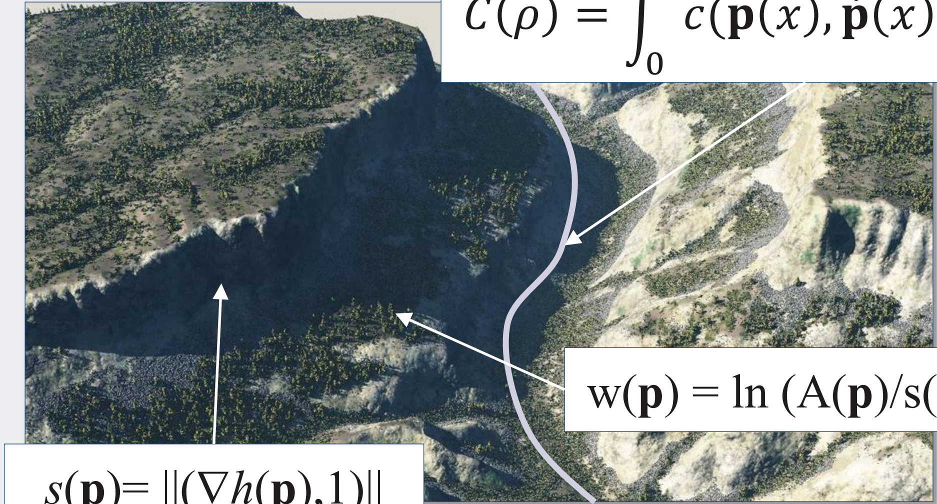
Digital World Modeling

From mathematics …



$$C(\rho) = \int_0^1 c(\mathbf{p}(x), \dot{\mathbf{p}}(x), \ddot{\mathbf{p}}(x))\,dx$$

$$w(\mathbf{p}) = \ln\left(A(\mathbf{p})/s(\mathbf{p})\right)$$

$$s(\mathbf{p}) = \|(\nabla h(\mathbf{p}), 1)\|$$

… to the screen

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Digital World Modeling

Data Structures

Procedural Modeling

Erosion Simulation

Procedural Road Generation

Vegetation and Ecosystems

Growth models

Aging and weathering

Procedural Modeling of Height Fields

Faulting

Algorithm

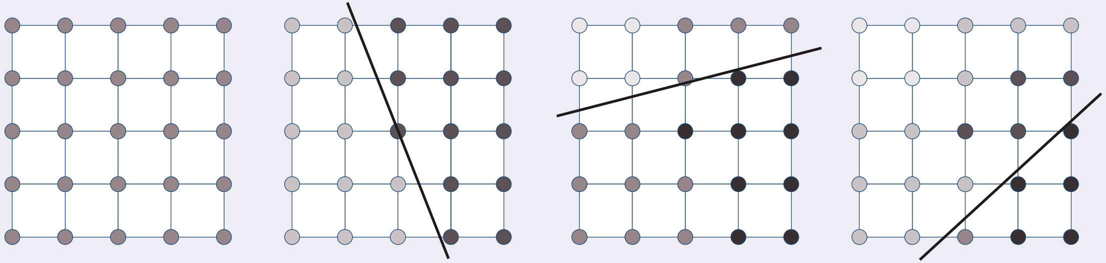
generate set of random faults F_i
Depth is a function of the distance $d\,(\mathbf{p}, F_i)$

Distance to lines $d(\mathbf{p}, \Delta_i)$
or circles $d(\mathbf{p}, C_i)$

$$h(p) = \sum_{k=0}^{n} \delta_i(\mathbf{p})$$

$$\delta_i(\mathbf{p}) = 1 - s \circ d(\mathbf{p}, F_i)$$

Any (compactly supported) smooth step function



B. Mandelbrot. The Fractal Geometry of Nature. 1982.
R. Voss. Random fractal forgeries. *Fundamental Algorithms for Computer Graphics*, **17**, 1991.

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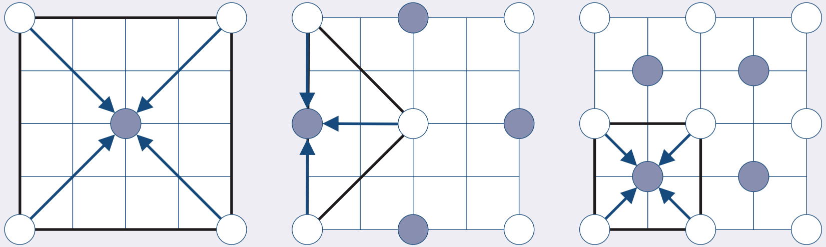
Recursive subdivision

Diamond Square

Height map of size $2^n + 1$
Requires explicit **storage**



```
// Initialise les valeurs aux coins
z[0][0] = z[0][n-1] = z[n-1][0] = z[n-1][n-1] = SEED;

double h = 200.0;
// Itération sur les niveaux, k = taille d'un carré
for (int k = n-1; k >= 2; k /= 2, h /= 2.0)
{
  int l = k/2; // Demi coté

  // Génération pour les carrés
  for (int x=0; x<n-1;x+=k)
  {
    for (int y=0; y<n-1; y+=k)
    {
      double a = (z[x][y] + z[x+k][y] +
        z[x][y+k] + z[x+k][y+k])/4.0;

      z[x+l][y+l] = a + random(-h,h);
    }
  }
```

```
  // Génération pour les losanges
  for (int x=0; x<n-1; x+=l)
  {
    for  (int y=(x+l)%k; y<n-1; y+=k)
    {
      double a = (z[(x-l+n)%n][y] +
        z[(x+l)%n][y] + z[x][(y+l)%n] +
        z[x][(y-l+n)%n])/4.0;

      z[x][y] = a+random(-h,h);

      // Cas spécial pour les arêtes
      if (x == 0) z[n-1][y] = a;
      if (y == 0) z[x][n-1] = a;
    }
  }
}
```

J. Lewis. Generalized stochastic subdivision. *ACM Transactions on Graphics*. **6**(3), 167–190, 1987.

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Function representation

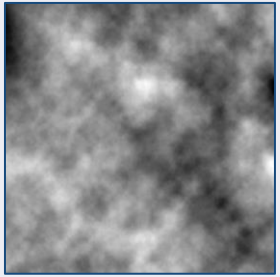
Global

Combination of scaled noise (frequency and amplitude)

$$h(\mathbf{p}) = \sum_{k=0}^{o-1} a_k n(f_k\,\mathbf{p})$$

Decreasing amplitude $a_k = 1/\alpha^k$

Increasing frequency $f_k = \varphi^k$



http://www.joshushund.com

Ebert *et al*. Texturing and Modeling: A Procedural Approach. *Academic Press Professional*, 1998.

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Global basis functions

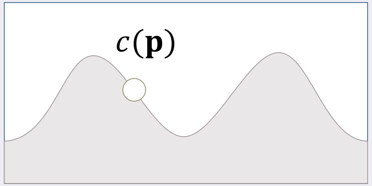
Trigonometric functions

Base **sine** or **cosine** $\mathbf{R}^2 \to [-1,1]$

Scaled cosine c characterized by amplitude and wavelength

$$c(\mathbf{p}) = a\cos(\mathbf{p}/w)$$

Amplitude Wavelength



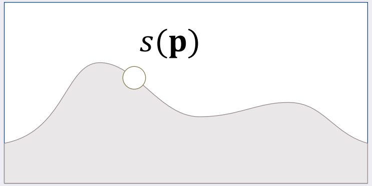
Noise

Base **noise functions** $n\colon \mathbf{R}^2 \to [-1,1]$

Scaled noise characterized by amplitude and wavelength

$$s(\mathbf{p}) = a\, n(\mathbf{p}/w)$$

Amplitude Wavelength



Ebert *et al*. Texturing and Modeling: A Procedural Approach. *Academic Press Professional*, 1998.

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Other basis functions

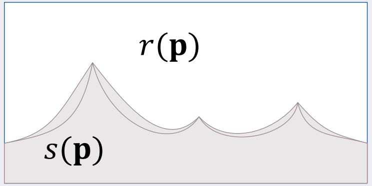
Modified noise functions

Basis function enhanced to generate ridges

$$\mathbf{r}(\mathbf{p}) = 2(1 - |n(\mathbf{p})|) - 1 = 1 - 2|n(\mathbf{p})|$$

Ridge noise

Noise



$$s(\mathbf{p}) = 2(1 - |n(\mathbf{p})|)^2 - 1$$

Sharpened ridge noise

$$m(\mathbf{p}) = \min_i(n, n \circ t_i)$$

Intersection ridge noise

t_i is a affine transformation

S. Worley. A cellular texture basis function. In Proceedings of SIGGRAPH ’96, 291–294, 1996

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More complex functions

Fractional Brownian motion

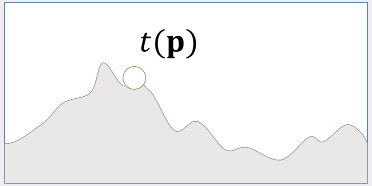
Combination of **noise functions** $\mathrm{n}: \mathbf{R}^2 \rightarrow [-1,1]$ to obtain fractal Brownian motion ***fBm***

Noises are computed using transformed points to avoid grid artefacts

$$h(\mathbf{p}) = t(\mathbf{p}) = \sum_{k=0}^{n} a_k n(\mathbf{T}_k(\mathbf{p})/w_k)$$

Turbulence

Transformed point



In general, $a_k = a_0 2^{-k}$ and $w_k = w_0 2^{-k}$

Lacunarity [Ebert1998]

$$\mathbf{T}_k(\mathbf{p}) = \mathbf{R}_k \mathbf{p} + \mathbf{o}_k$$

Random offset

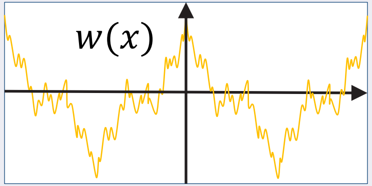
Random rotation matrix $\mathbf{R}_k = \mathbf{R}_0^k$

Note

Le value noise n ressemble à la fonction de Weierstrass (1872)

Avec des nombres aléatoires dans $\{-1,1\}$ et une interpolation en cosinus :

$$n \equiv \sum_{i=1}^{\infty} \frac{1}{a^i} \cos\left(b^i \pi x\right)$$



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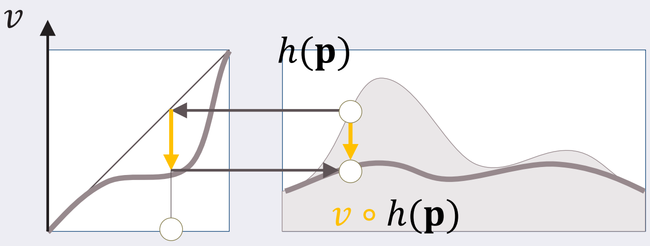
Deformations

Vertical deformations

Any function $v: \mathbf{R} \rightarrow \mathbf{R}$ that modifies the elevation



$$\tilde{h} = v \circ h$$

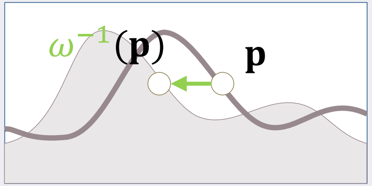
Deformation applied to the elevation

Horizontal deformations

Any warping $\omega^{-1}: \mathbf{R}^2 \rightarrow \mathbf{R}^2$ can be used to introduce small perturbations

$$\tilde{h} = h \circ \omega^{-1}$$

Space deformation applied to the point



$$\omega^{-1}(\mathbf{p}) = \mathbf{p} - \mathbf{t}$$

Translation

$$\omega^{-1}(\mathbf{p}) = \mathbf{p} + n(\mathbf{p})$$

Noise displacement

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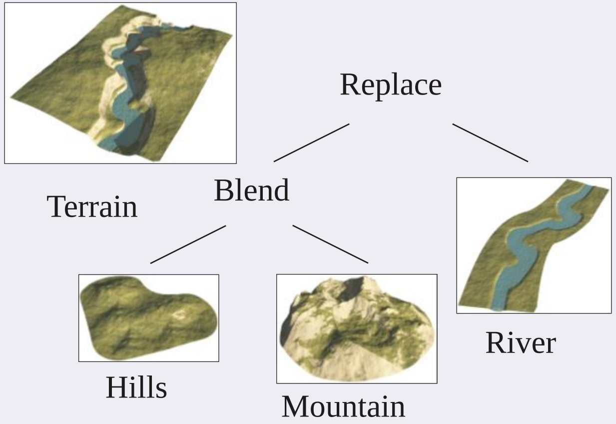
Procedural Authoring of Height Fields

Construction tree

Local function representation

- **Hierarchical** representations using primitives organized in a tree [Genevaux 2015]
- Sparse combination of **landforms**
- Combination of an elevation functions $h: \mathbf{R}^2 \rightarrow \mathbf{R}$ with a weighting function $\alpha: \mathbf{R}^2 \rightarrow [0,1]$

Control the influence of the primitives



Génevaux *et al*. Terrain modeling from feature primitives. *Computer Graphics Forum*, 2015.
Guérin *et al*. Sparse representation of terrains for procedural modeling. *Computer Graphics Forum*, **35**, 2, 2016

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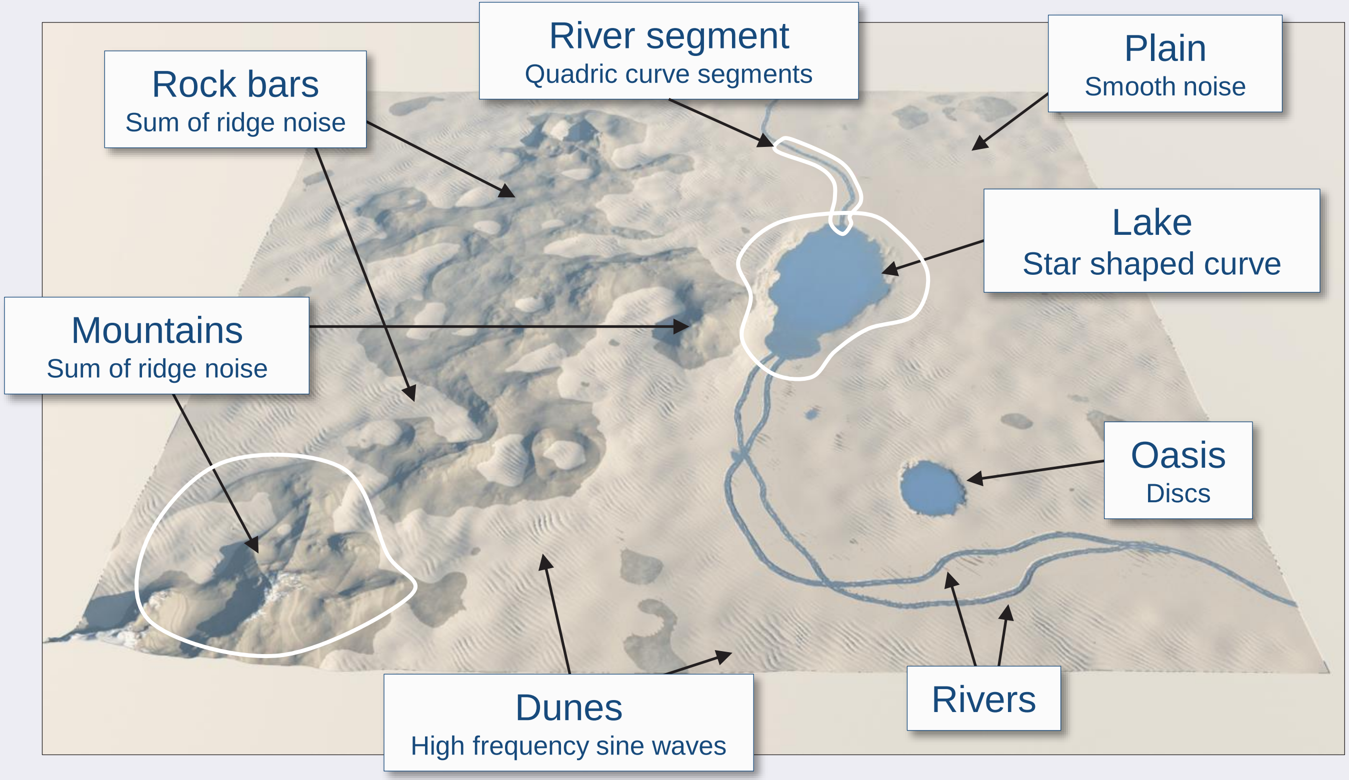
Control

Vector model

Terrain representation is function based, independent of any resolution



Rock bars
Sum of ridge noise

River segment
Quadric curve segments

Plain
Smooth noise

Lake
Star shaped curve

Mountains
Sum of ridge noise

Oasis
Discs

Dunes
High frequency sine waves

Rivers

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Fundamentals

Local function representation

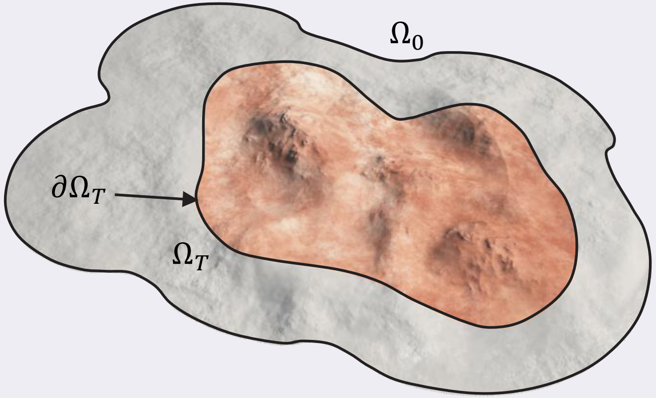
Nodes define two functions $h\colon \mathbf{R}^2 \rightarrow \mathbf{R}$ and $\alpha\colon \mathbf{R}^2 \rightarrow \mathbf{R}^+$

$$\boxed{\Omega_0 = \{\mathbf{p} \in \mathbf{R}^2, \alpha(\mathbf{p}) > 0\}}$$

Compact support of the primitive

$$\boxed{\Omega_T = \{\mathbf{p} \in R^2, \alpha(\mathbf{p}) > T\}}$$

Domain where the terrain is defined



Regions of influence allow for:
- **Continuity** of the terrain
- **Compact** support
- Local **Lipschitz** property

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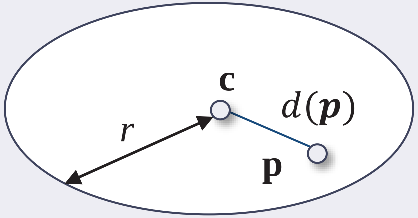
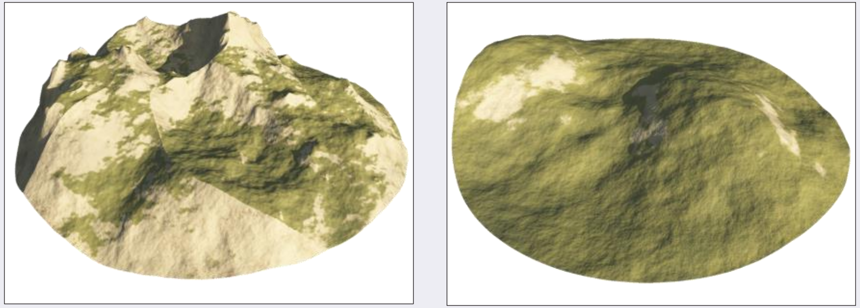
Disc primitive

Mountain and hills over a disc

- Elevation $\mathrm{h}(\mathbf{p})$ as a combination of noises with amplitude and wavelength
- Domain of influence over a disc $\alpha(\mathbf{p}) = g \circ |\mathbf{p} - \mathbf{c}|$

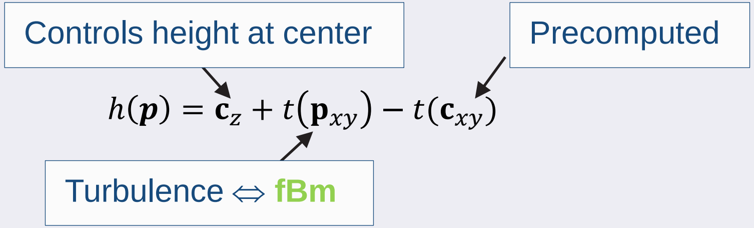




$g(x) = \left(1 - \frac{x^2}{r^2}\right)^3$ if $x < r$ and 0 otherwise

Implementation details

- Elevation function



Controls height at center

Precomputed

$$h(\boldsymbol{p}) = \mathbf{c}_z + t(\mathbf{p}_{xy}) - t(\mathbf{c}_{xy})$$

Turbulence ⇔ fBm

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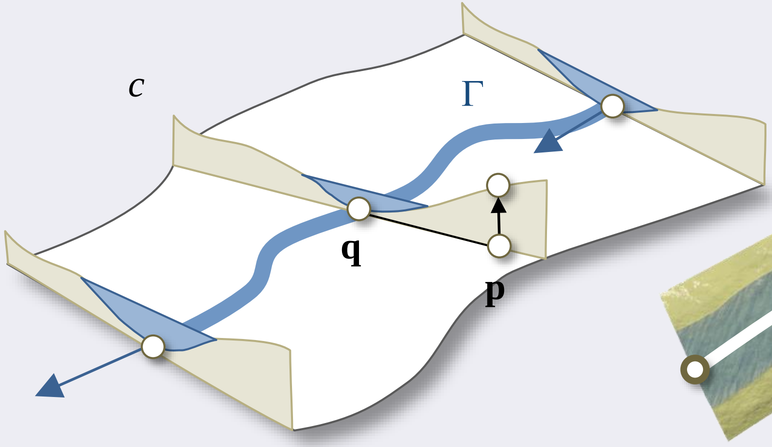
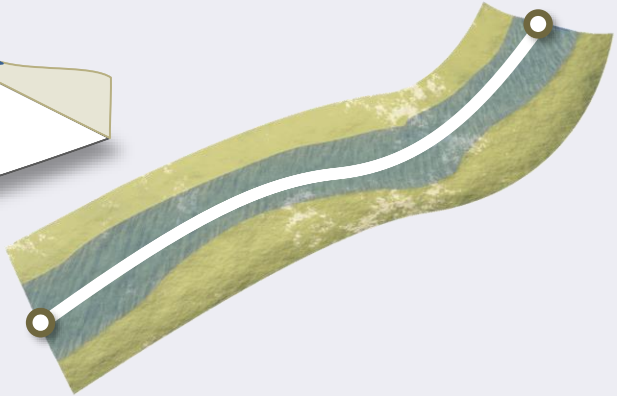
Curve primitives

River segments

Elevation $\mathrm{h}(\mathbf{p})$: combination of cross section profile along a curve
Domain of influence around curve $\alpha(\mathbf{p}) = g \circ d(\mathbf{p}, \boldsymbol{\Gamma})$





Compute the projection $\mathbf{q} = \pi_\Gamma(\mathbf{p})$ of $\mathbf{p}$ on the curve
Elevation is defined as $\mathrm{h}(\mathbf{p}) = \mathbf{q}_z + c \circ d(\mathbf{p}, \boldsymbol{\Gamma})$

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Blending

Blending two primitives with their own domain

Aggregation of landforms, yields new elevation with new domain

Elevation $h = (\alpha_a h_a + \alpha_b h_b)/(\alpha_a + \alpha_b)$

New influence $\alpha = \alpha_a + \alpha_b$


Mountain Hill Smooth blending

The contour $\partial\Omega_T$ of the new domain is the contour of the implicit equation $\alpha - T = 0$

$$\partial\Omega_T = \{\mathbf{p} \in R^2, \alpha(\mathbf{p}) = \mathrm{T}\}$$

Challenge: compute the Lipschitz constant of h inside Ω_T

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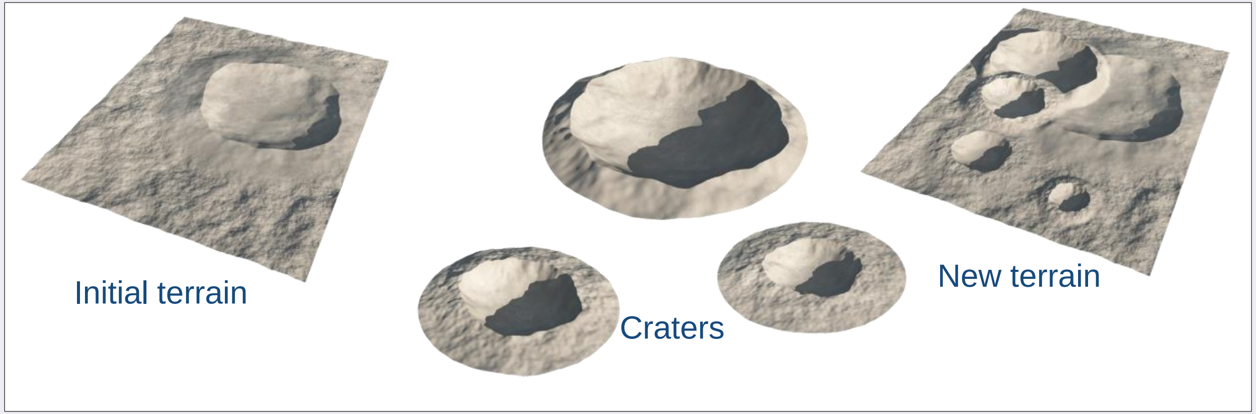
Replacement operator

Replace a part of a terrain with another one

- Asymmetric operator
- Elevation $h = (1 - \alpha_b)h_a + \alpha_b h_b$
- Preserve influence of the left argument $\alpha = \alpha_a$



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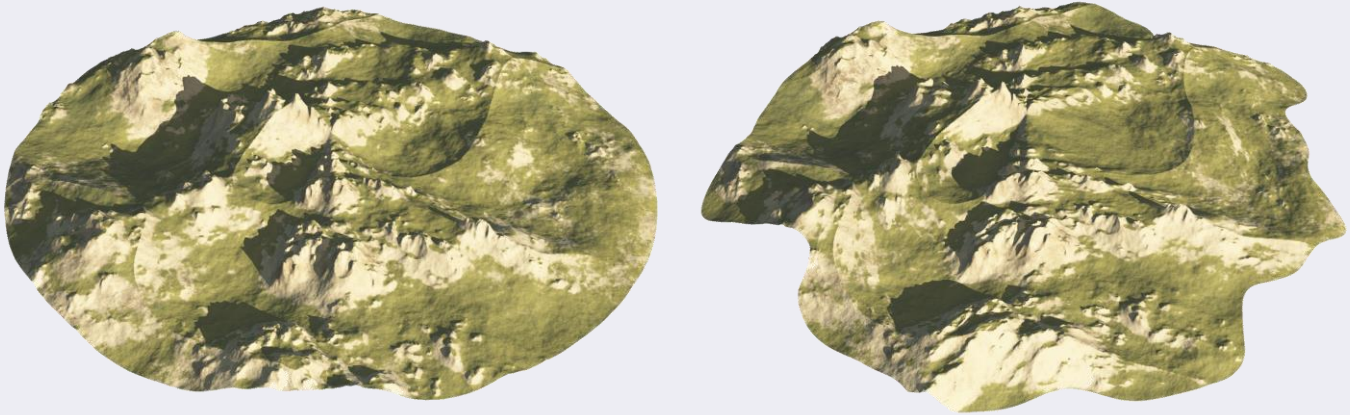
Warping operators

Deformation of space

- Any deformation $\omega^{-1}: \mathbf{R}^2 \rightarrow \mathbf{R}^2$ can be used as warping
- Elevation $h = h_a \circ \omega^{-1}$ and coefficient $\alpha = \alpha_a \circ \omega^{-1}$



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Scenery

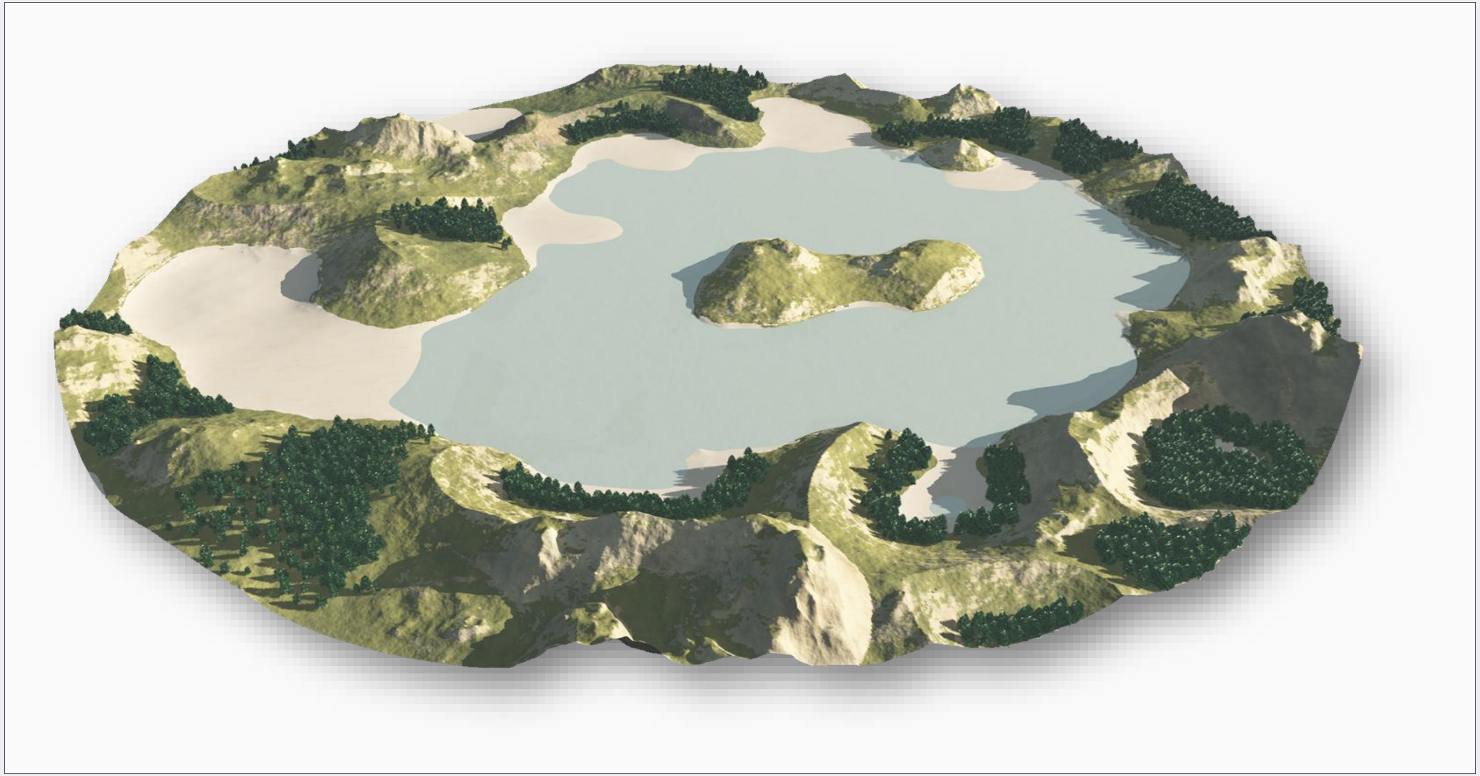
From example

Procedural

Local primitives

Implicit modeling

Appendix



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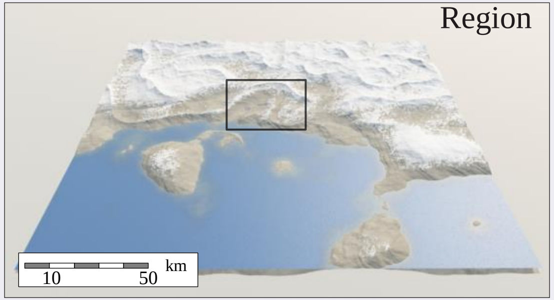
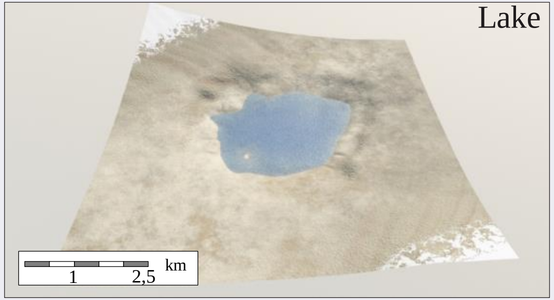
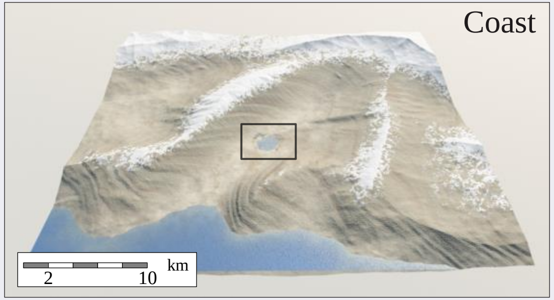
Scenery

Large scenery

- Function representation
- $200 \times 200\,\mathrm{km}^2$ with specific localized details
- Blended primitives authored manually
- Tree size **81 kB**


Region


Lake


Coast

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Procedural Modeling of Volumetric Landforms

Implicit modeling

Implicit surface representation

Elevation functions can be easily converted to an implicit form
Scalar field representation

Elevation function
$z = h(x,y)$

→

Scalar field function
$f(x,y,z) = z - h(x,y)$

f is **not** a distance bound

Distance bound

Recall that if f is $\lambda -$ Lipschitz, then f/λ is a distance bound to the surface [Hart1995]

$$\lambda_f = \sqrt{1+\lambda^2}$$

λ is the Lipschitz bound of h

This global bound can be optimized when evaluating along a ray [Galin2020]

f is a distance bound →

Scalar field function
$f(x,y,z) = (z - h(x,y))/\lambda_f$

J. Hart. Sphere Tracing. The Visual Computer 12(10), 1995
E. Galin, E. Guérin, A. Paris, A. Peytavie. Segment Tracing Using Local Lipschitz Bounds. *Computer Graphics Forum*, **39**(2), 2020.

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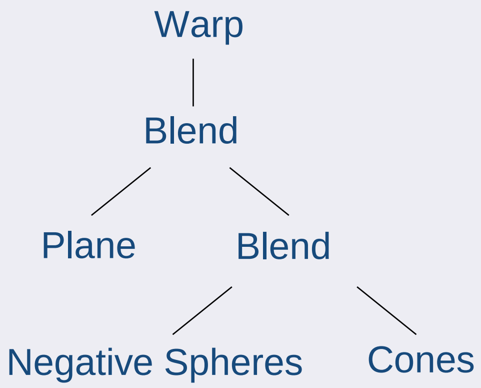
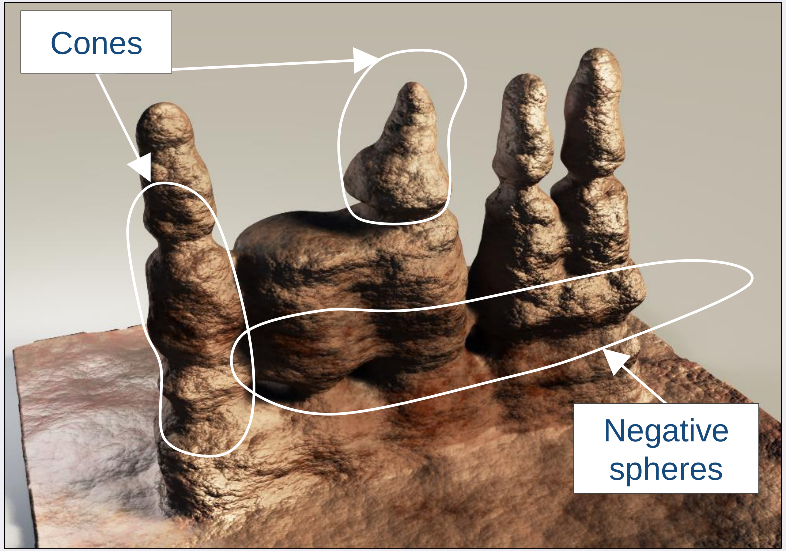
Construction tree

Scalar field

Volumetric primitives with roughness created from spheres, curves [Paris2019]
Operators : blending, intersection, difference, warping [Wyvil1999]


Warp
Blend
Plane
Blend
Negative Spheres
Cones



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B. Wyvill, A. Guy, E. Galin, The Blob Tree. *Computer Graphics Forum* ***18****(2)* 1999
A. Paris, E. Galin, A. Peytavie, E. Guérin, J. Gain. Terrain Amplification with Implicit 3D Features. *ACM Transactions on Graphics*, **38**(5), 2019

Primitives with details

Roughness

Enhance smooth primitives with noise

Adding noise may cause holes or floating elements

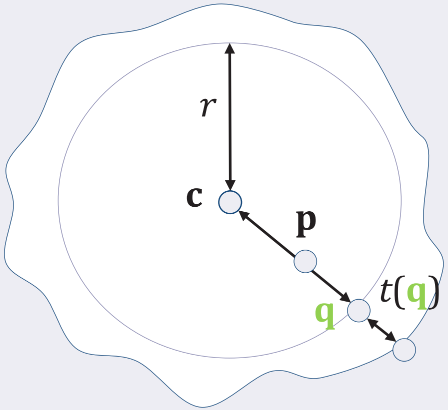
If noise if too intense

Star shaped primitive

$$\mathbf{q} = \mathbf{c} + r\,\frac{\mathbf{p} - \mathbf{c}}{\|\mathbf{p} - \mathbf{c}\|}$$

$$f(\mathbf{p}) = \|\mathbf{p} - \mathbf{c}\| - (r + t(\mathbf{q}))$$

Turbulence



Point skeleton

Lipschitz bound

Formally, f should be $1-$Lipschitz

Computation the Lipschitz bound

$$\lambda = 1 + \lambda_t$$

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Instantiation

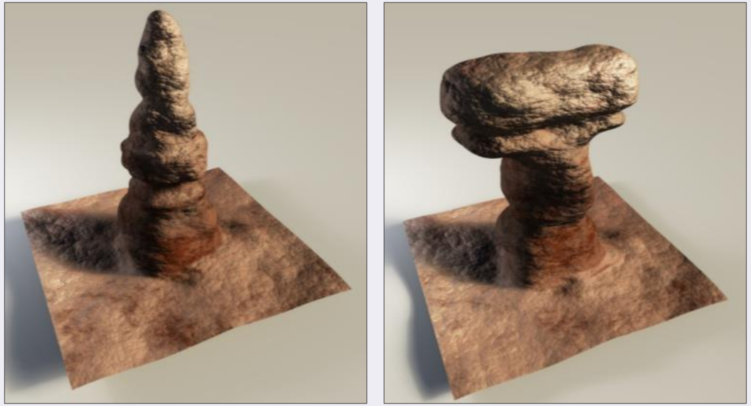
Blocks

Aggregate primitives to create an atlas $A = \{a_i\}$ of complex shapes
Hierarchically reuse shapes


Box **d**


Block **b**


Cone **c**



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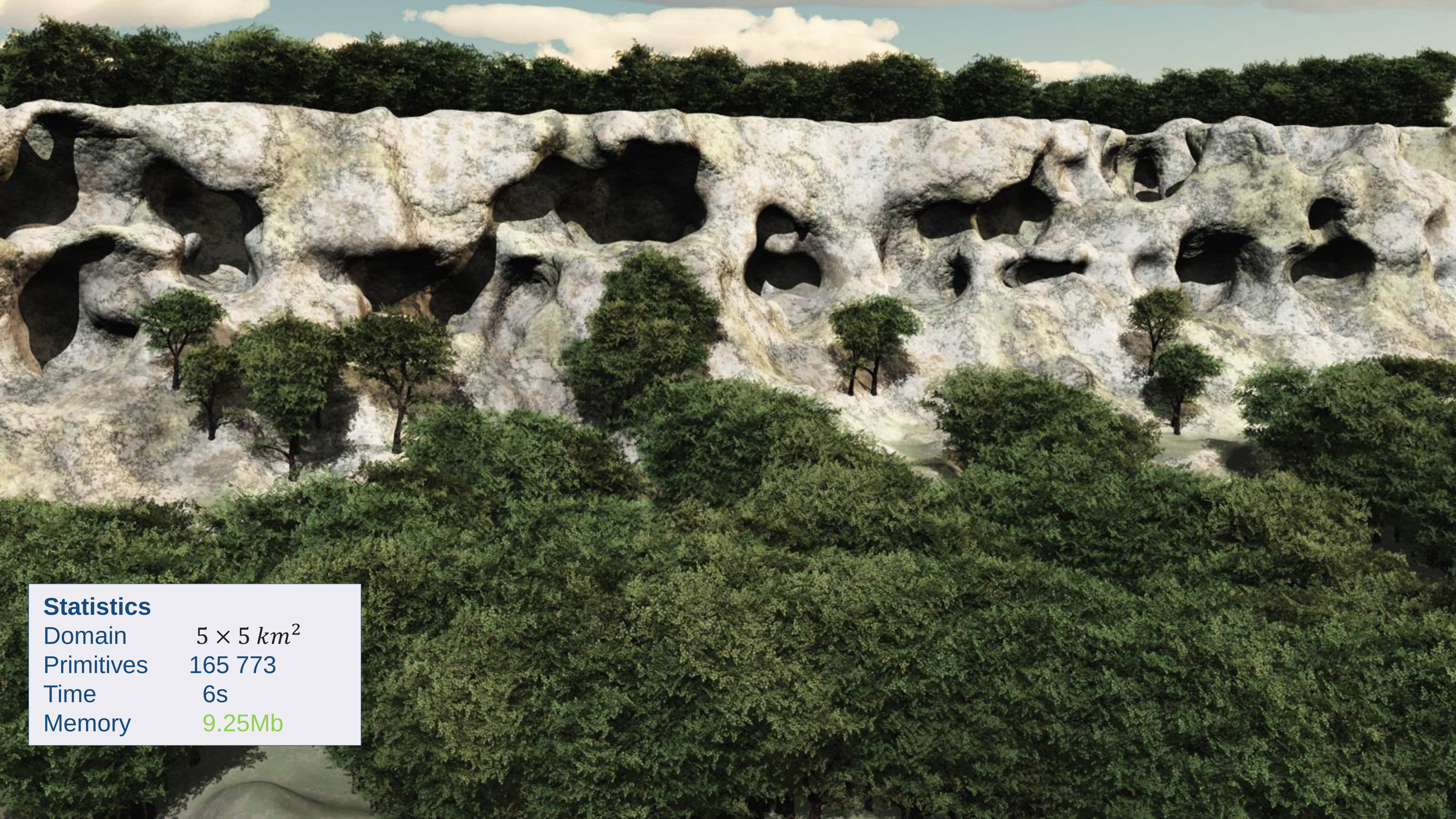
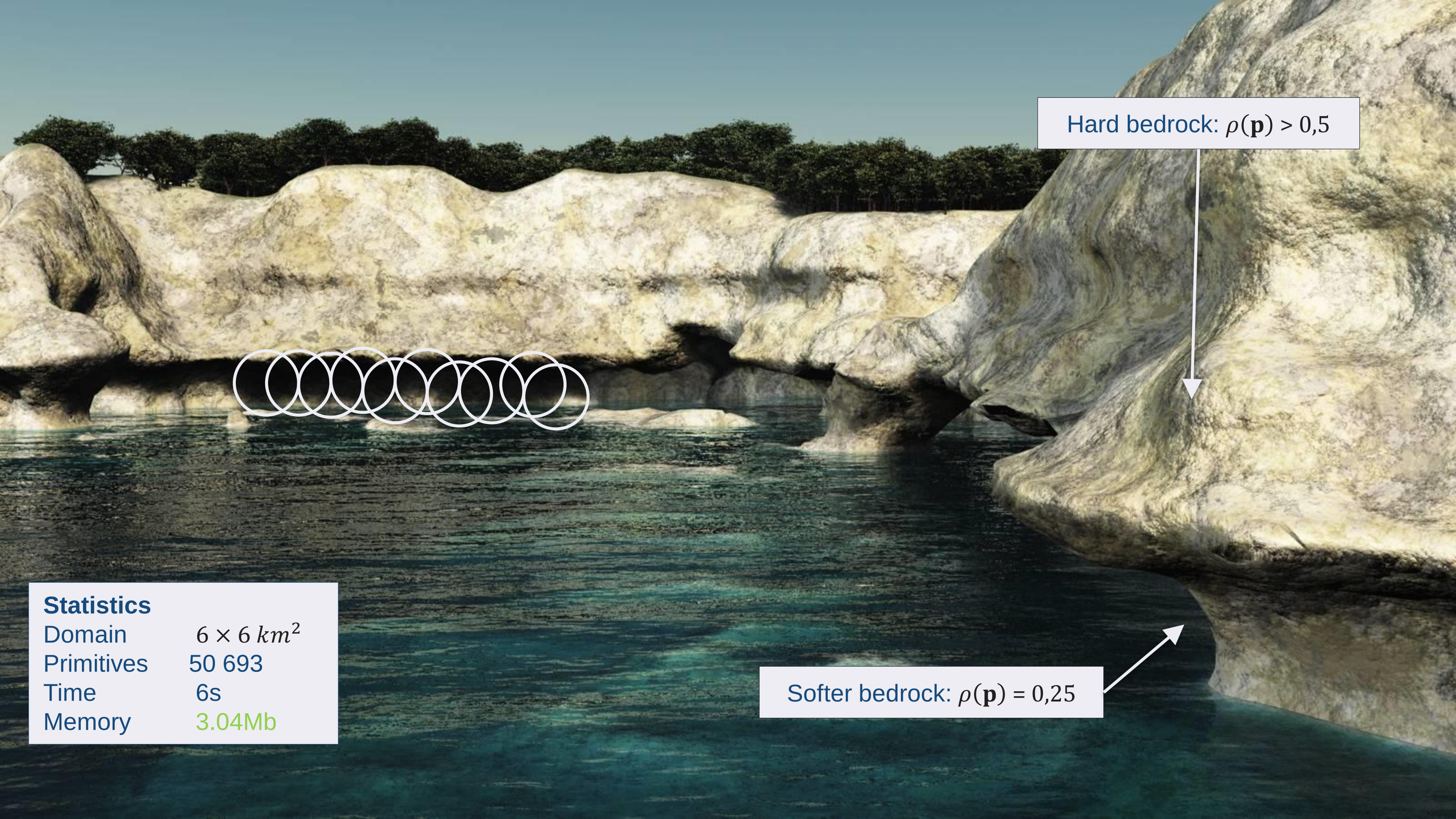


Statistics

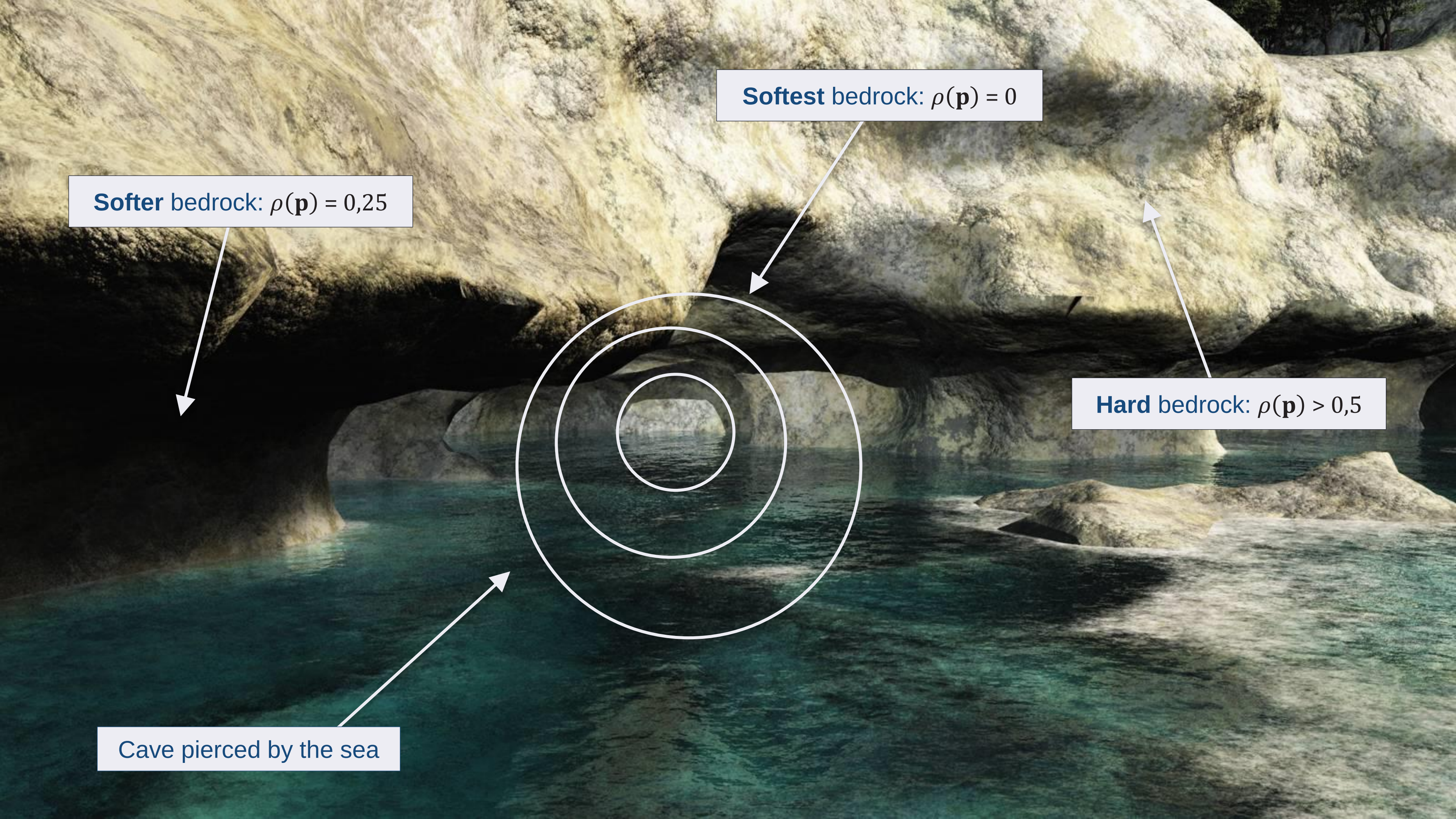
Domain	$5 \times 5\ km^2$
Primitives	165 773
Time	6s
Memory	9.25Mb



Hard bedrock: $\rho(\mathbf{p}) > 0{,}5$

Softer bedrock: $\rho(\mathbf{p}) = 0{,}25$

Statistics	
Domain	$6 \times 6\ km^2$
Primitives	50 693
Time	6s
Memory	3.04Mb



Softest bedrock: $\rho(\mathbf{p}) = 0$

Softer bedrock: $\rho(\mathbf{p}) = 0{,}25$

Hard bedrock: $\rho(\mathbf{p}) > 0{,}5$

Cave pierced by the sea

Supplementary material

Conclusion

Height fields and layered height fields

- **Conspicuous** in terrain modeling
- **Versatile** for a variety of generation methods

Function-based models

- Useful for modeling some **specific landforms**
- Modeling large landscapes with a high resolution

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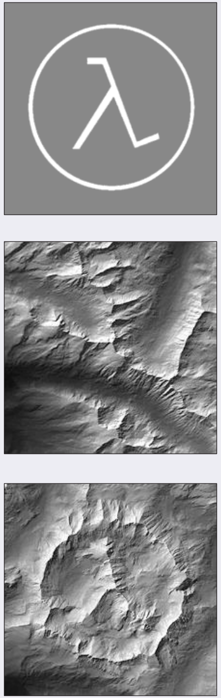
Synthesis from example

Patch matching

Création

Recherche et combinaison de motifs [Zhou2007]
Similaire aux méthodes de synthèse de texture



H. Zhou, J. Sun, G. Turk, J. Rehg. Terrain Synthesis from Digital Elevation Models, *IEEE Transactions on Visualization and Computer Graphics*, **13** (4), 834-848, 2007

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