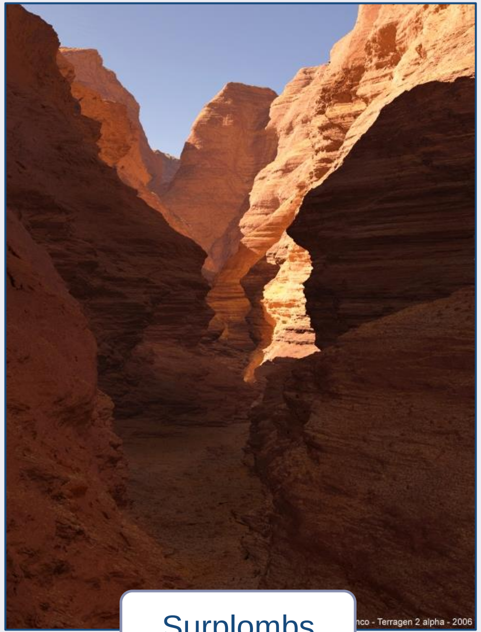
Digital World Modeling

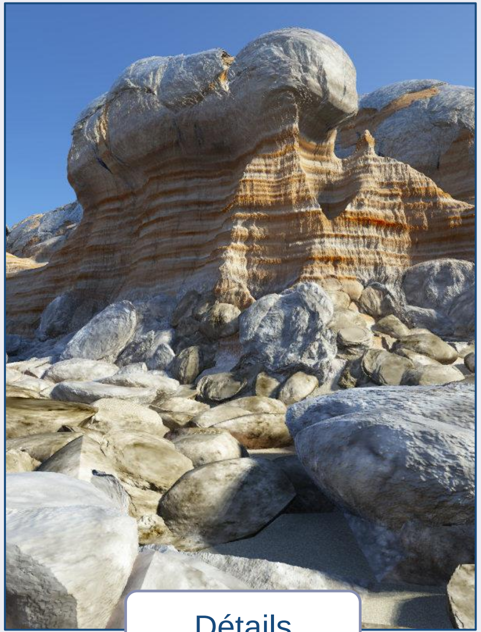
Data Structures
Procedural Modeling
Erosion Simulation
Procedural Road Generation
Vegetation and Ecosystems
Growth models
Aging and weathering

Verrous scientifiques et techniques

- Terrains de très grande dimensions à différentes résolutions
- Terrain géologiquement corrects
- Prise en compte des différents matériaux
- Détails géométriques (rochers, cailloux)



Surplombs



Détails

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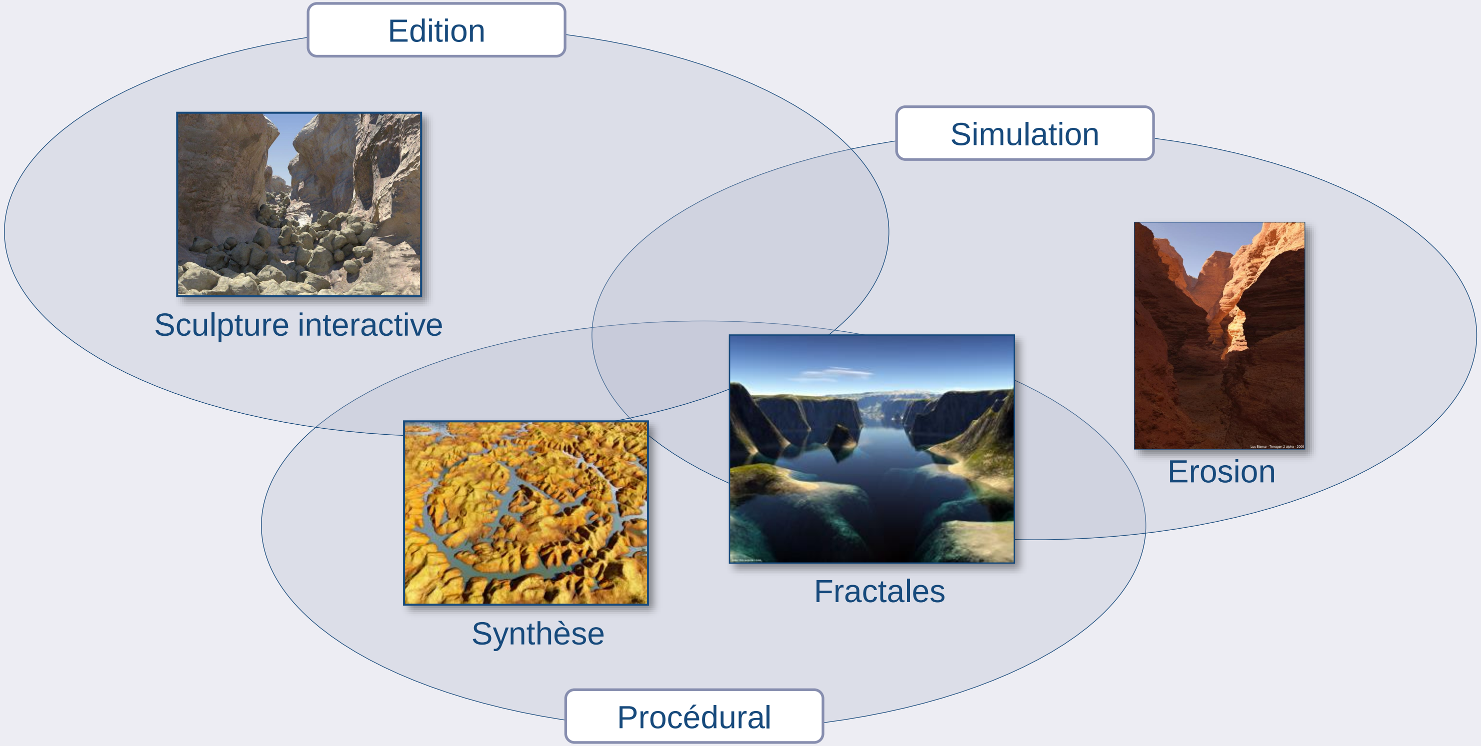
Introduction

Techniques de création

Méthodes de génération et de reconstruction



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Challenges
Realism
Variety of landforms
Range of scale
Control and authoring
Constraints
Memory
Speed

Classification

Criteria

Categories of landforms
Model

Versatile Simulation

Memory intensive

Authoring landforms

Compact

Automatic

Data	Hybrid	Functions

Elevation	Height maps	Best of both worlds ?	Procedural
Volume	Voxels		Implicits

Overhangs Complex landforms

Even more **memory** intensive

Sculpting

Computationally demanding

Realism and **control** of the landforms?
As **compact** as possible representations?

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Classification

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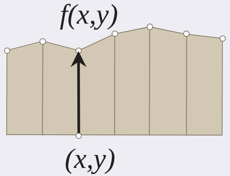
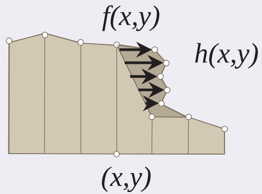
Structure de données

Représentation surfacique : hauteur du relief
Modèles volumiques : différents matériaux en surface et en profondeur

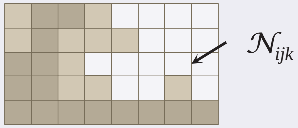
Modèles
- Surfaciques
  - Fonctions
  - Cartes
  - Maillages
- Volumiques
  - Voxels
  - Piles
  - Surfaces implicites



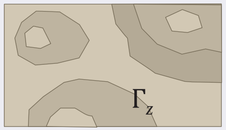
Carte d'élévation



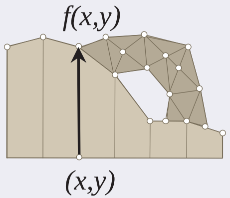
Carte de surplombs



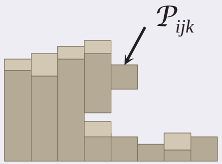
Enumération spatiale



Courbes de niveau



Hybrides



Piles de matière

Elevation functions

Function representation

Elevation functions

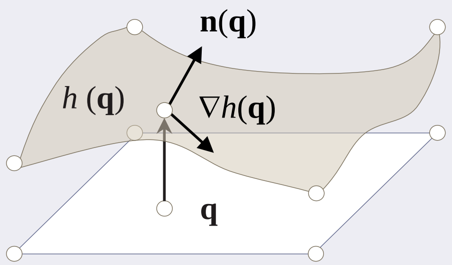
- Explicit function $h : \mathbf{R}^2 \rightarrow \mathbf{R}$, direct elevation $h(\mathbf{p})$
- Compact model representation
- Gradient ∇h defines the steepest slope vector

$$s(\mathbf{p}) = |\nabla h(\mathbf{p})|$$

Slope

$$\mathbf{n}(\mathbf{p}) = (-\nabla h(\mathbf{p}), 1)$$

Normal, normalize $\hat{\mathbf{n}} = \mathbf{n}/|\mathbf{n}|$



Elevation function h

Computing h (on the fly) may be **demanding**
Control of landforms, authoring

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Metrics

Curvature

For a general elevation function $z = h(x,y)$

Partial derivatives

$$\kappa = \frac{h_{xx}h_{yy} - h_{xy}^2}{\left(1 + h_x^2 + h_y^2\right)^2}$$

Gaussian curvature

$$\mu = \frac{\left(1 + h_y^2\right)h_{xx} - 2h_x h_y h_{xy} + \left(1 + h_x^2\right)h_{yy}}{2\left(1 + h_x^2 + h_y^2\right)^{3/2}}$$

Mean curvature

Profile and planform curvature

Profile curvature κ_{pro} is in the direction of the steepest slope ∇h

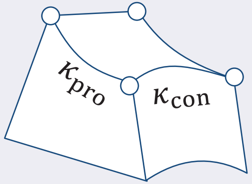
Planform or contour curvature κ_{con} is orthogonal to ∇h

$$\kappa_{\text{pro}} = -\frac{h_{xx}h_x^2 + 2h_{xy}h_x h_y + h_{yy}h_y^2}{\left(h_x^2 + h_y^2\right)\left(1 + h_x^2 + h_y^2\right)^{3/2}}$$

Rate of change of slope

$$\kappa_{\text{con}} = -\frac{h_{xx}h_y^2 - 2h_{xy}h_x h_y + h_{yy}h_x^2}{\left(h_x^2 + h_y^2\right)^{3/2}}$$

Divergence of a flow on the surface



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Heightfields

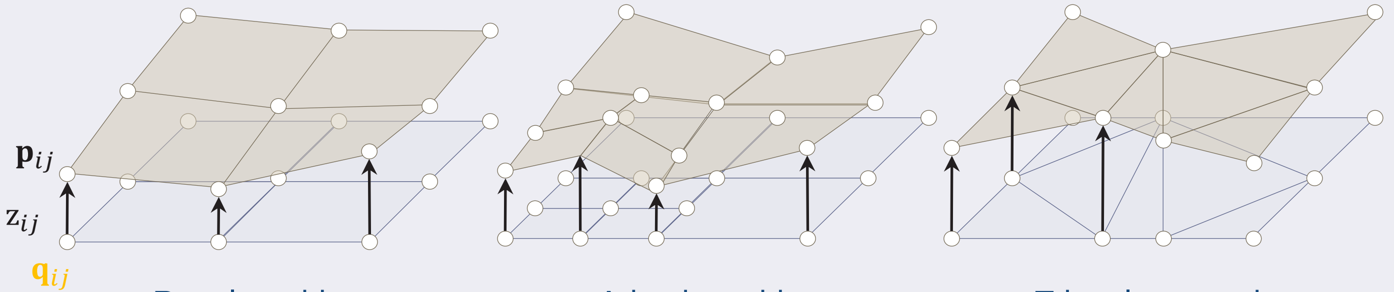
Height fields

Structure

Versatile model (generation, simulation)
Elevation reconstructed from elevation $\mathbf{z}_{ij}$
Discrétisation régulière ou irrégulière (adaptative) de Ω
On définit pour $\mathbf{q}_{ij} \in \Omega = B(\mathbf{a}, \mathbf{b})$ les élévations $z_{ij} = h(\mathbf{q}_{ij})$

$$\mathbf{p}_{\text{ij}} = \left(\mathbf{q}_{ij}, z_{ij} = h(\mathbf{q}_{ij})\right)$$

$$\mathbf{q}_{ij} = \mathbf{a} + \left(\frac{(\mathbf{b}_x - \mathbf{a}_x)\, i}{n_x - 1}, \frac{(\mathbf{b}_y - \mathbf{a}_y)\, j}{n_y - 1}\right)$$



Regular grid | Adaptive grid | Triangle network

L'élévation $h(\mathbf{p})$ à l'intérieur des cellules de Ω est définie par interpolation des valeurs aux sommets

Note

Memory demanding model : storage increases in $O(n^2)$
Algorithms **do not scale** for large terrains with a high resolution

1k × 1k
4Mb, $\leq$ 1s

16k × 16k
1Gb, $\geq$ 2h

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Profile and planform curvature

Formulas can be derived from the general equations $z = h(x, y)$
Recall that partial derivatives can be approximated as

$$h_x \approx \frac{h_{i+1,j} - h_{i-1,j}}{2\varepsilon}$$

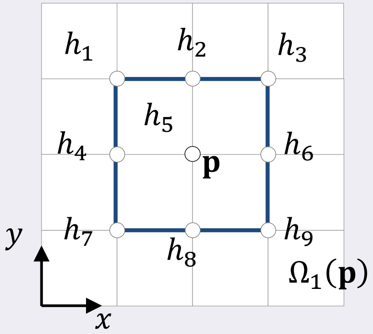
$$h_{xx} \approx \frac{h_{i+1,j} + h_{i-1,j} - 2h_{ij}}{\varepsilon^2}$$

$$h_{xy} \approx \frac{h_{i+1,j+1} + h_{i-1,j-1} - h_{i+1,j-1} - h_{i-1,j+1}}{4\varepsilon^2}$$

Surface S can be approximated [Evans 1980] by a bi-variate quadratic surface Q

$$z = a_{20}x^2 + a_{02}y^2 + a_{11}xy + a_{10}x + a_{01}y + a_{00}$$

$$\begin{aligned}
a_{20} &= \big(h_1 + h_3 + h_4 + h_6 + h_7 + h_9 - 2(h_2 + h_5 + h_8)\big)/6\delta^2 \\
a_{02} &= \big(h_1 + h_2 + h_3 + h_7 + h_8 + h_9 - 2(h_4 + h_5 + h_6)\big)/6\delta^2 \\
a_{11} &= (h_3 + h_7 - h_1 - h_9)/4\delta^2 \\
a_{10} &= (h_3 + h_6 + h_9 - h_1 - h_4 - h_7)/6\delta \\
a_{01} &= (h_1 + h_2 + h_3 - h_7 - h_8 - h_9)/6\delta \\
a_{00} &= (2(h_2 + h_4 + h_6 + h_8) - (h_1 + h_3 + h_7 + h_9) + 5h_5)/9
\end{aligned}$$



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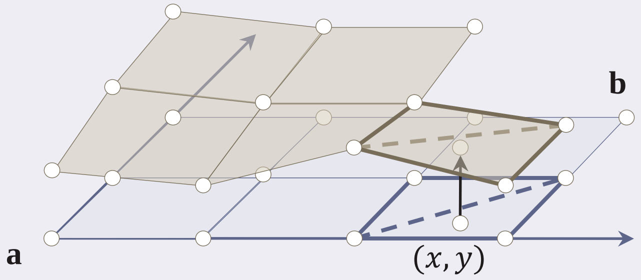
Height fields

Elevation computation

- Elevation z_{ij} is stored for $\mathbf{q}_{ij} \in \Omega$
- Compute $h(\mathbf{p})$ by bilinear interpolation




```
class HeightField
{
protected:
  Vector a, b;        // Bounding box
  int nx, ny;         // Discretization
  double z[];         // Array of heights
public:
  double Height(const double&,const double&);
  double HeightGrid(int,int);
  // …
};
```

```
double HeightField::Height(const double& x,
  const double& y)
{
  // Local coordinates
  double u=(x-a[0])/(b[0]-a[0]);
  double v=(y-a[1])/(b[1]-a[1]);

  // Cell location within grid
  int nu=int(u*nx);
  int nv=int(v*ny);

  // Local coordinates within cell
  u=u-nu*(b[0]-a[0])/nx;
  v=v-nv*(b[1]-a[1])/ny;

  if (u+v<1)
  {
    return (1-u-v)*HeightGrid(i,j)
     +u*HeightGrid(i+1,j)
     +v*HeightGrid(i,j+1);
  }
  else
  {
   return (u+v-1)*HeightGrid(i+1,j+1)
     +(1-v)*HeightGrid(i+1,j)
     +(1-u)*HeightGrid(i,j+1);
  }
}
```

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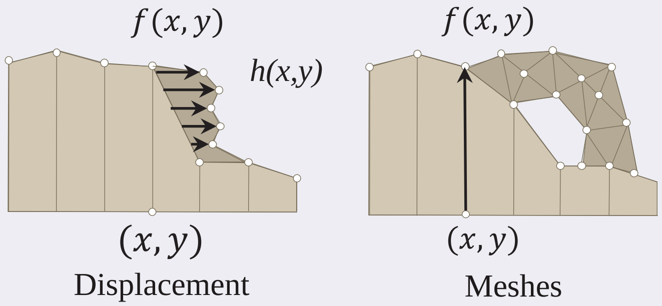
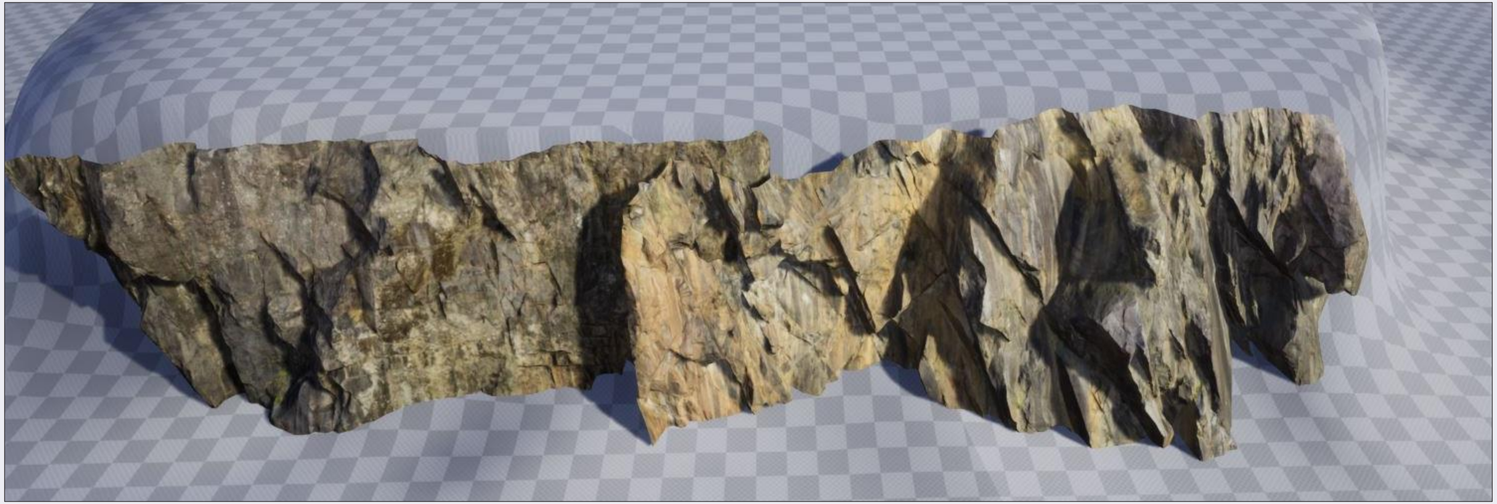
Limitations

Limitations of elevation models

- Overhangs and cliffs
- Specific displacement or meshes for vertical parts





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Ghost Recon Wildlands

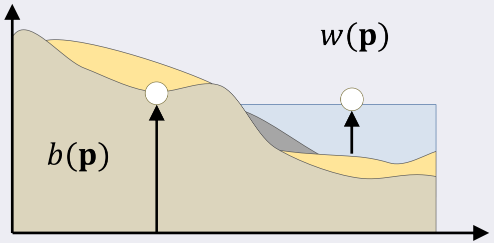
Layered Models

Layered representations

Concept

Bedrock elevation b is augmented with the thickness t_i of other material layers

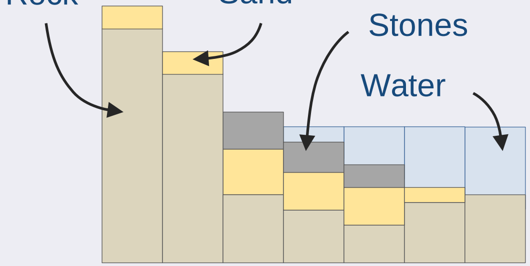
$$h(\mathbf{p}) = b(\mathbf{p}) + \sum_{i \in L} t_i(\mathbf{p})$$



Elevation functions

Applications

Erosion simulation produce fallen rocks or sediments
Ecosystems may use material layers



Discrete layer stacks

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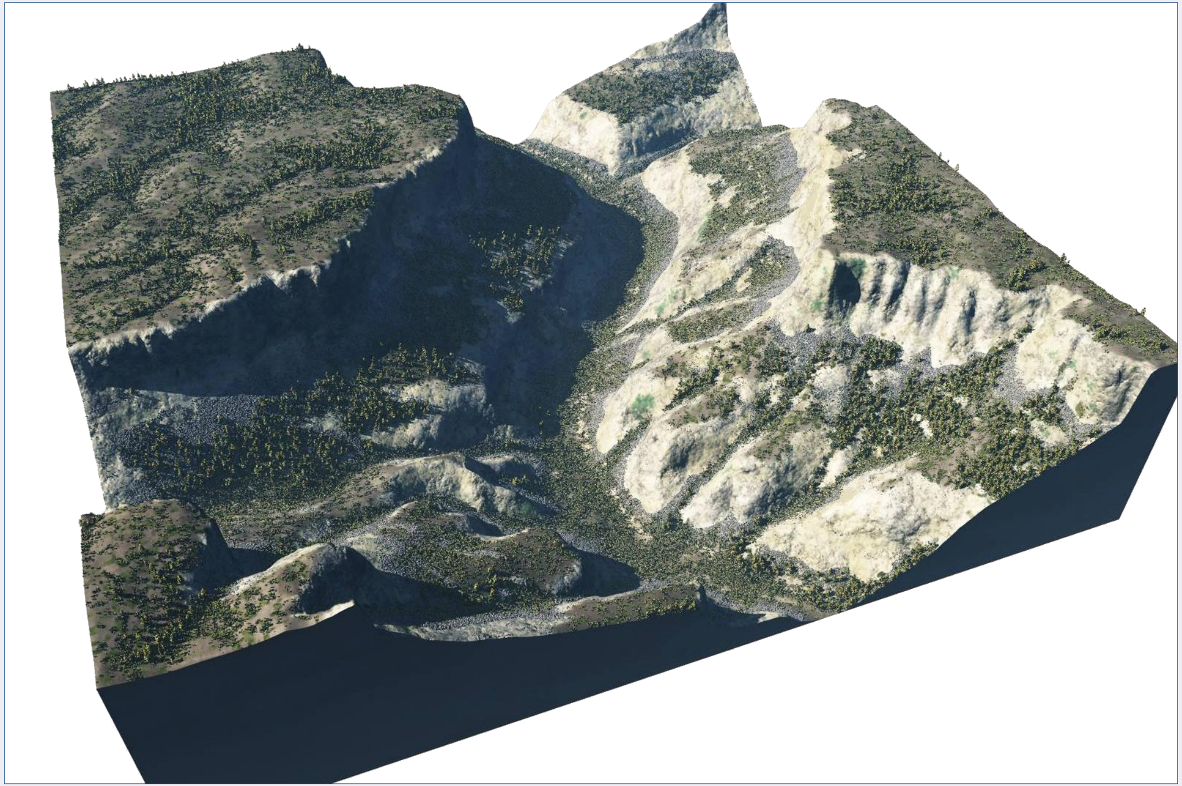
Benes *et al.* Layered data representation for visual simulation of terrain erosion. *Spring Conference on Computer Graphics*, 2001.
Musgrave et al. The synthesis and rendering of eroded fractal terrains. *Computer Graphics*, **23**, 3, 1989.

Layered representations

Combined terrain ecosystem simulation

Interacting layers



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Volumetric models

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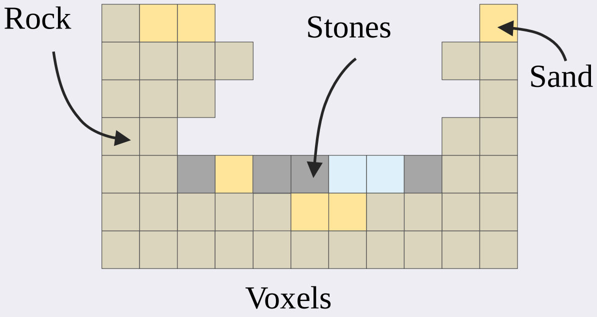
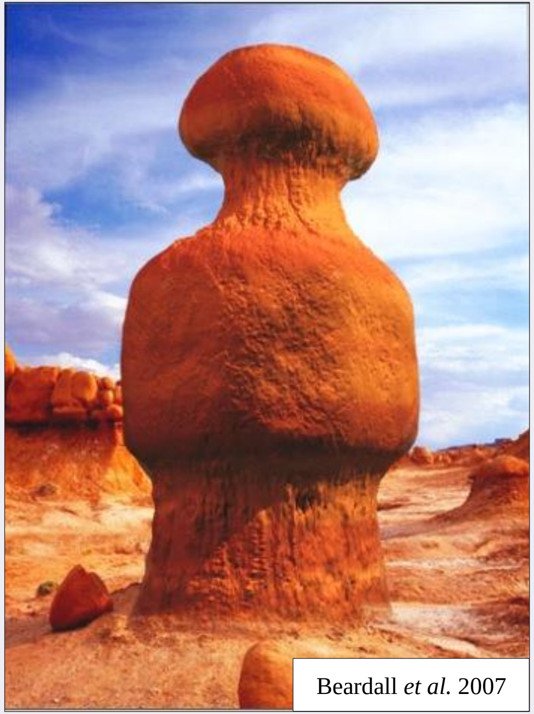
Model

Material function $\mu(\mathbf{p})$ defined in a voxel grid [Jones2010]



Allows **sculpting** and **simulations**

Memory demanding

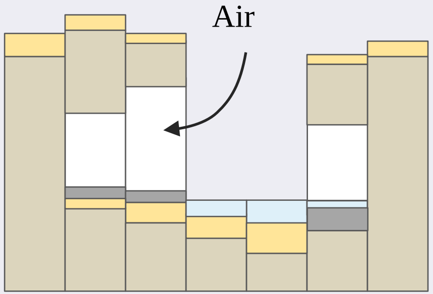


Jones *et al*. Directable weathering of concave rock using curvature estimation. *Transactions on Visualization and Computer Graphics*, **16** (1), 2010
Beardall *et al*. Goblins by spheroidal weathering. *Eurographics Workshop on Natural Phenomena*, 2007

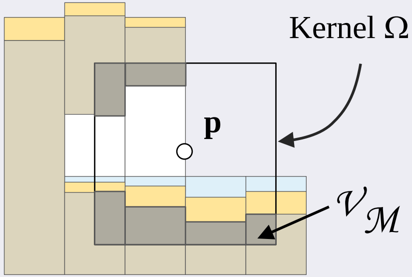
Hybrid representations

Smoothed layered stack

Layered materials with air allow the creation of overhangs and cavities [Peytavie2009]
Implicit model : convolution $\mu(\mathbf{p}) = 0$ for air, $\mu(\mathbf{p}) = 1$ for material



Layer stacks with overhangs



Layer stacks

Convolution $f = 2\mu * k - 1$

Fast computation $\mu * k = \frac{V_M}{V_\Omega}$

Peytavie *et al*. 2009

Peytavie *et al*. Arches: a framework for modeling complex terrains. *Computer Graphics Forum*, **28**, 2, 2009

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Hybrid representations

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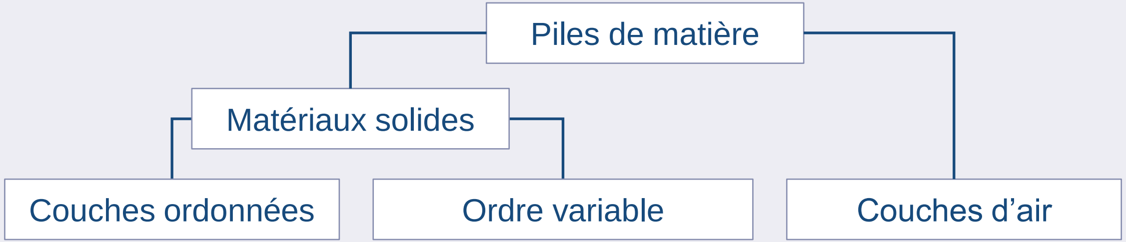
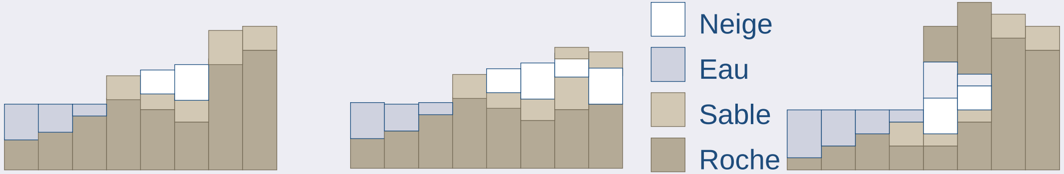
Hybrid representations

Principe

Un terrain est une grille de piles de matériaux
L'élévation finale correspond à la hauteur totale de la pile
Lorsque l'ordre est fixe, on simplifie la structure en une superposition de cartes de matière

$$h = \sum_{0 \le i \le n} h_i \quad h_i : \Omega \subset \mathbf{R}^2 \to \mathbf{R}$$





Ordre des matériaux fixe   Ordre variable   Terrains avec surplombs

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Structures de données

Comparaison entre plusieurs cartes d'élévations et une structure dynamique

```
class LayerField
{
protected:
  Vector a, b;       // Bounding box
  int nx, ny;        // Discretization
  double rock[];     // Array of heights for rock layer
  double sand[];     // Array of heights for sand
  double water[];    // Array of heights for water
public:
  double Height(const double&,const double&);
  double HeightGrid(int,int);
  // …
};
```

```
class Material       // Definition of a single cell
{
protected:
  double h;          // Height of material in cell
  int type;          // Material type
public:
// …
};

class MaterialStack  // Stack of cells
{
protected:
  Material stack[];  // Array of different materials
  int n;             // Size of array
public:
// …
};

class MaterialStackField // Array of stacks of cells
{
protected:
 Vector a, b;        // Bounding box
  int nx, ny;        // Discretization
  MaterialStack array[];// Array of  material stacks
public:
// …
};
```


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Hybrid representations

Structure

Modèle hybride combinant surface implicite – modèle discret
Combinaison pour l'édition et la simulation

	Implicite	Discret
Roche	Sculpture Fissures	
Matériaux granuleux		Dépôt Erosion Stabilisation
Liquides		Remplissage Ecoulement Phases

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Hybrid representations

Structure de données

Caractérisation volumique en piles de matière g
Lissage de la surface par convolution par un noyau h à support compact

$$S = \{\mathbf{p} \in R^2, f(\mathbf{p}) = 0\}$$

$$f(\mathbf{p}) = \frac{i(\mathbf{p})}{4\sigma} - 1$$

$$i(\mathbf{p}) = g * h(\mathbf{p}) = \int_{\mathbf{R}^3} g(\mathbf{p}) h(\mathbf{p} - \mathbf{q}) d\mathbf{q}$$

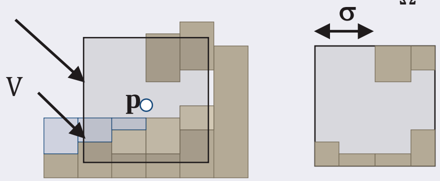
$$g(\mathbf{p}) = \begin{cases} 1 \text{ si } \mathbf{p} \in M \\ 0 \text{ sinon} \end{cases}$$

Squelette

$$h(\mathbf{p}) = \begin{cases} 1 \text{ si } |\mathbf{q}|_\infty \leq \sigma \\ 0 \text{ sinon} \end{cases}$$

Noyau

Support de convolution Ω de volume V_Ω



$$f(\mathbf{p}) = \frac{2V}{V_\Omega} - 1$$

A. Peytavie, E. Galin, J. Grosjean, S. Mérillou. Arches: a Framework for Modeling Complex Terrains. *Computer Graphics Forum (Proceedings of Eurographics)*, **28**(2), 457-467, 2009.

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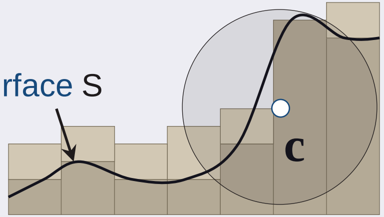
Hybrid representations

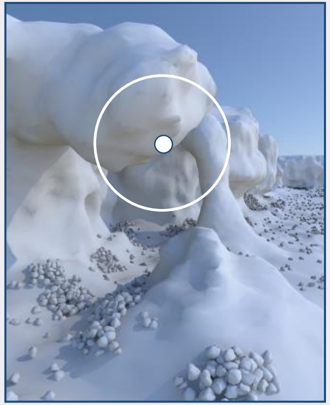
Combinaison des modèles implicites et discrets

- Créer une primitive avec un centre **c**
- Mélange avec la primitive implicite
- Discrétisation de la surface implicite en piles
- Stabilisation



Couches initiales

Couches modifiées



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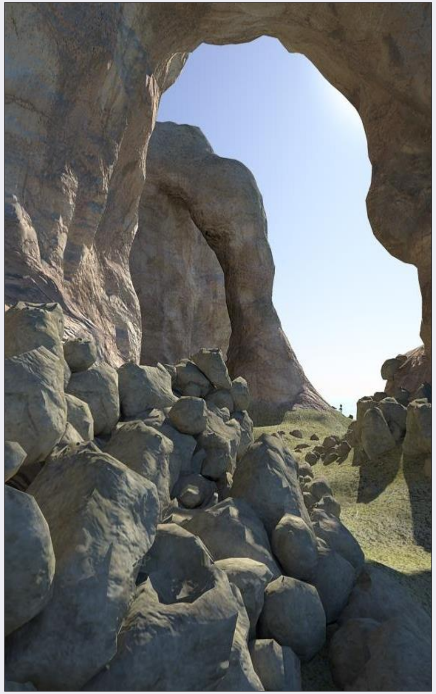
Hybrid representations

Smoothed layered stack

Layered materials with air allow the creation of overhangs and cavities
Implicit model created by a convolution



Peytavie *et al*. Procedural generation of rock piles using aperiodic tiling. *Computer Graphics Forum* **28**, 7, 2009

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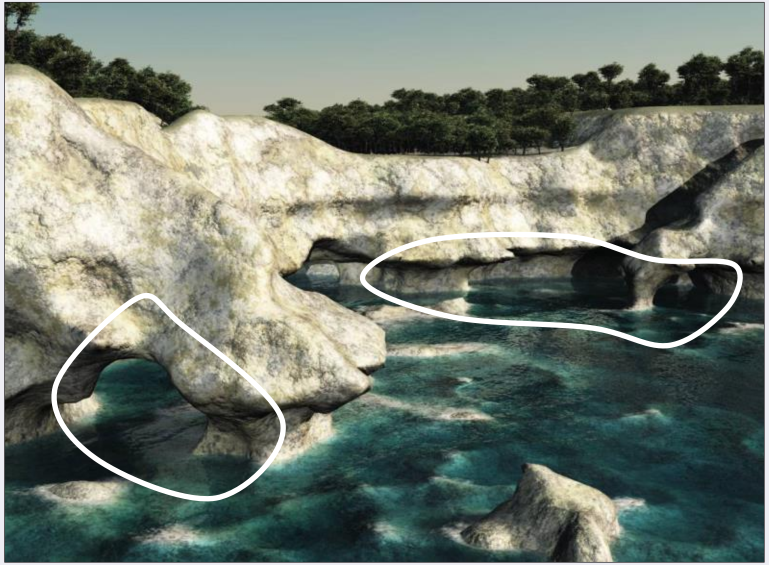
Implicit Surfaces

Field function

Find good primitives and operators for carving landforms

Cliffs, overhangs, caves



Need for effective **operators** and **primitives**

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Conclusion

Height fields and layered height fields

- **Conspicuous** in terrain modeling
- **Versatile** for a variety of generation methods

Function-based models

- Useful for modeling some **specific landforms**
- Modeling large landscapes with a high resolution

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