When Should You Wait Before Updating?

Toward a Robustness Refinement

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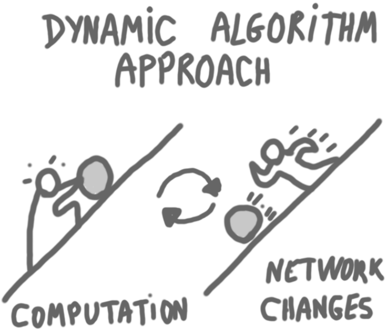
Dynamic algorithms and our approach



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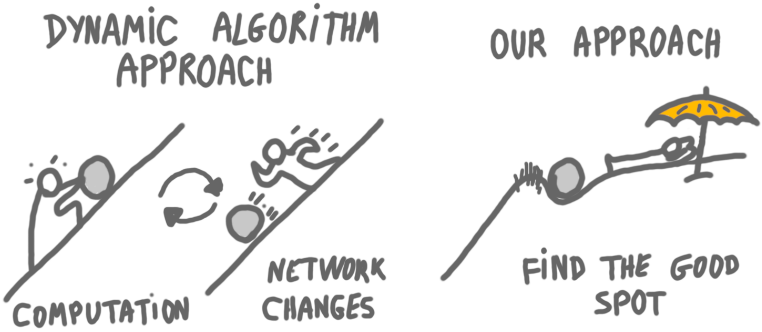

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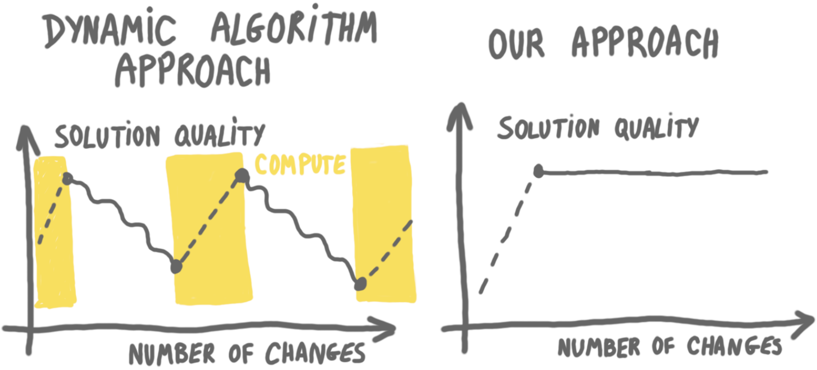


General goal

Identify instances and solutions
that are more robust than others

Necessary restrictions

In some situations there is no hope to get any sort of robustness.

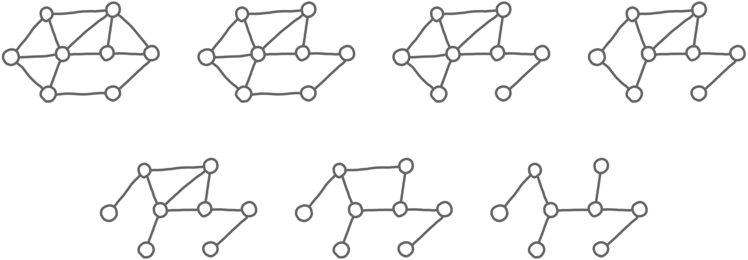
→ Very sensitive problems, eg maintain the number of edges

→ Arbitrary dynamic.

Restriction of the dynamic

Assumption on the dynamic:

- Edge removals only (static vertex set, no edge additions)
- Keeping connectivity.



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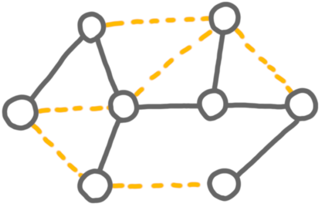
Motivations:

- Decaying networks
- More promising
- Eventual footprint.

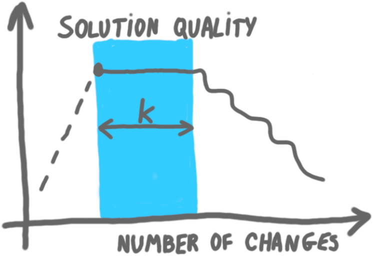


Robustness: A new form of heredity motivated by dynamic networks by Casteigts, Dubois, Petit, Robson. 2020.

Granularity of edge-removals

New parameter: k = number of edges removed.

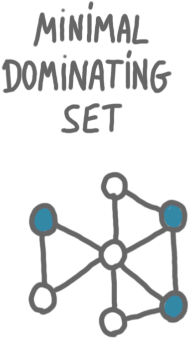
A solution is k-robust if it is still correct after k edge removals.

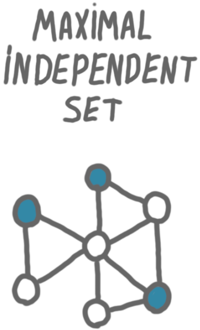


Focus on local problems

Locally checkable problems (LCL): the solution is correct if and only if it is locally correct everywhere.

Our three problems:





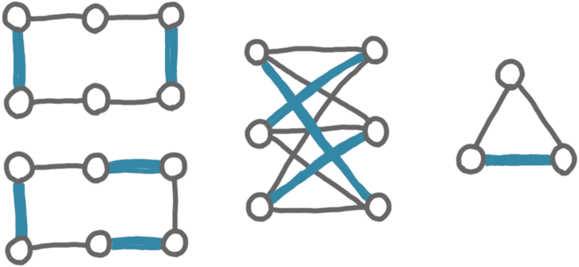
Two basic questions

For a problem P and a parameter k:

Universal question: What is the set of graphs such that all solutions are k-robust, $\mathcal{U}_k^P$?

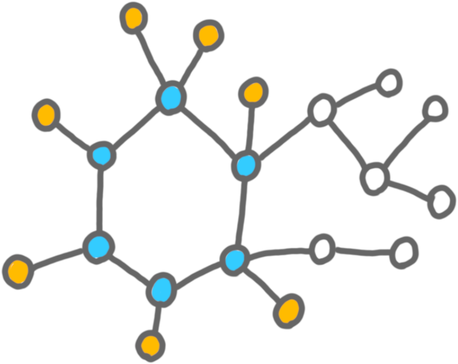
Existential question: What is the set of graphs such that there exists a solution that is k-robust, $\mathcal{E}_k^P$?

$\rightarrow$ Examples with $P =$ maximal matching and $k = 1$.



Basic building blocks

Trees

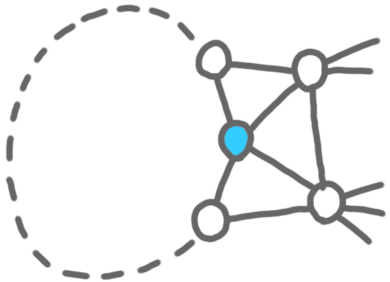


Sputnik graphs

Minimal dominating set results

Theorem: $\forall k,\ \mathcal{U}_k^{MDS} = \{\text{Sputnik graphs}\}$.

Proof idea:



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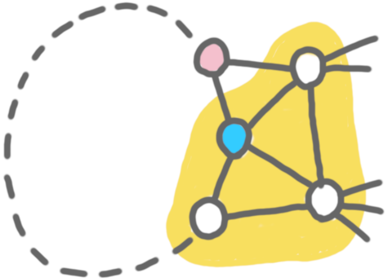
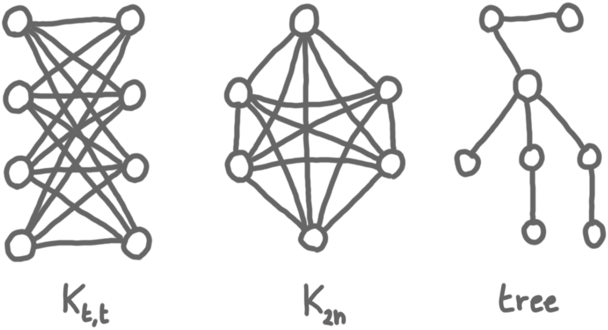


Maximal matchings results

Theorem:

$\mathcal{U}_1^{MM} =$ balanced bicliques ($K_{t,t}$), even cliques (K_{2n}), and trees.

$\mathcal{U}_{k>1}^{MM} =$ cycle C_4, and trees.



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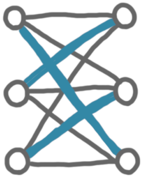
Core of the proof: proving that "$G \in \mathcal{U}_1^{MM}$" is equivalent to "all maximal matchings in G are perfect matchings" (except for trees).



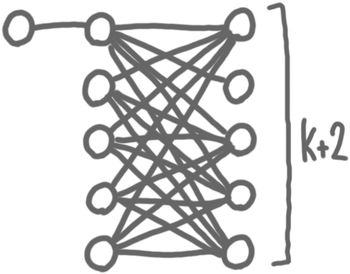
Theorem [Summer79]: Randomly matchable graphs $= \{K_{tt}, K_{2n}\}$.

Maximal independent set results

$\rightarrow$ No exact characterization of $\mathcal{U}_k^{MIS}$. :-(

Theorem: A strict hierarchy: $\mathcal{U}_{k+1}^{MIS} \subsetneq \mathcal{U}_k^{MIS}$.

Proof: $G_k \in \mathcal{U}_k^{MIS} \setminus \mathcal{U}_{k+1}^{MIS}$:



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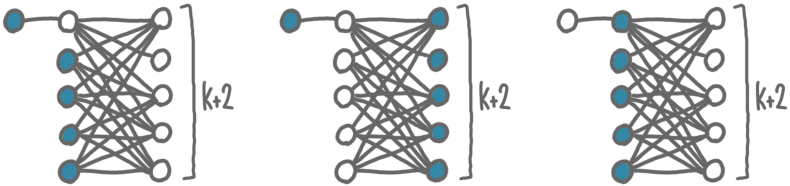

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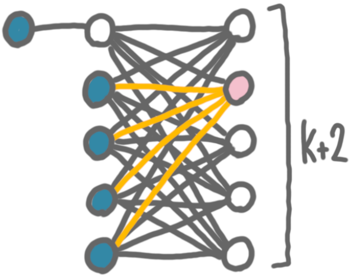
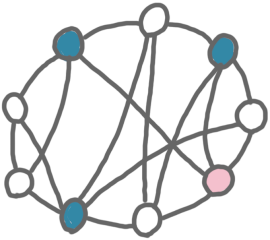


About the existential question

Theorem [CDPR20]: There exists a polynomial time algorithm to decide if a graph belongs to $\mathcal{E}_{\infty}^{MIS}$.

Theorem: Deciding $\mathcal{E}_{1}^{MIS}$ is NP-hard.

Proof: Focus on the 2-connected case where is it equivalent to "perfect stable" (= every node not in the set is dominated at least twice).



Theorem [Croitoru-Suditu 83] Deciding if there exists a perfect stable is NP-hard.

Conclusion

- Universal question: Good understanding, but small classes.
- Existential question: Large classes but difficult to recognize and use.
- Maybe the right question is in between: graphs such that there exist robust solutions that are easy to find/maintain.
- Even better: new criteria to quickly measure the robustness of subgraphs, in order to guide algorithms.

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