How long it takes for an ordinary node with an ordinary ID to output ?

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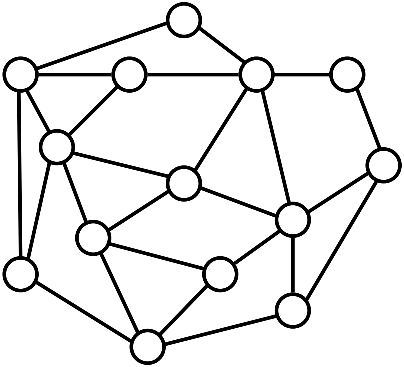
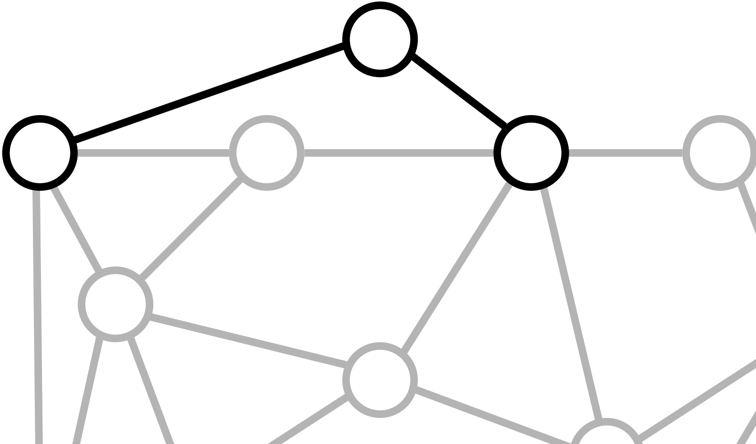
Université Paris Diderot

SIROCCO · June 2017

The network



Computation in rounds



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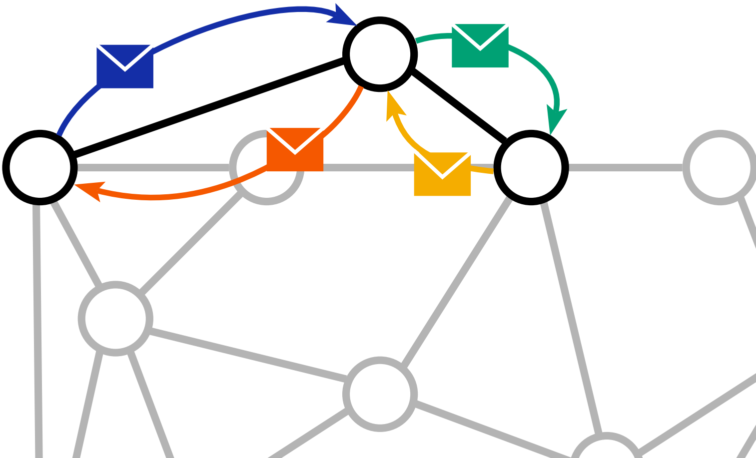

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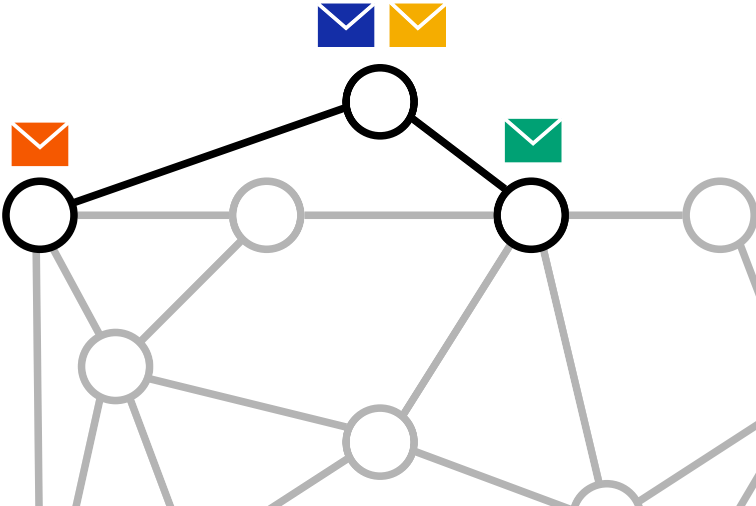

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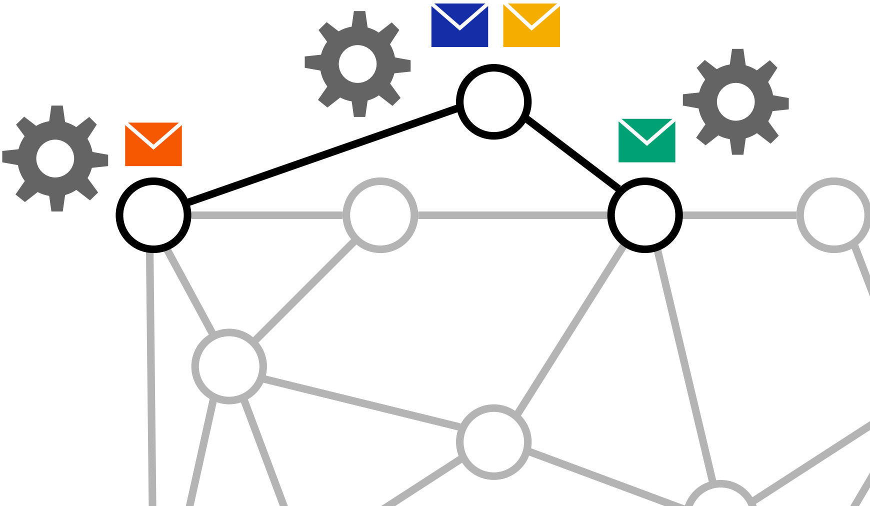

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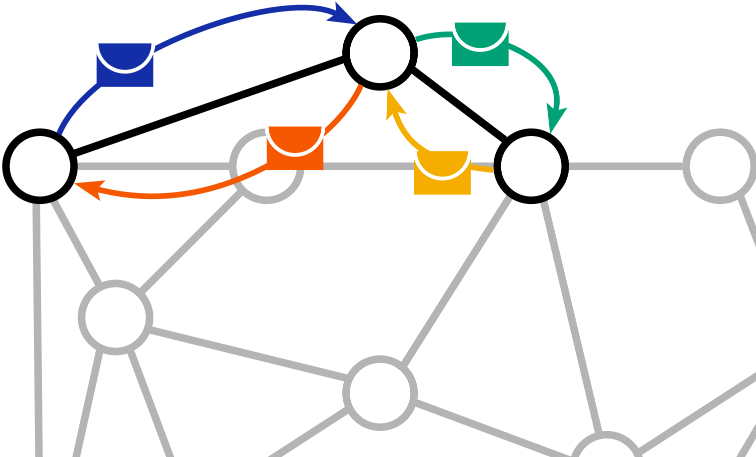

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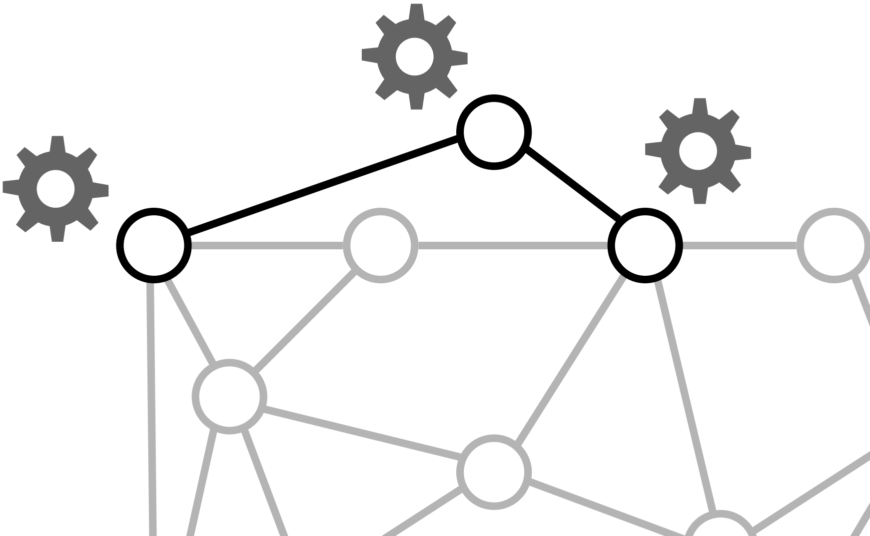

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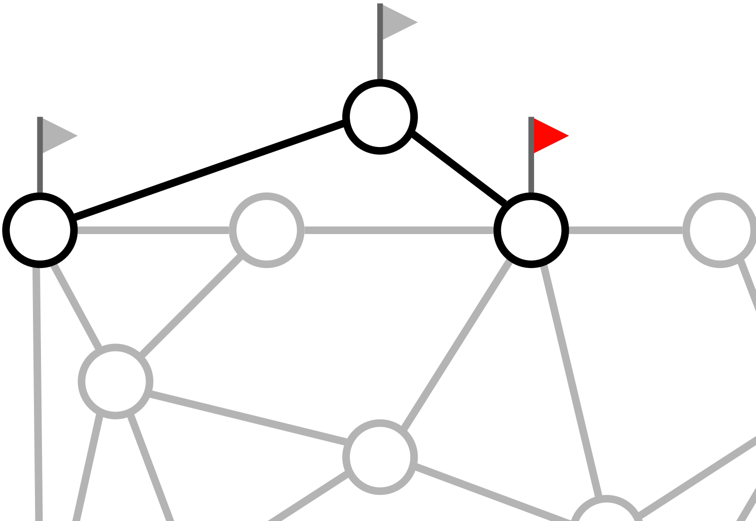

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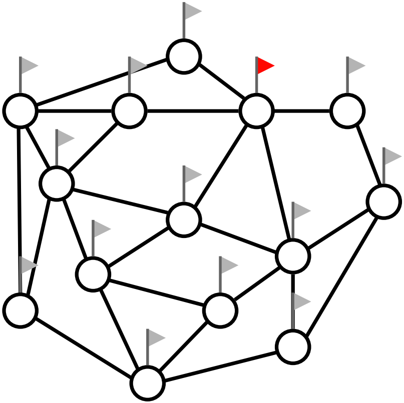
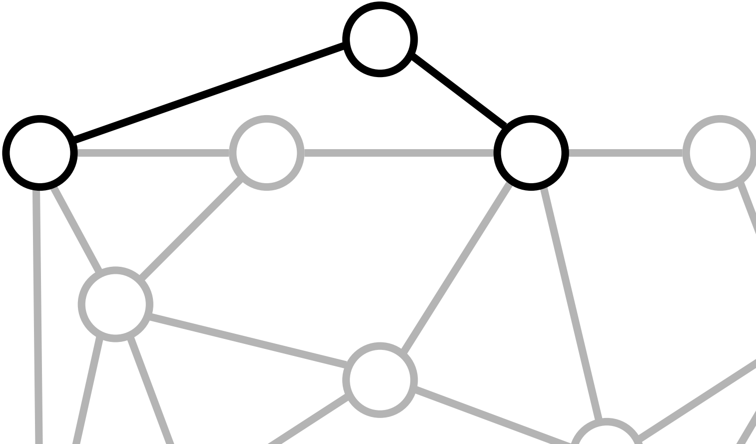


Local knowledge



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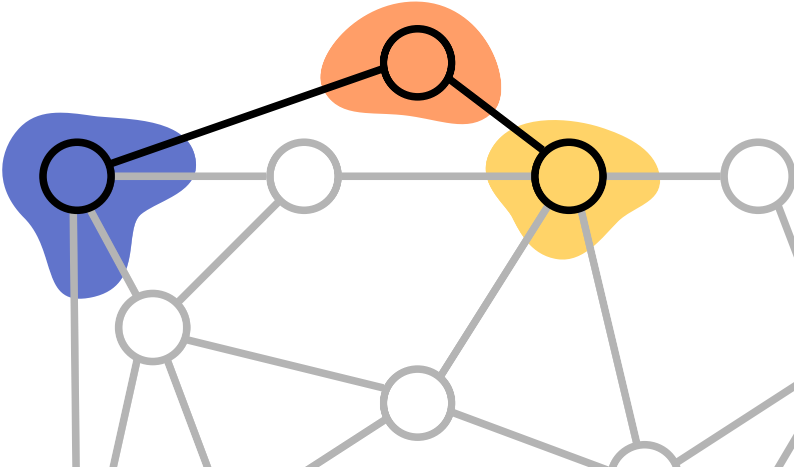

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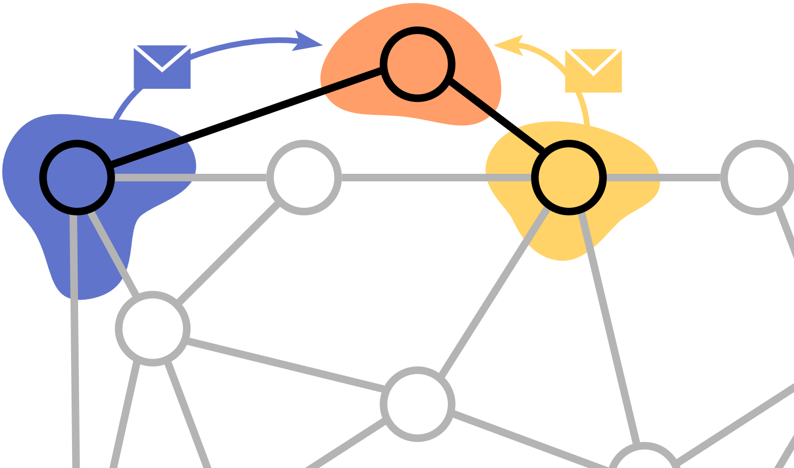

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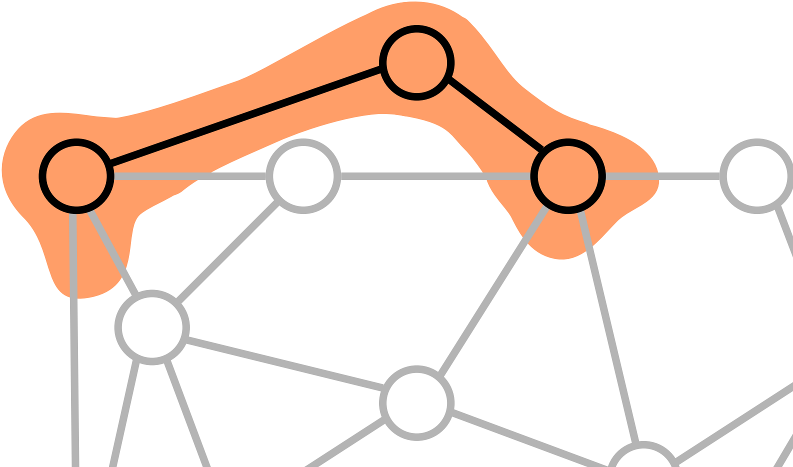

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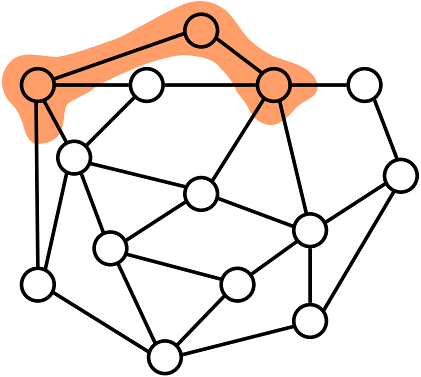

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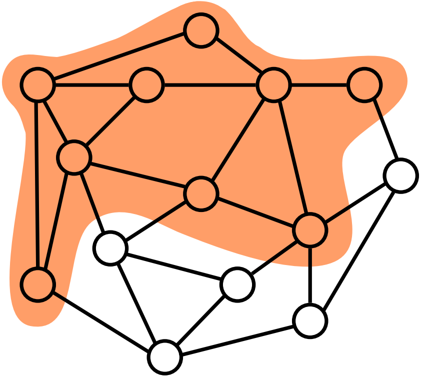

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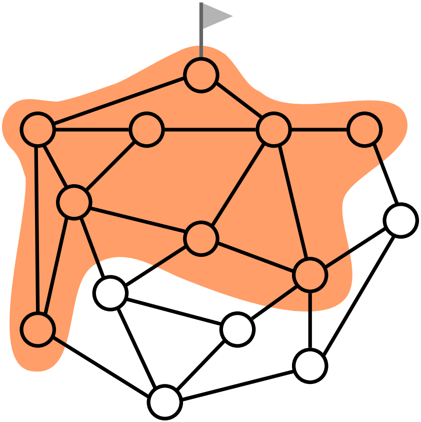
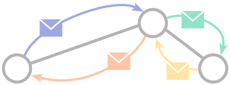
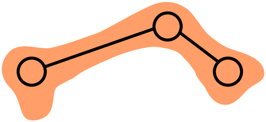


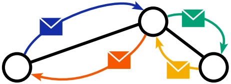
When to output ?

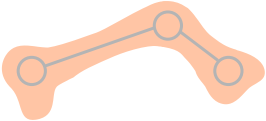




Outputs	Simultaneous	Non-simultaneous
n known	Yes	Not required
Algorithm	Compute $t(n)$, check $t(n)$-view, output.	Increase radius, until enough info, output.

When to output ?





Outputs	Simultaneous	Non-simultaneous
n known	Yes	Not required
Algorithm	Compute $t(n)$, run $t(n)$ rounds, output, stop.	Run until enough info, output, continue to run, stop.

Complexity measures

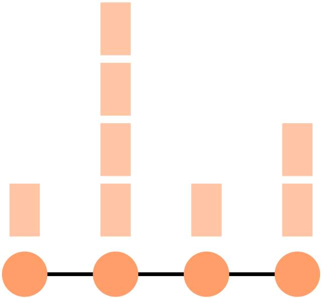
Given a graph G, and an ID assignment :

Slowest node :

$$\max_{v \in G} t(v)$$

Ordinary node :

$$\frac{1}{n} \sum_{v \in G} t(v)$$



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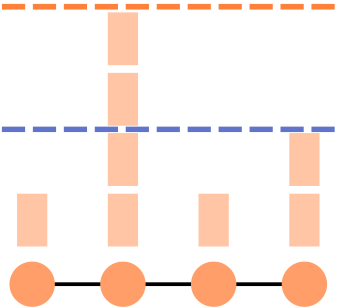


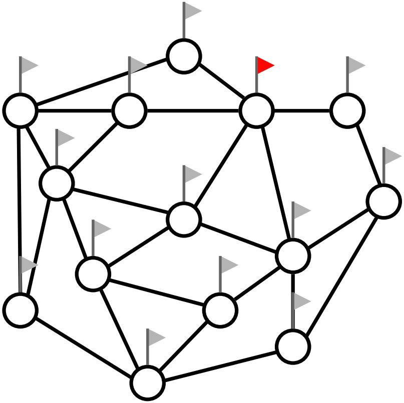
Complexity measures

Complexity of a problem :

$$\max_{G} \max_{IDs} \frac{1}{n} \sum_{v \in G} t(v)$$

Leader election

Leader election : exactly one node is selected.



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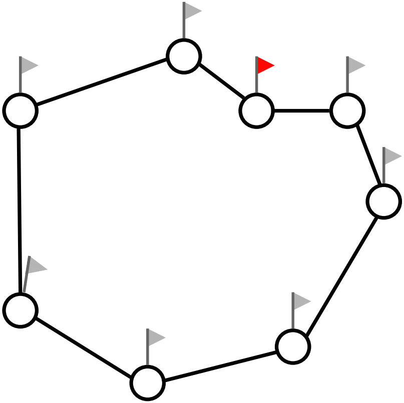
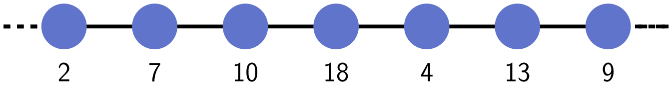


An algorithm for L.E.

For each node :

- increase the radius until you see a larger ID
- if such an ID exists, then output *non-leader*
- else output *leader*.

An algorithm for L.E.



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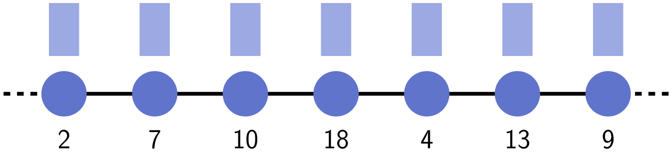

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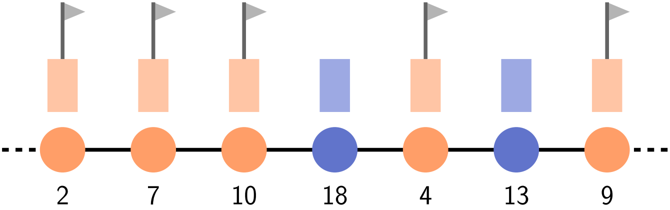

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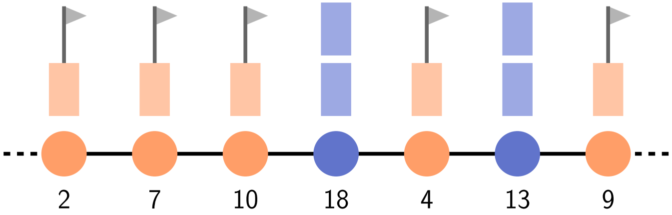

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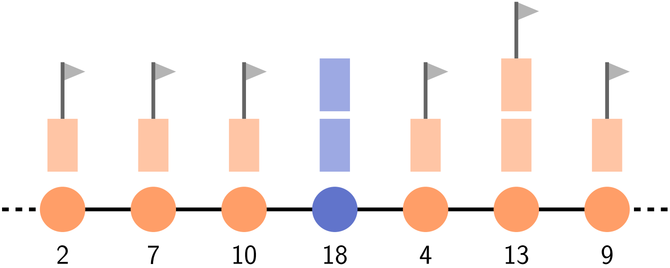

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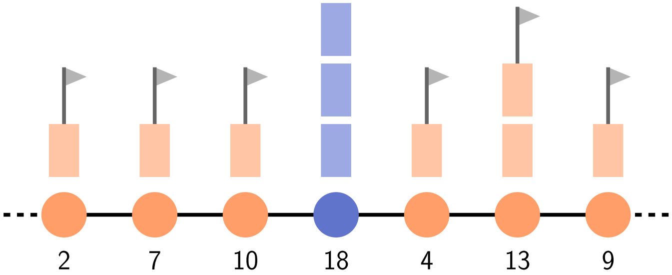

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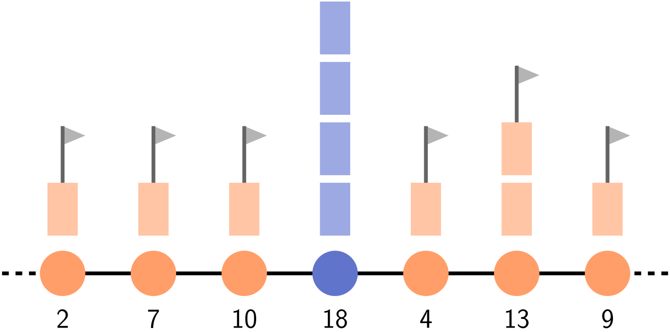

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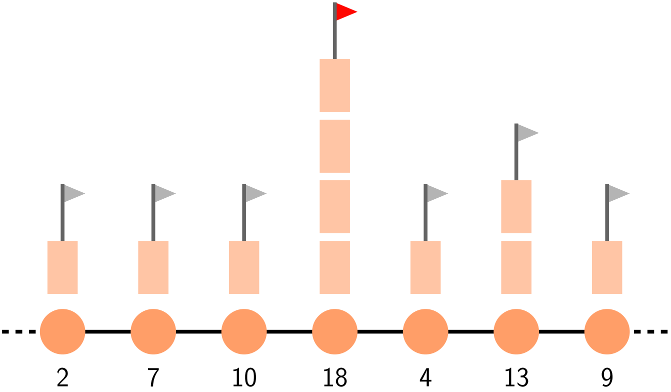

An exponential gap

Ordinary node best ID assign.	Ordinary node worst ID assign.	Slowest node any ID assign.
$O(1)$	$O(\log n)$	$\Omega(n)$

Local problems

Local problem :

Locally correct
everywhere

Local problems

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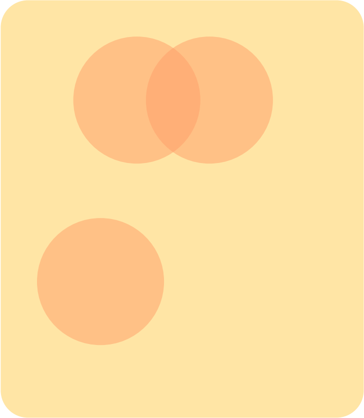

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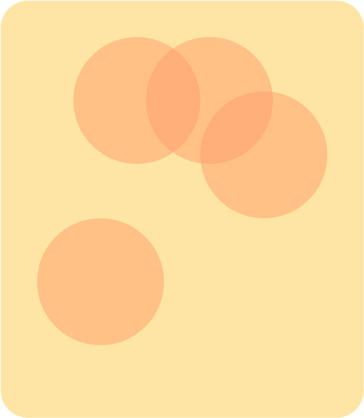


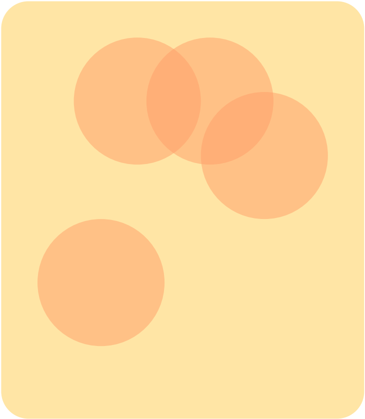
Local problems

Typical local problem :

Colouring

Typical global problem :

Leader election



No gap for local problems

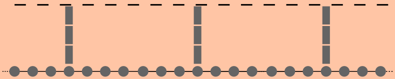
Example of application :

Thm [Linial] : For 3-colouring a cycle, the slowest node requires $\Omega(\log^* n)$ rounds.

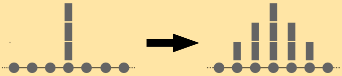
Thm [this paper] : For 3-colouring a cycle, an ordinary node requires $\Omega(\log^* n)$ rounds.

Roadmap of the proof

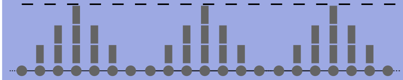
Many high radiuses



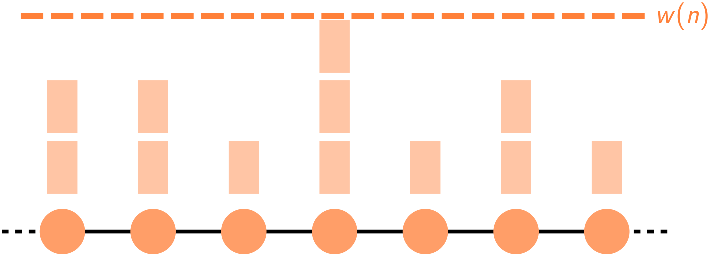
Smoothness



No gap



Proof : many high radiuses



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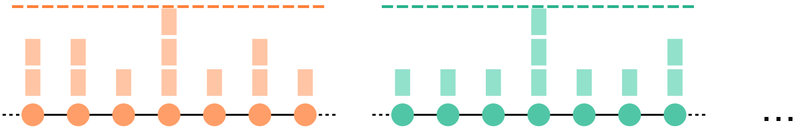

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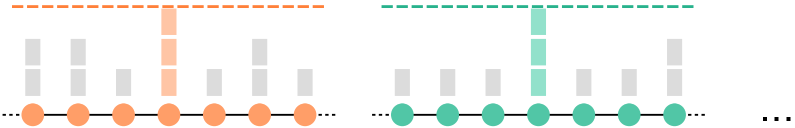

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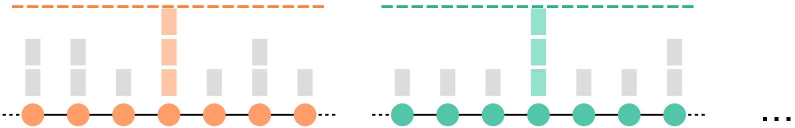

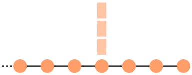

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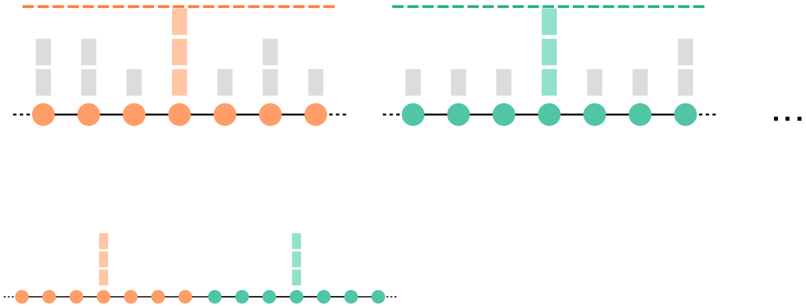

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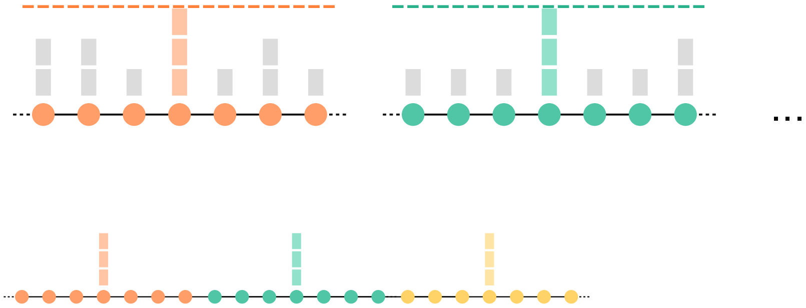

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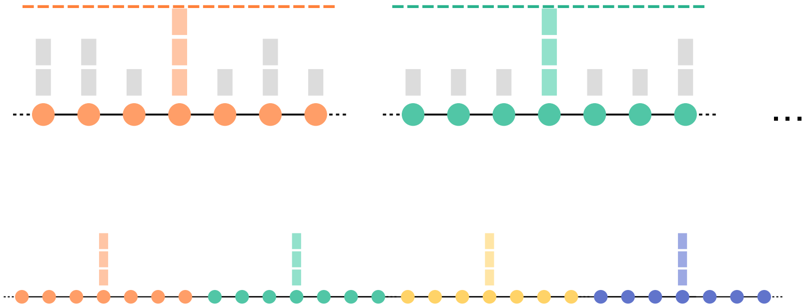
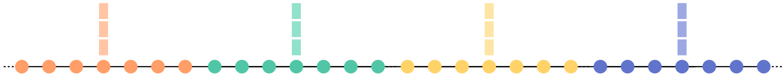
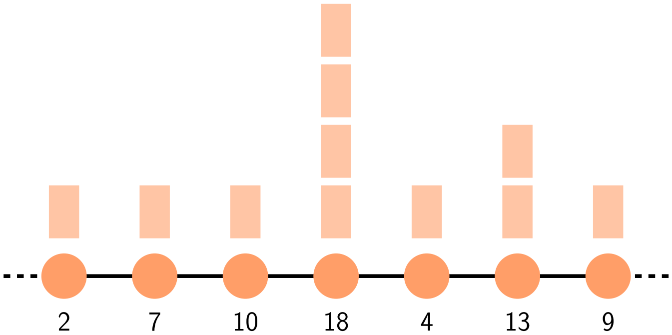


Proof : many high radiuses

$\rightarrow \frac{n}{w(n)}$ nodes with $w(n)$ radius



Proof : smoothness



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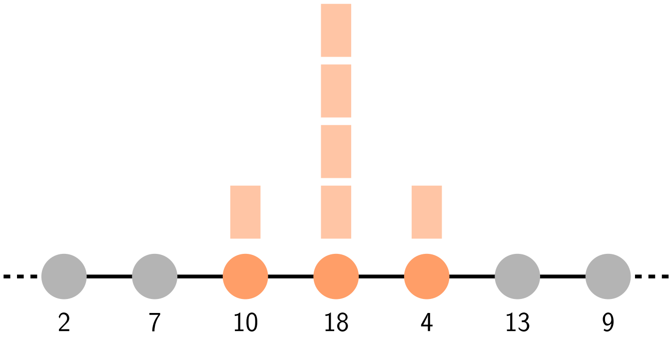

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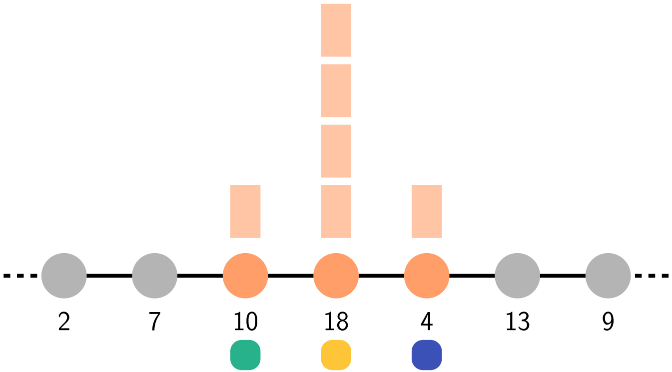

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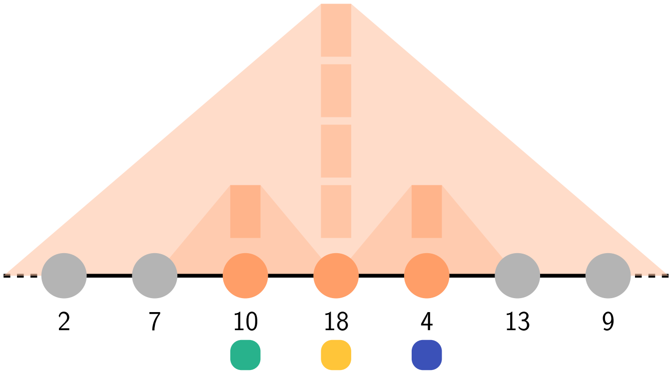

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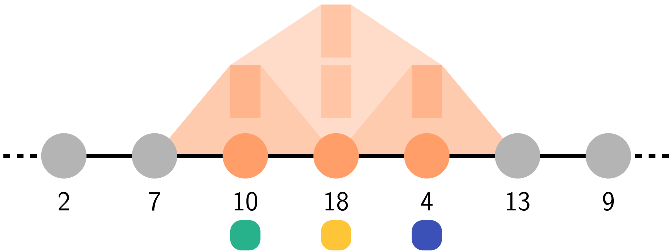

Complexity measures

Complexity of a problem :

$$\max_{G} \max_{IDs} \frac{1}{n} \sum_{v \in G} t(v)$$

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Random IDs

Complexity of a problem :

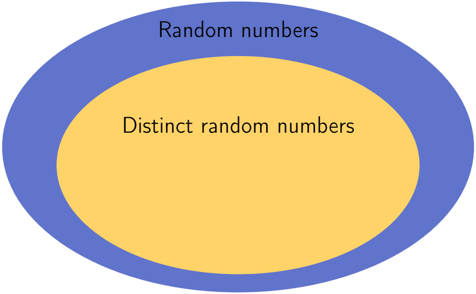
$$\max_G \frac{1}{|IDs|} \sum_{IDs} \max_{v \in G} t(v)$$

Random IDs

Random ID Assignment	Randomized algorithm
Random $O(\log n)$ ID assign.	Random $O(\log n)$ numbers
Uniqueness	Independance

Random IDs

Whp. n random numbers from $[n^4]$ are distinct.



Ordinary node and ID

Complexity of a problem :

$$\max_{G} \frac{1}{|IDs|} \sum_{IDs} \frac{1}{n} \sum_{v \in G} t(v)$$

Ordinary node and ID

3-colouring of a cycle	Worst ID assign.	Random ID assign.
Slowest node	$\Theta(\log^* n)$	$\Theta(\log^* n)$
Ordinary node	$\Theta(\log^* n)$	$\Theta(1)$