TS-Reconfiguration of Dominating Sets in circle and circular-arc graphs

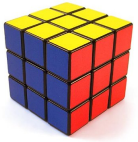
Nicolas Bousquet

joint work with
Alice Joffard

FCT 2021

Reconfiguration

A one-player game is a puzzle : one player makes a series of moves, trying to accomplish some goal.



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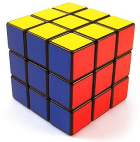

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- Applications to random sampling, bioinformatics, discrete geometry...etc...

Main questions

- **Reachability problem.** Given two configurations, is it possible to transform the one into the other ?
- **Connectivity problem.** Given any pair of configurations, is it possible to transform the one into the other ?
- **Minimization.** Given two configurations, what is the length of a shortest sequence ?

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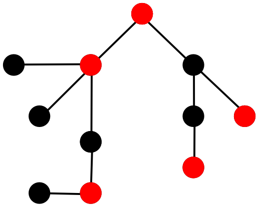
Today : Reachability + Dominating Set Reconfiguration.

Dominating Set Reconfiguration

A dominating set is a subset X of vertices such that $N[X] = V$.

$\Leftrightarrow$ A set of tokens whose (closed) neighborhood is V.



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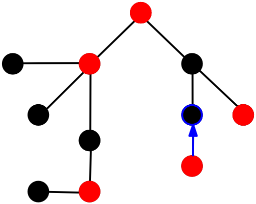

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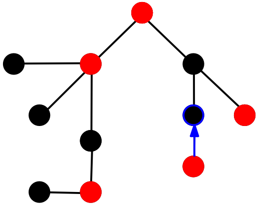

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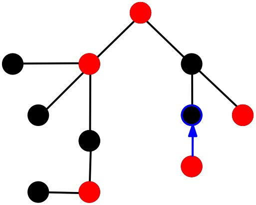

Dominating Set Reconfiguration (DSR)
Input : A graph G, two independent sets S, T.
Output : YES iff $S \rightsquigarrow T$

Our results I

Theorem (Bonamy, Dorbec, Ouvrard '20)

DSR is PSPACE-complete even restrited to bipartite graphs, split graphs and planar graphs.

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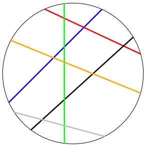
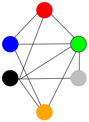
Candidate : circle graphs.

Intersection of chords of a circle.





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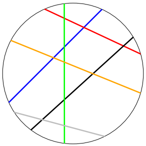

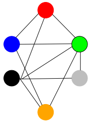

[Our result] PSPACE-complete on circle graphs.

Our results II

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There exists a transformation between any pair of dominating sets for interval graphs.

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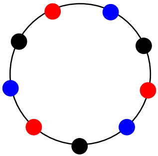

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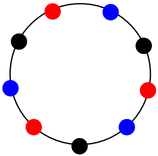

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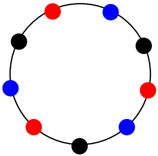

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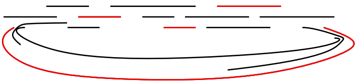

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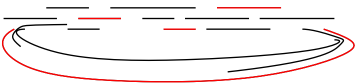

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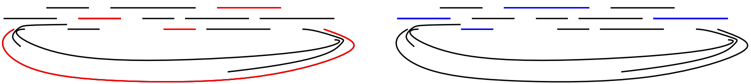

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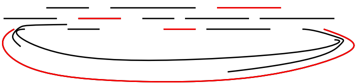

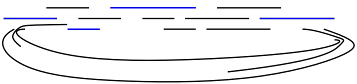

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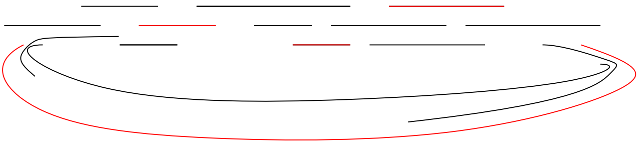

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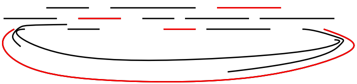

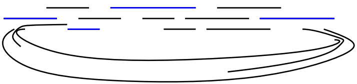

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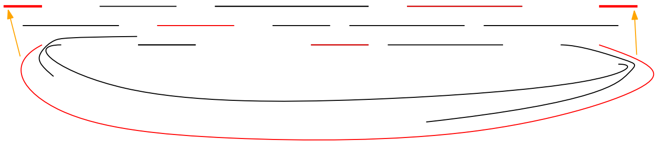

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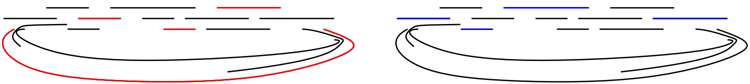

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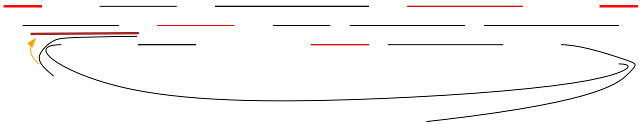

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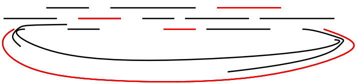

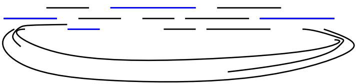

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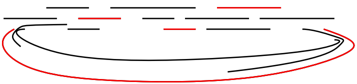

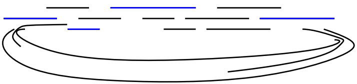

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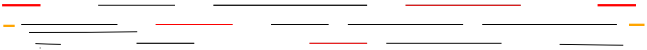

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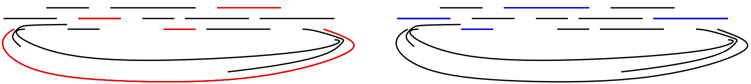

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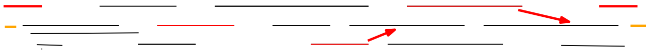

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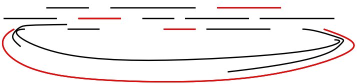

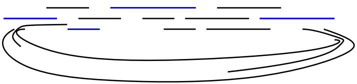

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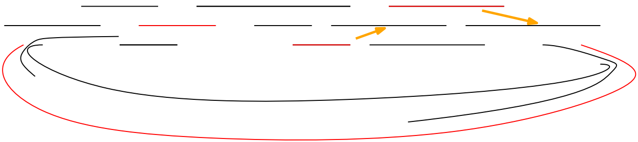

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Main technicalities :

Minimum Dominating Sets (of size k) are rigid.
$\rightarrow$ We need to consider DS of size $k+1$.
$\rightarrow$ Create gadgets to control the structure of these DS to guarantee that there are still "close" from an assignment of the SAT formula.

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- Does there exist a (hereditary) graph class where minimum DS is NP-complete and DSR is in P ?
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