# Domination in circle graphs

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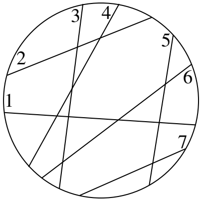
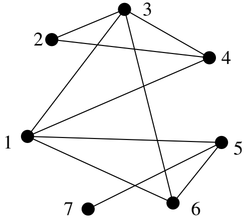
# Circle graphs

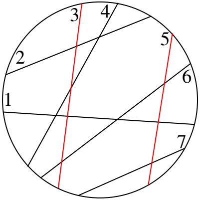
## Circle graph

A circle graph is a graph which can be represented as an intersection graph of chords in a circle.



# Dominating set




## Dominating set

Set of chords which intersects all the chords of the graph.

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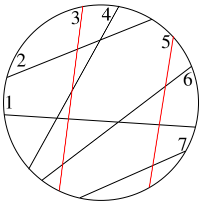


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Set of chords which intersects all the chords of the graph.

- Independent dominating sets.

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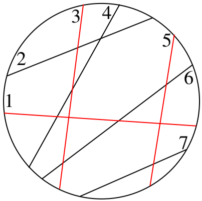


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Set of chords which intersects all the chords of the graph.

- Independent dominating sets.
- Connected dominating sets.

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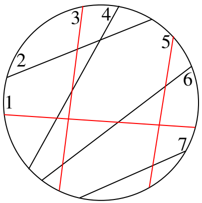


## Dominating set

Set of chords which intersects all the chords of the graph.

- Independent dominating sets.
- Connected dominating sets.
- Total dominating sets.

All these problems are NP-complete

# Parameterized complexity

## FPT

A problem parameterized by $k$ is *FPT* (Fixed Parameter Tractable) iff it admits an algorithm which runs in time $Poly(n) \cdot f(k)$ for any instances of size $n$ and of parameter $k$.

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## $W[1]$-difficulty

Under some algorithmic hypothesis, the $W[1]$-hard problems do not admit FPT algorithms.

### Theorem (B., Gonçalves, Mertzios, Paul, Sau, Thomassé)

Dominating set parameterized by the size of the solution is $W[1]$-hard.

### $k$-colored clique

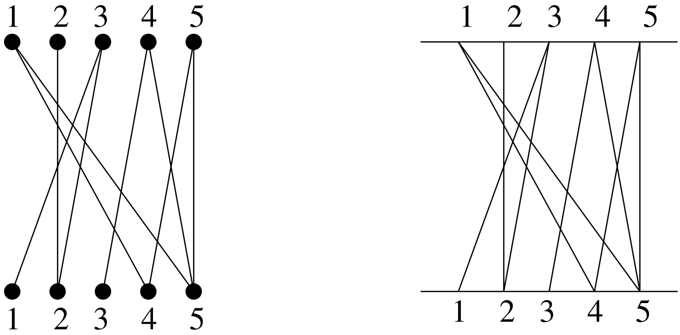
**Input :** $G$ colored with $k$-colors. $n$ vertices of each color.
**Parameter :** $k$.
**Output :** YES iff there is a clique of size $k$ with one vertex of each color.

### Theorem

$k$-colored clique is $W[1]$-hard parameterized by $k$.

# Reduction from $k$-colored clique

Idea : Simulate the behaviour of the vertices of each color.



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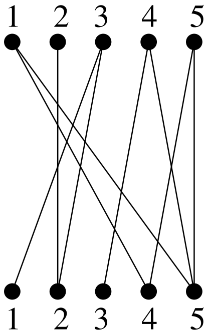
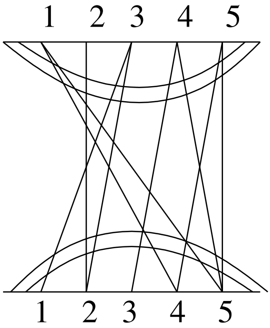
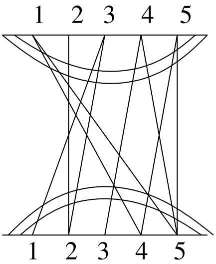
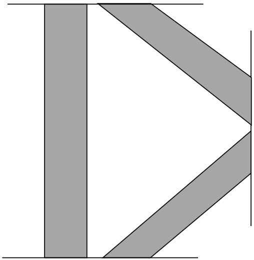


# Transformation into a circle graph



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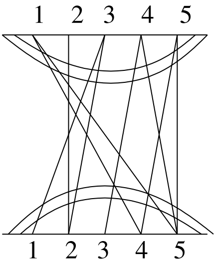


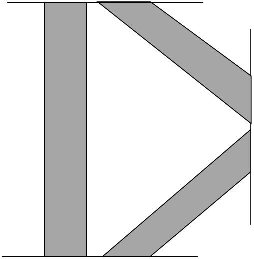

Assume that we only use chords of the bipartite graphs.

- At least $k(k-1)/2$ chords in the dominating set.

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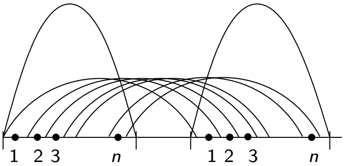


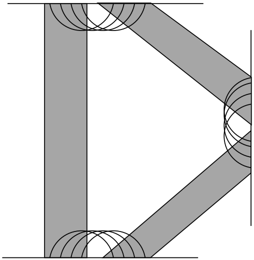

Assume that we only use chords of the bipartite graphs.

- At least $k(k-1)/2$ chords in the dominating set.
- The "value" can only decrease.

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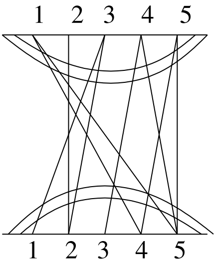


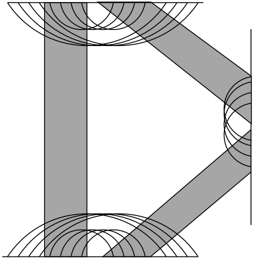

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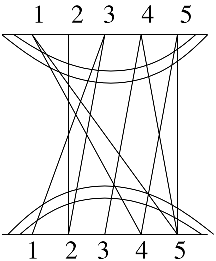


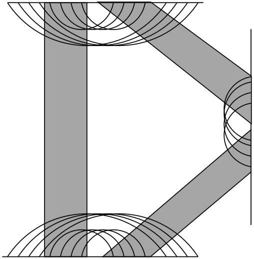

Assume that we only use chords of the bipartite graphs.

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- The "value" can only decrease.

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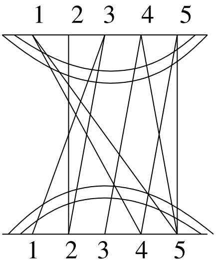


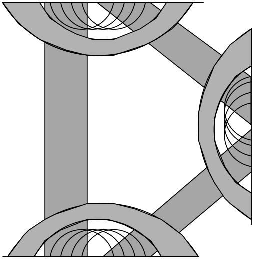

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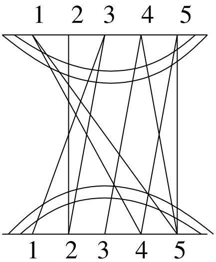
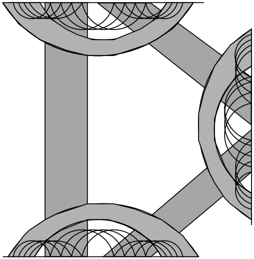
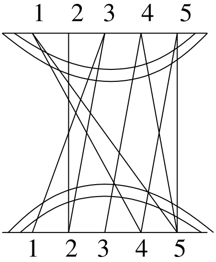


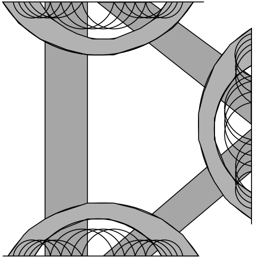



Assume that we only use chords of the bipartite graphs.

- At least $k(k+1)/2$ chords in the dominating set.
- The "value" can only decrease.
- The first and the last "values" are the same.

# Form of a dominating set



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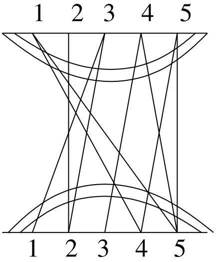
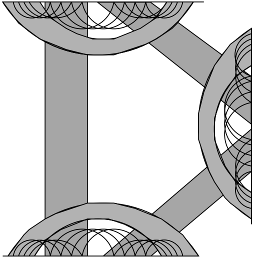
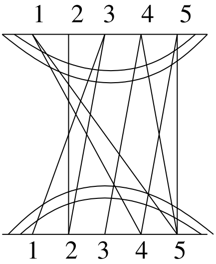
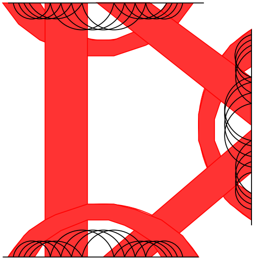


- If there is a multicolored clique, there is a dominating set of size $k(k+1)/2$.
- A dominating set has size at least $k(k+1)/2$.
- A dominating set of such a size has one endpoint in each "part".

# Form of a dominating set



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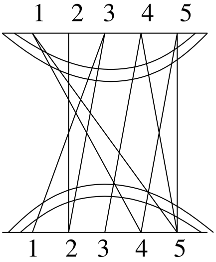


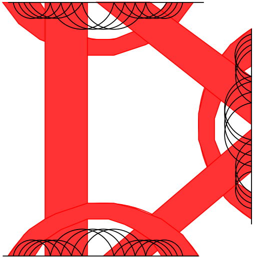

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- A dominating set of such a size has one endpoint in each "part".

$\Rightarrow$ The possible chords are the red chords.

$\Rightarrow$ There is a dominating set of size $k(k+1)/2$ iff there is a $k$-colored clique.

# Corrolaries

**Theorem**

Connected Dominating set is $W[1]$-hard in circle graphs parameterized by the size of the solution.

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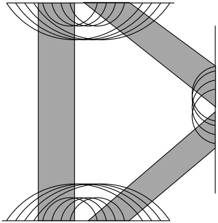

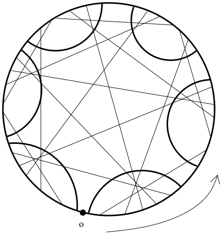

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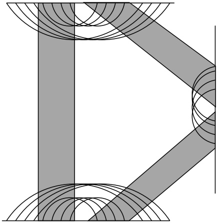

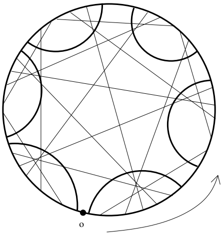


**Theorem**

Total Dominating set is $W[1]$-hard in circle graphs parameterized by the size of the solution.

# Independent Dominating set

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The independent dominating set problem is $W[1]$-hard for circle graphs parameterized by the size of the solution.

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The independent dominating set problem is $W[1]$-hard for circle graphs parameterized by the size of the solution.

**Theorem**

The acyclic dominating set problem is $W[1]$-hard parameterized by the size of the solution.

# Some positive results

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**Input :** A circle graph $G$, an integer $k$.
**Output :** YES iff there exists a dominating path of length $k$.
This problem is in $P$.

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**Input :** A circle graph $G$, an integer $k$.
**Output :** YES iff there exists a dominating tree of size $k$.
This problem is in $P$.

# An FPT result

**Theorem**

**Input** : A circle graph $G$, a tree $T$ of size $k$.
**Parameter** : $k$
**Output** : YES iff there exists a dominating tree isomorphic to $T$.
This problem is in NP-complete and FPT.

# Open problems

**Conjecture**

The Bounded Treewidth Dominating Set problem is polynomial in circle graphs.

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The Bounded Treewidth Dominating Set problem is polynomial in circle graphs.

## Open problems

Does the domination problem in circle graphs admits a polynomial kernel parameterized by treewidth ? by vertex cover ?

# Thanks for your attention

Any question ?