Graph classes and forbidden patterns on three vertices

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Teaser talk

Forbidden patterns in graphs
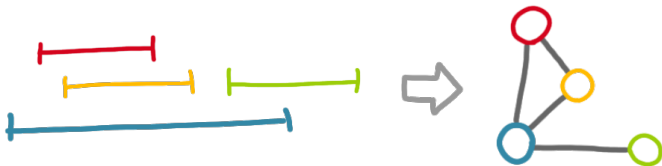
Definition : A graph is an interval graph if it is the intersection graph of a set of intervals.

Characterization : A graph is an interval graph if and only if, there exists an ordering of its vertices such that for every $u < v < w$, if (u, w) is an edge then (u, v) is also an edge.
$\rightarrow$ In other words the following pattern is forbidden :

Main theorem

Theorem : Up to a few simple operations, the non-trivial classes defined by a set of pattern (on three nodes) are :

1. forests
2. linear forests
3. stars
4. interval
5. split
6. bipartite
7. chordal
8. comparability
9. triangle-free
10. permutation
11. threshold
12. proper interval
13. caterpillar
14. trivially perfect
15. bipartite chain
16. 2-star
17. 1-split
18. augmented clique
19. bipartite permutation
20. triangle-free ∩ co-chordal
21. clique
22. complete bipartite

Why is it interesting ?

1. All these classes are well-known.*
2. There are much fewer classes than one could expect.
3. Not just a list, a lot of structure.
4. All these classes can be recognized very efficiently (thanks to a strong link between ordering and traversals).