From auctions to graph coloring

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Journées du G-SCOP 2017

2011-2014 :

Thèse en théorie et algorithmique des graphes à Montpellier.

2014-2015 :

Post-doctorat en théorie des jeux économiques à Montréal.

2015-2016 : ATER en combinatoire et théorie des graphes à Lyon.

What is an auction ?

Auctions today

Ad auctions.

Google
YAHOO!
bing

≈ 150 billions a year.

Spectrum auctions.

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Spectrum auctions : a seller (the state) sells frequencies to telecommunication companies trying to maximizing the revenue (of the state) and, if possible global welfare.

Let's design an auction !

Your valuation : 1000$.
Opponent's valuation :
Between 950$ and 1000$.

Item + discount	**Item + no discount**	**No item**
		

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First price auction

The bidder with the higher price has the item and he pays the price he announces for the item.

Efficency ?

You bid 1000$ and your opponent bids 950$.

 instead of 

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You bid 970$ and your opponent bids 980$.

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Even worse from the seller ! He does not maximize his profit !

Truthfulness and efficiency

An auction is truthful if no bidder has any incentive to lie.
(His welfare can only decrease if he is lying on his valuation)

Informal claim

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The ultimate goal :

Design the best possible truthful auction...
... that can be explained to human beings...
... and whose "proof" is simple otherwise they won't trust you.

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Second price auction

The bidder with the higher price has the item and he pays the price of **the second highest bid** for the item.

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Second price auction

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$\Rightarrow$ This auction is truthful !

Proof on an example

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The seller maximizes his profit (under reasonable conditions).

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How best to allocate bandwidth dates back 100 years.
Since the 1990s, auctions have become the standard way to allocate bandwidth.

Two main auctions used worldwide :

- SMRA (Simultaneous Multi-Round Auction).
- CCA (Combinatorial Clock Auction).

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  Each bidder bids on her favorite set.
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  If an item is in several bids, its price increases.
Return the "best possible" allocation.

SMRA and CCA

Item vs package bidding :

- Package bidding in the CCA : all or nothing bid at price $p(S)$.
  $\Rightarrow$ The bidder receives either all or none of the items.
- Item bidding in the SMRA : a bid for S at price $p(S)$ is the union of single item bids for s at price $p(s)$ for $s \in S$.
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Drawback :

No market clearing $\Rightarrow$ usually market clearing helps for finding guarantees.

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These auctions :

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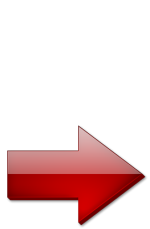
Theorem (B., Cai, Hunkenschröder, Vetta)

The CCA has a polylogarithmic guarantee (under technical assumptions).
Almost tight.

New auctions



- Buy TV and audio useless frequencies.
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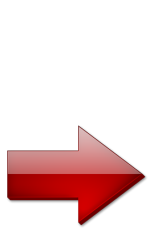

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First auction of that type (April 2017) :

- 19.8 billions of revenue.
- More than 200 companies bought or sold frequencies.
- Second-highest grossing spectrum auction in FCC history.

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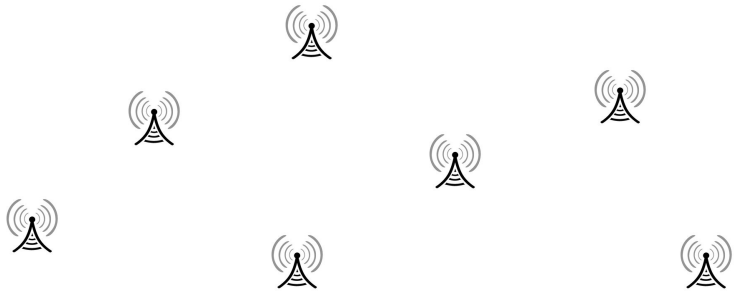

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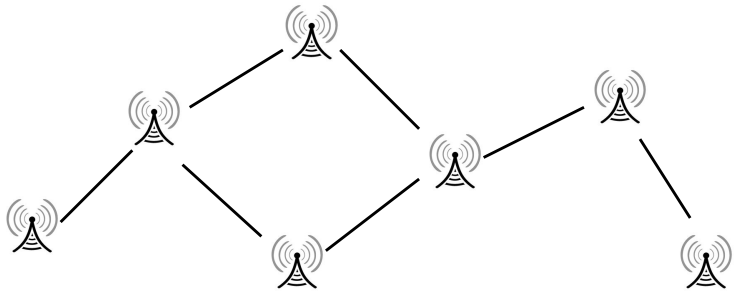

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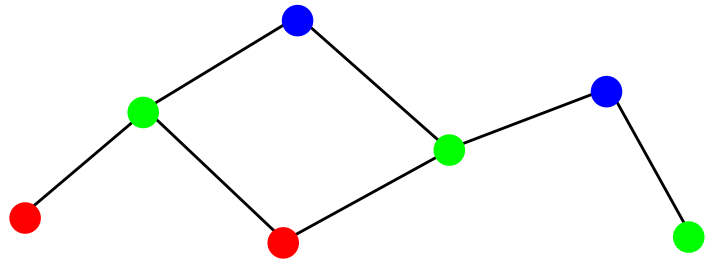

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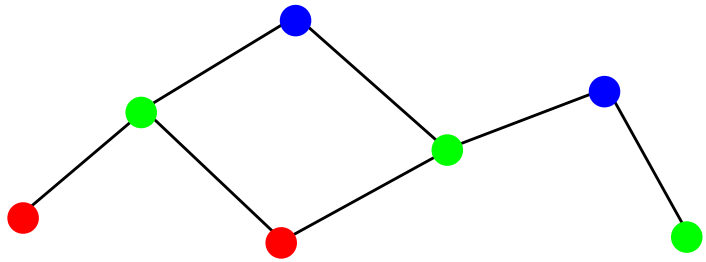

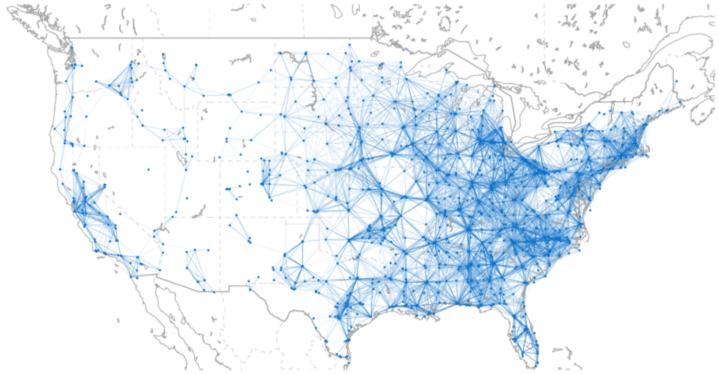

(source : FCC)

A problem

Problem

Coloring a graph is NP-hard and hard to approximate.

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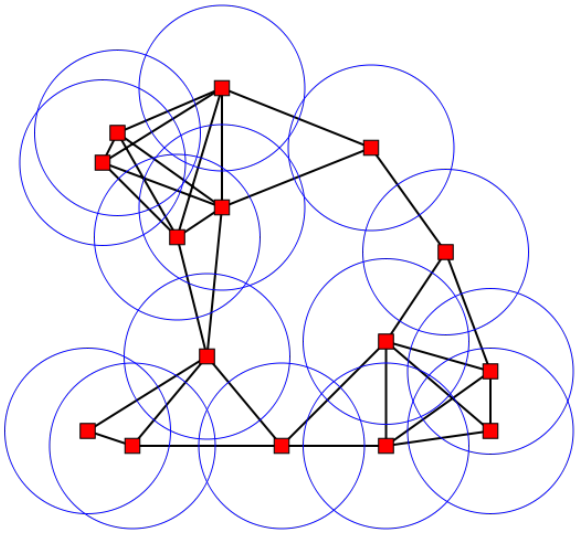
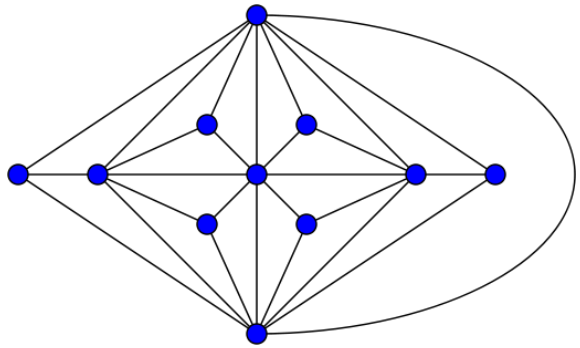
What can we do ?

Use the structure of the graph to derive efficient (approximation) algorithm to color graphs.





Conclusion

Questions

- Why is it working ?

  Understand the shape of valuation functions.

- Improve the "truthfulness process" of spectrum auctions.

  Implementations are messy...

- Improve coloring algorithms on geometric classes.

  Hard problems open for decades.

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Thanks for your attention !