Modèles statistiques et fréquentiels pour l'image - Master ID3D

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LIRIS - CNRS

04/09/2024

Organisational notes

- The course slides are available on my webpage on the day of the course: http://liris.cnrs.fr/julie.digne/cours/cours_image_stats.html

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- The course slides are available on my webpage on the day of the course: `http://liris.cnrs.fr/julie.digne/cours/cours_image_stats.html`
- Practical work ("TP"): in python with numpy and pillow.

Course schedule

- September 4th: Markovian model and texture synthesis.
- September 9th: Classification Methods and dimensionality reduction. (+TP)
- September 11th: Choosing a Model, Regression Problems, Considerations on Norms.
- September 16th: Image histograms and histogram specification, half-toning (+TP)
- September 18th: Patch-based image processing and editing (+TP)
- Exam on November 6th (to be confirmed)

Project and evaluation

The assignment is available on my webpage. Evaluation will be done through one-to-one interviews.

- September 9th - 1h30 session
- September 16th - 1h30 session
- September 18th - 2h session

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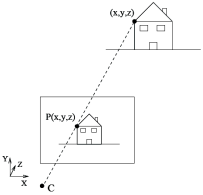
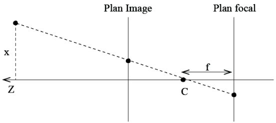
157	159	163	163	165	165	165	165	168	168
157	163	163	165	168	165	168	168	170	170

"The Eiffel tower" "A blue sky" …

Acquisition of digital images

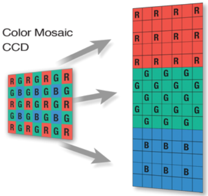
- Projection of a 3D scene on a 2D plane
- Numerically: only a table of numbers
- Black and white: $I : \Omega \subset \mathbb{R}^2 \to \mathbb{R}$; $I(x, y) = i$
- Color: $I : \Omega \subset \mathbb{R}^2 \to \mathbb{R}^3$; $I(x, y) = (r, g, b)$





Acquisition of digital images

- CCD Matrix (*Charged Coupled Device*): integrates the quantity of photons arriving at each cell
- Each pixel integrates a given color.



Bayer Pattern - demosaicking

What we'll see in this course

- Model an image as a *distribution* of colors
- Detect objects by *model regression* (least squares...)
- *Classify* objects by their similarities
- *Compare* textures, images

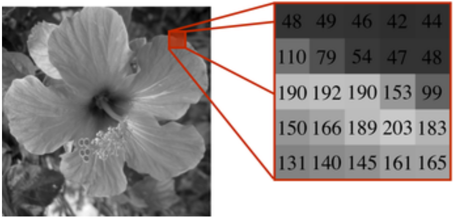
Plan

Grayscale image

Each pixel encodes a light intensity.

For an 8-bits image, a pixel can take 256 integer values ($0 \leq I(p) \leq et255$).

0 encodes a **black** pixel, 128 encodes a **gray** pixel and 255 for a white pixel.



Color Image

- A table (matrix) in which all pixels are triplets (R, V, B) corresponding to the color decomposition on the three primary colors red, green and blue.
- $(0, 0, 255)$ blue, $(0, 255, 0)$ green, $(255, 0, 0)$ red.

A color image

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Red channel

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green channel

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blue channel

The three channels are highly correlated..

From color values to gray scale values

Compute the image luminance using the channel values:

$$L = \frac{R + V + B}{3}$$

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Other (better?) color representations

HSV colorspace

- H indicates the color hue (red, yellow, green)
- Saturation S expresses the fact that the color is more or less pure
- Lightness value V indicates the luminosity of a pixel

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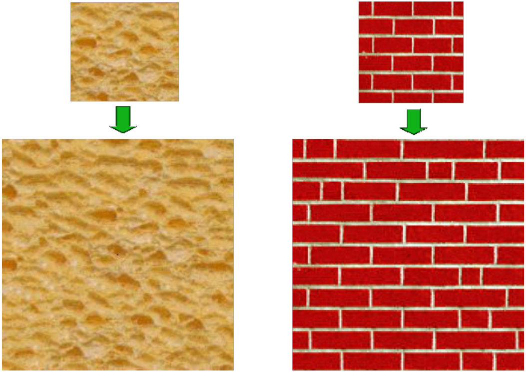
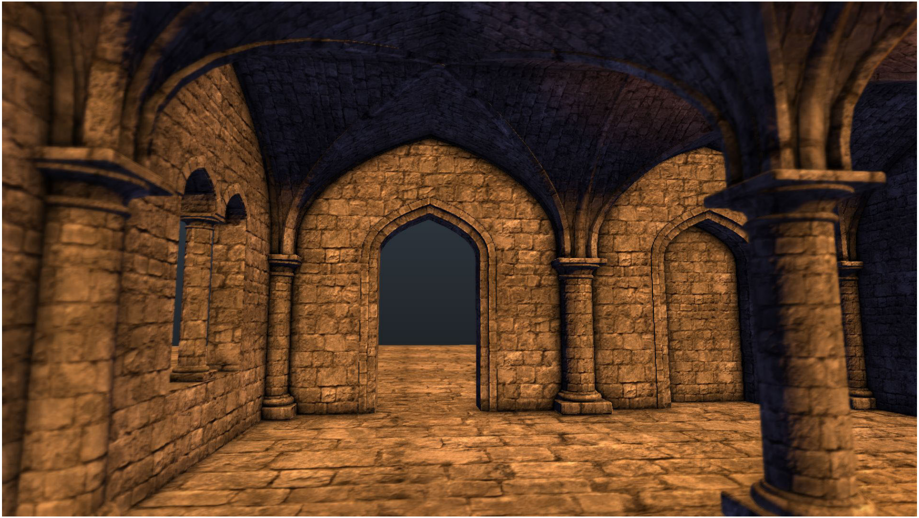
Plan

Texture Synthesis

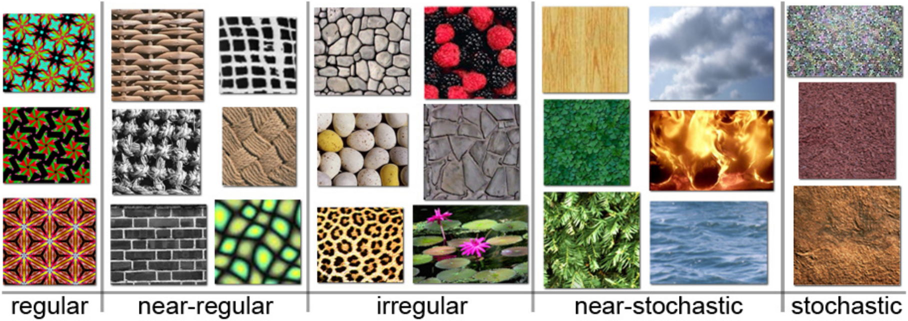


Goal



Copyright http://www.castlebuilder3d.com/

Textures are difficult



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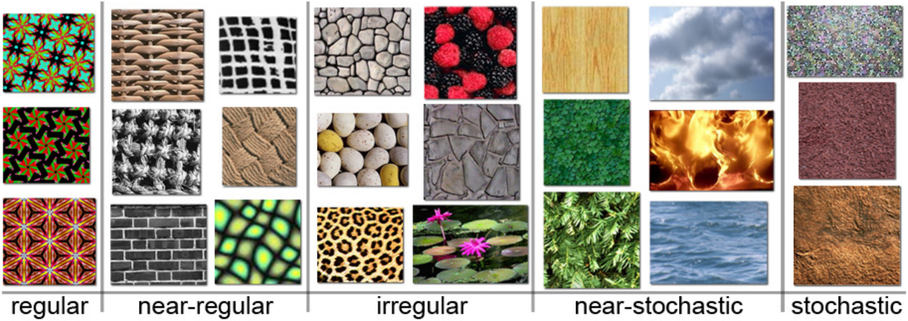

- Copy-pasting an image patch would work for regular textures but not for stochastic textures.
- Drawing pixel values from a probability distribution would work for stochastic textures but not for regular ones.

Plan

To begin with: Markov Chains

An example

Predicting the weather as “sunny”, “cloudy” or “rainy” for each day. The simplest approximation is to assume that the weather on day i only depends on the weather on day $i-1$.

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Order

This is a first order Markov Chain

Wheather Markov Chain

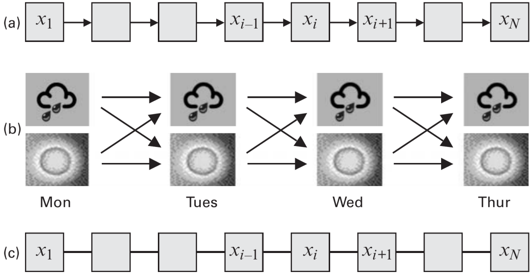


Image from Blake et al. 2011

Transition probabilities

Transition probabilities

The transition probability matrix between two states is

$$P(X_i = a | X_{i-1} = b) = M_i(a, b)$$

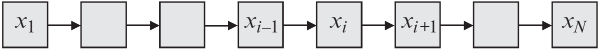
The Markov Chain is said to be stationary if the transition probabilities are independent of i, *i.e.*

$$M_i(a, b) = M_{i-1}(a, b) = M(a, b)$$

- One can devise Markov chains with higher orders (dependencies on $i-1$, $i-2$...): In that case, the value of X_i depends on a limited number of previous states $X_{i-1}, \cdots, X_{i-n}$.

Markov chains as graphs

- The chain can be represented as a graph:
  - Each node corresponds to one X_i
  - An edge exists between X_i and X_{i-1} to model $P(X_i|X_{i-1})$.

(a) 

[Blake 2011]

A graph corresponding to a markov chain of order 1

Markov Random Fields

Definition

A Markov Random Field (MRF) is defined as a probabilistic model over an undirected graph $(\mathcal{V}, \mathcal{E})$

$$P(x_i|(x_j)_{i \neq j}) = P(x_i|\{x_j|(i,j) \in \mathcal{E}\})$$

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- Consequence: $P((x_i)_i) = \prod_{(i,j) \in \mathcal{E}} F_{i,j}(x_i, x_j)$

Modeling an image as a graph

Graph of an image

A graph can model the relationship between each pixel (or super-pixel) and its neighbors.

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Markov Random Fields on graphs: Local Markov Property

The random variable at a node depends solely of the random variables in its neighborhood (Markov blanket).

Modeling an image as a graph

Graph of an image

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Markov Random Fields on graphs: Local Markov Property

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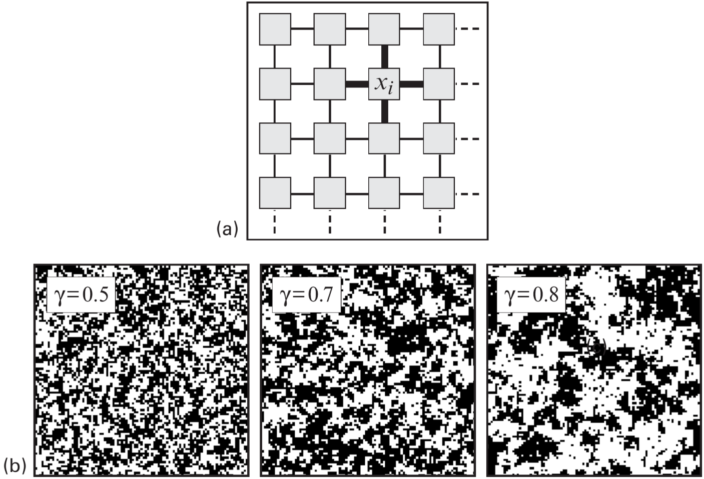
- From now on we will assume that the distribution is positive (in that case local Markov property $\Leftrightarrow$ global Markov property)

Energy

Energy

Since P is a positive distribution, we can rather rely on an energy $E(x)$ such that $P(x) = \prod_{c \in \mathcal{C}} \phi(x) = \exp(-E(x))$ and $E(x) = \sum_{c \in \mathcal{C}} \Phi_c(x_c)$.

Ising Model



[Blake 2011]

Ising Model

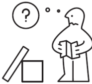
Ising Model

Each variable X_i takes values in $\{0, 1\}$. The cliques have size 2 (two neighboring pixels).

$$\phi_{ij} = \gamma |x_i - x_j|$$

where γ is a parameter of our method. The total energy is

$$E(x) = \sum_{(i,j) \in \mathcal{E}} \gamma |x_i - x_j|$$

- What does this γ model ?

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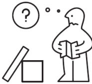
- What does this γ model ? When two neighboring variables have different values the energy increases of amount γ

Gibbs Sampling

Sampling from a MRF

At each visit to a site i, x_i is sampled from the local conditional distribution $P(x_i|x_j, (i,j) \in \mathcal{E}, j \neq i)$. Start with random values for the pixels and traverse all sites in random order until convergence.

- local conditional distribution:

$$p = \frac{P(X_i = 1)}{P(X_i = 0)} = \frac{\exp -\gamma E(x_0, \cdots, X_i = 1, \cdots, x_n)}{\exp -\gamma E(x_0, \cdots, X_i = 0, \cdots, x_n)} = \exp -\gamma \Delta E$$

with ΔE the difference of energy between the two states.

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  with ΔE the difference of energy between the two states.
- The process converges
- ... But can be extremely slow.

Plan

Texture synthesis as a MRF Problem

- A texture is modeled by a Markov Random Field

[Efros Leung 1999], **[Wei Levoy 2001]**

Texture synthesis as a MRF Problem

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Texture synthesis as a MRF Problem

- A texture is modeled by a Markov Random Field
- Each pixel value depends on the pixel values of its neighbors
- The size of the neighborhood encodes how stochastic the texture is.

[Efros Leung 1999], **[Wei Levoy 2001]**

Synthesizing one pixel [Efros Leung 1999]

Synthesis

Let I_{smp} be the texture sample image, I_{real} be the infinite texture $I_{smp} \subset I_{real}$ and I the image being synthesized. Assume all pixels are known except p. Let $w(p)$ be its neighborhood, then:

$$P(I(p) = a|I) = P(I(p) = a|w(p))$$

- Let $d(w_1, w_2)$ be a distance between two patches (usually SSD of $SSD * G$)

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- Let $d(w_1, w_2)$ be a distance between two patches (usually SSD of $SSD * G$)
- $\Omega(p) = \{w \in I_{real} | d(w, w(p)) = 0\}$
- I_{real} is unavailable

Synthesizing one pixel

Heuristic

Replace $\Omega(p)$ by $\Omega'(p)$ containing patches that are close to $w(p)$. Let $w_{best} = argmin_{w \in I_{smp}} d(w, w(p))$ then:

$$\Omega'(p) = \{w \in I_{smp} | d(w, w(p)) \leq (1+\varepsilon) d(w_{best}, w(p))\}$$

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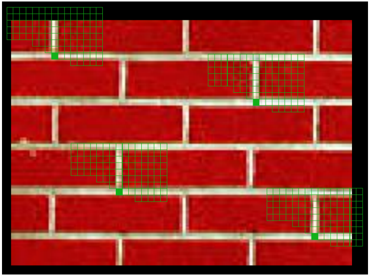
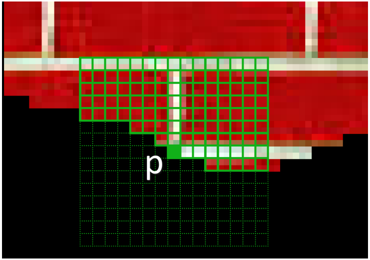
Finally

p is taken to be the average of the values of all center pixels in $\Omega'(p)$ (variation: the value of one uniformly drawn patch in $\Omega'(p)$.

Texture Synthesis Algorithm

- Assuming Markov property, compute $P(p|N(p))$
  - Search the input image for all similar neighborhoods $\Omega'(p)$
  - Pick one match at random





Texture synthesis Algorithm

1. Start from a $\ell \times \ell$ seed from the input

Texture synthesis Algorithm

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2. The new pixel p to fill is randomly picked among the ones that have the larger number of *filled* neighbors in their neighborhood.

Partial distance

Let $w(p) \in I$ and $w'(p') \in I_{smp}$ be two neighborhoods partially filled. $\tilde{N}$ is the set of all v such that $I(p+v)$ and $I_{smp}(p'+v)$ are defined. The distance between w and w' is

$$d(w, w') = \frac{\sum_{v \in \tilde{N}} \|I(p+v) - I_{smp}(p'+v)\|_2^2 G_\sigma(\|v\|)}{\sum_{v \in \tilde{N}} G_\sigma(\|v\|)}$$

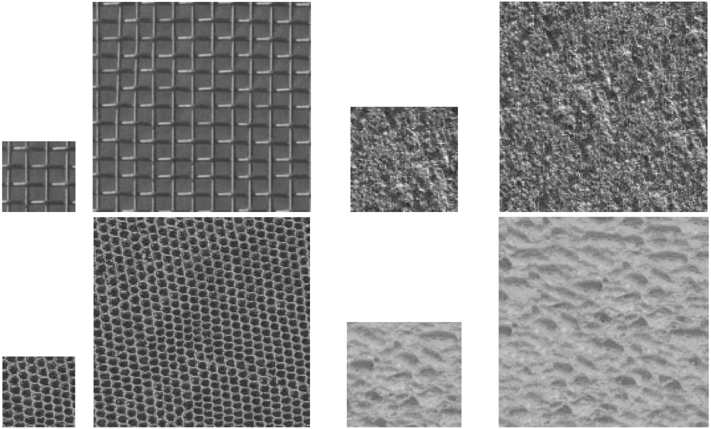
with G_σ a centered Gaussian with standard deviation σ.

Full Algorithm [Efros Leung 1999]

Input: image I_{smp}, output size, neighborhood size,

1. Initialize with $\ell \times \ell$ random seed of I_{smp}
2. While output I not filled
    1. Pick a pixel p not yet filled with a maximal number of filled neighbors
    2. Compute the distance of $w(p)$ to all patches of input I_{smp}
    3. Build $\Omega'(p)$
    4. Pick randomly one of the similar neighborhood $w(p')$ in $\Omega'(p)$
    5. Set $I(p) = I_{smp}(p')$ (= Fill p with central value)

Results



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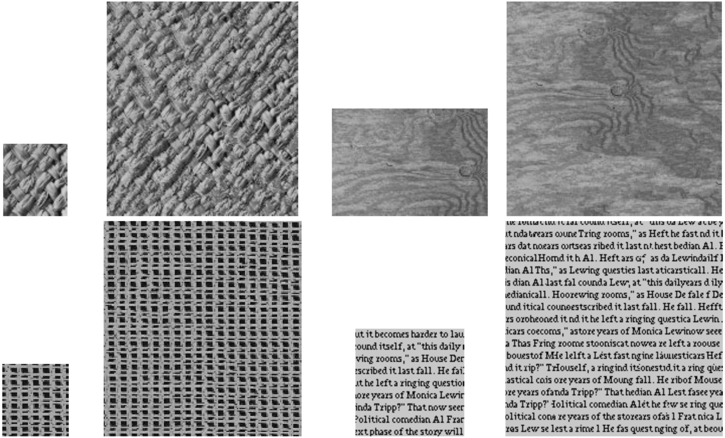

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scribed it last fall. He fai
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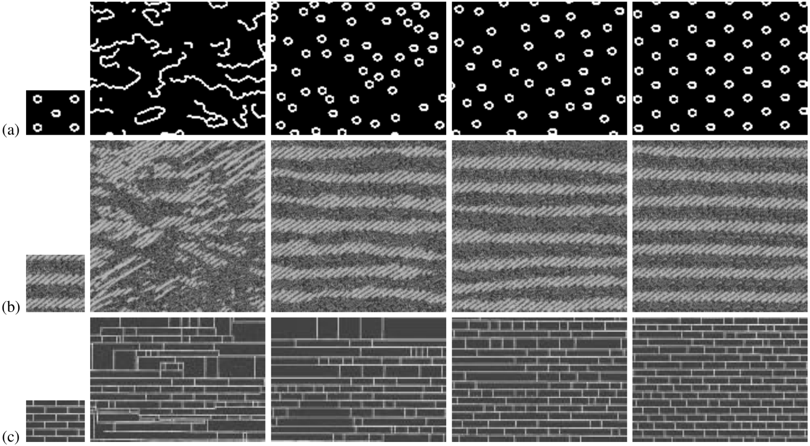
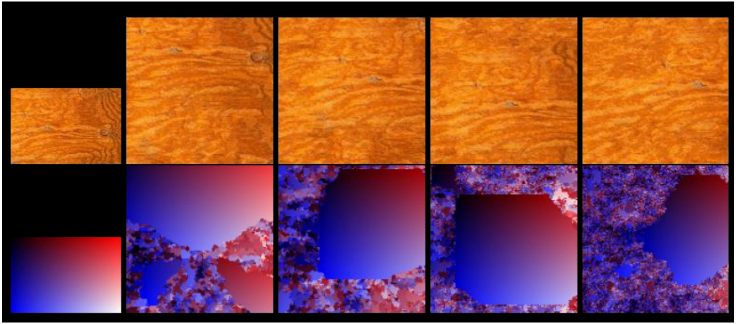
ore years of Monica Lewir
inda Tripp?" That now seer
Political comedian Al Frar
ext phase of the story will

Varying neighborhood size



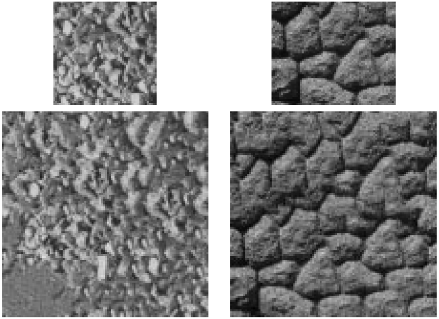
Patch sizes: $5, 11, 15, 23$

Varying epsilon ε



$$\varepsilon = 0.05, 0.1, 0.2, 0.5$$

Failure Cases



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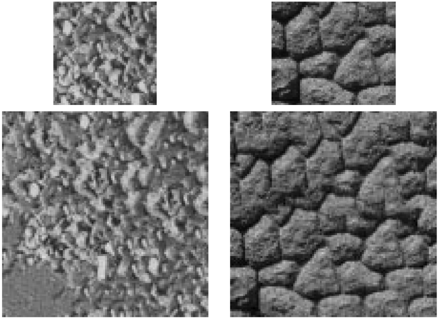

- Growing garbages

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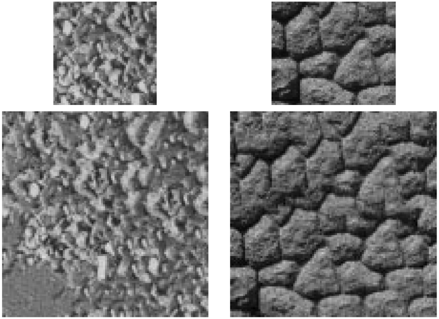
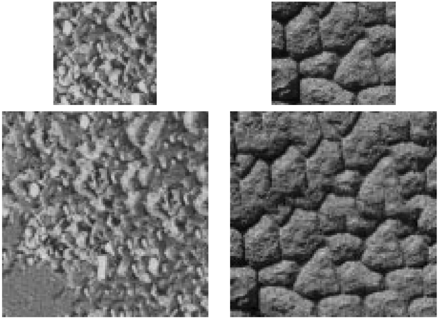


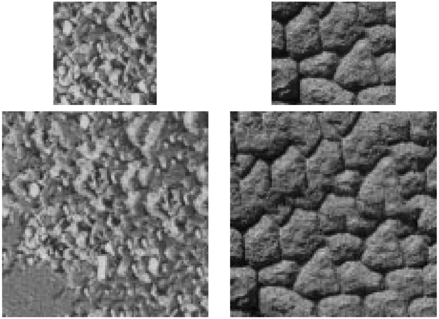
- Growing garbages (ε too large)

Failure Cases



- Growing garbages (ε too large)
- Verbatim Copy

Failure Cases

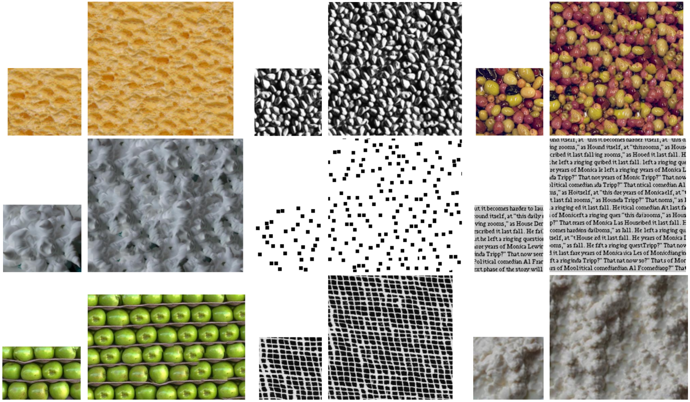


- Growing garbages (ε too large)
- Verbatim Copy (ε too small)

Plan

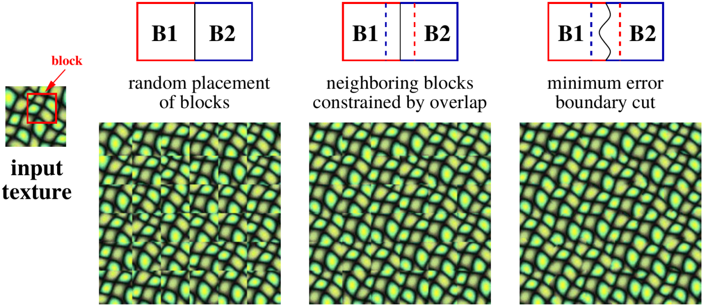
Image Quilting [Efros Freeman 2001]

- Focuses on boundary optimization between neighboring patches



[Efros Freeman 2001]

Patch placement



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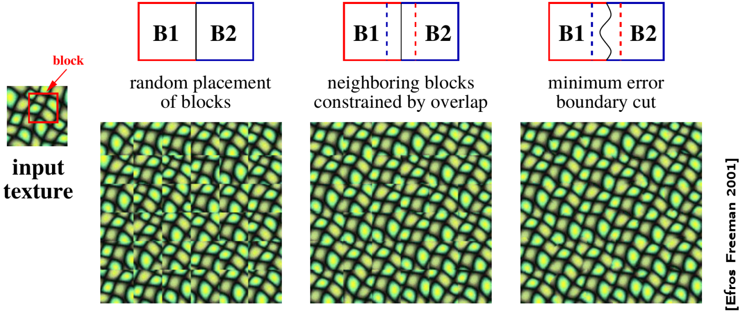
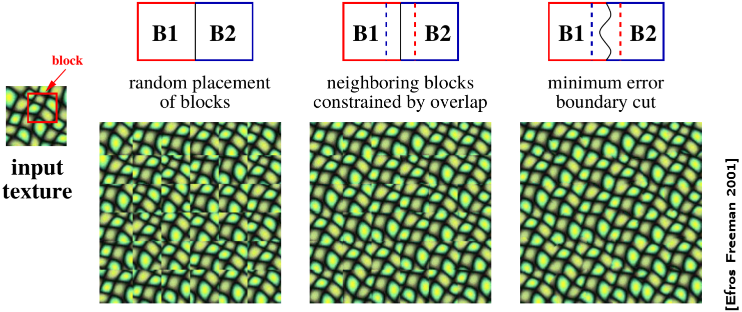


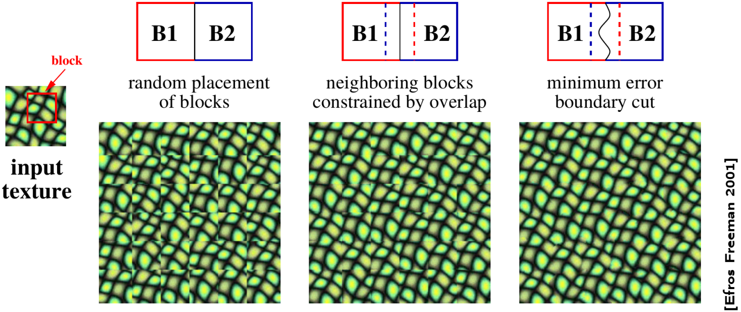
- Paste patches from the sample texture randomly (left)

Patch placement



- Paste patches from the sample texture randomly (left)
- Better: Paste patches from the sample texture randomly among those that fit approximation with their neighbors (middle)

Patch placement



- Paste patches from the sample texture randomly (left)
- Better: Paste patches from the sample texture randomly among those that fit approximation with their neighbors (middle)
- Optimize the boundary so that the patches blend *seamlessly* (right)

Overlapping criterion

- When a set of patches are placed, the next one should fit the set of already pasted patches
- The measure is similar to the partial distance as before.

Overlap error

Let B_1 and B_2 be two overlapping blocks, with overlaps B_1^{ov} and B_2^{ov}. The error image is then $e = \|B_1^{ov} - B_2^{ov}\|_2^2$ (e has the size of the overlap).

Boundary Optimization

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Vertical Boundary Path

For each pixel $(i, j) \in e$ store:

$$E_{i,j} = e_{i,j} + \min(E_{i-1,j-1}, E_{i-1,j}, E_{i-1,j+1})$$

When we reach the bottom we can take the minimum and roll back to get the path of pixels.

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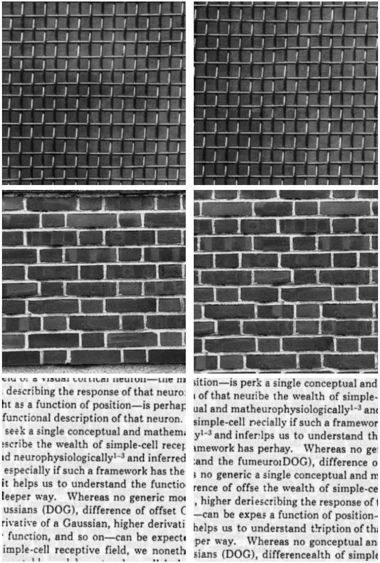
[Raad, Galerne 2017]

Results



[Efros Freeman 2001]

Results

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Plan

Graph Cuts

- Minimize energies by casting the problem as a min-cut problem

Graph Cuts

- Minimize energies by casting the problem as a min-cut problem
- Applications to segmentation, stereo, denoising energies

Problem Statement [Boykov Jolly 2001]

Given an image I, we look for a label A_p for each pixel p of an image. A_p can take values 0 or 1. There are two special sets of pixels O and B containing pixels that we know belong to class O or class B

Segmentation Energy

Segmentation Energy

$$E = \lambda \sum_p R_p(A_p) + \sum_{p,q \text{ neighbors}} B_{p,q}\delta_{p,q}$$

Where

- $R_p(A_p)$ encodes the probability for a pixel p to have label A_p
- $\delta(p,q) = 0$ if $A_p = A_q$ and 1 otherwise.
- $B(p,q)$ is the energy that two pixels have different classes given their color values.

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- $R_p(bkg)$, $R_p(obj)$ class models
- $B(p,q)$ likelihood for a boundary to cross edge p, q

Graph Construction

Graph Topology

The graph $G = (V, E)$ corresponding to the image is built as follows:

- All pixels are vertices of the graph, two special nodes S and T are also added to V
- Edges are added between nodes corresponding to neighboring pixels in the image
- Edges are also added between all pixels and S and all pixels and T

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- Edges are also added between all pixels and S and all pixels and T

- A neighborhood should be defined

Graph Construction

Graph Edge Weights

- Edge between pixels p, q: weight $B(p, q)$
- Edge between pixel p and node S: weight $\lambda R_p(bkg)$
- Edge between pixel p and node T: weight $\lambda R_p(obj)$

Graph Construction

Graph Edge Weights

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- $K = 1 + max_p \sum_{q \in \mathcal{N}(p)} B(p, q)$

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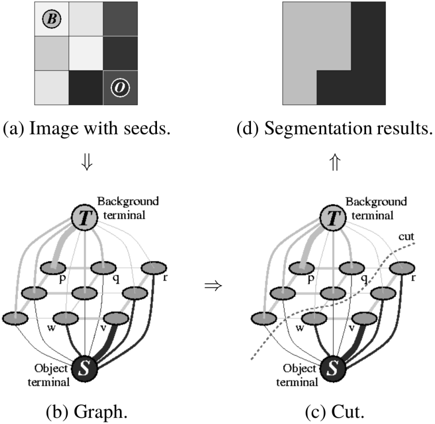
- If $p \in O$: edge p, S has weight K, edge p, T has weight 0
- If $p \in B$: edge p, S has weight 0, edge p, T has weight K

Equivalence to a min cut problem

Energy minimization

The minimium of the segmentation energy is obtained by finding the minimum cost cut on graph G separating S from T

Toy example



[Boykov and Jolly 2001]

- Some hard constraints are created using seeds $p \in O$ or $p \in B$

Proof

Lemma

The minimum cut $\hat{C}$ on graph G is feasible *ie*

- $\hat{C}$ severs exactly one t-link for each p (either $p - S$ or $p - T$)
- if $(p, q) \in \hat{C}$, then one of them is linked to S and the other is linked to T after the cut
- if $p \in O$, then $(p, T) \in \hat{C}$
- if $p \in B$, then $(p, S) \in \hat{C}$

- Proof: Bear in mind that $\hat{C}$ is minimal.

Link with the segmentation

Correspondence

Any feasible cut C corresponds to a segmentation $A(C)$ such that:

$$A_p(C) = \begin{cases} obj \text{ if } (p, T) \in C \\ bkg \text{ if } (p, S) \in C \end{cases}$$

Optimal segmentation

Theorem

Among all segmentation A satisfying $A_p = obj$ if $p \in O$ and $A_p = bkg$ if $p \in B$, the one defined by the minimal cut $\hat{C}$ minimizes the segmentation energy.

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Among all segmentation A satisfying $A_p = obj$ if $p \in O$ and $A_p = bkg$ if $p \in B$, the one defined by the minimal cut $\hat{C}$ minimizes the segmentation energy.

- Proof: link the energy with the cost of the cut.

What are $R_p(A_p)$ and $B(p, q)$?

Regional term

The regional term is a log-likelihood term

$$R_p(A_p) = \begin{cases} -\log P(I_p|O) \text{ if } A_p = obj \\ -\log P(I_p|B) \text{ if } A_p = bkg \end{cases}$$

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Boundary penalty

$$B(p,q) = \frac{1}{dist(p,q)} \exp - \frac{\|I_p - I_q\|_2^2}{2\sigma^2}$$

How do we compute the log-likelihoods?

MRF formulation

The sets O and B give histograms of pixel values for the object and background yielding in turn $P(I_p|O)$ and $P(I_p|B)$

Result

(a) Original image

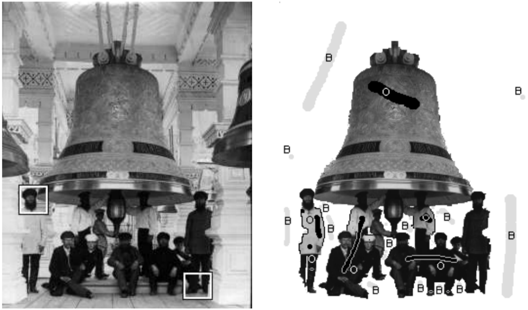
(b) Result for $\lambda = 7$—43

(c) Result for $\lambda = 0$

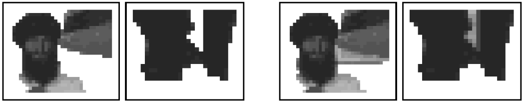
(d) Result for $\lambda = 60$

[Boykov and Jolly 2001]

Result



(a) Original B&W photo (b) Segmentation results



(c) Details of segmentation with regional term (d) Details of segmentation without regional term

[Boykov and Jolly 2001]

Plan

Texture Synthesis using Graph Cuts

Kwatra et al. 2004

Principle

Idea

Copy irregularly shaped patches on the image and arrange the boundaries between the copied patches

Candidate patch selection

- A candidate rectangular patch (or patch offset) is selected by performing a comparison between the candidate patch and the pixels already in the output image.

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- An optimal (irregularly shaped) portion of this rectangle is computed and only these pixels are copied to the output image.

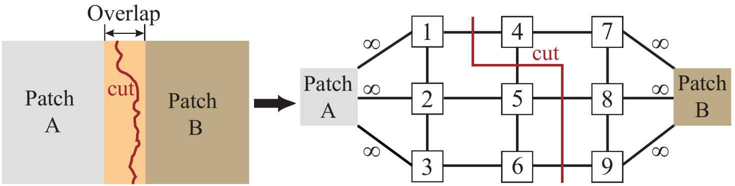
Candidate patch selection

- A candidate rectangular patch (or patch offset) is selected by performing a comparison between the candidate patch and the pixels already in the output image.
- An optimal (irregularly shaped) portion of this rectangle is computed and only these pixels are copied to the output image. This is where we use graphcuts

Matching quality

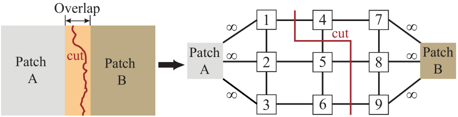
s and t two adjacent pixel positions in two copied patches overlap region. $A(s)$ and $B(s)$ pixel colors at s in the two patches. *Matching quality cost* M between s and t:

$$M(s,t,A,B) = \|A(s) - B(s)\| + \|A(t) - B(t)\|$$



[Kwatra et al. 2004]

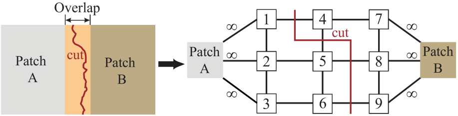
Graph Cut between two patches



[Kwatra et al. 2004]

- Goal Find a minimal path separating A from B

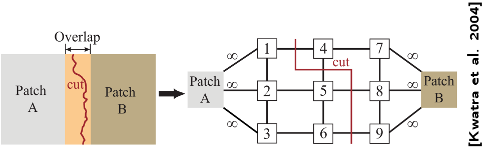
Graph Cut between two patches



[Kwatra et al. 2004]

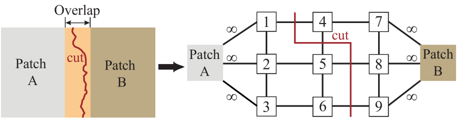
- Goal Find a minimal path separating A from B
- Connect neighboring pixels by an edge with weight $M(s, t, A, B)$

Graph Cut between two patches



- Goal Find a minimal path separating A from B
- Connect neighboring pixels by an edge with weight $M(s,t,A,B)$
- Add two terminal nodes corresponding to A and B

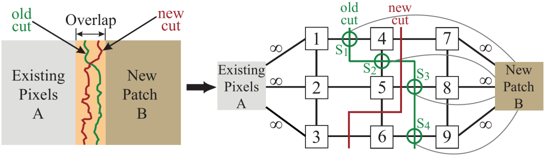
Graph Cut between two patches



[Kwatra et al. 2004]

- Goal Find a minimal path separating A from B
- Connect neighboring pixels by an edge with weight $M(s, t, A, B)$
- Add two terminal nodes corresponding to A and B
- min-cut algorithm yields the best boundary

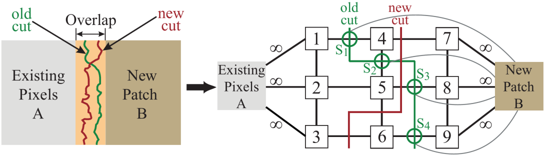
Between more than 2 patches



[Kwatra et al. 2004]

- Assume we have already copy-pasted several patches yielding existing pixel values

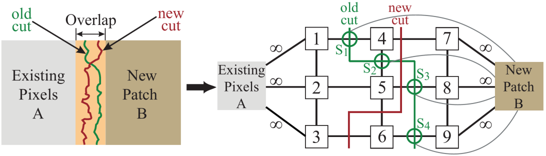
Between more than 2 patches



[Kwatra et al. 2004]

- Assume we have already copy-pasted several patches yielding existing pixel values
- Copying a new patch B

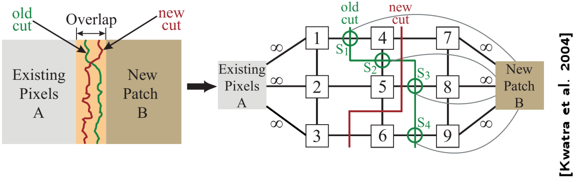
Between more than 2 patches



[Kwatra et al. 2004]

- Assume we have already copy-pasted several patches yielding existing pixel values
- Copying a new patch B
- Graph cuts used to find the new seam

Between more than 2 patches

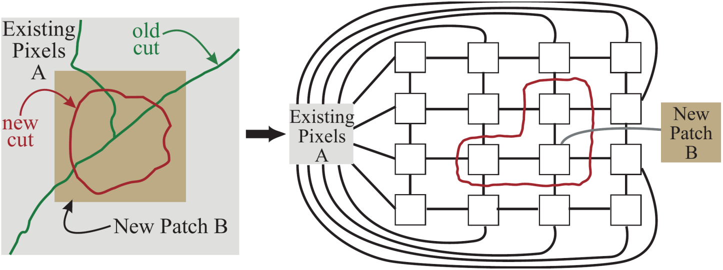


- Assume we have already copy-pasted several patches yielding existing pixel values
- Copying a new patch B
- Graph cuts used to find the new seam

Multiple Seams

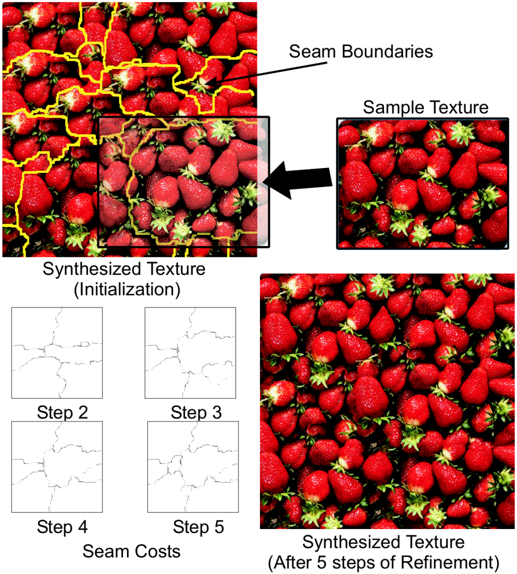
Each pixel s keeps track of the patch A_s it originated from, then the weights on graph edges between two neighboring pixels s and p originating from patches A_s, A_p is simply: $M(s, p, A_s, A_p)$

Surrounded regions



[Kwatra et al. 2004]

Algorithm


Seam Boundaries
Sample Texture
Synthesized Texture
(Initialization)
Step 2
Step 3
Step 4
Step 5
Seam Costs
Synthesized Texture
(After 5 steps of Refinement)

Patch placement

Three strategies are possible:

- Random patch placement

Patch placement

Three strategies are possible:

- Random patch placement
- Entire patch matching

Patch placement

Three strategies are possible:

- Random patch placement
- Entire patch matching
- Subpatch matching

Random patch placement

- The new patch (entire sample texture) is translated to a random offset location. The graph cut algorithm selects a piece of this patch to lay down into the out- put image and the process is repeated

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Pros & Cons

Fastest synthesis method, good result for random textures.

Entire patch matching

- Search for translations of the input image that match well with the currently synthesized output.

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Pros & Cons

Best results for structured and semi-structured textures.

Sub-patch matching

- First pick a small sub-patch in the output texture.
- Look for a sub-patch in the input texture that matches this output-sub-patch well.

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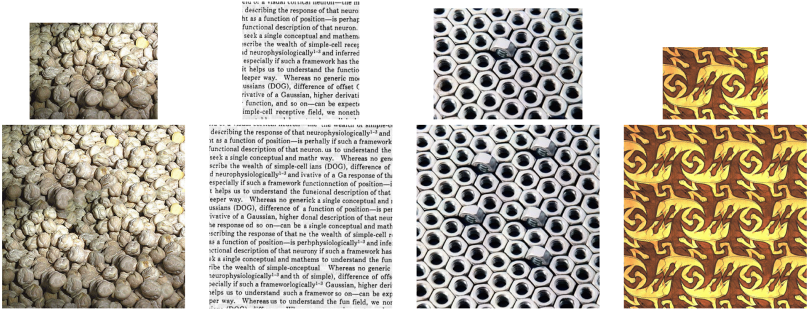
Pros & Cons

most general method.

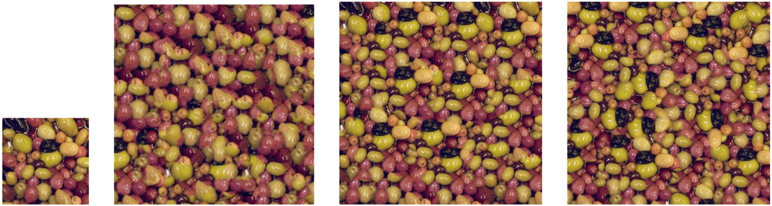
Results



[Kwatra et al. 2004]

Results

Input Image Quilting Graph cut



Input Image Quilting Graph cut Rotation & Mirroring

[Kwatra et al. 2004]

Results

[Kwatra et al. 2004]

Conclusion

- Other methods for Texture Synthesis: Gabor Noise, variational methods ... A vast literature on the subject exists
- Now: Machine learning methods (Gatys et al. 2015 and so on) beyond the scope of this course.

[Zhou et al. 2018]