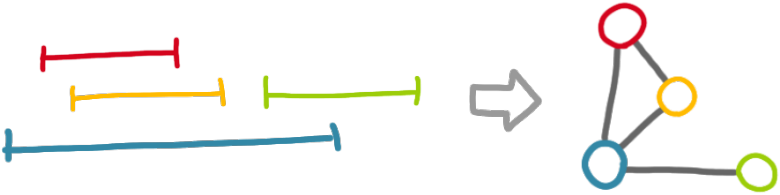
Graph classes and forbidden patterns on three and four vertices

Laurent Feuilloley and Michel Habib

Based on
Graph classes and forbidden patterns on three vertices
(to appear in SIDMA)
and on on-going work.

Short version · November 2020

Interval graphs : geometry



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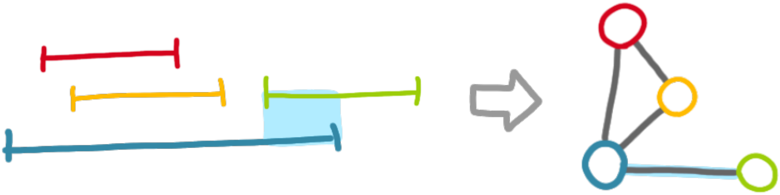

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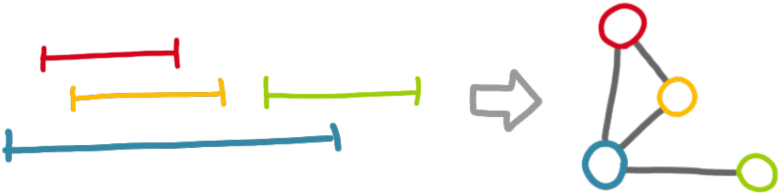

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Interval graphs : with an ordering

Characterization : A graph is an interval graph if and only if, there exists an ordering of its vertices such that for every $u < v < w$, if (u, w) is an edge then (u, v) is also an edge.

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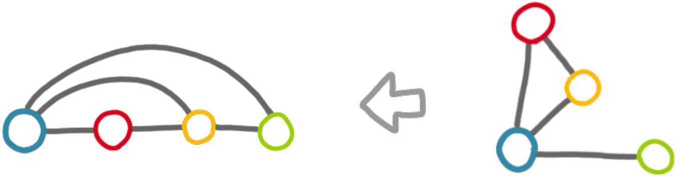

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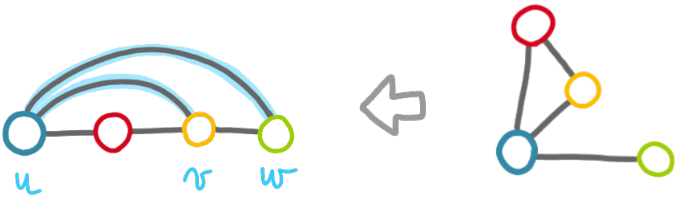

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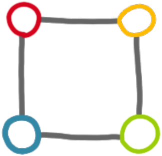

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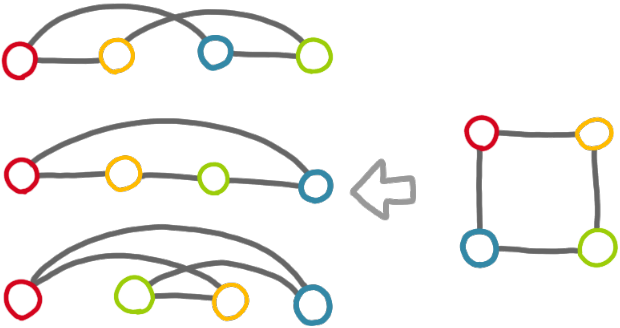

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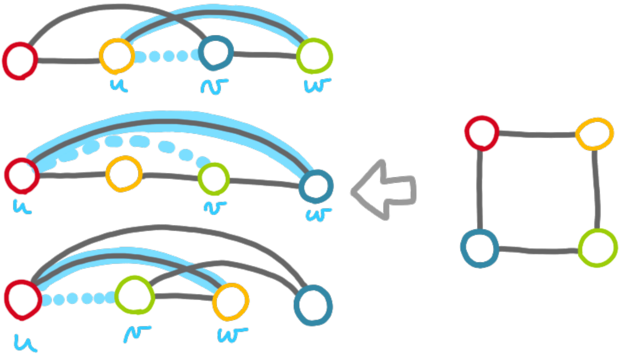
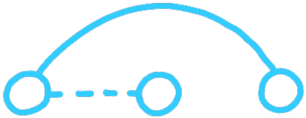


Pattern characterization

Characterization : A graph is a XXX if and only if, there exists an ordering of its vertices such that the following pattern does not appear :



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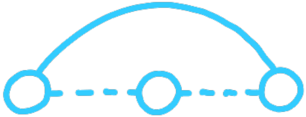

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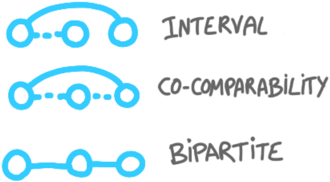

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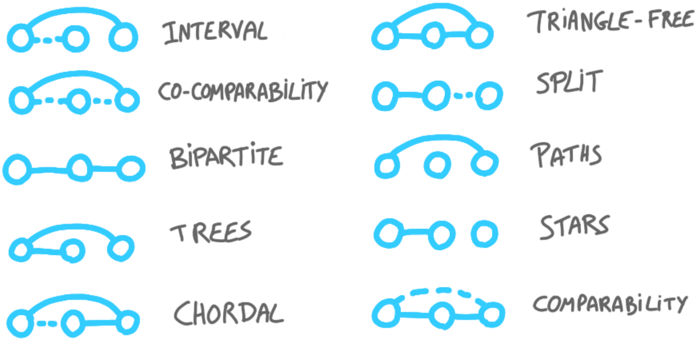

Already noted by Skrien in 82 and Damashke in 90.

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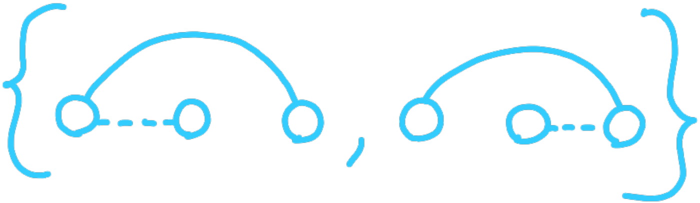

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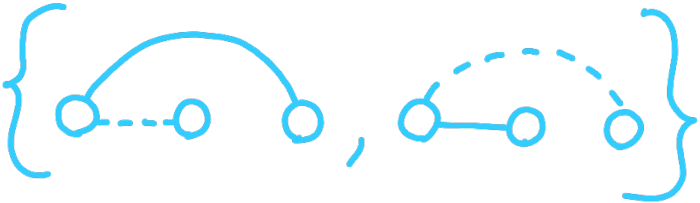

Theorem

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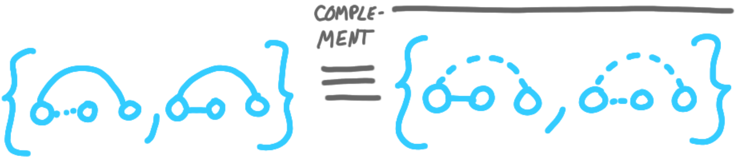
1. forests
2. linear forests
3. stars
4. interval
5. split
6. bipartite
7. chordal
8. comparability
9. triangle-free
10. permutation
11. threshold
12. proper interval
13. caterpillar
14. trivially perfect
15. bipartite chain
16. 2-star
17. 1-split
18. augmented clique
19. bipartite permutation
20. triangle-free $\cap$ co-chordal
21. clique
22. complete bipartite

Structure

- Mirror patterns



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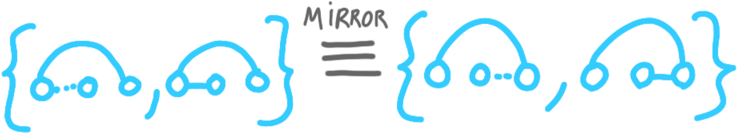
- Mirror patterns
- Complementary patterns



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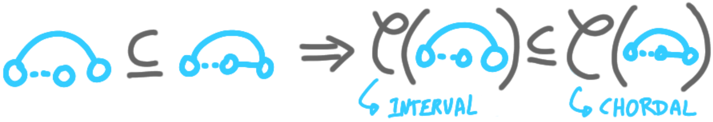
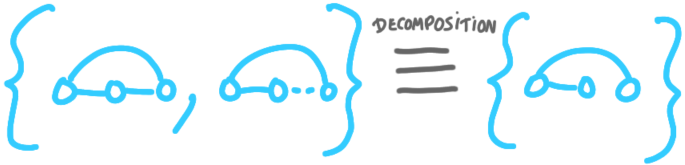


Structure

- Mirror patterns
- Complementary patterns
- Inclusions of patterns $\rightarrow$ classes inclusions
- “Splitting patterns”



Algorithms

Theorem (Hell, Mohar and Rafiey) : Every class defined by a set of patterns on three nodes can be recognized in time $O(n^3)$.

New theorem : Every class defined by patterns on three nodes can be recognized in **linear time** except two of them (in time $O(n^{2,37})$), and mostly thanks to **graph traversals**.

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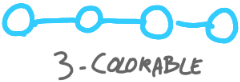

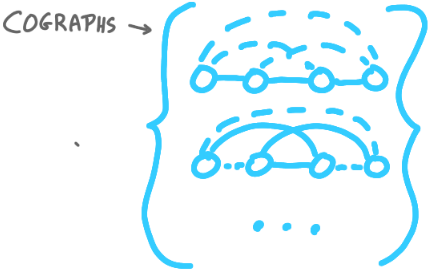

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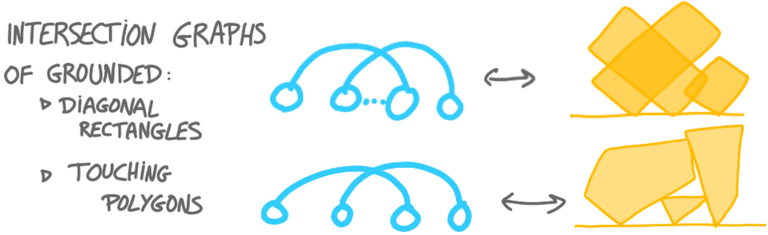
- A lot more cases, much less is known.
- A few examples : 3-colorable graphs, cographs, trivially perfect.
- On the trail of grounded intersection graphs.



What about 4 vertices ?

A few concrete open problems :

- Complexity of recognition of grounded rectangle graphs : P or NP ?
- List of the classes that have both a pattern characterization and a grounded intersection model.
- Find a criterion for deciding the complexity of the class based on its patterns.