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STUDY OF VOLUME VARIATION OF IMPLICIT OBJECTS

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We propose studying the variations of volume of implicit objects during an animation according to several points of view: choice of the function of density, variations of parameters such as the iso-value and the radius of influence for a given function, variations of the parameters inherent in a particular function. Modification of parameters of the function of density must be carried out with care. There are no rules concerning these variations. To avoid the non-monotonous variations, it is necessary to choose a function of density beforehand and study the intervals of variation of its parameters. A new discretization makes it possible to locate these variations for a later use in a process of control of these variations.

Keywords: Implicit object; volume computation; control of volume variation.

1. Introduction

Implicit surfaces appear in many applications such as medical imaging, computer-aided design, and computer graphic. Implicit surfaces have proved to be a well-suited and efficient model for animating and morphing shapes of arbitrary topology. We are interested here in a particular kind of implicit surfaces, which are equipotential implicit surfaces[a] called blobs.[1]

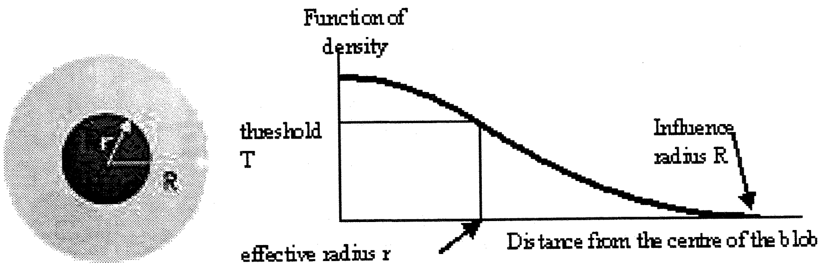
A field function $f(x, y, z)$ defines an implicit surface and assigns a scalar value to each point in space. An implicit surface is then $\{(x, y, z)/f(x, y, z) = T\}$ where T is called the threshold. For instance Fig. 1 is representing a blob (a) and a density function of a blob (b), where R is the radius of influence, which defines the part of

[a]An equipotential surface in 3D, defined with $f(x, y, z)$ is such that the value of $f(x, y, z)$ is a constant.

the space where f influences points of space; r is the effective radius defining the visible part of the blob; T, the threshold that regulates the size of the blob.

If there are n blobs in space, they are merging to construct an object. This object is then defined with $F(M) = \sum_{i=1}^{n} \alpha_i f_i(M)$ where f_i is the function of density of blob i, α_i is a negative or positive factor and M a point in space. For example, hereafter Fig. 2 is made up of blobs:

During an animation, volume of the implicit object composed with several blobs is varying as seen Fig. 3. In this figure, two blobs are moving away. We have represented the curves of the implicit functions f_1 (for blob at left) and f_2 (for blob at



(a) Cup of a blob (b) Function of density

Fig. 1. A blob.

(a) Three blobs: one of them with a negative factor (b) Two blobs are merging

Fig. 2. Three blobs on the left and two blobs on the right.

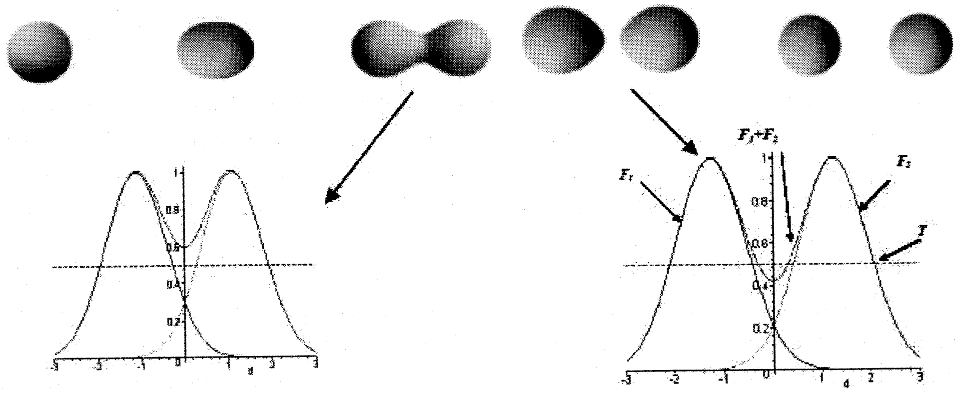


Fig. 3. A blob.

right), and the curve of $f_1 + f_2$ (which represents the implicit object). We notice that the curve of the sum is passing over the threshold if there is just one object and under the threshold otherwise. Volume is an integration of this curve. Then, we have to conclude that volume is varying along this animation.

We would like to compute volume variations of a 3D implicit object during an animation, to detect pathology of the heart for instance in medical imaging. Another application may be the modelization of objects with a high deformable material such as clay for virtual sculpture[2] that implies control of these variations.

The variation of volume may be 400% of the initial volume.[3] This variation is dependent on the function of density, parameters of this function, value of threshold, and value of radius of influence and so on. Unfortunately, just a few previous works compute the volume of blobs. The most famous is a voxel-based method subdividing space in small cubes. An other one is a method using territories based on seeds[4,5] and a third one proposes an analytical way to compute the volume.[6]

Some papers propose methods to control volume variations: for instance, Tong *et al.*,[7] proposes a method to preserve volume during an animation to generate water flow; or Desbrun *et al.*,[4] uses the seeds and territories method.

In this paper, we will speak on volume variations depending on iso-value and radii of influence of the function of density, localization of these variations and a new method to control the variations. We will study them using Murakami's function[b,8] and we will extend then to many other implicit functions too. We will also use simple examples with two blobs to help better understand the phenomenon (the phenomenon develops if one increases the number of primitives).

Our algorithm may be employed for instance in Ref. 9, where an active surface model, based on an implicit formulation in order to allow any topological changes and fast inside/outside detection is introduced. It could be used in a reconstruction process based on the implicit model, as in Ref. 10, to create an implicit surface model that can deform in tandem with an explicit surface[11] or perhaps in a construction process using radial basis functions as in Ref. 12.

2. Total Behavior of the Variation

We use two implicit primitives two blobs[c] defined thanks to the function of density of Murakami. Volume is computed with the voxels method. This method consists of arbitrarily subdividing space including the object in a great number of parallelepipeds, all of identical size called voxels. Hereafter, Fig. 4 accounts for two

[b] $$f(r) = \begin{cases} \left(1 - \left(\frac{r}{R_i}\right)^2\right)^2 & \text{if } 0 \le r < R_i \\ 0 & \text{otherwise} \end{cases}$$

[c] We will use the word "blob" instead of implicit primitives all over this paper.

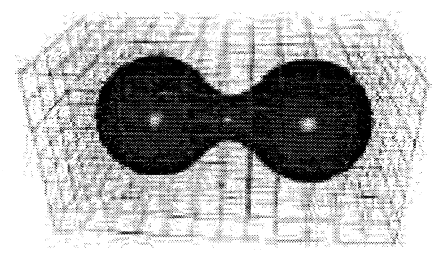


Fig. 4. Decomposition of space in voxels.

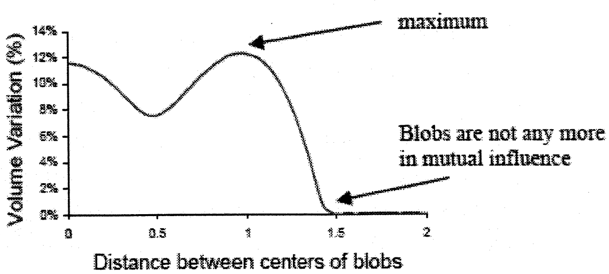


Fig. 5. Volume variation with the density function of Murakami.

blobs plunged in a space discretized in voxels. The volume of the implicit objects is calculated by summing all voxels contained in implicit volume.

We obtain the curve of Fig. 5 representing volume variations of the object obtained with two blobs when they are moving away, the iso-value is $T = 0.5$ effective radii are 0.5 and the radii of influence are $R_1 = R_2 = 1$.

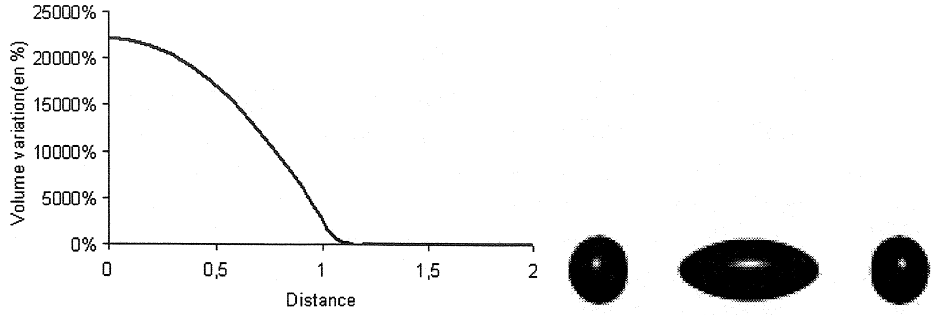
We note a significant variation of volume (the variations of volume are measured compared to initial volume. The initial volume is the sum of volume of blobs without deformation, as they are considered as spheres). Volume decreases initially and then increases until a maximum. The curve then decreases to reach one minimum when the blobs are not any more in influence.

In the borderline case of Fig. 6(a), we raise a reduction much more significant of volume (22130% of the initial volume) when the iso-value $T = 0.99$. Figure 6(b) shows us that a third object is present between the two implicit objects; the distance between their centers is $r_1 + r_2$. When the two blobs move away, this matter contribution disappears, that implies the reduction in volume.

We will now study these variations more in detail in the following:

3. Variations of the Volume for the Function of Density of Murakami

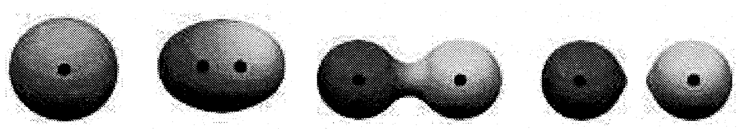
Murakami's function[8] is very simple to compute and is defined over a finite support. Other functions like that of Tsingos[13] or Blanc[14] would oblige us to study



(a) Curve representing much more variation

(b) A third object appears between the two primitives during merging

Fig. 6. Appearance of matter between the two implicit objects.



(a) $d = 0.48$ (b) $d = 0.98$ $d = 1.38$ $d = 1.46$

Fig. 7. Representation of two implicit objects for different values of distance d between centers.

the variations of parameters like thickness that would make this study even more complex.

We observe the curve for an iso-value $T = 0.5$ and the same radii of influence ($R_1 = R_2 = 1$). At the beginning of the animation, the objects are placed on a same center and they move away until they are not any more in influence (distance between the centers is then equal to the sum of the radii of influence).

In Fig. 7, we have represented the separation of two blobs whose centers are black. $R_1 = R_2 = 1; r_1 = r_2 = 0.5$.

The minimum of volume is obtained with a distance $d = 0.48$ (Fig. 7a.) From the point of topological view, only one object is built with two blobs. From this distance, volume increases to reach a maximum with $d = 0.98$. The increase in the distance allows an increase of the volume. The primitives are not any more in total mutual influence. The object is of ovoid type (Fig. 7b). The volume decreases from this distance and becomes less and less significant. The ovoid shape of the object disappears and we observe a more dug object (Fig. 7c), then two separated objects (topologically) (Fig. 7d). Then the curve forms a step from $d = 1.54 = \max(r_1 + R_2, r_2 + R_1)$. The primitives are no more under influence. The volume remains the same as the centers are moving away.

In the following, we will modify the parameters of the primitives to study their influence on the curve.

4. Variation of the Iso-Value and Radii of Influence for the Function of Density of Murakami

The modification of the iso-value implies the variation of the size of implicit primitives. The decrease in the iso-value implies an increase in volume[d] The curve of the variation of volume as a function of distance (Fig. 5) is modified depending on this iso-value. Figure 8 is representing three curves with three values of the threshold T. We notice that volume is varying according to the distance between blobs. The variation of volume is expressed as a percentage of the volume computed if the blobs are considered as spheres.

If the iso-value is near 0, ($T = 0.1$ for instance), the increase in volume is significant compared to the variation obtained when the iso-value is much higher than 0.1. If the iso-value is close to 1 ($T = 0.7$ for example), the curve is decreasing in a continuous way as the primitives move away.

Figure 9(a) represents the functions of density of the two implicit primitives as well as the sum of these ones. Figure 9(b) represents an enlargement of a part of the curve of the summation. If the iso-value (or threshold) T is close to 1 (here $T = 0.97$), the line representing the iso-value divides the curve in three pieces. That supposes the appearance of a third object between the two implicit primitives (remember Fig. 4). The distance between the primitives is then $d = 1.09$. In fact,

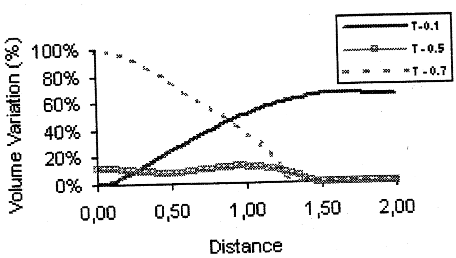
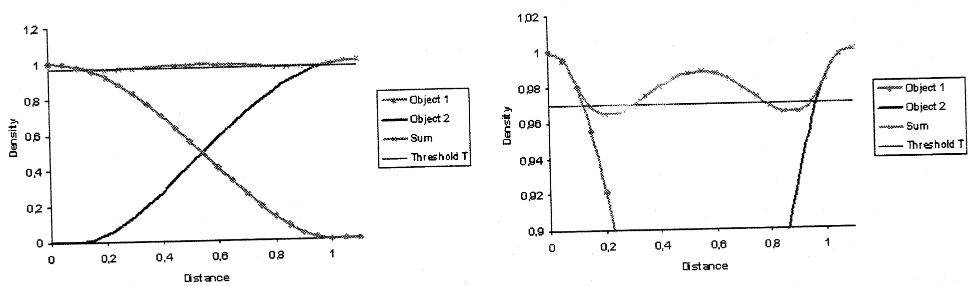


Fig. 8. Different values of the threshold.

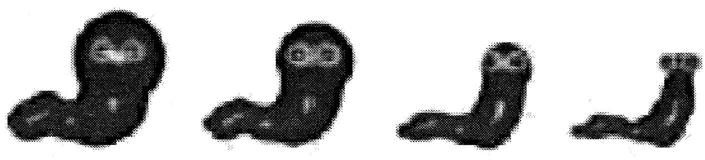


(a) Sum passes three times over the threshold

(b) Zoom of the curve

Fig. 9. Representation of two implicit objects for different values of distance d between centers.

[d]The visible part of a blob is over the threshold.



(a) $d = 0.48$ (b) $d = 0.98$ $d = 1.38$ $d = 1.46$

Fig. 10. The same scene composed with several blobs with different values of the threshold.

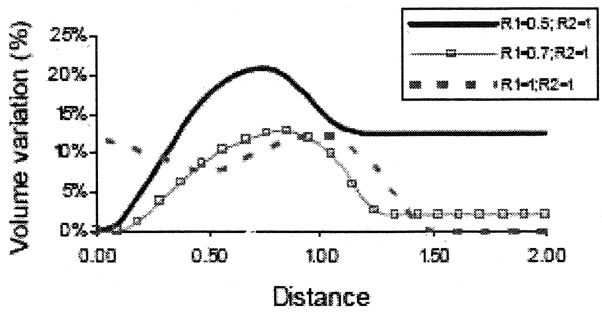


Fig. 11. Various values of the radii of influence.

the curve of the sum represents the field along the segment joining the two skeleton points (center of blobs).

We will find in Fig. 10 an illustration of variation of the threshold.

The variation of the radii of influence of the implicit primitives implies a variation of volume (see Fig. 11).

On Fig. 11, the reduction of the radii of influence induces a reduction in volume. On this curve, the iso-value T is 0.5. We choose to vary the radius of influence of one of the two blobs, the other one remaining constant. Volume decreases as the radius of influence of an implicit object is decreasing. We note that the modification of the radius influences the shape of the curve of variation of volume.

We have just shown that the variation of the iso-value or of the radius of influence modifies the shape of the curve of variation of volume. Handling of these parameters must be carried out with precaution if one wants to avoid great variations of volume.

5. Extension to Other Functions

A lot of functions of density contains parameters such as the thickness or the stiffness.[15] The choice of the value of these parameters can also influence the variations of volume. We are interested in functions of Blinn,[1] Tsingos,[15] Nishimura[8] or Wyvill.[16,17,e] On Fig. 12, we compare the curves of volumes obtained for these various functions.

[e] We will explain them in appendix.

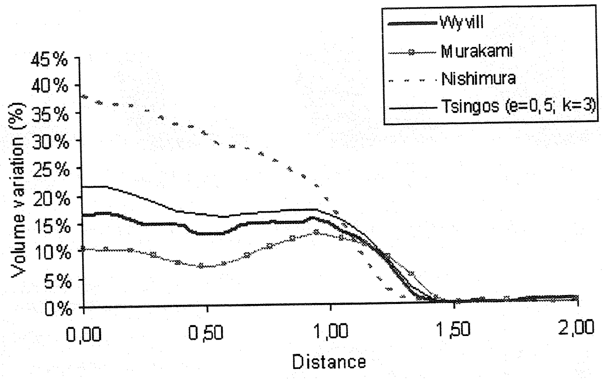
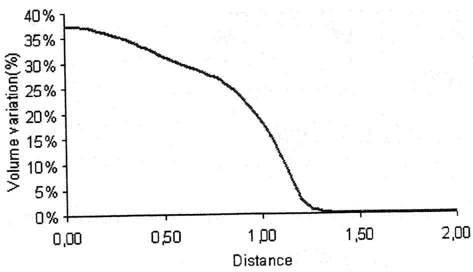
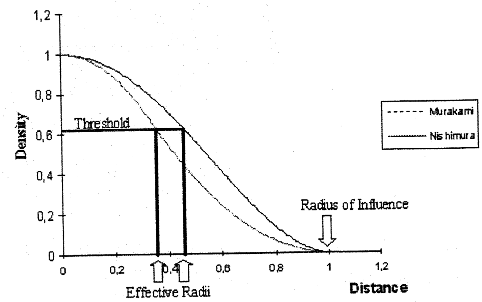


Fig. 12. Calculus of volume using many functions of density.



(a) Variation of volume, Nishimura's function

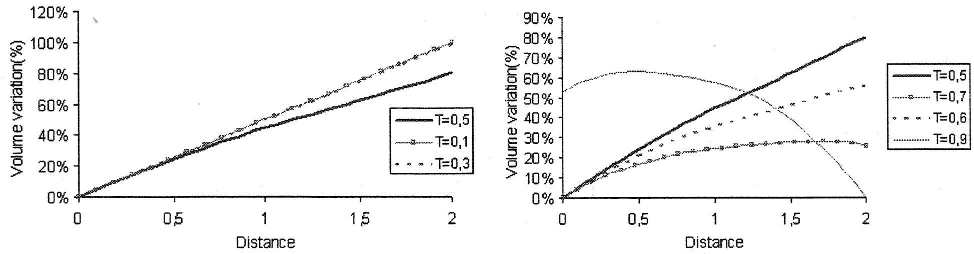


(b) Functions of Murakami and Nishimura

Fig. 13. Function of density of Nishimura.

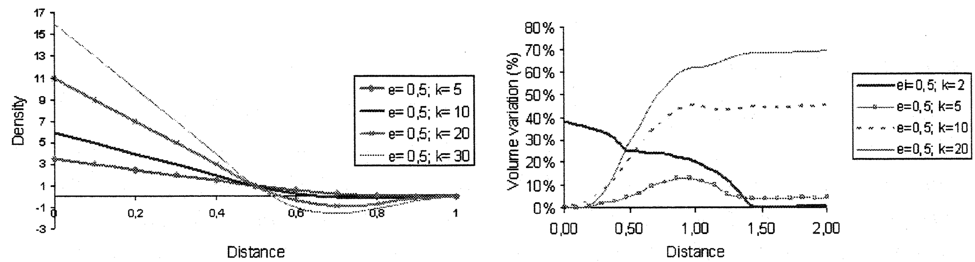
The function of Blinn is not represented. Since its behavior is different, it will be studied separately. The tendency of the curve of volume obtained for functions of density of Murakami, Wyvill and Tsingos is rather similar. The curve of volume obtained with the function of density of Nishimura is monotonous decreasing (Fig. 13a).

The difference in shape of the functions of density explains this change of tendency compared to the curve of volume obtained with the function of Murakami. The function of Nishimura decreases more quickly than Murakami's when the distance from the origin of the curve increases (Fig. 13b). The slope is then softer, because the value of the tangent to the curve is closer to 0. One obtains a monotonous reduction in volume more progressive. For the function of density of Blinn, Fig. 14, we notice that for a lower or equal iso-value to 0.5, the volume is always increasing when the implicit primitives move away. The primitives are always in influence because the exponential functions, which govern them, act all over the space. (The function is not defined on a finished support.) When the threshold T becomes higher than 0.5, the behavior of the curve changes. We observe an increase in volume, which can be followed by a reduction (Fig. 14b).



(a) $T \leq 0.5$ (b) $T \geq 0.5$

Fig. 14. Function of Blinn.



(a) Variation of density (b) Variation of volume

Fig. 15. Variation of stiffness k for the function of Tsingos.

The function of density of Tsingos gives results very close to those obtained with the functions of Murakami or Wyvill. The shapes of the curves representing volume according to the distance between the centers are similar. This shape is modified using the values of the parameters thickness e_i and stiffness k_i (Fig. 15b).

We use identical parameters for the two primitives (thickness e, stiffness k and radius of influence R). We choose to vary the parameters thickness and stiffness. The parameter thickness is used to modify the effective radius of the object. The parameter of stiffness regulates the slope of the function (Fig. 15a). The modification of this parameter is not easy when k becomes large ($k > {}^{3}/_{e-R}$). The function of density gives negative values, which brings to withdraw matter from the other primitives in influence.

The use of negative values does not have an influence on the variations of volume. The form of the function used when the distance to the center of the primitive is lower than the thickness can be defined thanks to a parameter of linearity. The variation of this part of the function of density does not influence the shape of the curve of variation of volume since this part is over the top of the iso-value.

In the calculation of volume, we test if the influence is higher than the iso-value (to determine if the point belongs to implicit volume). The value of the influence thus does not have any importance for us.

We conclude that the variation of volume depends on the function of density. The slope is modified by variation of the parameters of the functions. The variations of volume can thus be controlled by modifying these parameters during an animation.[18]

6. Localization of the Variation

We studied the variations of implicit volumes of objects according to the parameters and the function of density, which is used. It is now interesting to locate these variations on the surface of the implicit objects. We chose to implement the modified method of the seeds.[2,19] With this method, we can compute the volume of the implicit object very quickly and locate the variations. With the classical octree method, the volume may be computed but variations not localized.

6.1. *Seeds method*

This method breaks up into two phases. The first one is initialization, which consists of placing seeds (points of sampling) around the implicit object. These seeds are then moving towards the surface of the implicit object that allows a sampling of the object. The seeds are placed on the border of an including box (including each primitive). The side of the box is equal to twice the thickness of the object. On Fig. 16, we represent a 2D primitive in blue surrounded with an including box, sampled by seeds. The step of discretization is ΔS. One seed is migrating from its initial position I to the center O of the primitive along a vector $\overrightarrow{IO}$.

Each seed allows the positioning of a pyramid, whose apex is located at the center of the primitive and height is the distance from the center of the implicit object to the seed. The base of the pyramid is perpendicular to the height. The dimension of the base is the same one for all seeds; it depends on the smoothness of discretization chosen at the initialization of seeds. The volume of the object is equal to the sum of volume of each pyramid.

6.2. *A new discretization of the implicit objects*

We have expressed the seeds in spherical co-ordinates. The origin of the spherical frame of reference is the center of the primitive to which the seeds belong. A g_{ij}

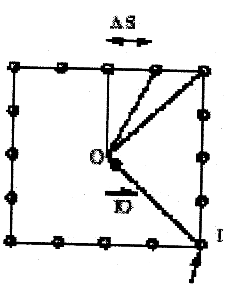


Fig. 16. Initialization of seeds on the surface of each primitive.

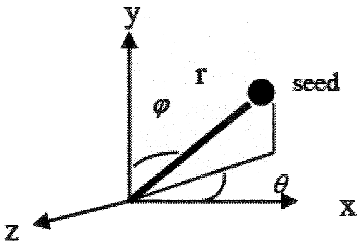


Fig. 17. Definition of a seed with spherical coordinates.

seed is defined by a radius r_{ij} and two angles i and j (as seen Fig. 17). As an initialization r_{ij} is fixed by approximating the effective radius of the blob regarded as a sphere. i and j are the angles calculated according to the number of seeds. Space is divided into angular portions. Two seeds are close if $|\theta_i - \theta_j|$ or $|\varphi_i - \varphi_j|$ is less than an epsilon.

For the moment, it seems suitable for point skeleton only. But we expect a future extension for other skeletons.

6.3. *Localization*

We seek to locate the places of variation using the pyramids. When the implicit primitives are not in influence, all of the heights of pyramids have the same values. If there is mutual influence, the heights of the pyramids in the zone of inter-influence will increase on the axis of the centers, there is creation of matter on this axis. The places of variations are in the zones of inter-influence. When the centers of the primitives move away, volume decreases because the parts furthest away from the primitives are not any more in influence. The volume of the pyramids located at these places decreases. We can then compensate for the possible volume expansion between the centers.

After a minimum, the reduction in volume on the external border (zone where there is no more mutual influence) is less significant. That induces an increase in total volume since the size of the pyramids located between the centers always grows.

Volume grows until a distance $d = 0.98$ between the centers (Fig. 7). For lack of mutual influence between the centers of the primitives, implicit volume grows hollow. The volume of the pyramids located perpendicular to the axis of the centers decreases. The pyramids, which were in contact, separate to form two topologically distinct primitives. There is no more mutual influence as soon as the distance becomes equal to $\max(r_1 + R_2, r_2 + R_1)$.

The volume then remains constant.

A 2D cut of the preceding animation enables us to better visualize the localization of the variations of volume (Fig. 18). The pyramids are then triangles. The localization of the variation of the surface of the triangles is easy. At a given

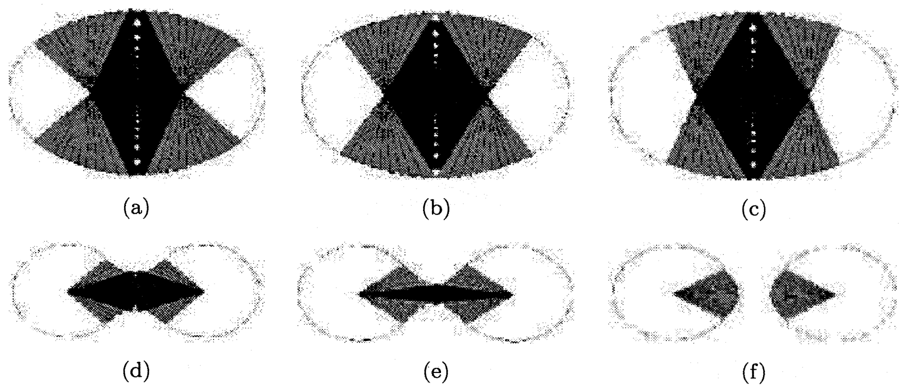


Fig. 18. Places of variation (in 2D during the separation of two primitives). For the black parts, the triangles of which volume increases; for the grey parts the triangles whose volume decreases.

moment, we calculate the total surface of the triangles composing the object. Each triangle is related to a seed by its base. It is located by its height on the basis of the center of the implicit object to which it belongs. The centers of the primitives are moving away. The size of the triangles changes. Figure 18 shows the places of the variations of the surface of the triangles. From Figures (a) to (f), the distance between the centers increases. If the surface of the triangle decreases compared to the preceding image in animation, the triangles are represented in light. If it is the opposite, (the surface increases), the triangles are represented in dark. The triangles whose surface is constant do not appear.

7. Control Volume Variation

The goal of the previous study is to control volume variation during an animation or to represent non-compressible material or very deformable material like clay with implicit surface.

A first method to control volume consists of decreasing the distance between the center of a blob and points in space.[18] The value of the density function is modifying with the distance. This method may be seen as a local control of the implicit surface. An other method[7] uses a space partition and a graph of influence in which nodes represent blobs and an edge between two blobs means that there is influence between both. This graph is transformed to isolate surfaces.

We have already proposed a new global volume control to preserve the volume of the implicit surface.[2] We chose to modify the threshold using a proportional derived controller. Modifications of the threshold follow the following equation: $T^t = T^{t-1} + \varepsilon^t$ where T^t is the threshold at time t and ε variation. We establish

$$V^t = \sum_{i=1}^{n} V_i^t, \quad V^{t-1} = \sum_{i=1}^{n} V_i^{t-1}, \quad V^0 = \sum_{i=1}^{n} V_i^0,$$

and

$$\Delta^t = \frac{V^t - V^0}{V^0}, \quad \dot{\Delta}^t = \frac{V^t - V^{t-1}}{V^0}, \quad \varepsilon^t = \alpha\Delta^t + \beta\dot{\Delta}^t,$$

where V_i^t is the volume of blob i at time t. We chose to compute initial volume at time $t = 0$ as the sum of volumes of blobs considered without influence. That is to say

$$V^0 = \sum_{i=1}^{n} \frac{4}{3}\pi \left(d\left(s, P_i\right)\right)^3,$$

where $d(s, P_i)$ is the distance between point P_i and the center s of the blob. (s may be a seed on the skeleton if it is not any more a point but another geometrical figure as segment.)

Initial value of threshold is initialized to 0.5. The dark gray curve in Fig. 19(a) represents volume variation without control and the other one represents volume variation with our global control Volume computation is always fewer than 5% of the total volume when the distance between blobs is increasing. Figure 19b represents variation of the threshold to control the volume variation of the previous figure.

8. Use of Other Blending Functions

In the preceding paragraphs, we chose to use a traditional operation of blending. We can use other operations. For example, in Fig. 20 we compare the variations of volumes obtained with an addition and an union of blobs.[20] The shape of the curves is different.

If the operation of union is used, volume grows in a monotonous way when the primitives move away. The computation of the density on a point of space, for this operation, does not consider anymore the sum of the influences but corresponds to the maximum value of the densities of the primitives. When the implicit objects have the same centers, volume is minimal. The height of the pyramids and then the

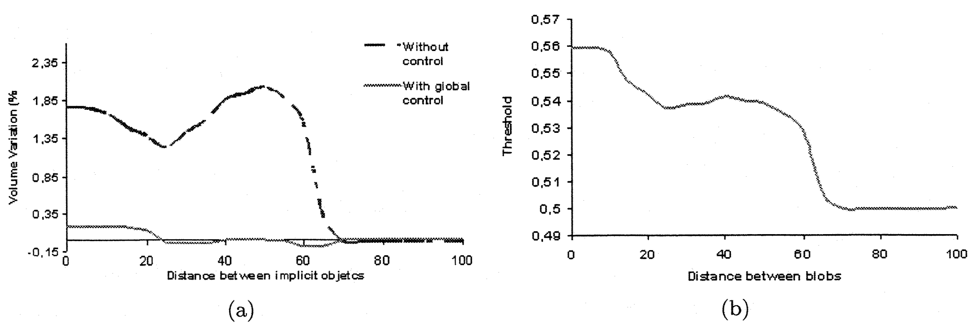


Fig. 19. (a) Comparison between volumes computed with and without global control. (b) Threshold's variation with distance between blobs.

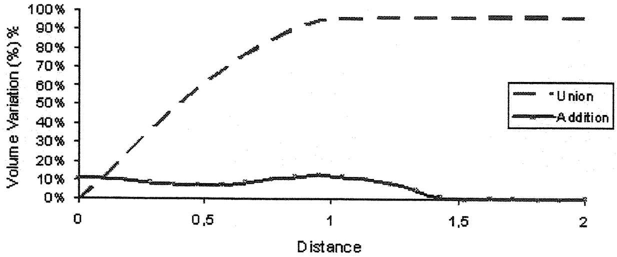


Fig. 20. A lot of merging operations.

volume are increasing as the implicit objects are moving away. This increase in the volume of the pyramids perpendicular to the axis of the centers is not compensated by a reduction in volume. When the two objects are not anymore in influence, the volume is at its maximum (the heights of the pyramids are maximum).

It is noticed that the variations of volume are minimal, close to zero, throughout animation. This is due to the fact that the volume of the object is equal to the sum of volumes of the primitives, which are not deformed.

9. Conclusion and Future Works

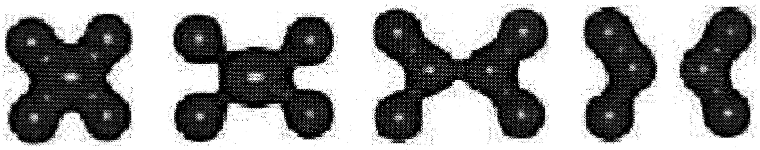
We studied, in this paper, the variations of volume of blobs during an animation. We noticed that these variations are due to the choice of the function of density, of the value of the threshold, the radii of influence and the value of the parameters inherent in many functions. The choice of the value of the parameters is significant. It can imply great variations of volume. The modification of the iso-value must be made with precaution. If control volume during an animation is required, it is necessary to take into account this last remark.

We also located the variations of volume on the surface of the primitives thanks to the use of pyramids. The increase in volume is done between the primitives, but also on their external parts when the mutual influence is total.

This study has several possible applications. The first one is the control of variations of volume of the primitives during an animation, as seen in Sec. 7. Indeed, these variations can be very significant what involves a very disturbed animation, and even the spontaneous appearance of non-desired primitives.

Another application is the modeling of soft objects like clay for virtual sculpture. We have tested our control of volume variation on the sequence below composed with 6 blobs (Fig. 21). We remark that volume is constant as T is varying and blobs are moving away.

An extension of this study is of course the use of other skeletons rather than points. For instance we would like to compute volume and control volume variations on implicit objects composed with primitives defined with anisotropic functions.[21,22]



(a) T = 0.491 V = 39.25 (b) T = 0.446 V = 39.25 (c) T = 0.464 V = 39.25 (d) T = 0.453 V = 39.25

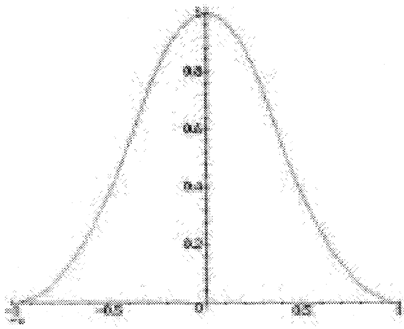
Fig. 21. Modification of a block of clay.

Appendix A. Appendices

Function of Nishimura:

$$d(P) = \begin{cases} d_i\left(1 - 3\left(\frac{r}{R_i}\right)^2\right) & \text{if } 0 \le r \le \frac{R_i}{3} \\ \frac{3d_i}{2}\left(1 - \frac{r}{R_i}\right)^2 & \text{if } \frac{R_i}{3} \le r \le R_i \\ 0 & \text{else} \end{cases}$$

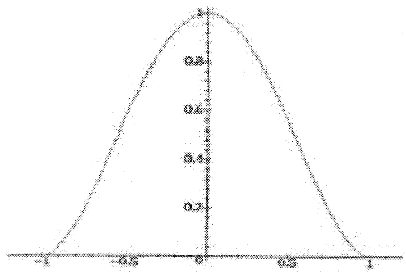
r is the distance of a point P to the center of a blob i, d_i a factor, b_i the radius of the blob.



Function of Murakami:

$$f(r) = \begin{cases} \left(1 - \left(\frac{r}{R_i}\right)^2\right)^2 & 0 \le r < R_i \\ 0 & \text{else} \end{cases}$$

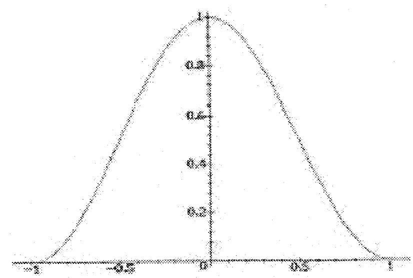
r is the distance of a point P to the center of a blob i, d_i a factor, b_i the radius of the blob.



Function of Wyvill:

$$f(r) = \begin{cases} \text{if } 0 \le r < R_i \\ -\frac{4}{9}\left(\frac{r}{R_i}\right)^6 + \frac{17}{9}\left(\frac{r}{R_i}\right)^4 - \frac{22}{9}\left(\frac{r}{R_i}\right)^2 + 1 \\ \text{else} \\ 0 \end{cases}$$

r is the distance of a point P to the center of a blob i, d_i a factor, b_i the radius of the blob.



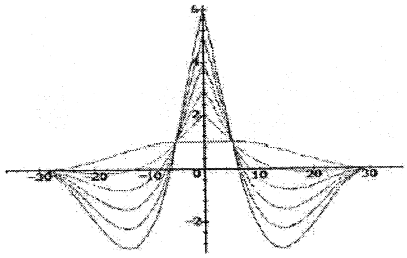


Fig. 22. Function of Tsingos: k varies from 0.1 to 1 and r_0 remains constant.

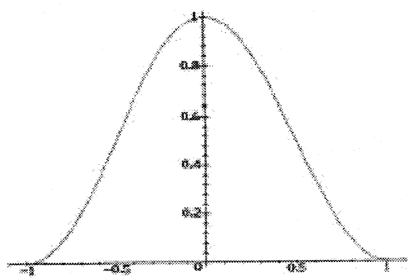


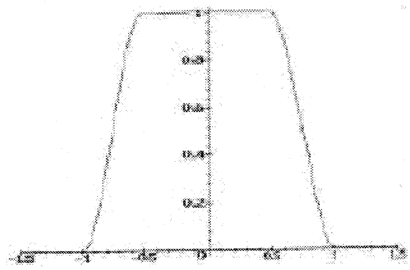
Fig. 23. Function of Tsingos: k remains constant and r_0 is varying from 0 to 1.

Function of Tsingos:

$$f(r) = \begin{cases} ar^2 + br + c & \text{if } 0 \leq r < r_0 \\ (r - R_i)^2\,(dr + e) & \text{if } r_0 \leq r \leq R_i \\ 0 & \text{else} \end{cases}$$

$$\text{with} \begin{cases} d = -\frac{k(r_0 - R_i) + 2}{(r_0 - R_i)^3} \\ e = \frac{kr_0(r_0 - R_i) + 2r_0 - R_i}{(r_0 - R_i)^3} \end{cases}$$

r is the distance of a point P to the center of a blob i, d_i a factor, b_i the radius of the blob r_0 is the thickness and k the stiffness.



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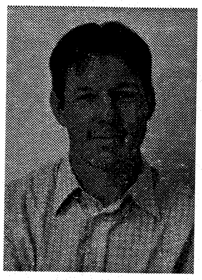

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