Digital World Modeling
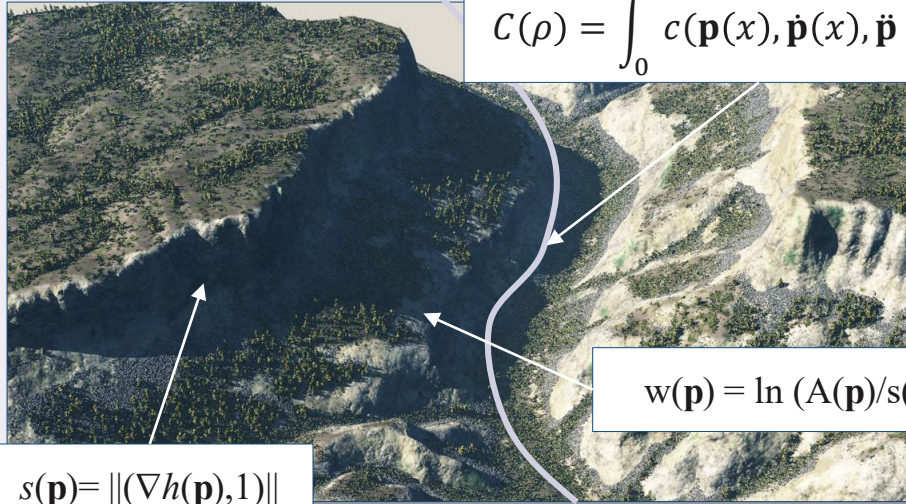
From mathematics …

$$C(\rho) = \int_0^1 c(\mathbf{p}(x), \dot{\mathbf{p}}(x), \ddot{\mathbf{p}}(x))dx$$

$$w(\mathbf{p}) = \ln(A(\mathbf{p})/s(\mathbf{p}))$$

$$s(\mathbf{p}) = \|(\nabla h(\mathbf{p}), 1)\|$$

… to the screen

E. Galin

Université Lyon 1

Digital World Modeling

Data Structures
Procedural Modeling
Erosion Simulation
Procedural Road Generation
Vegetation and Ecosystems
Growth models
Aging and weathering

Verrous scientifiques et techniques

Terrains de très grande dimensions à différentes résolutions

Terrain géologiquement corrects

Prise en compte des différents matériaux

Détails géométriques (rochers, cailloux)

Surplombs

Détails

Techniques de création

Méthodes de génération et de reconstruction

Edition

Simulation

Sculpture interactive

Erosion

Synthèse

Fractales

Procédural

eric.galin@liris.cnrs.fr

http://liris.cnrs.fr/~egalin
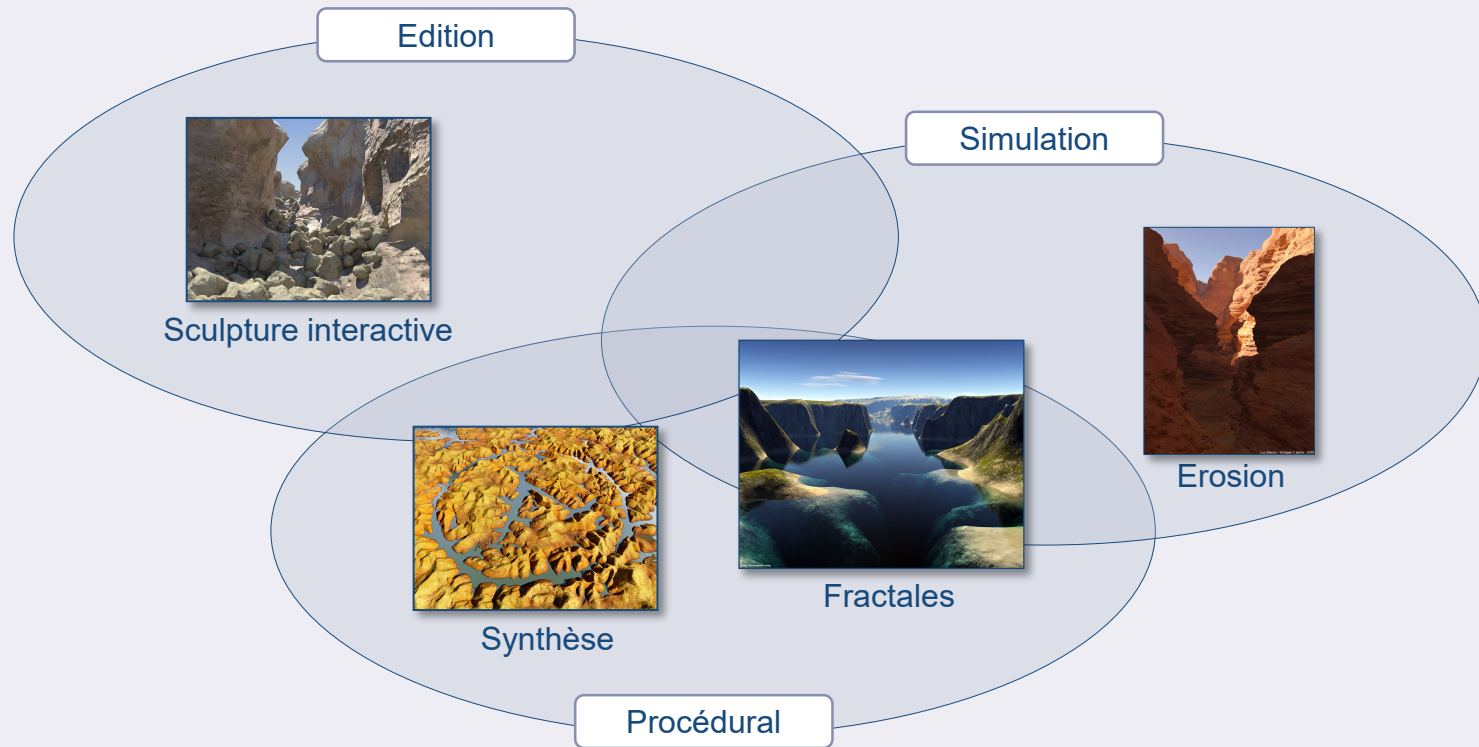
Challenges
Realism
Variety of landforms
Range of scale
Control and authoring

Constraints
Memory
Speed

Phénomènes naturels

Tectonique des plaques, formation des montagnes, érosion
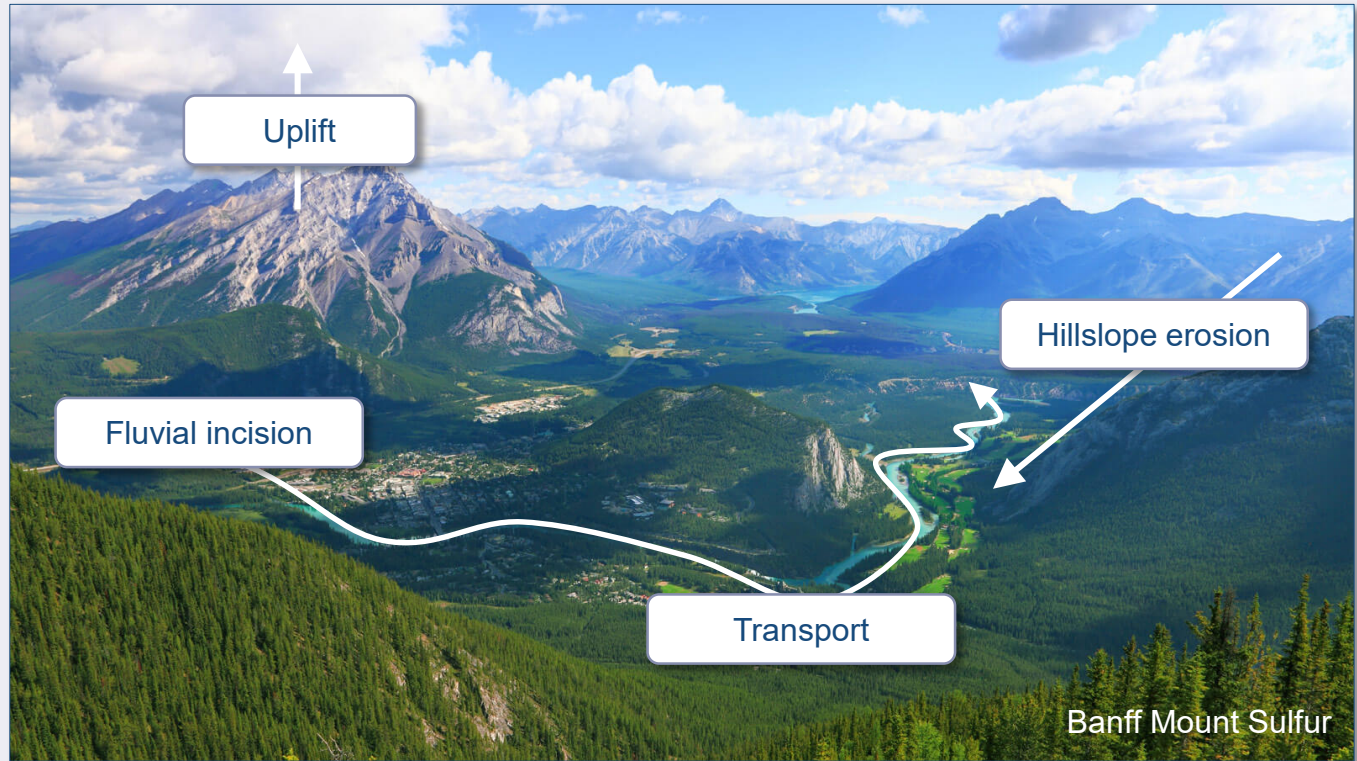
Uplift

Hillslope erosion

Fluvial incision

Transport

Banff Mount Sulfur

eric.galin@liris.cnrs.fr

http://liris.cnrs.fr/~egalin

Classification

Criteria

Categories of landforms
Model
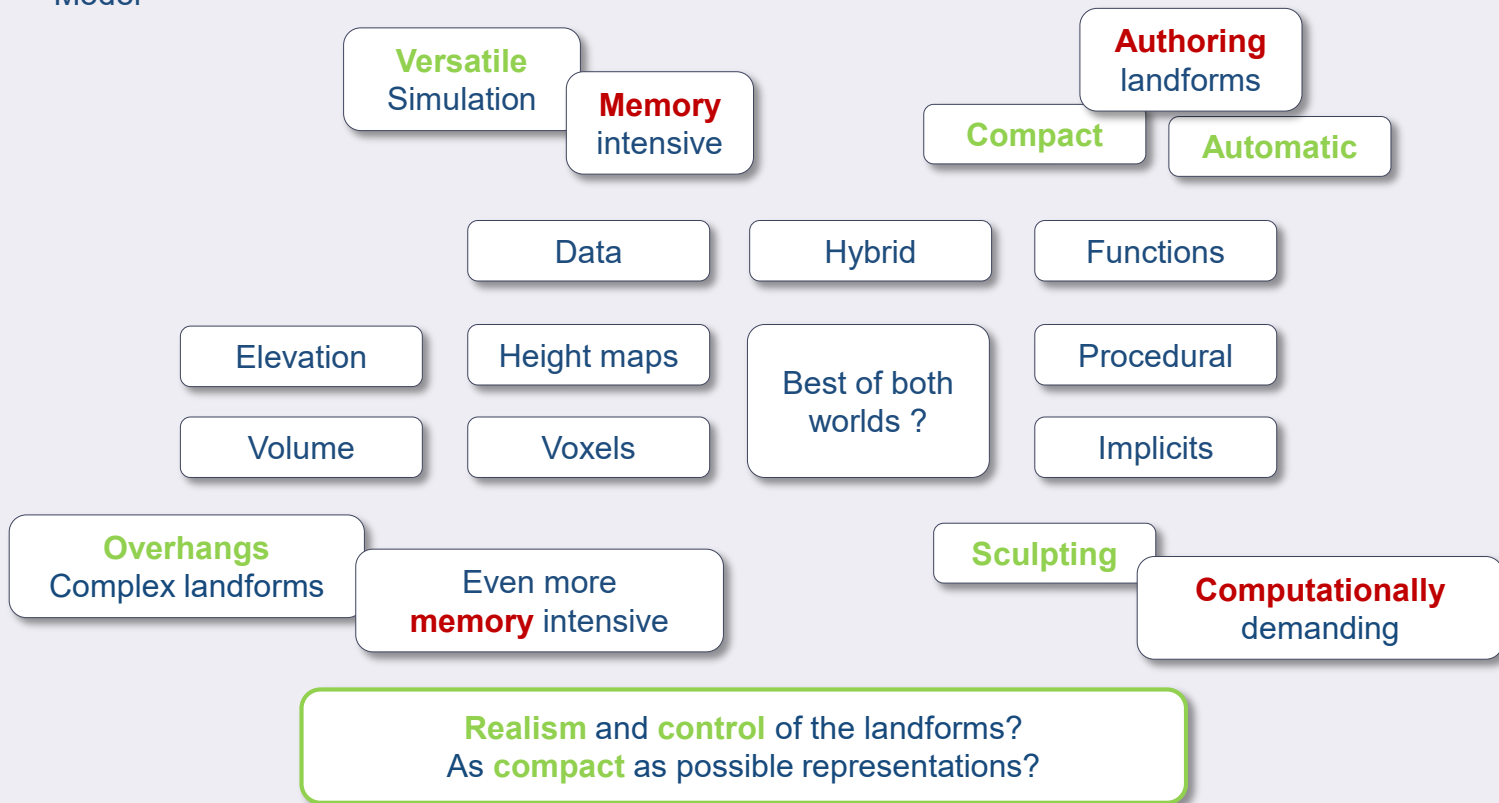
Versatile
Simulation

Memory
intensive

Authoring
landforms

Compact

Automatic

Data

Hybrid

Functions

Elevation

Height maps

Best of both worlds ?

Procedural

Volume

Voxels

Implicits

Overhangs
Complex landforms

Even more
memory intensive

Sculpting

Computationally
demanding

Realism and **control** of the landforms?
As **compact** as possible representations?

eric.galin@liris.cnrs.fr

http://liris.cnrs.fr/~egalin
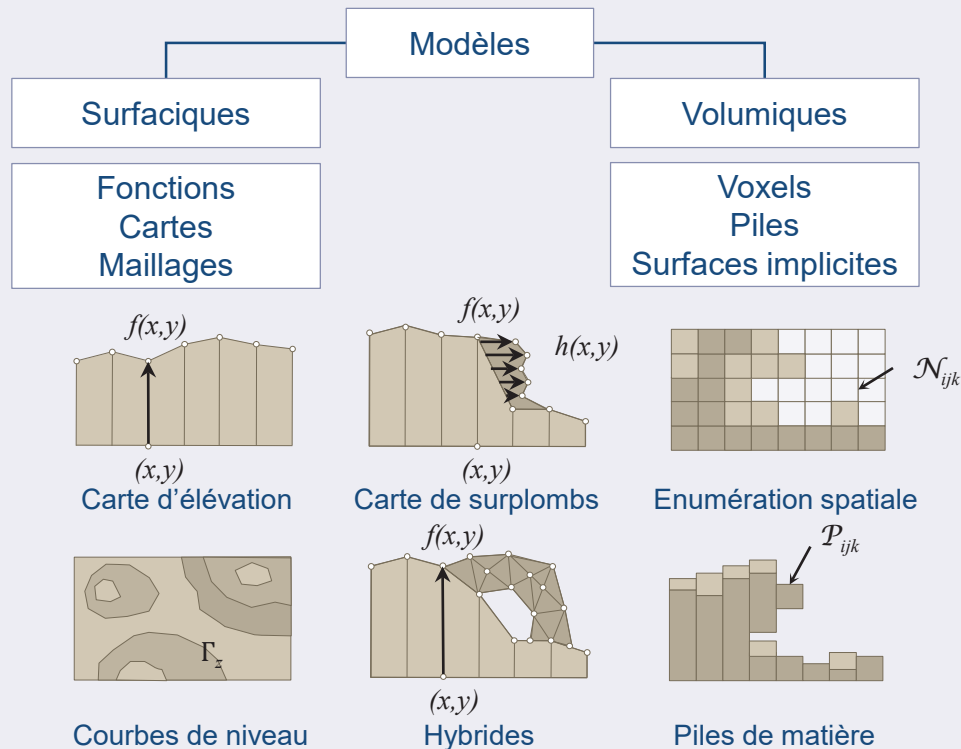
Structure de données

Représentation surfacique : hauteur du relief

Modèles volumiques : différents matériaux en surface et en profondeur

Carte d'élévation

Carte de surplombs

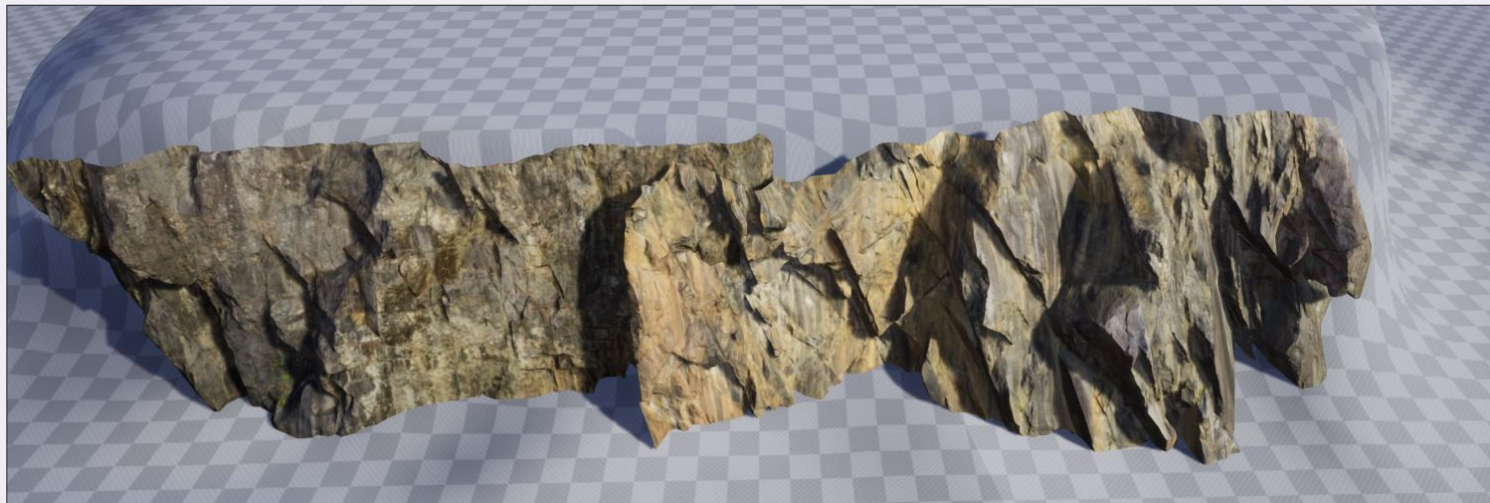Enumération spatiale

Courbes de niveau

Hybrides

Piles de matière

Surfaces d'élévation

Réduction de la complexité : représenter le relief de $S \subset \mathbf{R}^3$ à $\mathbf{R}^2$

Pas de surplombs

Maillages spécifiques sur les portions verticales

eric.galin@liris.cnrs.fr

http://liris.cnrs.fr/~egalin

Elevation functions

Function representation

Elevation functions

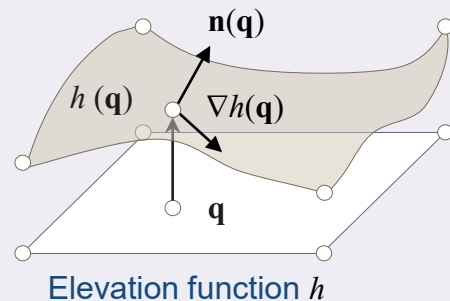Explicit function $h : \mathbf{R}^2 \to \mathbf{R}$

Gradient ∇h defines the steepest slope vector

$$s(\mathbf{p}) = |\nabla h(\mathbf{p})|$$

Slope

$$\mathbf{n}(\mathbf{p}) = (-\nabla h(\mathbf{p}), 1)$$

Normal, normalize $\hat{\mathbf{n}} = \mathbf{n}/|\mathbf{n}|$

Elevation function h

Direct evaluation of $h(\mathbf{p})$

Compact model $\sim 1 - 10^2$ kb

Computing h (on the fly) may be **demanding**
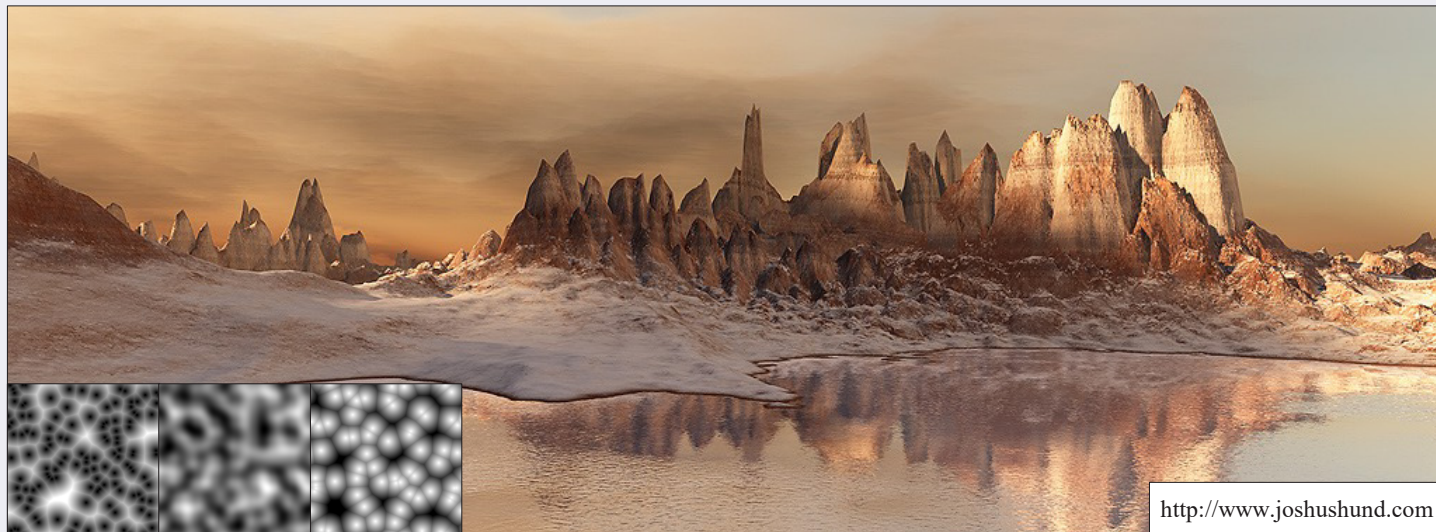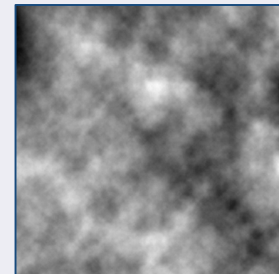**Control** of landforms, authoring

Function representation

Global

Combination of scaled noise (frequency and amplitude)

$$h(\mathbf{p}) = \sum_{k=0}^{o-1} a_k n(f_k\, \mathbf{p})$$

Decreasing amplitude $a_k = 1/\alpha^k$ Increasing frequency $f_k = \varphi^k$

http://www.joshushund.com

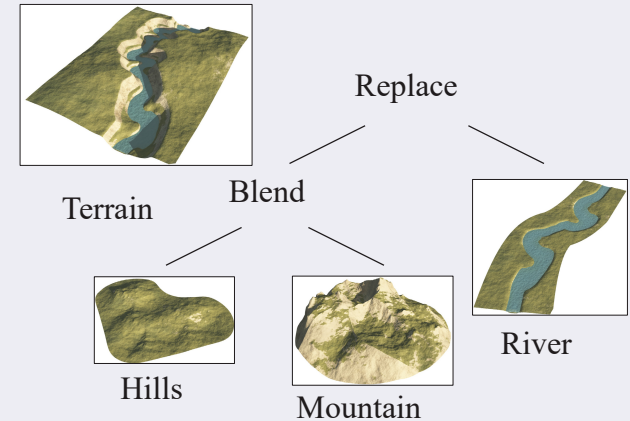Ebert *et al*. Texturing and Modeling: A Procedural Approach. *Academic Press Professional*, 1998.

Function representation

Local

Hierarchical representations using primitives organized in a tree
Sparse combination of **landform functions**

Génevaux *et al.* 2013

Replace

Terrain

Blend

Hills

Mountain

River

Génevaux *et al*. Terrain modeling from feature primitives. *Computer Graphics Forum*, 2015.
Guérin *et al*. Sparse representation of terrains for procedural modeling. *Computer Graphics Forum,* **35**, 2, 2016

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Université Claude Bernard Lyon 1

eric.galin@liris.cnrs.fr

http://liris.cnrs.fr/~egalin

Heightfields

Height fields

Structure

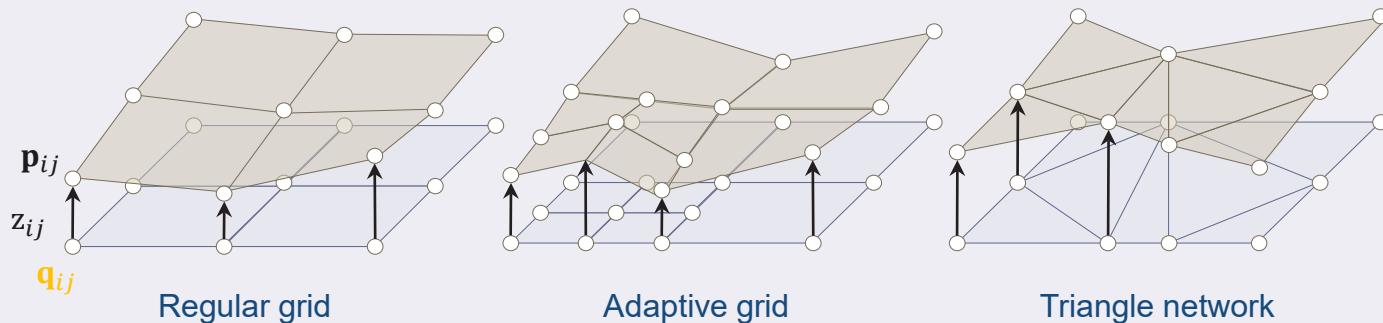**Versatile** model (generation, simulation)
Elevation reconstructed from samples
Discrétisation régulière ou irrégulière (adaptative) de Ω
On définit pour $\mathbf{q}_{ij} \in \Omega = B(\mathbf{a}, \mathbf{b})$ les élévations $z_{ij} = h(\mathbf{q}_{ij})$

$$\mathbf{p}_{ij} = \left(\mathbf{q}_{ij}, z_{ij} = h(\mathbf{q}_{ij}) \right)$$

$$\mathbf{q}_{ij} = \mathbf{a} + \left(\frac{(\mathbf{b}_x - \mathbf{a}_x)\, i}{n_x - 1}, \frac{(\mathbf{b}_y - \mathbf{a}_y)\, j}{n_y - 1} \right)$$

Regular grid   Adaptive grid   Triangle network

L'élévation $h(\mathbf{p})$ à l'intérieur des cellules de Ω est définie par interpolation des valeurs aux sommets

Note

Memory demanding model : storage increases in $O(n^2)$
Algorithms **do not scale** for large terrains with a high resolution

1k × 1k
4Mb, ≤ 1s

16k × 16k
1Gb, ≥ 2h

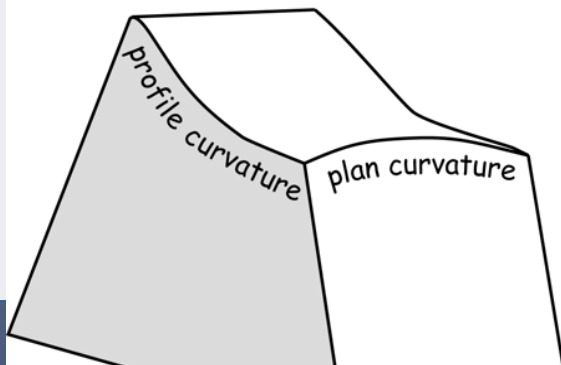eric.galin@liris.cnrs.fr

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$$D = [(Z_4 + Z_5)/2 - Z_0] / C^2$$

$$E = [(Z_2 + Z_7)/2 - Z_0] / C^2$$

$$F = (Z_3 - Z_1 + Z_6 - Z_8) / 4C^2$$

$$G = (Z_5 - Z_4) / 2C$$

$$H = (Z_2 - Z_7) / 2C$$

plan curvature

$$\frac{2 (DH^2 + EG^2 - FGH)}{G^2 + H^2}$$

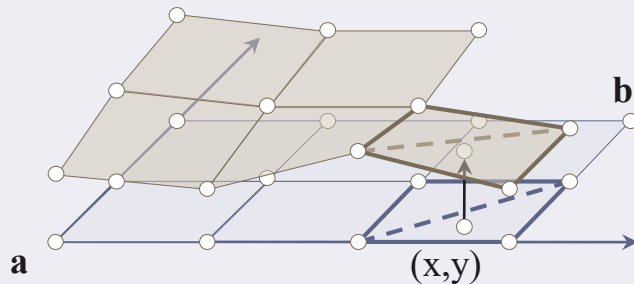profile curvature

$$\frac{-2 (DG^2 + EH^2 + FGH)}{G^2 + H^2}$$

Elevation computation

Elevation z_{ij} is stored for $\mathbf{q}_{ij} \in \Omega$

Compute $h(\mathbf{p})$ by bilinear interpolation


```
class HeightField
{
protected:
  Vector a, b;        // Bounding box
  int nx, ny;         // Discretization
  double z[];         // Array of heights
public:
  double Height(const double&,const double&);
  double HeightGrid(int,int);
  // …
};
```

```
double HeightField::Height(const double& x,
  const double& y)
{
  // Local coordinates
  double u=(x-a[0])/(b[0]-a[0]);
  double v=(y-a[1])/(b[1]-a[1]);

  // Cell location within grid
  int nu=int(u*nx);
  int nv=int(v*ny);

  // Local coordinates within cell
  u=u-nu*(b[0]-a[0])/nx;
  v=v-nv*(b[1]-a[1])/ny;

  if (u+v<1)
  {
    return (1-u-v)*HeightGrid(i,j)
      +u*HeightGrid(i+1,j)
      +v*HeightGrid(i,j+1);
  }
  else
  {
    return (u+v-1)*HeightGrid(i+1,j+1)
      +(1-v)*HeightGrid(i+1,j)
      +(1-u)*HeightGrid(i,j+1);
  }
}
```

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Université Claude Bernard Lyon 1

eric.galin@liris.cnrs.fr

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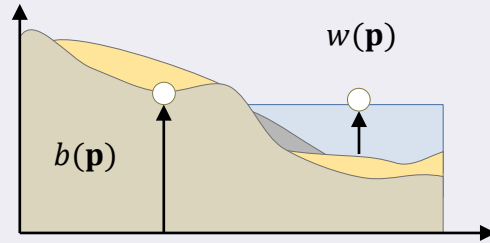
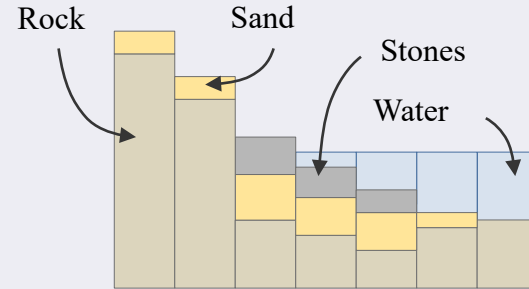Terrain models

14 février 2024

18

Layered representations

Concept

Bedrock elevation is augmented with the thickness of other material layers

Elevation functions

Discrete layer stacks

Consistency of layers ? How define h_S from $h_{\mathcal{B}}$?

Allows **complex simulations**

Applications

Erosion simulation produce fallen rocks or sediments

Ecosystems may use material layers as environmental conditions

Benes *et al.* Layered data representation for visual simulation of terrain erosion. *Spring Conference on Computer Graphics*, 2001.
Musgrave et al. The synthesis and rendering of eroded fractal terrains. *Computer Graphics*, **23**, 3, 1989.

eric.galin@liris.cnrs.fr

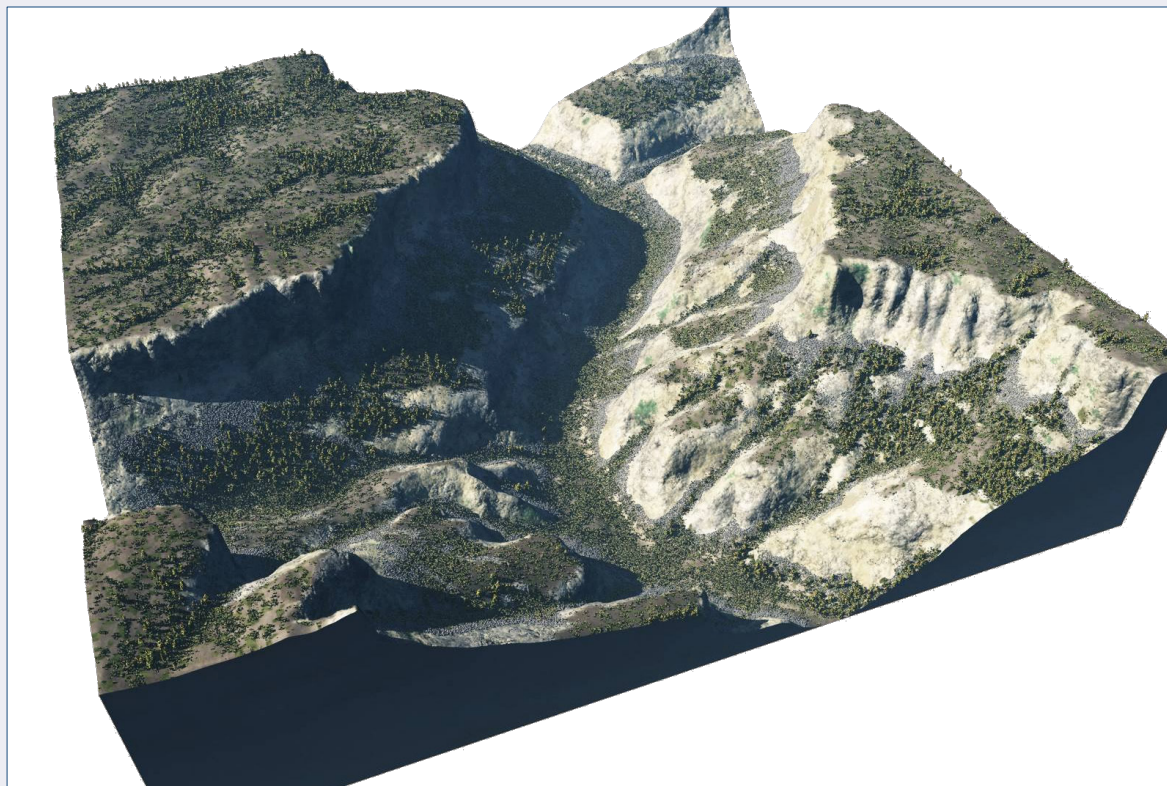http://liris.cnrs.fr/~egalin

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Classification

Surfaces

Volumes

Analysis

Combined terrain ecosystem simulation

Interacting layers

Volumetric models

Volumetric models

Limitations of elevation models

Overhangs and cliffs

Specific displacement or meshes

$f(x,y)$

$h(x,y)$

(x,y)

Displacement

$f(x,y)$
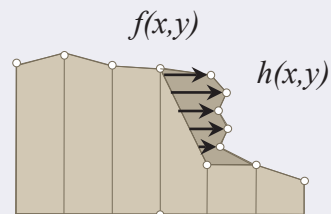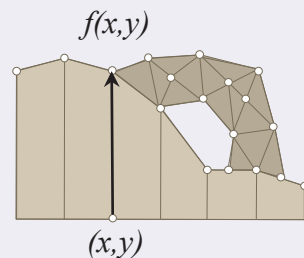
(x,y)

Meshes

Ghost Recon Wildlands

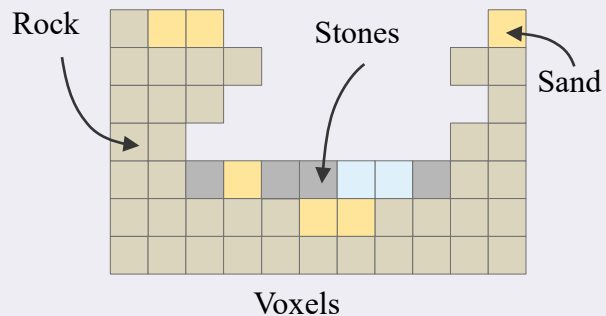Consistent modeling of **overhangs** and **cavities**?

eric.galin@liris.cnrs.fr

http://liris.cnrs.fr/~egalin

Université Claude Bernard Lyon 1

Classification

Surfaces

Volumes

Analysis

Model

Material function $\mu(\mathbf{p})$ defined in a voxel grid [Jones2010]

Rock

Stones

Sand

Voxels

Allows **sculpting** and **simulations**

Memory demanding
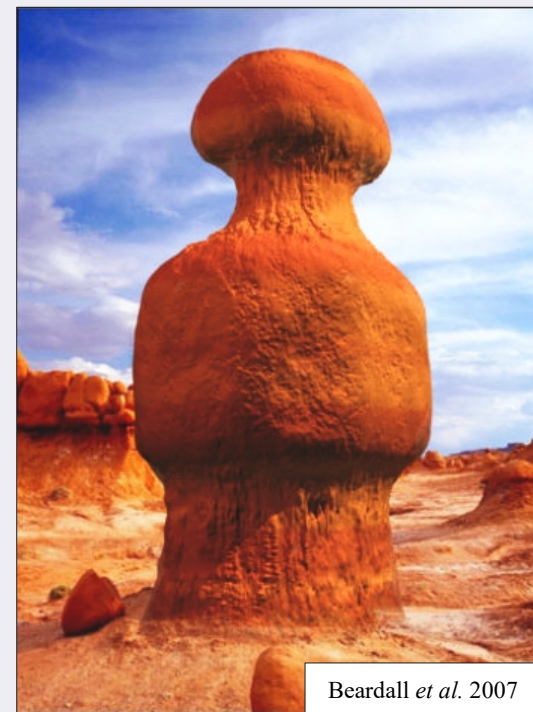
Beardall *et al.* 2007

Jones *et al*. Directable weathering of concave rock using curvature estimation. *Transactions on Visualization and Computer Graphics*, **16 (**1), 2010
Beardall *et al*. Goblins by spheroidal weathering. *Eurographics Workshop on Natural Phenomena*, 2007

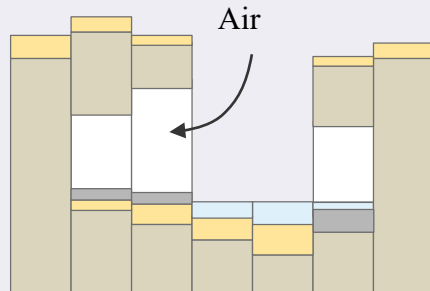eric.galin@liris.cnrs.fr

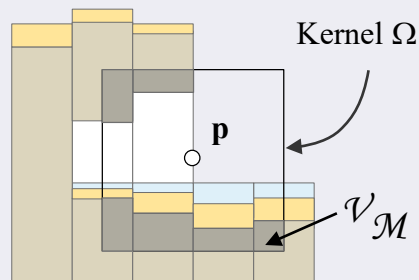http://liris.cnrs.fr/~egalin

Terrain models

Smoothed layered stack

Layered materials with air allow the creation of overhangs and cavities [Peytavie2009]

Implicit model : convolution $\mu(\mathbf{p}) = 0$ for air, $\mu(\mathbf{p}) = 1$ for material
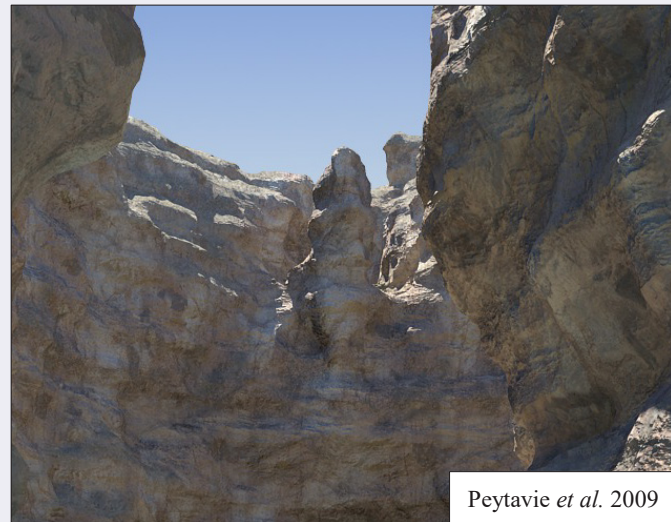

Layer stacks with overhangs

Layer stacks

Convolution $f = 2\mu * k - 1$

Fast computation $\mu * k = \dfrac{V_M}{V_\Omega}$

Peytavie *et al.* 2009

Peytavie *et al*. Arches: a framework for modeling complex terrains. *Computer Graphics Forum,* **28**, 2, 2009

eric.galin@liris.cnrs.fr

http://liris.cnrs.fr/~egalin

Classification

Surfaces

Volumes

Analysis

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eric.galin@liris.cnrs.fr

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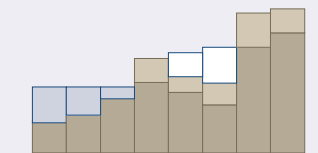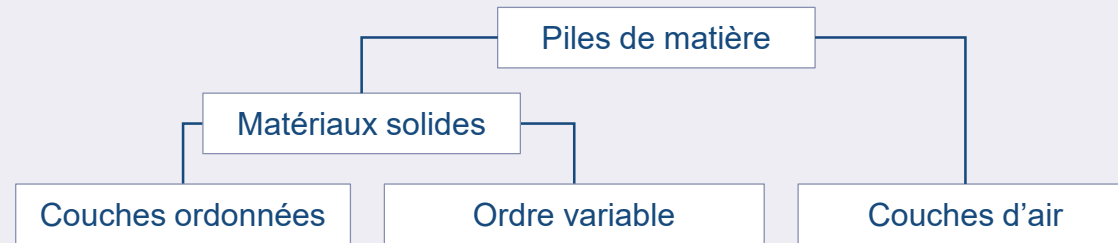
Hybrid representations

Principe

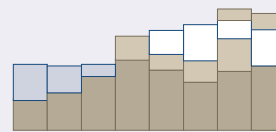Un terrain est une grille de piles de matériaux

L'élévation finale correspond à la hauteur totale de la pile

Lorsque l'ordre est fixe, on simplifie la structure en une superposition de cartes de matière
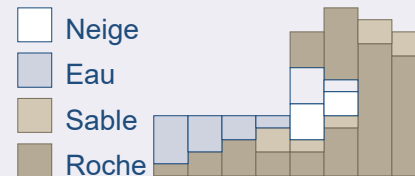
$$h = \sum_{0 \le i \le n} h_i \quad h_i : \Omega \subset \mathbf{R}^2 \to \mathbf{R}$$

Piles de matière

Matériaux solides

Couches ordonnées — Ordre variable — Couches d'air

Neige
Eau
Sable
Roche

Ordre des matériaux fixe — Ordre variable — Terrains avec surplombs

eric.galin@liris.cnrs.fr

http://liris.cnrs.fr/~egalin

Classification

Surfaces

Volumes

Analysis

Structures de données

Comparaison entre plusieurs cartes
d'élévations et une structure dynamique

```
class LayerField
{
protected:
  Vector a, b;        // Bounding box
  int nx, ny;         // Discretization
  double rock[];      // Array of heights for
rock layer
  double sand[];      // Array of heights for
sand
  double water[];     // Array of heights for
water
public:
  double Height(const double&,const
double&);
  double HeightGrid(int,int);
  // …
};
```

```
class Material       // Definition of a single cell
{
protected:
  double h;          // Height of material in cell
  int type;          // Material type
public:
// …
};

class MaterialStack  // Stack of cells
{
protected:
  Material stack[];  // Array of different materials
  int n;             // Size of array
public:
// …
};

class MaterialStackField // Array of stacks of cells
{
protected:
  Vector a, b;        // Bounding box
  int nx, ny;         // Discretization
  MaterialStack array[];// Array of  material stacks
public:
// …
};
```

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eric.galin@liris.cnrs.fr

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Terrain models

Classification

Surfaces
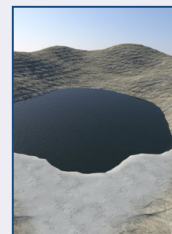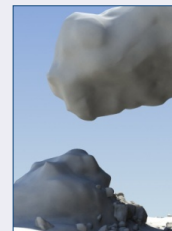
Volumes

Analysis

Structure

Modèle hybride combinant surface implicite – modèle discret

Combinaison pour l'édition et la simulation

	Implicite	Discret
Roche	Sculpture Fissures	
Matériaux granuleux		Dépôt Erosion Stabilisation
Liquides		Remplissage Ecoulement Phases

eric.galin@liris.cnrs.fr

http://liris.cnrs.fr/~egalin

Terrain models

Hybrid representations

Structure de données

Caractérisation volumique en piles de matière g

Lissage de la surface par convolution par un noyau h à support compact

$$S = \{\mathbf{p} \in R^2, f(\mathbf{p}) = 0\}$$

$$f(\mathbf{p}) = \frac{i(\mathbf{p})}{4\sigma} - 1$$

$$i(\mathbf{p}) = g * h(\mathbf{p}) = \int_{\mathbf{R}^3} g(\mathbf{p})h(\mathbf{p} - \mathbf{q})d\mathbf{q}$$
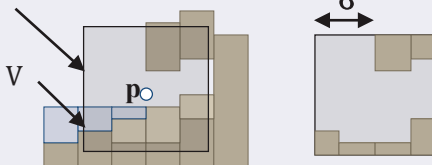
$$g(\mathbf{p}) = \begin{cases} 1 \text{ si } \mathbf{p} \in M \\ 0 \text{ sinon} \end{cases}$$

$$h(\mathbf{p}) = \begin{cases} 1 \text{ si } |\mathbf{q}|_\infty \leq \sigma \\ 0 \text{ sinon} \end{cases}$$

Squelette

Noyau

Support de convolution Ω de volume V_Ω

$$f(\mathbf{p}) = \frac{2V}{V_\Omega} - 1$$
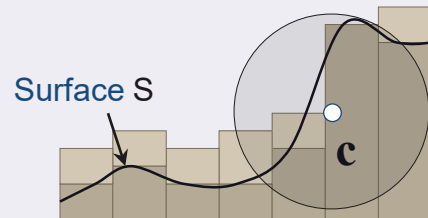
Classification

Surfaces

Volumes

Analysis

Combinaison des modèles implicites et discrets

Créer une primitive avec un centre **c**
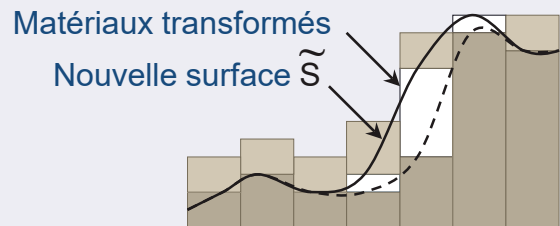Mélange avec la primitive implicite
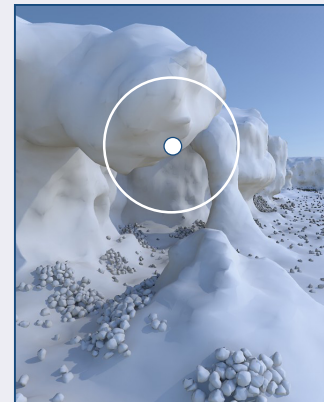Discrétisation de la surface implicite en piles
Stabilisation

Surface S

c

Couches initiales

Matériaux transformés

Nouvelle surface $\tilde{S}$

Couches modifiées

eric.galin@liris.cnrs.fr

http://liris.cnrs.fr/~egalin

Smoothed layered stack

Layered materials with air allow the creation of overhangs and cavities

Implicit model created by a convolution

Peytavie *et al*. Procedural generation of rock piles using aperiodic tiling. *Computer Graphics Forum* **28**, 7, 2009

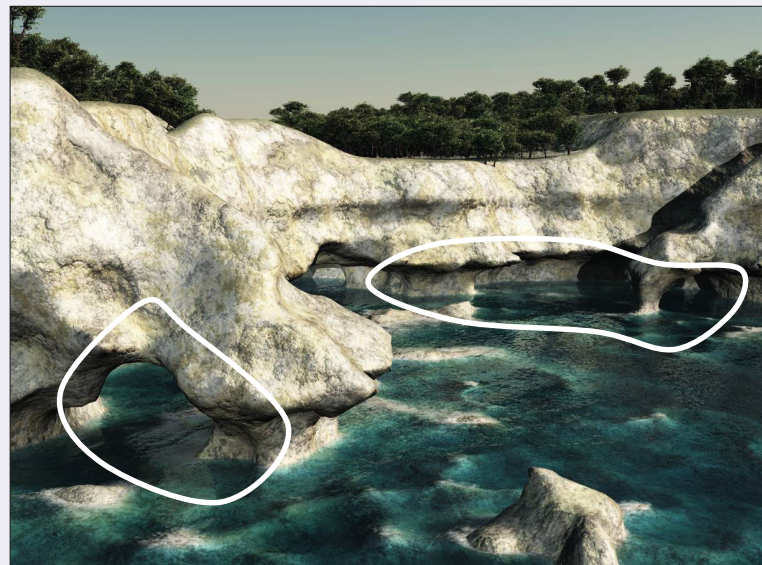eric.galin@liris.cnrs.fr

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Implicit Surfaces
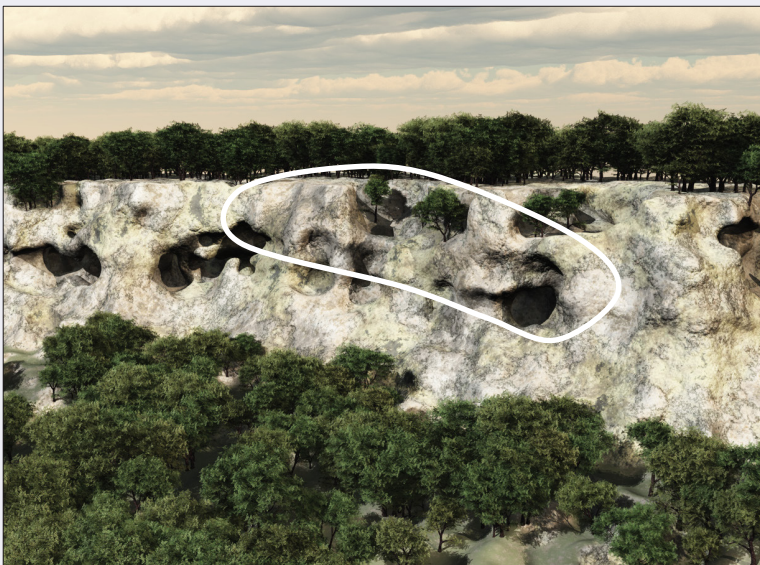
Field function

Find good primitives and operators for carving landforms
Cliffs, overhangs, caves

Need for effective **operators** and **primitives**

eric.galin@liris.cnrs.fr

http://liris.cnrs.fr/~egalin

Classification

Surfaces

Volumes

Analysis

Height fields and layered height fields
Conspicuous in terrain modeling
Versatile for a variety of generation methods

Function-based models
Useful for modeling some **specific landforms**
Modeling large landscapes with a high resolution

eric.galin@liris.cnrs.fr

http://liris.cnrs.fr/~egalin