Silent MST approximation for tiny memory

Lélia Blin, Swan Dubois and Laurent Feuilloley

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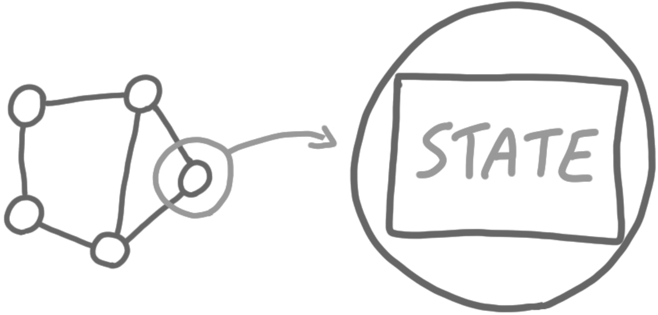
The state model and MST

1. Computation on a graph and every node has a state, with two parts : the mutable and the non-mutable memory.



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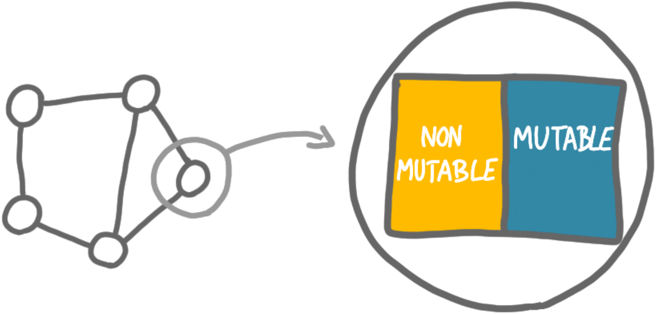

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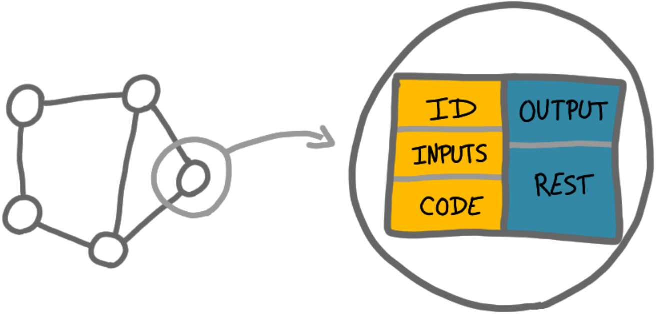
2. The non-mutable memory contains the ID, inputs and code.
3. At the end the mutable memory should contain the output.



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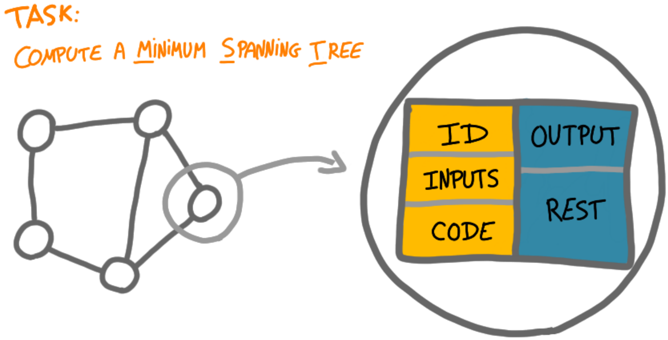

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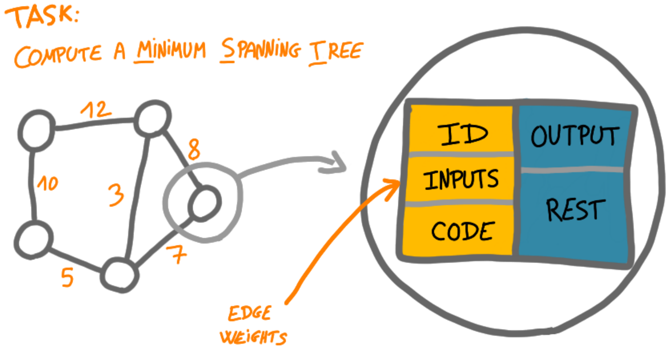

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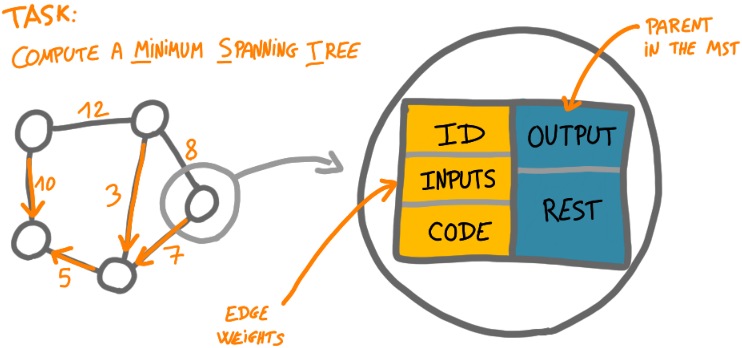
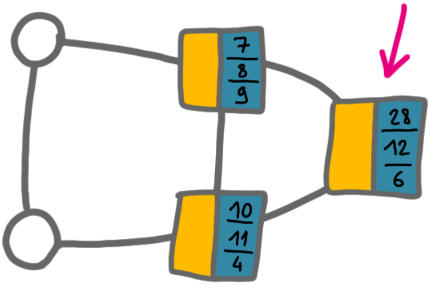
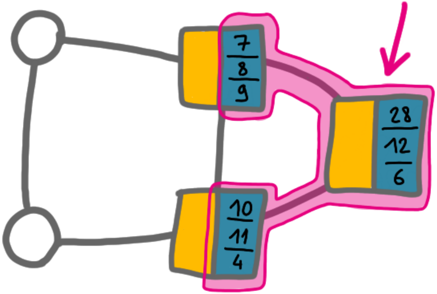


Computation in the state model



Computation in the state model

- Local view, modeling messages.



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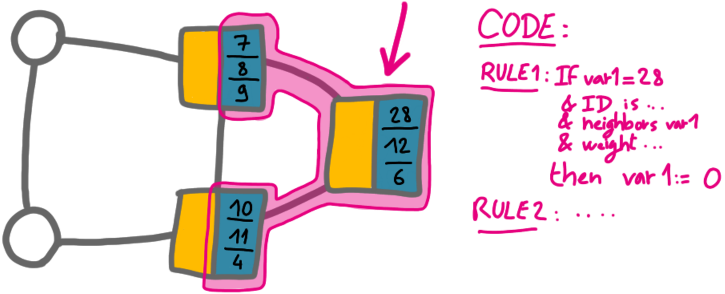

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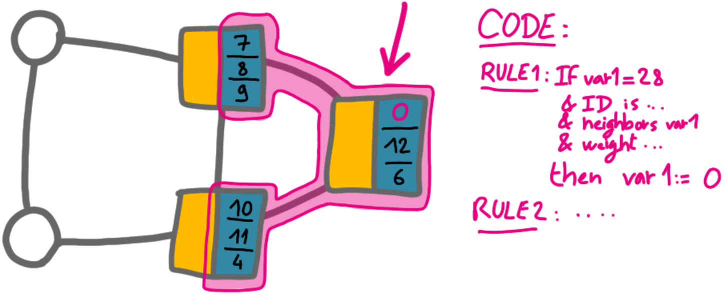

Computation in the state model

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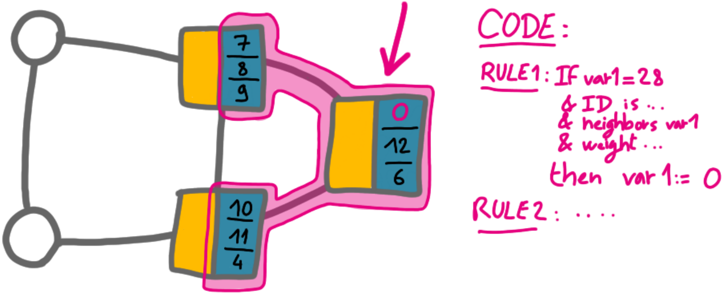
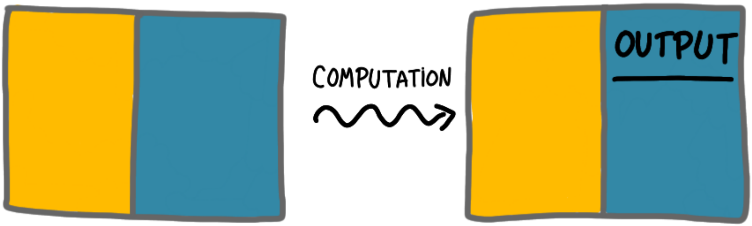
- Adversary scheduler : chooses the (active) nodes taking steps.



Silent self-stabilization

- Fault-free setting vs. self-stabilizing setting



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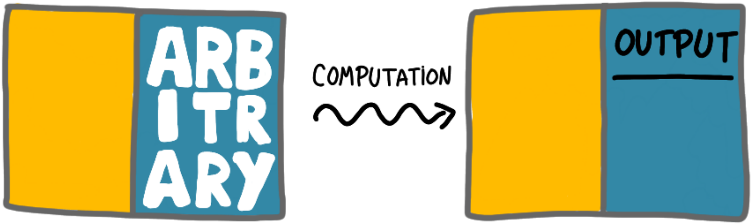

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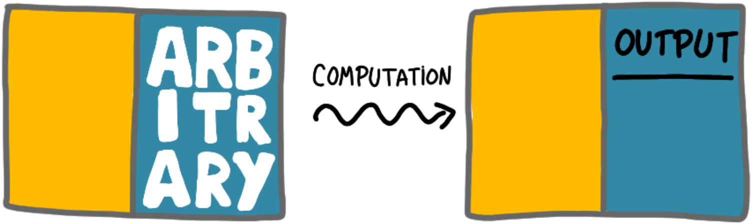

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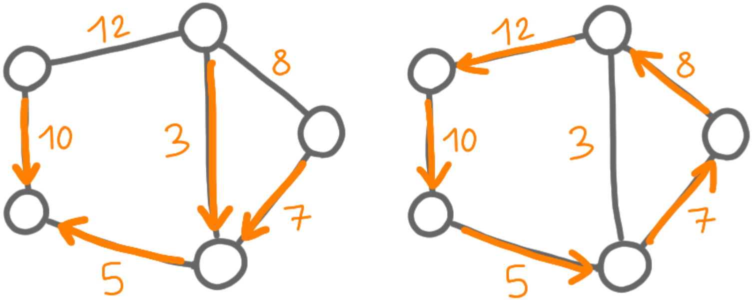
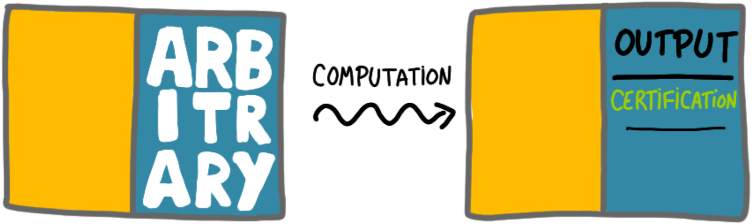


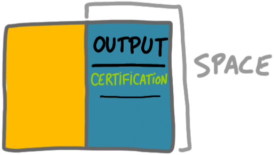
Silent self-stabilization

- Fault-free setting vs. self-stabilizing setting
- Silent self-stabilization : at the end, no activated nodes
- The output is not enough (for most tasks).
- Certification has been studied on its own (*Proof-labeling schemes* Korman, Kutten and Peleg)



Certification size and MST

- Focus on the space (keeping polynomial time).
- $\rightarrow$ minimize size of certification and do not use more space.



Theorem [Korman, Kutten] : The optimal certification size for minimum spanning tree is $\Theta(\log n \times s)$, when weights are encoded on s bits.
$\rightarrow$ For weights in a $poly(n)$ range $s = \log n$, and the optimal size is $\Theta(\log^2 n)$.

Our results

Theorem 1 : A (full) self-stabilizing MST algorithm with optimal space $O(\log n \times s)$.

Proof : First : build a non-certified MST. Second : build the optimal certification on top.

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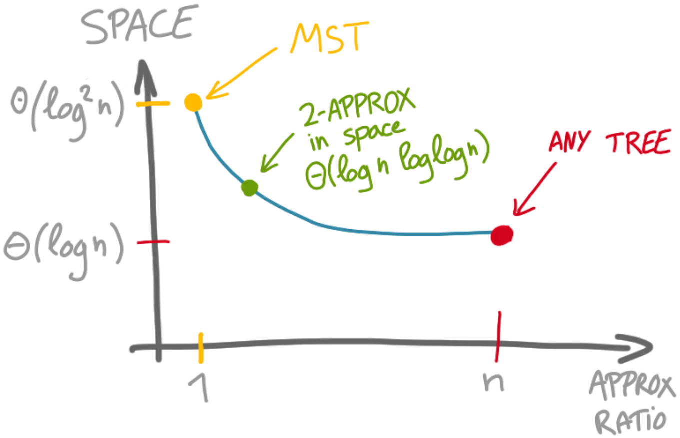
Theorem 2 : We can parameterize the algorithm such that there is tradeoff between space complexity and quality of the solution.

Approximation and trade-off



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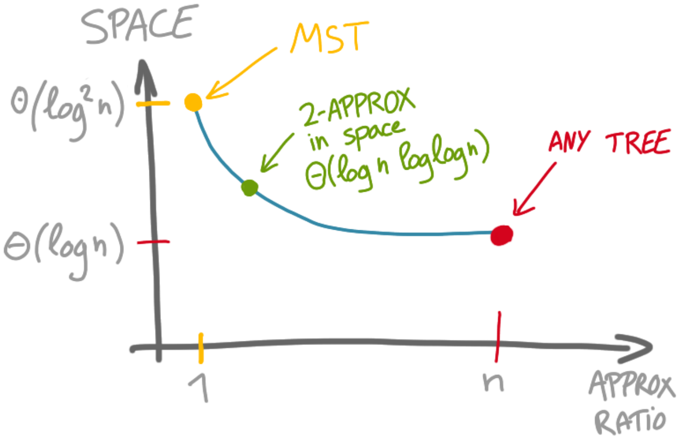
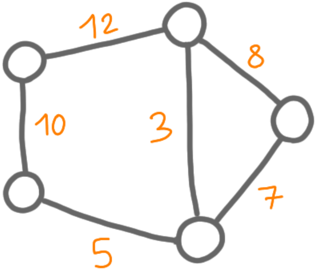


Approximation in certification : Censor-Hillel, Paz and Perry (2017), and Emek and Gil (2020).

Example : 2-approximation

- Round every weight to the next power of 2.



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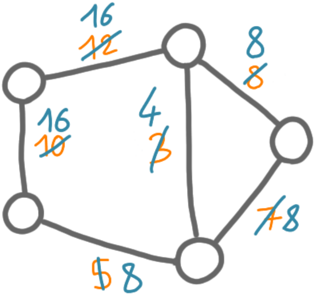

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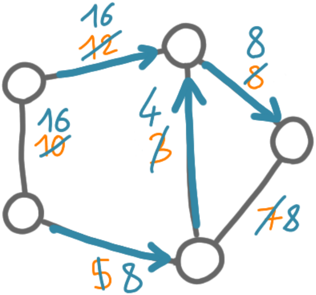

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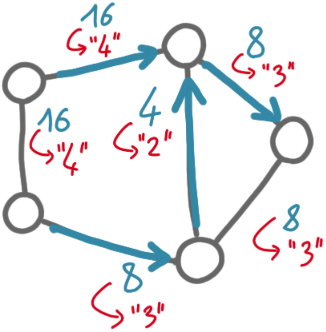
- Round every weight to the next power of 2.
- An MST on these new weights is a 2 -approximation.
- Weights can be encoded in a compact way : 2^p encoded as p.
- This reduces the weight exponentially $\log n \to \log\log n$.
- Using Theorem 1, we get a 2-approx in space $\log n \log\log n$.



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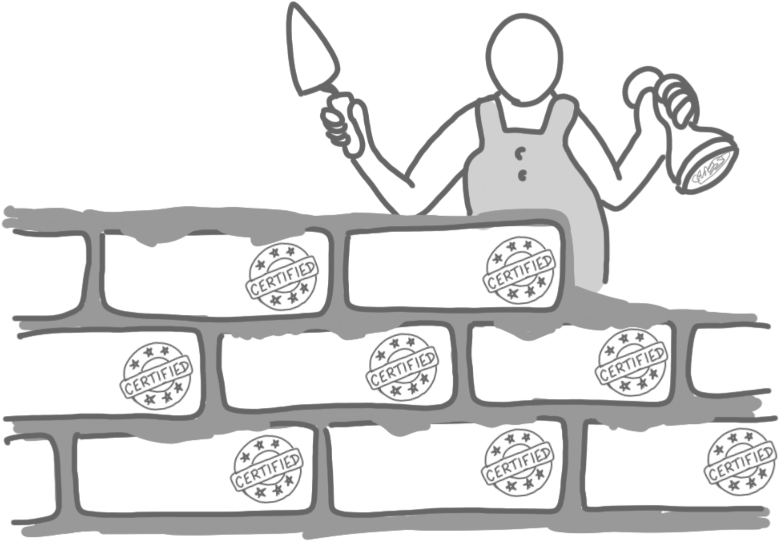

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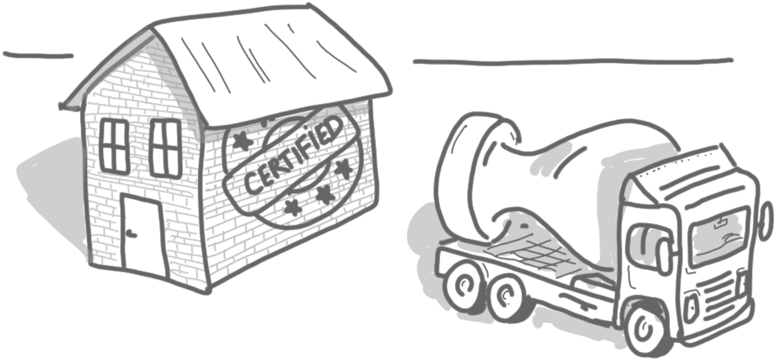

Thanks for your attention !