Resolution of Non-Linear Problems In Realistic-Lung-Inflating Simulation with Finite Element Method

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ABSTRACT

Hadrontherapy treatment needs accurate tumour targeting, which is difficult for lung cancer due to breathing motions. We propose to quantify lung deformations and displacements by a simulation technique based on the geometrical and mechanical properties of organs. Thereby, we model lung behaviour by a 3D dynamic deformable model derived from continuous mechanics, computed with finite elements method (FEM).

1 Introduction

1.1 State of the Art

Various approaches have been explored to globally model lungs in 3D :

- Zordan *et al.* [2] developed a model of thorax breathing for animation purpose. Lung motions are driven by spring-muscle elements.
- Kaye *et al.* [3] studied the kinematics of lung with a mass-spring system, assuming a uniform external pressure.
- Amrani [4] has developed a model based on a particle system that undergoes deformations due to inflation and obstacles.
- Grimal *et al.* [5] used a finite element model to study thoracic impact injuries. Breathing motion was not included.

The state of the art shows that the existing models do not use any personalised parameter or are not accurate enough to properly track tumours.

1.2 Our context

- We model lung in the framework of continuum mechanics.
- Computation is solved with FEM.
- This method implies non-linearity such like:
  - Stiffness evolution according to displacement.
  - Boundary condition.

⇒ This method needs appropriated convergence parameters.

1.3 Questions

- How to model physiological and anatomical lung behaviour?
- How to integrate The patient's customised data?
- How to compute the non-linear behaviours?
- How to fix parameters to have accurate results?

2 Our approach in few points

2.1 Lung Anatomy and Physiology

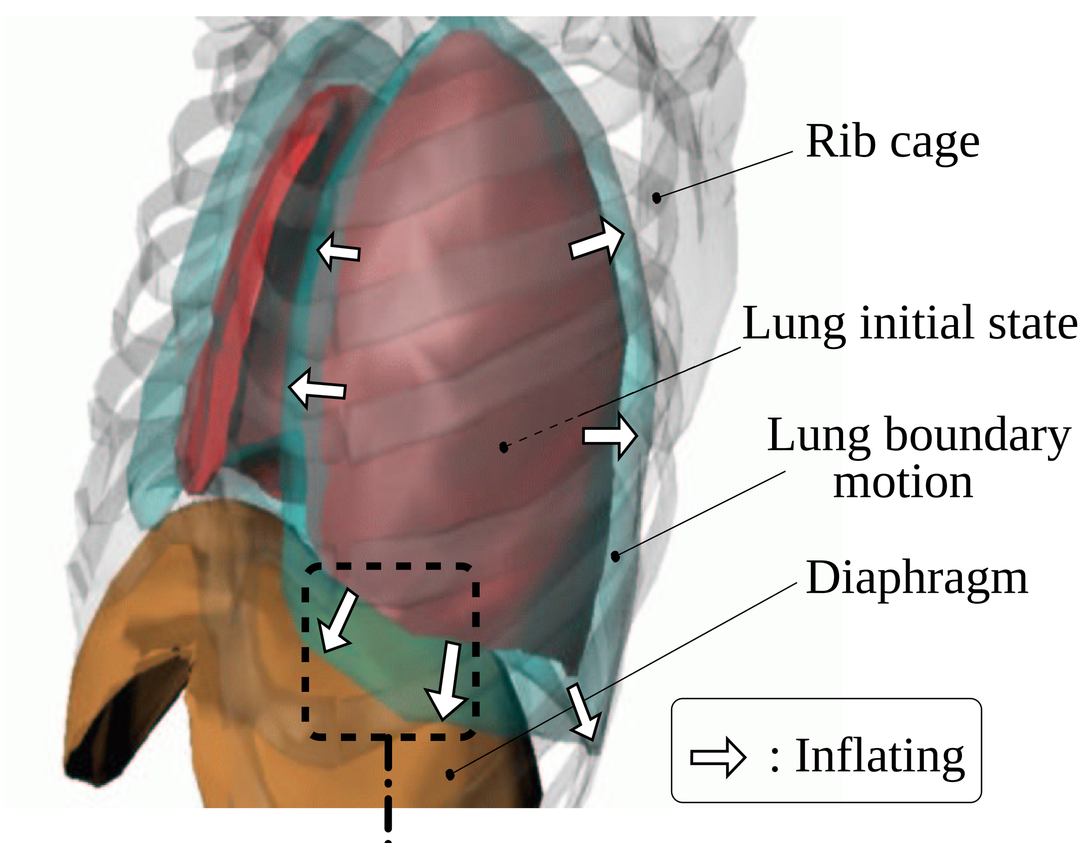
Lungs are wrapped in a fibrous membrane: the pleura, the very low pressure in the cavity making tissue remains in contact.
⇒ Diaphragmatic and rib-cage-muscle actions cause pressure changes inside pleura.
⇒ It steadily induces lungs inflating or deflating.

According to their location, Langen et al. [6] show that lung tumours can undergo large displacements.

2.2 Lung Inflation Modelling

- We simulate as precisely as possible the pleura behaviour:
  - The whole system is fixed to the trachea.
  - A uniform negative pressure is applied around the lung at its initial surface.
  - We check when the external surface matches the external surface of the lung at its boundary motion.
  - Sliding without friction (as pleura does) is allowed.

  ⇒ Surface to surface matching between two states.



3 Non-linearity problems

3.1 Finite Element Resolution

The FEM :

- is a numerical method;
- transforms the solution into a matrix representation;
- is based on space discretisation into small elements (meshing);
- consists here in searching displacements U to reduce as possible the residue R defined by (1).

$$R(U) = F - K(U).U \quad (1)$$

where K is the stiffness matrix and F is the load vector. For large strains there is strong modifications in shape and $F.K(U)$ is no linear.

This is the first source of non-linearity

The full system is then solved with the Newton-Raphson algorithm, an iterative method giving:

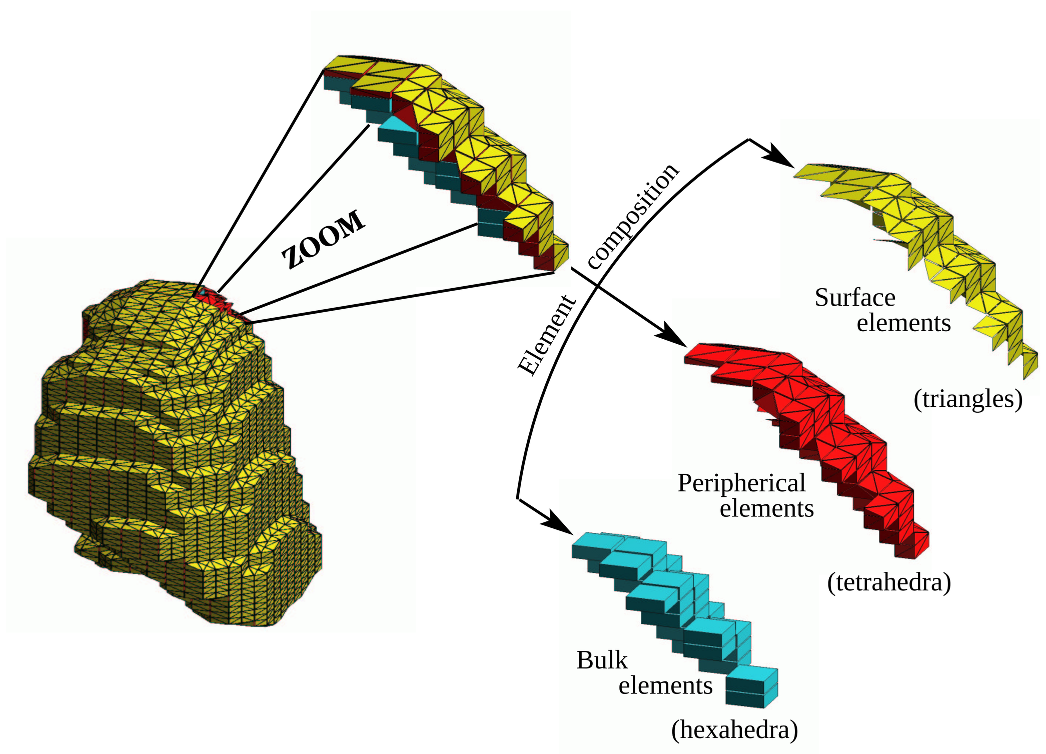
$$R(U) = R(U_{n-1}) + (U_n - U_{n-1})\frac{\partial R}{\partial U}(U_{n-1}) \quad (2)$$

(U_n) is defined by :

$$\begin{cases} K(U_{n-1}).\Delta U_n - R(U_{n-1}) = 0 \\ \Delta U_n = U_n - U_{n-1} \end{cases} \quad (3)$$

3.2 Meshing

Convergence rate directly depends on mesh accuracy.
⇒ Mesh is divided into three entities: triangles, tetrahedra and hexahedra:



3.3 Large-Strain Problem

Lung volume increases by a factor of two during a respiration cycle.
⇒ Large strains have then to be considered.
⇒ we employ the algorithm presented by J.C. Simo et al. [7].

This method uses the Cauchy-Green strain tensor ϵ_{cg} computed with the transformation gradient G of the geometrical deformation:

$$\epsilon_{cg} = 1/2(Id - (G.G^T)^{-1}) \quad (4)$$

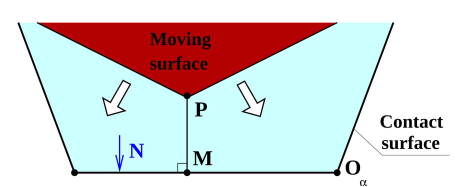
The stress tensor used is the Kirchoff tensor $\tau(X_f)$, computed like a "scaling" of the tensor $\sigma(X_i)$:

$$\tau(X_f) = det(G).\sigma(X_i) \quad (5)$$

where $\sigma(X_i)$ is the state of stress at the position X_i.
$K(U_{n-1})$ is evaluated with (4) and (5), then the residue $R(u_{n-1})$ is estimated according to (1), which gives U_n from (3).

3.4 Contact Problem

To handle contact conditions:



- Couples of points P and M potentially in contact are searched.
- Distance PM must be positive to satisfy the conditions of non penetration, *i.e.*:

$$PM_{n-1}.N + (U_M - U_P).N \leq 0 \quad (6)$$

- The equilibrium equation must be completed by a force to add compression and avoid a separation.

  This is the second source of non-linearity
- An equation must be added to express that this force only takes part when contact is reached and only corresponds to a compression force.

If the imposed negative pressure (with respect to atmosphere) is not sufficiently important, residue R of equation (1) will be reached before contact condition.
⇒ The pressure value must then be large enough.
⇒ Convergence is ensured by sub-iterations taking into account geometry reactualisations.

4 Numerical experiments

4.1 Experiments Parameters

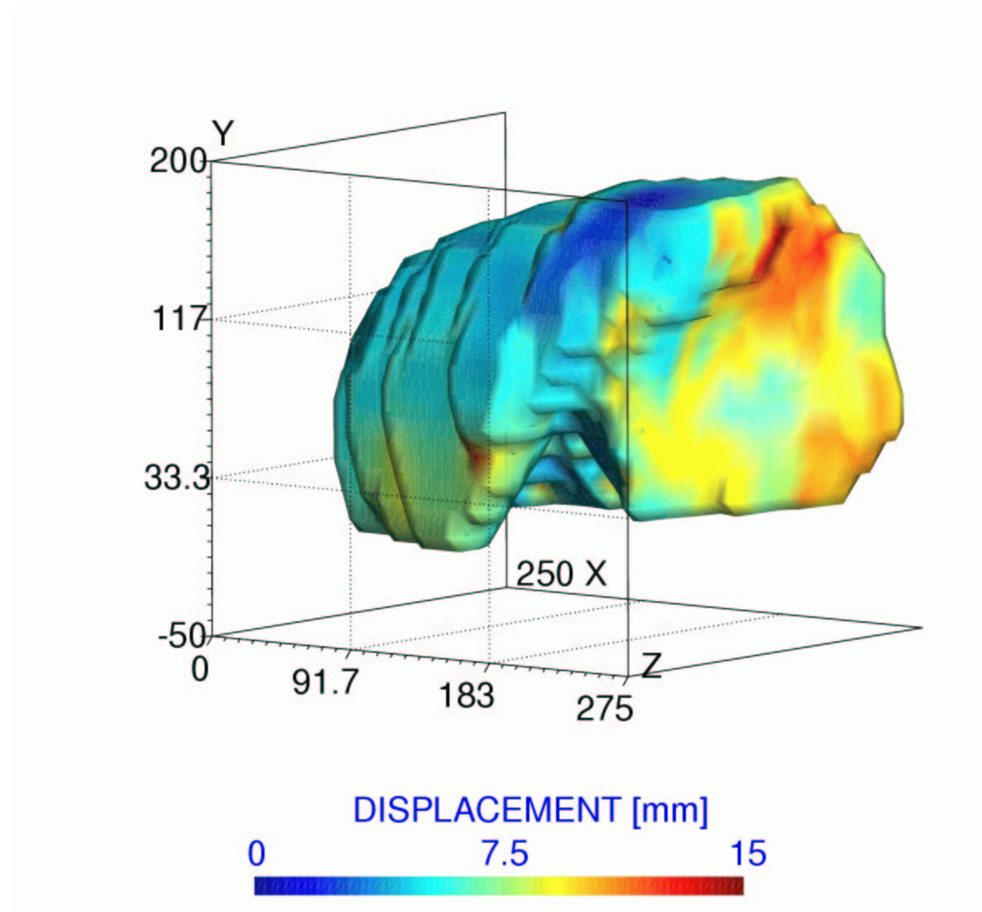
- Geometry is characterised by 10 778 points and 30 097 elements.
- Lung bounding box dimensions are 240 mm × 180 mm × 245 mm.
- Measured patient compliance (lung elsasticity) is 3.5 l/kPa.
- Minimum residue of equation 3 is set to $R = 10^{-6}$.
- Computation of lung motion have been calculated with several numbers of geometry reactualisation for the contact conditions.

Reactualisation number	Computing time	Average displacement$^\alpha$ [mm]
1	14h43m	5.66
3	15h15m	5.23
5	15h28m	5.1
7	15h48m	5.1

We observed a correct convergence can be obtained after 5 iterations.

α: Average displacement is computed with the displacement-vector norm of each mesh vertex.

4.2 Surface Deformation Results



- The displacement field is totally smooth.
- There is no convergence problem associated to mesh aberration.
- The final surface matches well with geometry extracted from CT scans (trial definition).

5 Conclusions

- A realistic way to model lung inflating is set.
- Convergence parameter related to non-linearity were managed.
- Results are convertible into 4D CT scan useful for physician and treatment planning soft.
- Do we need to relax any assumption (heterogeneity, anisotropy, ..) for hadrontherapy accuracy?

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