Semantic Web reasoning

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Outline

• **Summary**
  – Logics
  – Semantic Web Languages
  – Reasoning

• Web-based reasoning techniques
• Reasoning using SemWeb languages expressivity
• General aspects of reasoning and calculability
• Conclusion
Summary

• Formal logics

- First Order Logic
- Propositional Logic
- Predicate logic
- Modal Logic
- Description logics
- Fuzzy logic...

La logique est à l'origine la recherche de règles générales et formelles permettant de distinguer un raisonnement concluant de celui qui ne l'est pas. 

Wikipédia-Fr
Summary

• Description Logics (DL)

DL is a family of formal knowledge representation languages. Many DLs are more expressive than propositional logic but less expressive than first-order predicate logic. In contrast to the latter, the core reasoning problems for DLs are (usually) decidable, and efficient decision procedures have been designed and implemented for these problems. Wikipédia-En

– DLs model concepts, roles and individuals, and their relationships

– The fundamental modeling concept of a DL is the axiom - a logical statement relating roles and/or concepts.
Summary

• Description Logics (DL)
  – DLs provide various characteristics, corresponding to different types of *expressivity*
  – Each feature indicates “what you can say” with it
  – 3 basic logics:
    • Attributive Language (used in ontologies)
    • Frame based description language
    • Existential language
  – A set of extensions that provide specific constructs
Summary

• The Semantic Web
  – RDF
    • Triple: subject, predicate, object
    • Nodes
      – Subject, object
      – Possible types: IRI, literal, blank node
    • Predicate: link

Dogs $\rightarrow$ Meat

Dogs eat Meat
Summary

• The Semantic Web
  – RDF
    • Graph: set of (possibly interrelated) triples
The Semantic Web

- RDF
  - Blank node: resource without a definite IRI

Diagram:
- Blank node
- Meats
- Cows
- Dogs

Edges:
- ? hasSpecies Cows
- ? hasSpecies Dogs
- Meats eat
Summary

- The Semantic Web
  - RDF
    - Containers (Bag, Seq, Alt): provide access to a set of similar resources

![Diagram showing RDF containers and relationships]
Summary

• The Semantic Web
  – RDF
    • Resource: basically anything in RDF
Summary

• The Semantic Web
  – Structured Graph: RDF-S
    • Schema: Class, Property, hierarchy, data
    • Constraints: domain, range

![Semantic Web Diagram]
Summary

• The Semantic Web
  – Ontologies: OWL
    • Web Ontology Language
    • 2 versions
      – OWL (1), February 2004: Lite, DL, Full
      – OWL2, December 2012: 3 profiles ⇒ EL, RL, QL
    • Adds expressivity to the couple RDF-S + RDF
    • Allows defining vocabularies
    • Based on Description Logics
Summary

• The Semantic Web
  – Ontologies: OWL
    • OWL ontology components
      – Terminology (TBox): domain vocabulary
      – Assertions (ABox): to store actual data
      – Roles (RBox): term articulation
    • Vocabulary components
      
      | Classes       | Roles               |
      |---------------|---------------------|
      | Individuals   | Object properties   |
      | Literals      | Data properties     |
Summary

• The Semantic Web
  – OWL 2: building blocks

Source: https://www.w3.org/TR/owl-overview/
Summary

• The Semantic Web
  – OWL 2 : building blocks
    • Ontologies (conceptual structure): the structural elements, their roles and functionalities
      https://www.w3.org/TR/owl2-primer/
    • Concrete syntaxes: Functional-style, Manchester, RDF/XML, OWL/XML, Turtle...
    • Semantics: define the meaning of OWL 2 ontologies
      https://www.w3.org/TR/owl2-direct-semantics/
      https://www.w3.org/TR/owl2-rdf-based-semantics/
Summary

• The Semantic Web
  – **OWL2 profiles**
    • Expression Language (EL)
      – For classification tasks
      – Different expressivity levels
      – Query rewriting (SPARQL -> SPARQL)
    • Query Language (QL)
      – For querying large datasets
      – Relies on an internal RDB
      – Query rewriting (SPARQL -> SQL)
    • Rule Language (RL)
      – For entailing facts using rules
      – More to come...
The Semantic Web

- Ontologies: OWL

  - Supplementary semantic constructs
    - Expressions
      - Object Property
      - Data ranges
      - Class expressions
    - Axioms
      - Class expression axioms
      - Object properties
      - Data properties
      - Datatypes
      - Keys
      - Data assertions

An OWL 2 ontology $O$ is satisfied in an interpretation $I$ if all axioms in the axiom closure of $O$ are satisfied in $I$. 
Summary

• The Semantic Web
  – Ontologies: OWL
    • Cheatsheet reference for OWL 2 DL
      – Summary of OWL constructs:
        https://www.w3.org/TR/owl-quick-reference/
      – Detailed definitions:
        https://www.w3.org/TR/owl2-syntax/
      – Formal definitons:
        https://www.w3.org/TR/owl-direct-semantics/
      – DLs names vs. expressivity levels:
        https://en.wikipedia.org/wiki/Description_logic#Nomenclature
Summary

• Ontology limitations
  – Made by non-experts
    • inconsistent
  – Automatically generated
    • Not very useful without domain knowledge
  – Heterogeneous
  – Too high complexity
Summary

• The Semantic Web
  – Finally, what is it good for?
    • Describing domain vocabularies
    • Querying structured data sources (aka knowledge bases)
    • Agreeing on a common meaning of types of information
    • Interlinking data sources

  – But can’t we do better?...
Web-based reasoning

• Reasoning
  – The process of logically determining information that has not been explicitly been stated

  – Examples
    • Calculus
    • Syntactic analysis
    • Proofs
    • Verification
    • Inferences
    • Science?...
Web-based reasoning

• Inference
  – The process of entailing a fact based on rules
  – Basis of a reasoning process

Condition (head) \(\rightarrow\) conclusion (body)
(I finished my homework) AND (the weather is nice) \(\rightarrow\) I can go for a walk
Web-based reasoning

• Examples of inductive (logical) reasoning
  – Satisfiability (SAT)
  – Backward chaining (Prolog)
  – Forward chaining (rules)
  – Saturation (tableaux)
Outline

• Summary

• **Web-based reasoning techniques**
  – Rule-based reasoning
  – Tableaux-based reasoning
  – Comparison

• Reasoning using SemWeb languages expressivity

• General aspects of reasoning and calculability

• Conclusion
Rule-based reasoning

• Reasoning type
  – Inductive reasoning
  – Forward chaining

• Principles
  – Rules are applied to a set of facts
    • At the beginning: *explicit facts*
  – May infer new facts
    • *Implicit facts*
  – On which the rules are applied
  – Loop until nothing more happens
Rule-based reasoning

• Functioning with RDF graphs
  – Facts = triples
  – Order of rule application must not be relevant

• Usages
  – Datalog: query language for deductive DBs
  – View materialization: data duplication for creating views in DBs
Rule-based reasoning

• Use conjunctive rules
  – Only conjunctions in head clauses
  – To ensure commutativity
• Separate non-commutative operations in different queries
  – Deletions
  – Insertions
Incremental reasoning
(Motik, Horrocks, Kim 2012)

• Based on Gupta & Al. 1993, DRed
• Only designed for conjunctive rules
• Overdeletion
  – Evaluate the list of implicit facts inferred from explicit facts to suppress
• Rederivation
  – Evaluate the list of implicit facts that could have been derived from other explicit facts
Tableaux-based algorithms

• Explore all possible Aboxes (« tableaux »)
• Apply rules to search for unsatisfiabilities
  – Start: tableau is open
  – Apply rules (recursively)
  – If a clash is found: tableau is closed
  – If no clash is found and no more rule can be applied: tableau is complete
• If there is a complete tableau, the Tbox is satisfiable
Tableaux-based algorithms

- Tableaux calculus (aka truth tree)
  - Objective: explore all possible solutions for finite sets of formulas of various logics
    - Propositional Logic
    - First order predicate logic
    - Clause tableaux
  - Based on algorithms that break formulas into pieces
  - Close branches when a contradiction is found
  - Derivation process
    - Assess the derivation is complete
    - Store the derivation history
Hyper tableaux
(Baumgartner, Furbach, Niemelä 1996)

• Variant of the clausal normal form tableaux
  – Applies derivation in an efficient minimal model reasoning (*hyper derivation*)
  – Subsumption-based pruning and decision procedure
  – resolve away all negative literals of a clause in a single inference step

• Allows avoiding backtracking in reasoning mechanisms
  ➔ More “reasonably implementable” than regular clausal algorithms

Source:
Reasoning algorithms:  
Rule-based vs. Tableaux

• Rule-based instantiate variables (if they can)
• Tableaux make assumptions on all possibilities

• Example:

Do I know a woman who knows a man?
Outline

• Summary
• Web-based reasoning techniques
• Reasoning using SemWeb languages expressivity
  – RDF
  – RDF-S
  – OWL
• General aspects of reasoning and calculability
• Conclusion
RDF reasoning

• Only about facts, triple structure and datatypes

• RDF entailments
  – Simple entailments: structural matching by renaming blank nodes
  – RDF entailments: interpret RDF vocabulary
RDF reasoning

- Simple entailment rules

<table>
<thead>
<tr>
<th>Rule name</th>
<th>If E contains</th>
<th>then add</th>
</tr>
</thead>
<tbody>
<tr>
<td>sel</td>
<td>uuu aaa xxx .</td>
<td>uuu aaa __:nnn . where __:nnn identifies a blank node allocated to xxx by rule sel or se2.</td>
</tr>
<tr>
<td>se2</td>
<td>uuu aaa xxx .</td>
<td>__:nnn aaa xxx . where __:nnn identifies a blank node allocated to uuu by rule sel or se2.</td>
</tr>
<tr>
<td>lg</td>
<td>uuu aaa lll .</td>
<td>uuu aaa __:nnn . where __:nnn identifies a blank node allocated to the literal lll by this rule.</td>
</tr>
<tr>
<td>gl</td>
<td>uuu aaa __:nnn . where __:nnn identifies a blank node allocated to the literal lll by rule lg.</td>
<td>uuu aaa lll .</td>
</tr>
</tbody>
</table>

Source: Corno & Farinetti: Logic and Reasoning in the Semantic Web
RDF reasoning

- RDF entailment rules

<table>
<thead>
<tr>
<th>Rule name</th>
<th>If E contains</th>
<th>then add</th>
</tr>
</thead>
<tbody>
<tr>
<td>rdf1</td>
<td>uuu aaa yyy .</td>
<td>aaa rdf:type rdf:Property .</td>
</tr>
<tr>
<td>rdf2</td>
<td>uuu aaa lll . where lll is a well-typed XML literal .</td>
<td>_:nnn rdf:type rdf:XMLLiteral . where _:nnn identifies a blank node allocated to lll by rule lg.</td>
</tr>
</tbody>
</table>
RDF reasoning

• RDF axiomatic triples

```turtle
rdf:type       rdf:type       rdf:Property .
rdf:subject    rdf:type       rdf:Property .
rdf:predicate  rdf:type       rdf:Property .
rdf:object     rdf:type       rdf:Property .
rdf:first      rdf:type       rdf:Property .
rdf:rest       rdf:type       rdf:Property .
rdf:value      rdf:type       rdf:Property .
rdf:_1         rdf:type       rdf:Property .
rdf:_2         rdf:type       rdf:Property .
...            rdf:type       rdf:List .
rdf:nil        rdf:type       rdf:List .
```
RDF reasoning

• Interpolation Lemma
  – A set of graphs S entails a graph E if and only if every subgraph of S is an instance of E
  – Syntactic description of entailment
  – Satisfied by simple entailment rules
RDF reasoning

• RDF entailment lemma
  – S rdf-entails E if and only if there exists a graph
    • that can be derived
      – from S and the RDF axiomatic triples
      – by the application of rule lg and the RDF entailment rules
    • that entails E
  – Describes reasoning completeness
  – Satisfied by RDF entailment rules
RDF reasoning

• Usage
  – Structural graph mapping
  – Assertions about assertions (reification)

• Examples
  – Trust $\rightarrow$ recommendation
  – Comparison $\rightarrow$ adaptation
RDF-S reasoning

• Deduce new information on
  – Hierarchy (subclassOf, subPropertyOf)
  – Individuals (rdf-type)
RDF-S reasoning

- RDF-S entailment rules (1/2)

<table>
<thead>
<tr>
<th>Rule name</th>
<th>If E contains</th>
<th>then add</th>
</tr>
</thead>
<tbody>
<tr>
<td>rdfs1</td>
<td>uuu aaa lll. where lll is a plain literal (with or without a language tag)</td>
<td>_:nnn rdf:type rdfs:Literal where _:nnn identifies a blank node allocated to lll by rule rule lg.</td>
</tr>
<tr>
<td>rdfs2</td>
<td>aaa rdfs:domain xxx . uuu aaa yyy .</td>
<td>uuu rdf:type xxx .</td>
</tr>
<tr>
<td>rdfs3</td>
<td>aaa rdfs:range xxx . uuu aaa vvv .</td>
<td>vvv rdf:type xxx .</td>
</tr>
<tr>
<td>rdfs4a</td>
<td>uuu aaa xxx .</td>
<td>uuu rdf:type rdfs:Resource .</td>
</tr>
<tr>
<td>rdfs4b</td>
<td>uuu aaa vvv .</td>
<td>vvv rdf:type rdfs:Resource .</td>
</tr>
<tr>
<td>rdfs5</td>
<td>uuu rdfs:subPropertyOf vvv . vvv rdfs:subPropertyOf xxx .</td>
<td>uuu rdfs:subPropertyOf xxx .</td>
</tr>
</tbody>
</table>

Source: Corno & Farinetti: Logic and Reasoning in the Semantic Web
## RDF-S reasoning

### RDF-S entailment rules (2/2)

<table>
<thead>
<tr>
<th>Rule name</th>
<th>If E contains</th>
<th>then add</th>
</tr>
</thead>
<tbody>
<tr>
<td>rdfs6</td>
<td>uuu rdf:type rdf:Property .</td>
<td>uuu rdfs:subPropertyOf uuu .</td>
</tr>
<tr>
<td>rdfs7</td>
<td>aaa rdfs:subPropertyOf bbb uuu aaa yyy .</td>
<td>uuu bbb yyy .</td>
</tr>
<tr>
<td>rdfs8</td>
<td>uuu rdf:type rdfs:Class .</td>
<td>uuu rdfs:subClassOf rdfs:Resource .</td>
</tr>
<tr>
<td>rdfs9</td>
<td>uuu rdfs:subClassOf xxx . vvv rdf:type uuu .</td>
<td>vvv rdf:type xxx .</td>
</tr>
<tr>
<td>rdfs10</td>
<td>uuu rdf:type rdfs:Class .</td>
<td>uuu rdfs:subClassOf uuu .</td>
</tr>
<tr>
<td>rdfs11</td>
<td>uuu rdfs:subClassOf vvv . vvv rdfs:subClassOf xxx .</td>
<td>uuu rdfs:subClassOf xxx .</td>
</tr>
<tr>
<td>rdfs12</td>
<td>uuu rdf:type rdfs:ContainerMembershipProperty .</td>
<td>uuu rdfs:subPropertyOf rdfs:member .</td>
</tr>
<tr>
<td>rdfs13</td>
<td>uuu rdf:type rdfs:Datatype</td>
<td>uuu rdfs:subClassOf rdfs:Literal .</td>
</tr>
</tbody>
</table>

Source: Corno & Farinetti: Logic and Reasoning in the Semantic Web
RDF-S reasoning

- RDF-S axiomatic triples (1/3)

\[
\begin{array}{lll}
\text{rdf:type} & \text{rdfs:domain} & \text{rdfs:Resource} \\
\text{rdfs:domain} & \text{rdfs:domain} & \text{rdf:Property} \\
\text{rdfs:range} & \text{rdfs:domain} & \text{rdf:Property} \\
\text{rdfs:subPropertyOf} & \text{rdfs:domain} & \text{rdf:Property} \\
\text{rdfs:subClassOf} & \text{rdfs:domain} & \text{rdfs:Class} \\
\text{rdf:subject} & \text{rdfs:domain} & \text{rdf:Statement} \\
\text{rdf:predicate} & \text{rdfs:domain} & \text{rdf:Statement} \\
\text{rdf:object} & \text{rdfs:domain} & \text{rdf:Statement} \\
\text{rdfs:member} & \text{rdfs:domain} & \text{rdfs:Resource} \\
\text{rdf:first} & \text{rdfs:domain} & \text{rdf:List} \\
\text{rdf:rest} & \text{rdfs:domain} & \text{rdf:List} \\
\text{rdfs:seeAlso} & \text{rdfs:domain} & \text{rdfs:Resource} \\
\text{rdfs:isDefinedBy} & \text{rdfs:domain} & \text{rdfs:Resource} \\
\text{rdfs:comment} & \text{rdfs:domain} & \text{rdfs:Resource} \\
\text{rdfs:label} & \text{rdfs:domain} & \text{rdfs:Resource} \\
\text{rdf:value} & \text{rdfs:domain} & \text{rdfs:Resource} \\
\end{array}
\]

Source: Corno & Farinetti: Logic and Reasoning in the Semantic Web
RDF-S reasoning

- RDF-S axiomatic triples (2/3)

<table>
<thead>
<tr>
<th>rdf:type</th>
<th>rdfs:domain</th>
<th>rdfs:range</th>
<th>rdfs:Class .</th>
</tr>
</thead>
<tbody>
<tr>
<td>rdfs:range</td>
<td>rdfs:range</td>
<td>rdfs:Class .</td>
<td></td>
</tr>
<tr>
<td>rdfs:subPropertyOf</td>
<td>rdfs:range</td>
<td>rdfs:Property .</td>
<td></td>
</tr>
<tr>
<td>rdfs:subClassOf</td>
<td>rdfs:range</td>
<td>rdfs:Class .</td>
<td></td>
</tr>
<tr>
<td>rdf:subject</td>
<td>rdfs:range</td>
<td>rdfs:Resource .</td>
<td></td>
</tr>
<tr>
<td>rdf:predicate</td>
<td>rdfs:range</td>
<td>rdfs:Resource .</td>
<td></td>
</tr>
<tr>
<td>rdf:object</td>
<td>rdfs:range</td>
<td>rdfs:Resource .</td>
<td></td>
</tr>
<tr>
<td>rdfs:member</td>
<td>rdfs:range</td>
<td>rdfs:Resource .</td>
<td></td>
</tr>
<tr>
<td>rdf:first</td>
<td>rdfs:range</td>
<td>rdfs:Resource .</td>
<td></td>
</tr>
<tr>
<td>rdf:rest</td>
<td>rdfs:range</td>
<td>rdf:List .</td>
<td></td>
</tr>
<tr>
<td>rdfs:seeAlso</td>
<td>rdfs:range</td>
<td>rdfs:Resource .</td>
<td></td>
</tr>
<tr>
<td>rdfs:isDefinedBy</td>
<td>rdfs:range</td>
<td>rdfs:Resource .</td>
<td></td>
</tr>
<tr>
<td>rdfs:comment</td>
<td>rdfs:range</td>
<td>rdfs:Literal .</td>
<td></td>
</tr>
<tr>
<td>rdfs:label</td>
<td>rdfs:range</td>
<td>rdfs:Literal .</td>
<td></td>
</tr>
<tr>
<td>rdf:value</td>
<td>rdfs:range</td>
<td>rdfs:Resource .</td>
<td></td>
</tr>
</tbody>
</table>

Source: Corno & Farinetti: Logic and Reasoning in the Semantic Web
RDF-S reasoning

• RDF-S axiomatic triples (3/3)

```
rdf:Alt       rdfs:subClassOf       rdfs:Container .
rdf:Bag       rdfs:subClassOf       rdfs:Container .
rdf:Seq       rdfs:subClassOf       rdfs:Container .
rdfs:ContainerMembershipProperty rdfs:subClassOf rdf:Property .
rdfs:isDefinedBy rdfs:subPropertyOf rdfs:seeAlso .
rdf:XMLLiteral rdf:type            rdfs:Datatype .
rdf:XMLLiteral rdfs:subClassOf       rdfs:Literal .
rdfs:Datatype   rdfs:subClassOf       rdfs:Class  .
rdf:_1          rdf:type            rdfs:ContainerMembershipProperty .
rdf:_1          rdfs:domain         rdfs:Resource .
rdf:_1          rdfs:range           rdfs:Resource .
rdf:_2          rdf:type            rdfs:ContainerMembershipProperty .
rdf:_2          rdfs:domain         rdfs:Resource .
rdf:_2          rdfs:range           rdfs:Resource .
...```

Source: Corno & Farinetti: Logic and Reasoning in the Semantic Web
RDF-S reasoning

• RDF-S entailment lemma
  – S rdfs-entails E if and only if there is a graph
    • that can be derived
      – from S plus the RDF and RDFS axiomatic triples
      – by the application of rule lg, rule gl and the RDF and RDFS entailment rules
    • that either
      – simply entails E or
      – contains a clash
OWL reasoning

• General principle
  – Deducing new information from OWL axioms

• (Most used) principle: applying rules to
  – Enforce OWL constructors
  – Respond to queries

• In OWL2 RL, rules are specified in the spec:
  https://www.w3.org/TR/owl2-profiles/
  #Reasoning_in_OWL_2_RL_and_RDF_Graphs_using_Rules
OWL reasoning

- Based on OWL language constructs
- Different levels of expressivity

However, computing all interesting logical conclusions of an OWL ontology can be a challenging problem, and reasoning is typically multi-exponential or even undecidable.

- OWL2 profiles were created to
  - Be *tractable* (polynomial for tasks they are designed for)
  - Be easier to understand for practitioners
  - Make it easier to develop reasoning algorithms
Outline

• Summary
• Web-based reasoning techniques
• Reasoning using SemWeb languages expressivity
• General aspects of reasoning
  – Inference problems
  – Complexity
• Conclusion
Basic inference problems

• Subsumption
• Equivalence
• Consistency
• Instantiation

⇒ All these problems are reducible to KB satisfiability

Source: http://www.computational-logic.org/content/events/iccl-ss-2005/lectures/horrocks/part4b-tableaux.pdf
OWL inference problems

- Ontology Consistency
- Ontology Entailment
- Ontology Equivalence
- Ontology Equisatisfiability
- Class Expression Satisfiability
- Class Expression Subsumption
- Instance Checking
- Boolean Conjunctive Query Answering

Source: http://www.w3.org/TR/owl2-direct-semantics/
Closure and Reduction

• The closure of a graph is the graph defined by all triples that can be inferred by the reasoning algorithm
  – Using the closure of a graph for storing minimizes query processing time

• The reduction of a graph is the minimal subset of triples needed to compute its closure
  – Using the reduction of a graph for storing minimizes the storage space needed.
Hardness of reasoning

- Soundness
  - All computed inferences are really entailed
  - Necessary to be reliable
- Completeness
  - All computed inferences should actually be entailed
  - Not always necessary, if well documented

It is very easy to compute some OWL entailments – the challenge is to compute all entailments of a certain kind. Markus Krötzsch
A note about complexity

• **OWL 1 (OWL under RDF semantics)**
  – OWL DL was NP complete
  – OWL Full was undecidable

• **OWL 2 (OWL under direct semantics)**
  – In general, reasoning is N2ExpTime-complete
  – EL, RL and QL profiles have been designed to be *tractable* (i.e. polynomial) in their most important tasks
A note about complexity

• Practical considerations
  – A tractable reasoning process is polynomial
    • In worst case, but worst case is not always the case
  – In practice, polynomial reasoning is not necessarily achievable
    • What’s the use of a $n^{100}$-complex algorithm?
    • Even $n^4$ can be too complex on big data sets

→ Restraining the set of used constructs to the necessary ones is a good practice

→ Reducing the size of requests can also help...
Outline

• Summary
• Web-based reasoning techniques
• Reasoning using SemWeb languages expressivity
• General aspects of reasoning
• Conclusion
Conclusion

• Reasoning results can depend on
  – Representation language expressivity
  – KB
    • Size
    • Class hierarchy
    • Number and complexity of (actually used) relations
    • Presence of instances
  – Reasoning engine type
Conclusion

• Problems due to reasoning on the Semantic Web
  – Scalability
  – Distribution
  – Heterogeneity
  – Data quality
Conclusion

• Reasoning algorithms are quite simple
  – Building a reasoner is easy
  – Optimizing it is harder

• Complexity is always computed in worst case
  ➔ Make sure the worst case never happens
  ➔ Use different algorithms for different problems
Conclusion

• In any case, logical reasoning is a means to retrieve information « hidden » in the KB

  – It can solve complex queries in a generic, elegant manner

  – But it cannot produce new « knowledge »
References

• F. Corno, L. Farinetti - Politecnico di Torino: Logic and Reasoning in the Semantic Web (part I – RDF/RDFS)
  http://elite.polito.it/files/courses/01LHV/2012/4-RDFreasoning.pdf
• F. Corno, L. Farinetti - Politecnico di Torino: Logic and Reasoning in the Semantic Web (part II – OWL)
  http://elite.polito.it/files/courses/01LHV/2012/7-OWLreasoning.pdf
• M. Krötzsch: OWL 2 Profiles: An Introduction to Lightweight Ontology Languages
• RDF Semantics - W3C Recommendation 10 February 2004
  http://www.w3.org/TR/rdf-mt/
• RDF Schema 1.1 - W3C Recommendation 25 February 2014
  http://www.w3.org/TR/rdf-schema/
• OWL 2 Web Ontology Language Document Overview (Second Edition) - W3C Recommendation 11 December 2012
  http://www.w3.org/TR/owl2-overview/
  http://www.w3.org/TR/owl-direct-semantics
  http://www.w3.org/TR/owl-quick-reference