Domination in circle graphs

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Parameterized complexity

FPT

A problem parameterized by k is *FPT* (Fixed Parameter Tractable) iff it admits an algorithm which runs in time $Poly(n) \cdot f(k)$ for any instances of size n and of parameter k.

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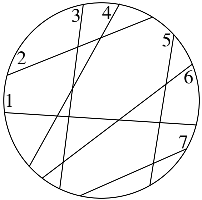
$W[1]$-hardness

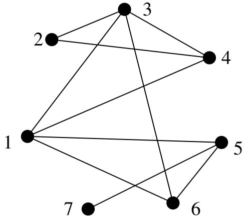
Under some algorithmic hypothesis, the $W[1]$-hard problems do not admit FPT algorithms.

Circle graphs

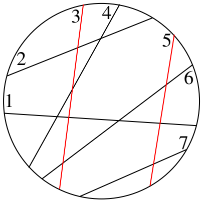
Circle graph

A circle graph is a graph which can be represented as an intersection graph of chords in a circle.





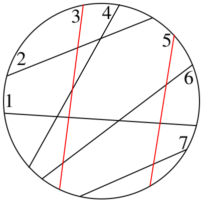
Dominating set



Dominating set

Set of chords which intersects all the chords of the graph.

Dominating set



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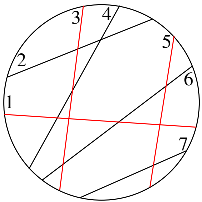
Set of chords which intersects all the chords of the graph.

- Independent dominating sets.

Dominating set

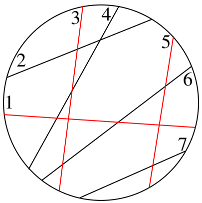


Dominating set

Set of chords which intersects all the chords of the graph.

- Independent dominating sets.
- Connected dominating sets.

Dominating set



Dominating set

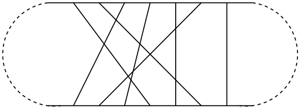
Set of chords which intersects all the chords of the graph.

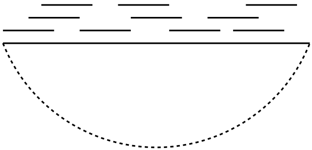
- Independent dominating sets.
- Connected dominating sets.
- Total dominating sets.

All these problems are NP-complete.

Hardness of DOMINATING SET

- DOMINATING SET is polynomial on proper interval graphs.
- DOMINATING SET is polynomial on permutation graphs.
- DOMINATING SET is $W[1]$-hard on general graphs.
- DOMINATING SET is NP-complete on circle graphs.





Main result

Theorem (B., Gonçalves, Mertzios, Paul, Sau, Thomassé)

DOMINATING SET parameterized by the size of the solution is $W[1]$-hard on circle graphs.

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k-COLORED CLIQUE

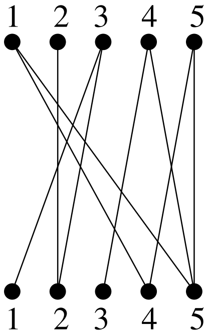
Input : G colored with k-colors. n vertices of each color.
Parameter : k.
Output : YES iff there is a clique of size k with one vertex of each color.

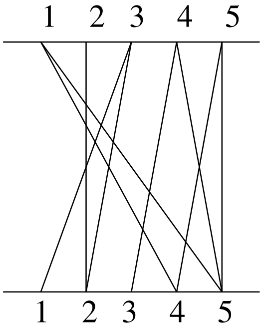
Theorem

k-COLORED CLIQUE is $W[1]$-hard parameterized by k.

Reduction from k-colored clique

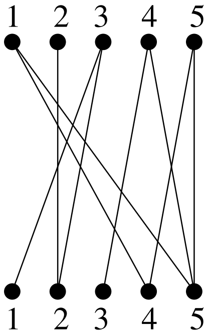
Idea : Simulate the behavior of the vertices of each color.

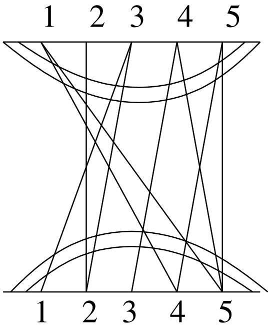




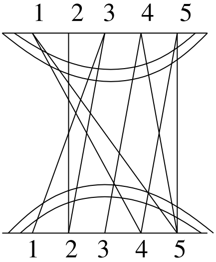
Reduction from *k*-colored clique

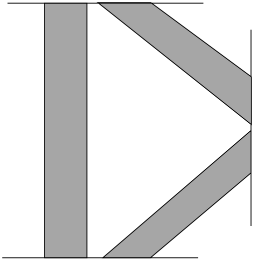
Idea : Simulate the behavior of the vertices of each color.





Transformation into a circle graph





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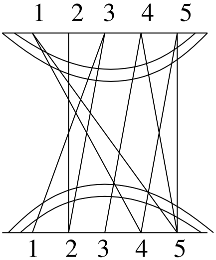

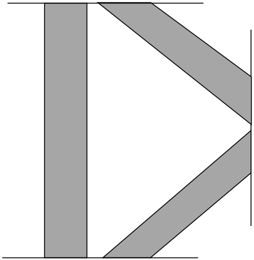

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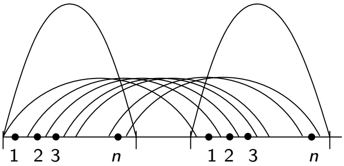

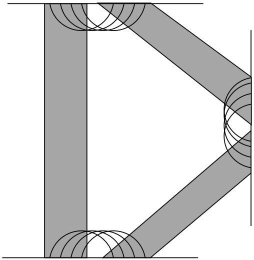

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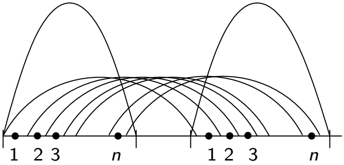
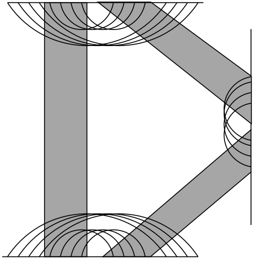




Corollaries

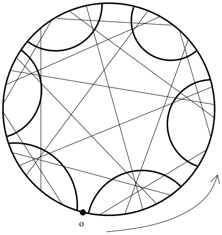
Theorem

CONNECTED DOMINATING SET is $W[1]$-hard on circle graphs parameterized by the size of the solution.

Corollaries

Theorem

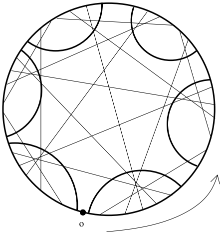
Connected Dominating Set is $W[1]$-hard on circle graphs parameterized by the size of the solution.



Corollaries

Theorem

CONNECTED DOMINATING SET is $W[1]$-hard on circle graphs parameterized by the size of the solution.



Theorem

TOTAL DOMINATING SET is $W[1]$-hard on circle graphs parameterized by the size of the solution.

Independent Dominating set

Theorem

INDEPENDENT DOMINATING SET is $W[1]$-hard on circle graphs parameterized by the size of the solution.

Independent Dominating set

Theorem

Independent Dominating Set is $W[1]$-hard on circle graphs parameterized by the size of the solution.

Theorem

Acyclic Dominating Set is $W[1]$-hard on circle graphs parameterized by the size of the solution.

Some positive results

Theorem

Input : A circle graph G, an integer k.
Output : YES iff there exists a dominating tree of size k.
This problem is in P.

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Theorem

Input : A circle graph G, an integer k.
Output : YES iff there exists a dominating tree of size k.
This problem is in P.

Theorem

Input : A circle graph G, an integer k.
Output : YES iff there exists a dominating path of length k.
This problem is in P.

An FPT result

Theorem

Input : A circle graph G, a tree T of size k.
Parameter : k
Output : YES iff there exists a dominating tree isomorphic to T.
This problem is in NP-complete and FPT.

Open problems

Conjecture

BOUNDED TREEWIDTH CONNECTED DOMINATING SET is polynomial in circle graphs.

Open problems

Conjecture

Bounded Treewidth Connected Dominating Set is polynomial in circle graphs.

- Dominating Set parameterized by treewidth is FPT.
- Dominating Set parameterized by treewidth does not have a polynomial kernel.

Open problem

Does the domination problem in circle graphs admits a polynomial kernel parameterized by treewidth ?

Thanks for your attention

Any question ?