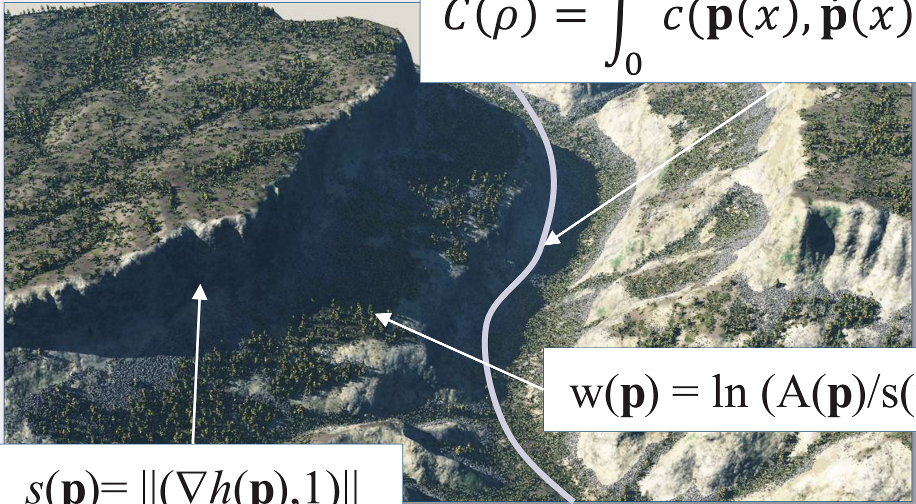
Digital World Modeling

From mathematics …



… to the screen

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Digital World Modeling

Data Structures

Procedural Modeling

Erosion Simulation

Procedural Road Generation

Vegetation and Ecosystems

Growth models

Aging and weathering

Diffusion Limited Aggregation

Diffusion Limited Aggregation

Fundamentals

Diffusion **L**imited **A**ggregation as a growth process

Could start with a set of seeds

Define an particle as a starting cluster $\mathrm{C}_0 = \{\mathbf{p}_0\}$
Launch a new particle $\mathbf{p}_k$ far away, move randomly
If $\mathbf{p}_k \cap \mathrm{C}_{\mathrm{k}-1} \neq \emptyset$, aggregate $\mathrm{C}_{\mathrm{k}} = \mathrm{C}_{\mathrm{k}-1} \cup \{\mathbf{p}_k\}$

Brownian motion

Collision detection

Statistics for aggregation

Improvements

Conditional aggregation : $\mathbf{p}_k \cap \mathrm{C}_{\mathrm{k}-1} \neq \emptyset$,
and conditions on $\mathrm{C}_{\mathrm{k}-1}$ and $f\big(B(\mathbf{p}_k, r)\big)$ are met

Function defining conditions

Neighborhood

Condition: few particles in the neighborhood

$$\#\big(\mathrm{C} \cap B(\mathbf{p}_k, r)\big) < \mathrm{N}$$

T. Witten, L. Sanders. Diffusion-limited aggregation, a kinetic critical phenomenon. Physical Review Letters 47 (1981)

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Optimization

Fundamentals

Global $O(n^2)$ complexity
Collision detection between particles is the most computationally intensive step

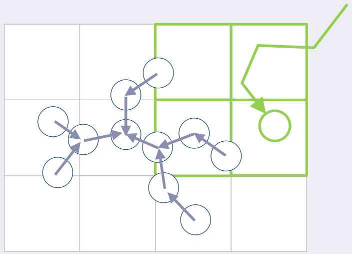
Define an particle as a starting cluster $C_0 = \{\mathbf{p}_0\}$
Launch a new particle $\mathbf{p}_k$ far away, move randomly
If $\mathbf{p}_k \cap C_{k-1} \neq \emptyset$, aggregate $C_k = C_{k-1} \cup \{\mathbf{p}_k\}$

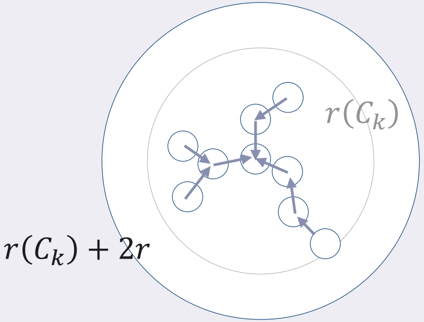
1 Escape radius

2 Avoid $O(n)$ per step with acceleration data structure

Accelerations

Seed radius $s = r(C_k) + 2r$ and escape radius $e = r(C_k) + 4r$
Loose grid reduce collision detection complexity to $O(1)$





Poisson Disc tiles [Lagae 2006] with precomputed connectivity could be used

Thesis

A. Lagae, P. Dutré. An Alternative for Wang Tiles: Colored Edges versus Colored Corners. *ACM Transactions on Graphics*. **4**(25), 2006.

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Results

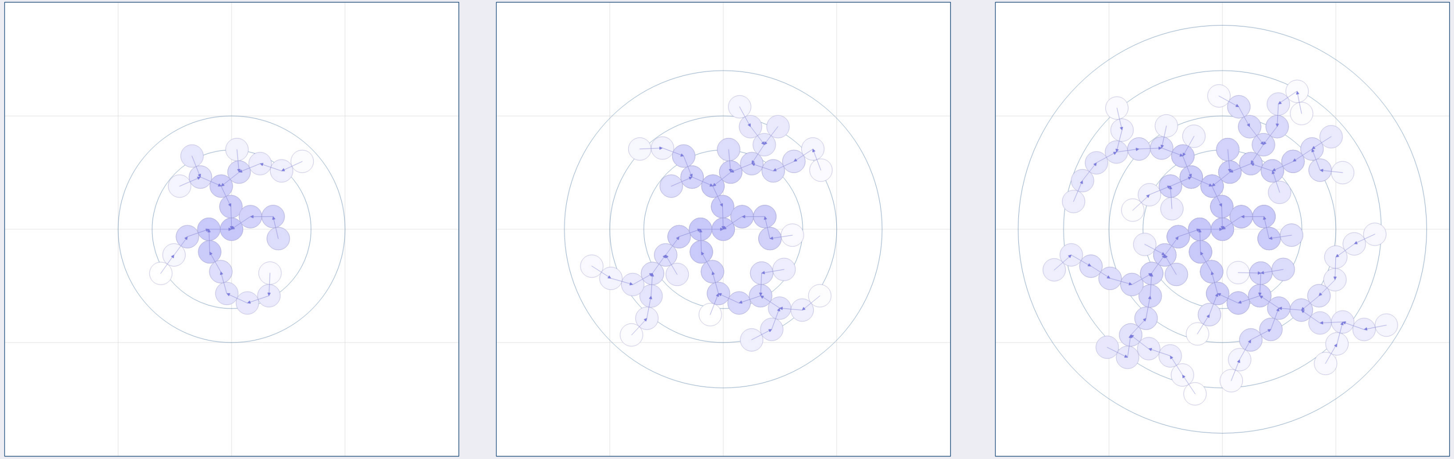
Aggregation

Growth

Invasion percolation

Growth

Model with 50, 100, and 200 particles



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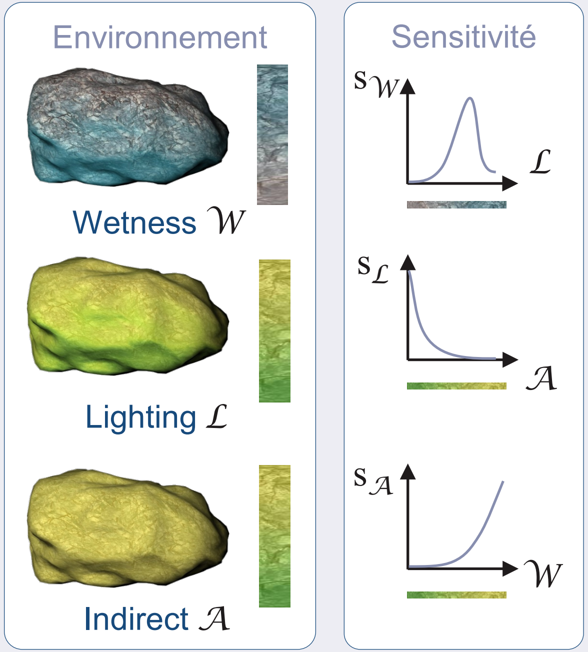
Lichen growth simulation

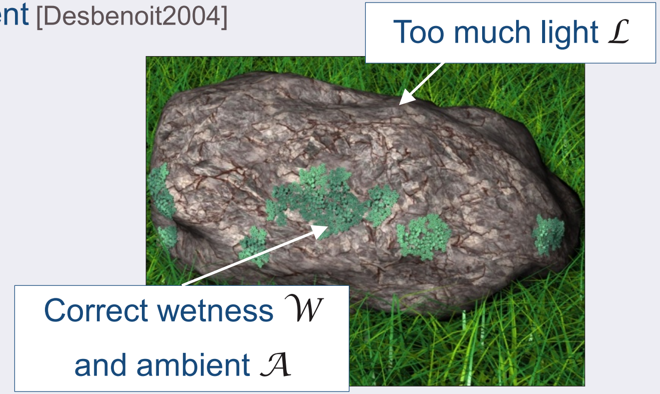
Lichens

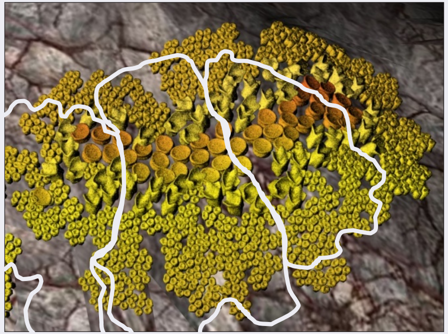
Open **D**iffusion **L**imited **A**ggregation

Agrégation fonction des caractéristiques de l'environnement [Desbenoit2004]

$$p\big(B(\mathbf{p}_k, r)\big) = \min(s_W \circ W, s_L \circ L, s_A \circ A)$$







B. Desbenoit, E. Galin and S. Akkouche, Simulating and Modeling Lichen Growth, *Computer Graphics Forum*, **23**(3), 2004

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Aggregation
Growth
Invasion percolation

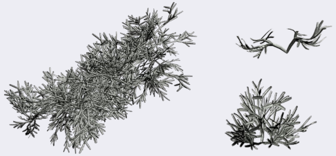
Lichens

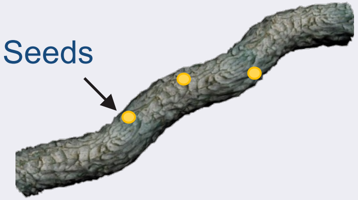
Croissance selon un algorithme de type **D**iffusion **L**imited **A**ggregation
Instanciation de modèles maillés texturés

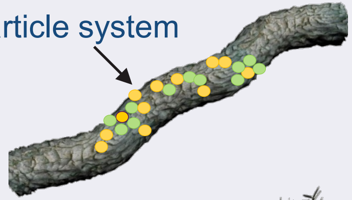
Mesh → **1. Seeding** Initialize particles
Accessibility α → **1. Seeding**

2. Propagation Open **DLA**, Particles system
Light $\mathcal{L}$, Wetness $\mathcal{W}$ → **2. Propagation**

3. Lichen creation Instantiation → Lichen model
Lichen atlas → **3. Lichen creation**







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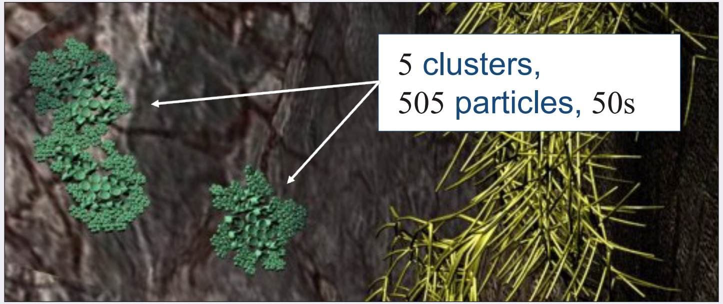
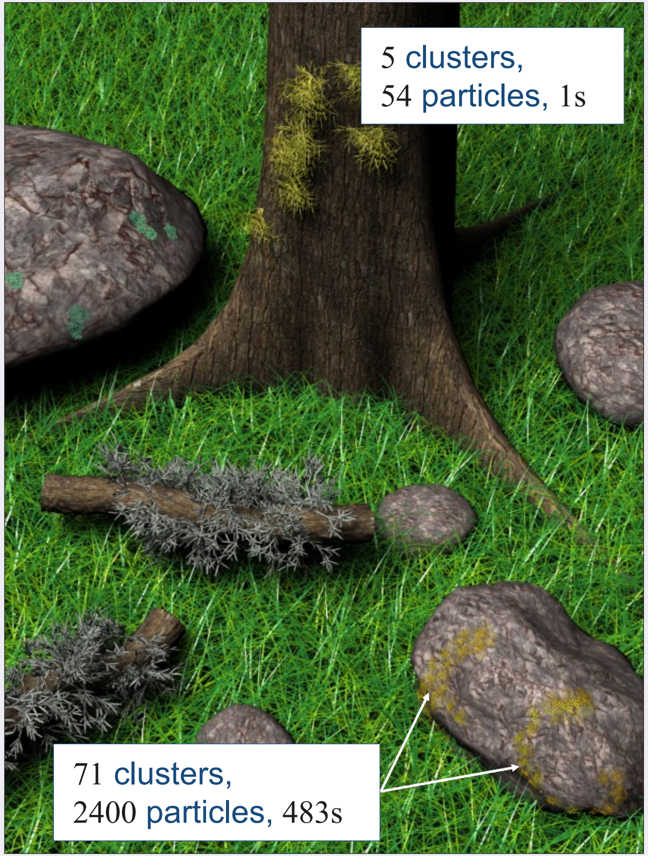
Lichens

Aggregation
Growth
Invasion percolation





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Eden Growth

Eden Growth

Aggregation
Growth
Invasion percolation

Fundamentals

Cluster growth by random accumulation of material on their boundary [Eden1961]

Could start with a set of seeds

Define an particle as a starting cluster $\mathrm{C}_0 = \{\mathbf{p}_0\}$
Randomly select a particle $\mathbf{p}_j$ from $\mathrm{C_k}$
Grow in a random direction such that $\mathbf{p}_{k+1} \cap \mathrm{C_k} \neq \emptyset$

$\mathbf{p}_{k+1}$ from $\mathbf{p}_j$

Collision detection
possibly $O(n)$ complexity

Improvements

Conditional growth according to a probability (environment) $f\big(B(\mathbf{p}_k, r)\big)$

M. Eden. A two-dimensional growth process. *Proceedings of Fourth Berkeley Symposium on Mathematics, Statistics, and Probability*. **4**. Berkeley: University of California Press. 223–239, 1961.

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Optimization

- Aggregation
- Growth
- Invasion percolation

Analysis

Global $O(n^2)$ complexity

Collision detection between particles is the most computationally intensive step

Define an particle as a starting cluster $\mathrm{C}_0 = \{\mathbf{p}_0\}$
Randomly select a particle $\mathbf{p}_j$ from C_k
Grow in a random direction such that $\mathbf{p}_j \cap \mathrm{C}_{\mathrm{k}-1} \neq \emptyset$

2 Keep track of free locations around cells

1 Avoid $O(n)$ per step with acceleration data structure

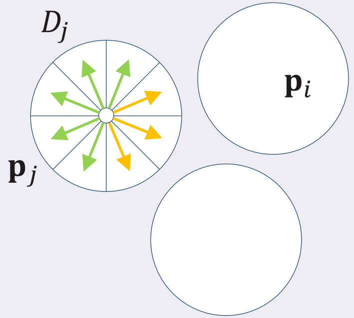
Accessibility for particles

Divide the neighborhood around a particle $\mathbf{p}_j$ in $s > 6$ candidate directions D_j with angle span $2\pi/s$

During growth step from $\mathbf{p}_j$, select direction in the candidate directions set D_j

After selection, update candidate directions D_i of particles $\mathbf{p}_i$ intersecting $\mathbf{p}_j$

A particle is alive if it still has a non empty set of directions $D_j \neq \emptyset$



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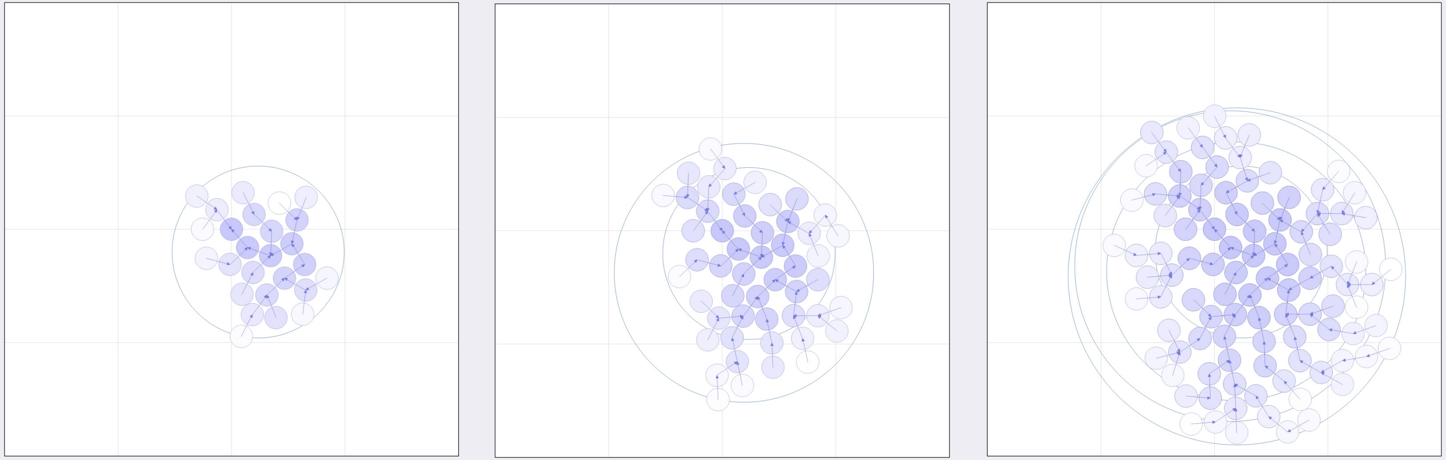
Results

Growth

Model with 25, 50, and 100 particles



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Invasion percolation

Invasion percolation

Aggregation
Growth
Invasion percolation

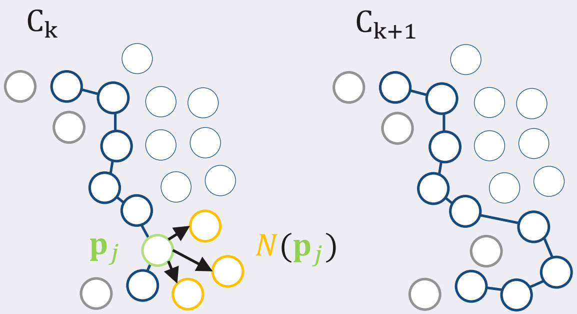
Fundamentals

Mathematical model for slow immiscible fluid invasion in porous medium [Wilkinson 1961]
Updates an evolving front C of sites according to a resistance function ρ

Deterministic

Define a set as a starting front (cluster) $\mathrm{C}_0 = \{\mathbf{p}_0\}$
Select a site $\mathbf{p}_j$ from C_k with the least resistance $\rho(\mathbf{p}_j)$
Mark $\mathbf{p}_j$ as visited
Remove $\mathbf{p}_j$ and update the front $\mathrm{C}_{k+1} = \mathrm{C}_k - \{\mathbf{p}_j\} + N(\mathbf{p}_j)$

Neighboring sites



D. Wilkinson and J, F Willemsen. Invasion percolation: a new form of percolation theory. *Journal of Physics A: Math. Gen*. **16**, 1983

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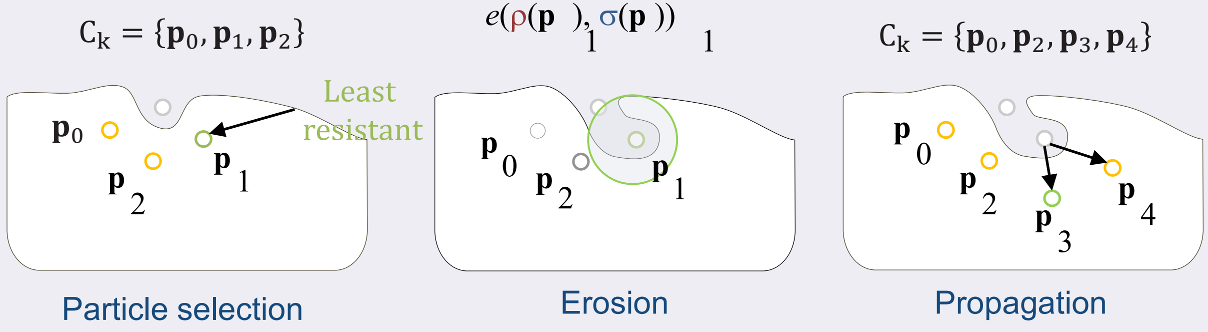

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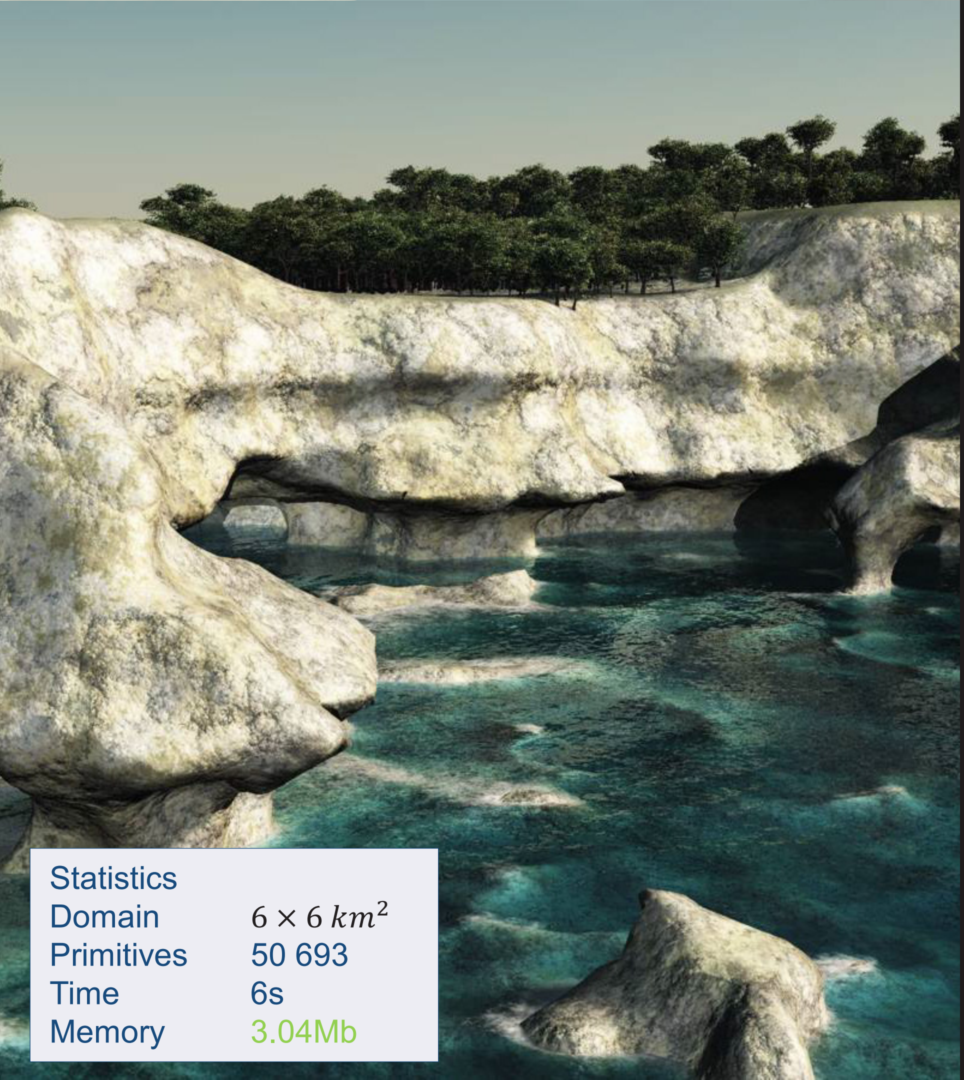
A. Paris, E. Galin, A. Peytavie, E. Guérin, J. Gain. Terrain Amplification with Implicit 3D Features. *ACM Transactions on Graphics*, **38**(5), 2019.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Time 6s
Memory 3.04Mb


Statistics
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Primitives 165 773
Time 6s
Memory 9.25Mb