Local certification of/on sparse graph classes

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joint works with
Laurent Feuilloley   Théo Pierron

Jagiellonian TCS seminar - November 2021

Disclaimer

All graphs are connected !

3-coloring and LOCAL model

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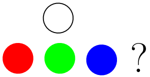

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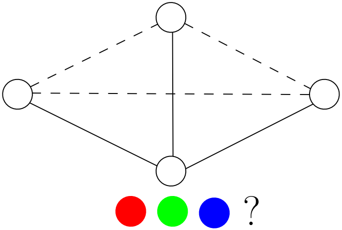

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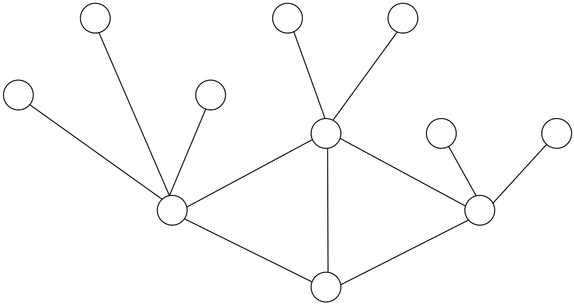

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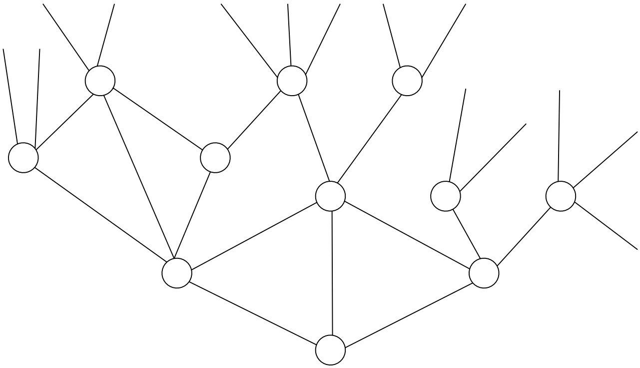

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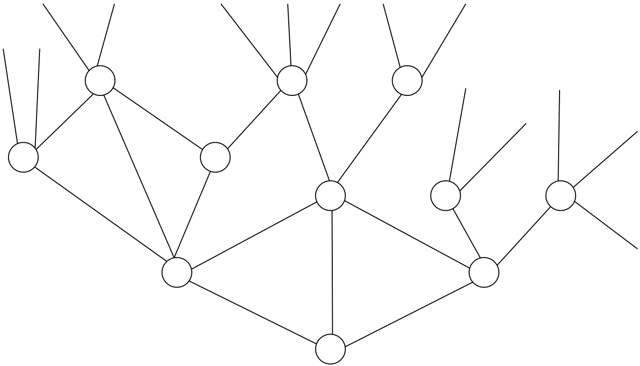
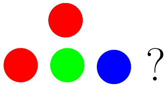


Question

Up to which distance do we have to look at to take a correc tdecision ?

Certify the 3-coloring



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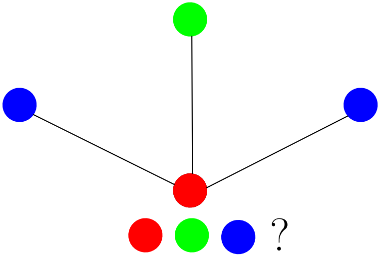

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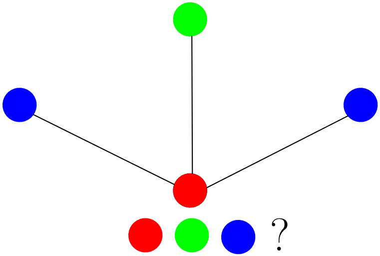

If vertices receive as a label their colors, we can check the coloring in the future by looking at distance 1 !

Local certification

- Every node has a unique ID in $[1, n]$. (It can be $O(n^c)$ (almost) for free)
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A property Π can be certified with $f(n)$ bits when :

- If Π is positive, there exists a certificate assignment to the nodes, each of size at most $f(n)$, such that all the nodes accept.
- If Π is negative, at least one node rejects for any possible certificate assignment.

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Give the entire graph with IDs to each vertex.

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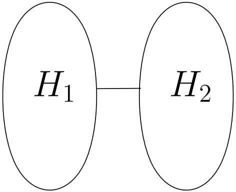
Give the entire graph with IDs to each vertex.

Questions :

- Can we improve $\Omega(n^2)$ in general ?
- What is a decent lower bound ?
  $\rightarrow \Omega(\log n)$ (size of labels).

What can't be certified with small certificate ?

[Göös, Suomela '16] **Non trivial automorphism**



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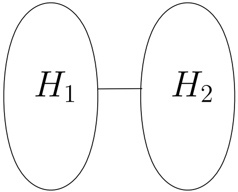

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[Censor-Hillel et al. '20] **Diameter 2.**
$\rightarrow \Omega(n)$ bits.

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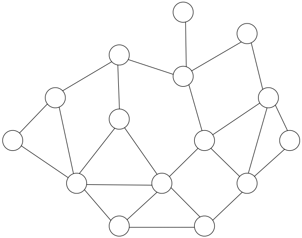

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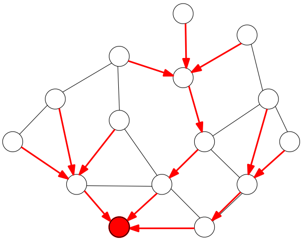

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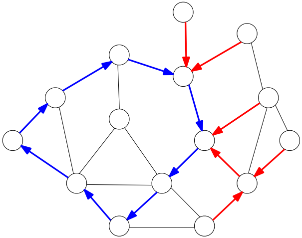

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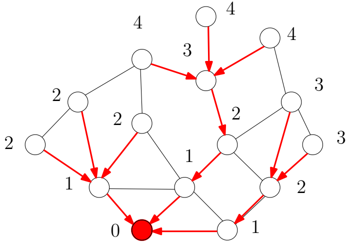

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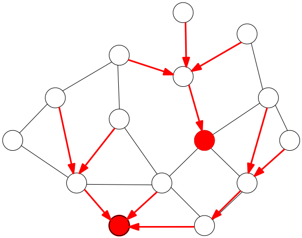

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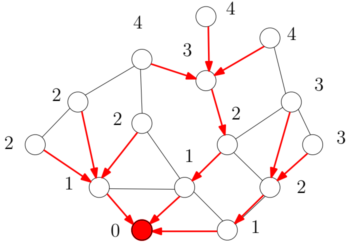

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H-minor free graphs can be certif. with $O(\log n)$ bits if $|H| \leq 4$.

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On the way :
Certification of 2 and 3-connectivity, block cut trees, development of new tools...

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Theorem (B, Feuilloley, Pierron '21+)

- $\mathrm{td}(G) \leq k$ can be certified with $O(k \log n)$ bits.
- Every MSO formula can be certified with $O(\log n)$ bits on bounded treedepth graphs.

Certifying K_4-minor free graphs (I)

Lemma (B., Feuilloley, Pierron)

If H is 2-connected :
Certification of 2-connected H-minor-free graphs with $O(\log(n))$ bits
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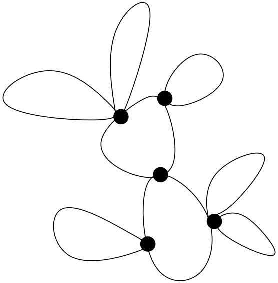

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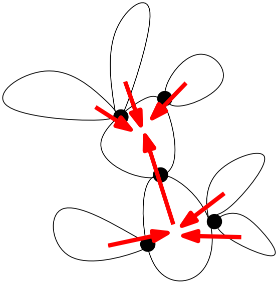

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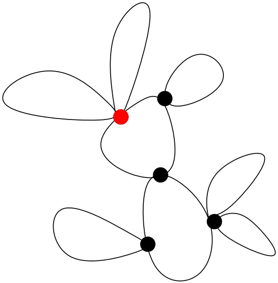

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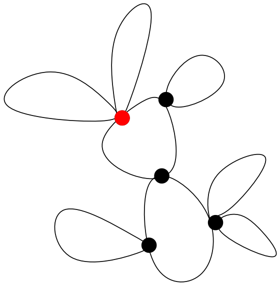

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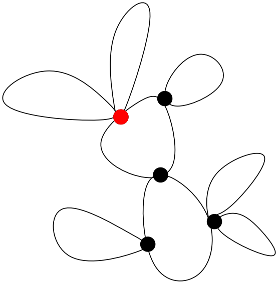
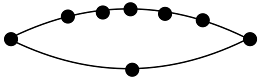
(Idea of the) Proof :

- A model of H should belong to a 2-connected component.
- Certify the block cut tree.
- Certify that 2-connected components are 2-connected.
- Give the H-certificate to each 2-connected component.



Ear decompositions $\Leftrightarrow$ 2-connected.

- Start from a cycle



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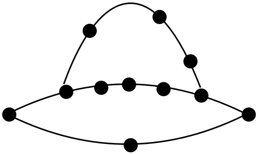

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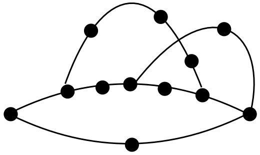

K_4-free graphs (II)

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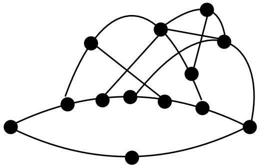

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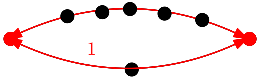
Certify ear decompositions.

- Certify the initial cycle and give label 1 to these vertices.



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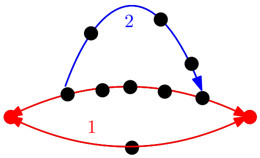

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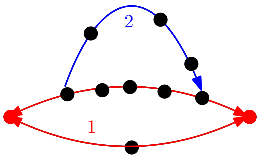

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Theorem (Eppstein)

The following are equivalent :

- G is a 2-connected K_4-minor-free graph,
- G is a 2-connected series-parallel graphs,
- G has a nested ear decomposition.

Question 1 - Next step ?

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Certification of H-minor free graphs with $O(\log n)$ bits
$\Rightarrow$ Certification of $(H + K_1)$-minor free graphs with $O(\log n)$ bits ?

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Can we certify (vertex) minimally non planar graphs ?

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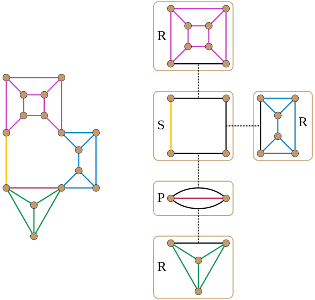
Certity F-minor free graphs for any forest F ?

Question 2 - SPQR trees partition



Source : wikipedia

- S node - Cycle of length at least 3.
- P node - Multigraph with 2 vertices and ≥ 3 edges.
- Q node - Single real edge.
- R node - 3-connected graph that is not S or P.

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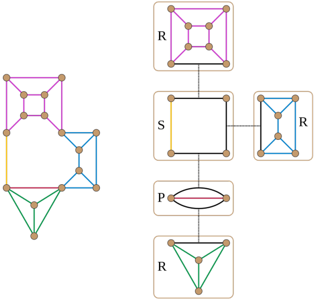

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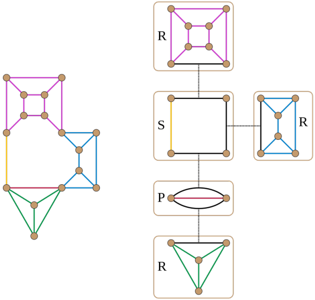

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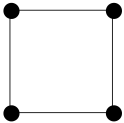

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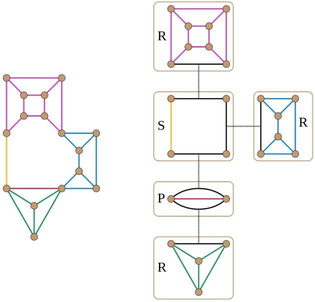

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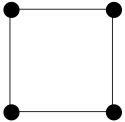

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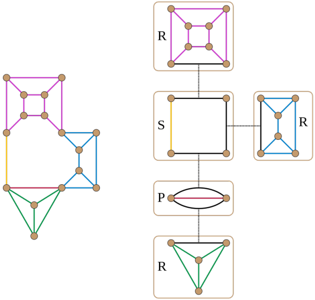

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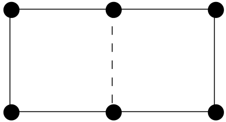

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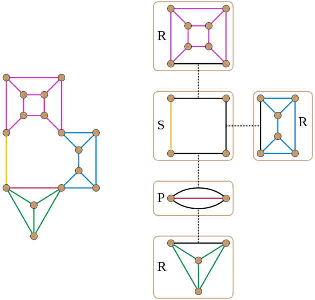

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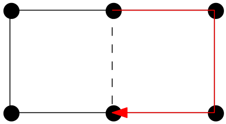

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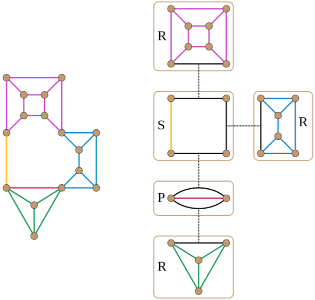

Source : wikipedia

- S node - Cycle of length at least 3.
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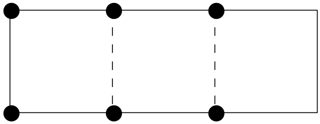

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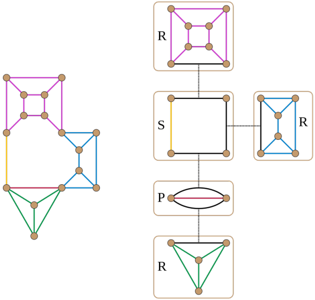

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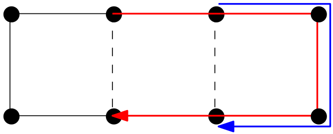

Question 3 - K_5-minor free graphs

Theorem (Wagner ''37)

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Remark : A similar result for $K_{3,3}$-graphs exist.

Question 4 - Connectivity

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Important question in network (robustness).

Monadic Second Order logic

First order (FO) :

- Quantifies on vertices
- Predicate $\rightarrow$ adjacency

$$\forall x \forall y, (x = y) \vee (x - y) \vee \exists z (x - z \wedge z - y)$$

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Monadic second order :

- Quantifies on sets of vertices (MSO_1) and edges (MSO_2)
- Predicates $\rightarrow$ adjacency $+$ membership

$$\begin{aligned} &\exists V_1 \exists V_2, \\ &\quad \forall v, (v \in V_1) \Leftrightarrow \neg(v \in V_2) \\ &\wedge \forall v \forall w, v - w \Rightarrow (v \in V_1 \Leftrightarrow w \in V_2) \end{aligned}$$

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- Excluding a minor is MSO_1
- Hamiltonian cycle is MSO_2

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Every MSO formula can be decided in FPT time parameterized by the treewidth.

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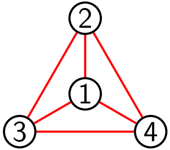
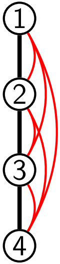
- ... And even if we assume that we have bounded treewidth, not clear how to do it !

One step back : Treedepth

Find a (rooted) tree T on $|V(G)|$ vertices such that each edge of G links a vertex with an ancestor.
$\mathrm{td}(G)$ = minimum depth of such a T





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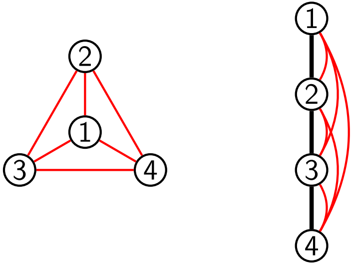

Bounded treedepth $\Rightarrow$ No long path

Zoology of depths

$$\mathrm{tw}(G) \leq \mathrm{pw}(G) \leq \mathrm{td}(G)$$

Advantages of treedepth :

- Diameter is bounded.
- [Gajarský and Petr Hlinený '16] For every graph of bounded schrubdepth we can construct a kernel that satisfies the same FO formulas.

  And bounded schrubdepth $\Rightarrow$ Bounded treedepth.
- For bounded schrubdepth graphs, (informally) FO = MSO.

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Theorem (B, Feuilloley, Pierron '21+)

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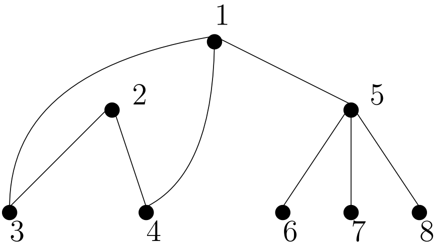

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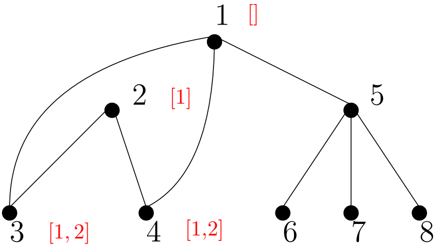

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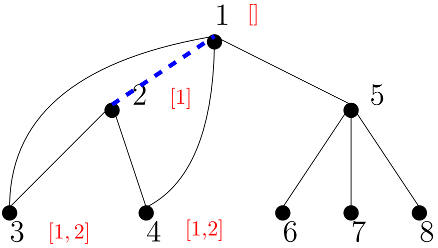

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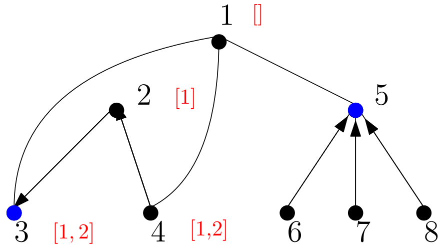

Certificate :

- List of ancestors
- Subtree rooted in a node connected to an ancestor for the graph below v for every v.

MSO certification

Theorem

Every MSO formula can be certified with $O(\log n)$ bits on bounded treedepth graphs.

- Existence of a (constructive and easy to certify) kernel for FO formulas on bounded treedepth graphs.

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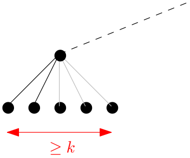

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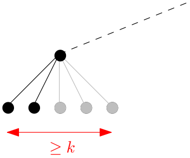

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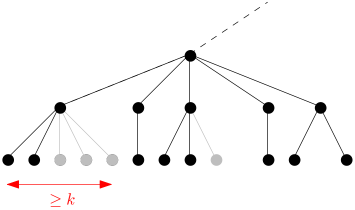

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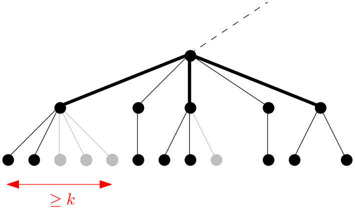

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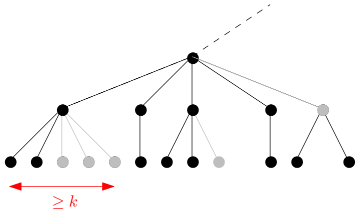

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Thanks for your attention !