Implicit Tensor-Mass solver on the GPU

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Abstract

The realist and interactive simulation of deformable objects has become a challenge in Computer Graphics. For this, the Tensor-Mass model is a good candidate: it enables local solving of mechanical equations, making it easier to control deformations from collisions or tool interaction. In this paper, a GPU implementation is presented for the implicit integration scheme, permitting to achieve robustness of the simulation at interactive time for linear and non-linear mechanical behaviors. Results show a notable speedup, making the parallel Tensor-Mass model a true alternative to Mass-Spring or Finite Element Methods, especially in the case of complex scenes.

Simulation of a deformable object

- Domain discretized into several elements
- Mechanical equations solved locally
- For each element (hexa, tetra, etc.):
  - Discretization of the displacement U_E
  - Computation of the deformation energy W_E
  - Derivation of W_E to obtain the elasticity force F_E
- Implicit integration scheme to obtain displacement
- Conjugate Gradient method to solve linear system

Energy of deformation

- Element of initial volume Vol_0
- Strain-tensor according to common elasticity models, within the element, yields at X:

$\epsilon_l(X) = \frac{1}{2}(\nabla U^T(X) + \nabla U(X))$ — Hooke

$\epsilon_{nl}(X) = \frac{1}{2}(\nabla U^T(X) + \nabla U(X) + \nabla U^T(X)\ \nabla U(X))$ — St-Venant Kirchhoff

- Deformation energy of elasticity models:

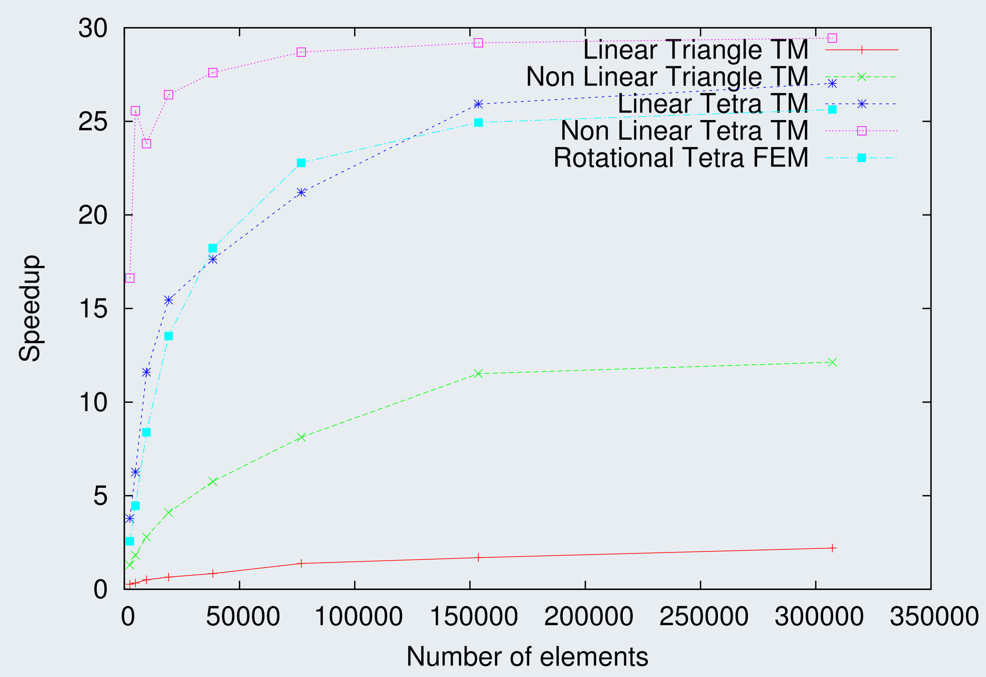
$$W_E(X) = \frac{\lambda}{2}(\text{tr}\ \epsilon(X))^2 + \mu\ \text{tr}\ \epsilon(X)^2$$

Force computation on the GPU

```
{N : number of elements}
{m : total number of nodes}
{n : number of nodes per element}
{N_n : max nb of neighbor elements for a node}
// Task 1- Computation of partial forces
for e = 0 to N − 1 do
  // Execution of N kernel1
  for v = 0 to n − 1 do
    PartialForce[ForceIndex[e][v]][index[e][v]]=Force();
  end for
end for
// Task 2 - Sum of partial forces
for i = 0 to m do
  // Execution of m kernel2
  for j = 0 to N_n − 1 do
    TotalForce[i] += PartialForce[i][j];
  end for
end for
```

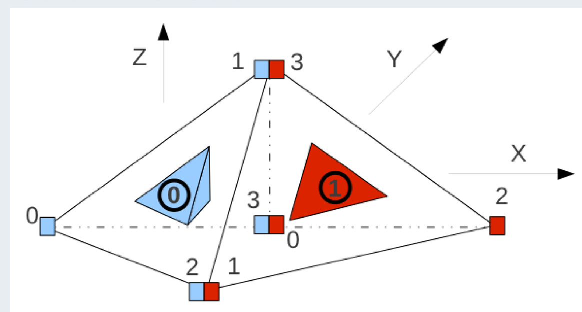
Speedup between GPU and CPU

- Beam composed of 307,200 elements
- CPU: Intel® Xeon® 4 cores @3.07 GHz
- GPU: GeForce GTX 560, 2047 MB, 56 cores @1.620 GHz
- Speedup of 25.5 for SOFA's FEM [1, 2]
- Speedup of 29.5 for our TM parallelization

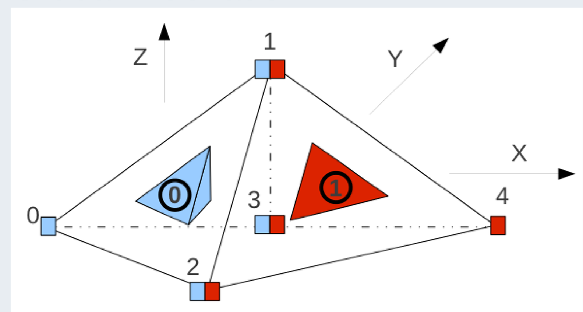


Data structures, 2 tetra

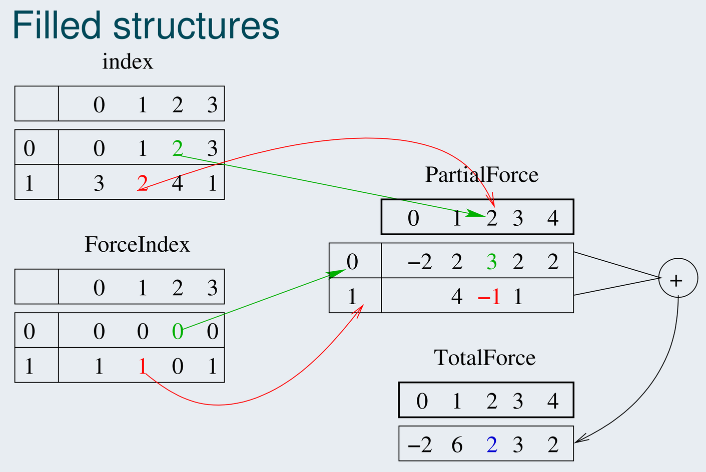
a) Local indexation



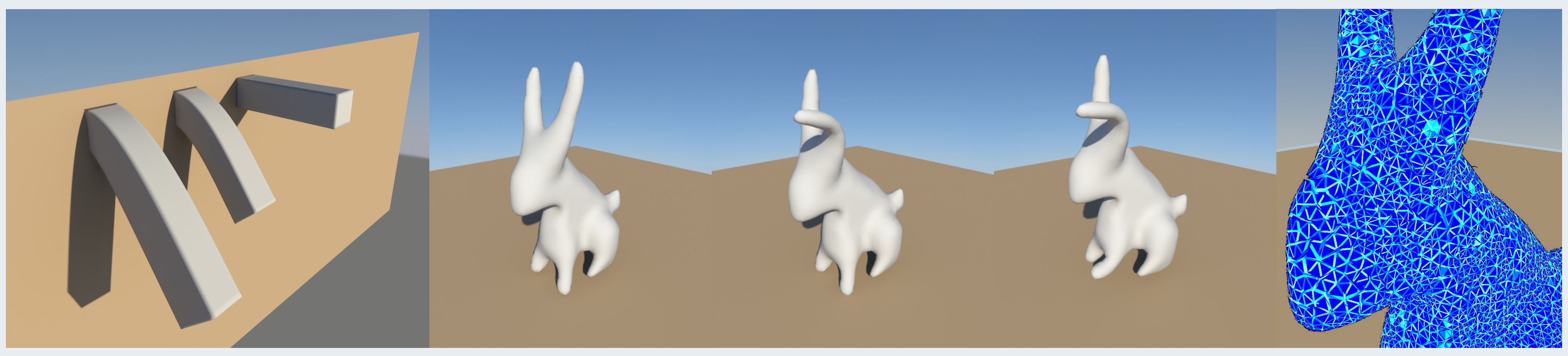
b) Global indexation



c) Filled structures



a) Rendered beam for different materials
b) Interactive deformation of a rabbit



Perspectives

- Add more geometrical elements
- Add more hyper-elastic behaviors
- Optimize data structure for Tensor-Mass
- Implement multi-resolution and adaptive simulation

⇒ Develop a complete simulation environment based on the Tensor-Mass model on the GPU

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