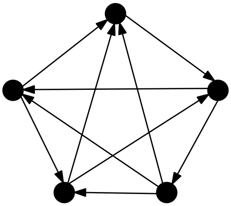
Domination in tournaments

Nicolas Bousquet

Birmingham, June 2017

Tournaments

A tournament is a digraph where either $x \to y$ or $x \leftarrow y$ for every pair of vertices x, y.



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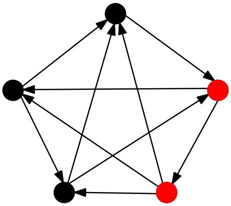

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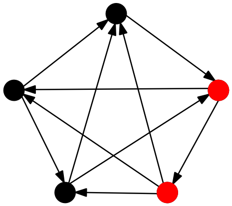
Notations :

$N^-(v)$: in-neighborhood of v excluding v.
$N^-[v]$: in-neighborhood of v including v.
$N^+[v]$: out-neighborhood of v including v.



Domination in tournaments

Theorem

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Question : What if add structure ?

Example : Transitive tournaments have domination number 1 !

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During the presentation today :

- k-majority tournaments. (Alon et al. '04)
- Union of k partial orders. (B., Lochet, Thomassé '17)

k-majority tournaments

$V = \{1, ..., n\}$. Let $\prec_1, \ldots, \prec_{2k-1}$ be total orders on V.
The tournament realized by $\prec_1, \ldots, \prec_{2k-1}$ has an arc $i \to j$ iff $i \succ j$ in at least k orders.

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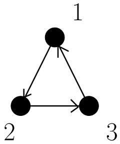

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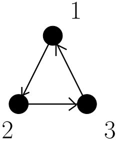

Definition (k-majority tournament)

Tournament realized by $2k-1$ total orders.

Example : 1-majority tournaments are transitive tournaments.

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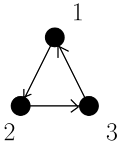

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Theorem (Alon, Brightwell, Kierstead, Kostochka, Winkler '04)

Every k-majority tournament has a dominating set of size $\mathcal{O}(k \cdot log(k))$.

Outline of the proof

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A tournament of "bounded VC-dimension" has a dominating set of bounded size.

1) Define VC-dimension for graphs.

2) Apply a result of Haussler and Welzl that implies that there is a dominating set of bounded size.

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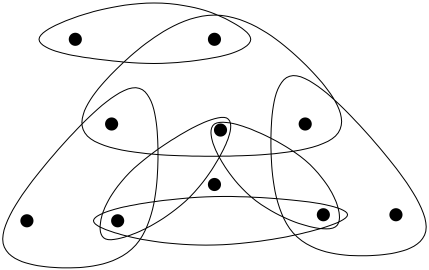
Show that k-majority tournaments have bounded VC-dimension.

"Double counting" on the possible traces of neighborhoods on a set of vertices.

Definitions

A hypergraph is a pair (V, E) where V is a set of vertices and E is a set of hyperedges (subsets of vertices).



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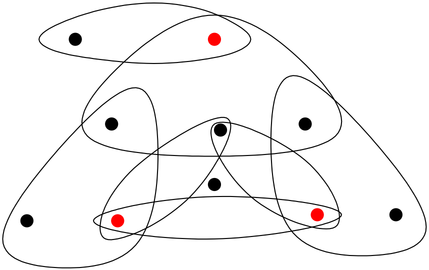
A hitting set is a subset of vertices intersecting all the hyperedges.
Transversality τ : minimum size of a hitting set.



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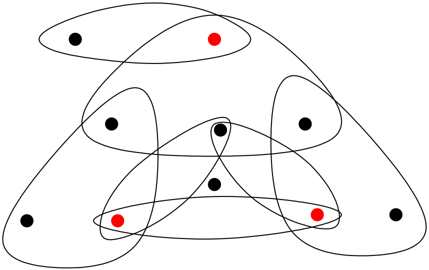

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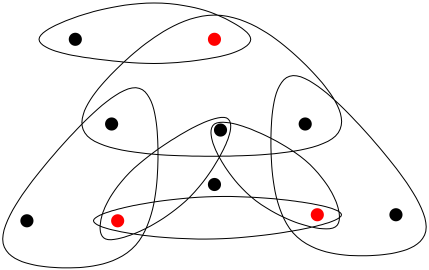

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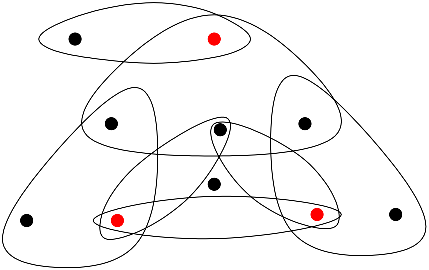
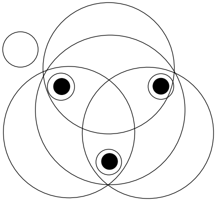
Remark : $\tau \geq \tau^*$. **No converse function in general !**



VC-dimension



A set $Y \subseteq X$ is a trace on X if there exists $e \in E$ such that $e \cap X = Y$.

A set $X \subseteq V$ is shattered iff all the traces on X exist.

The VC-dimension of a hypergraph is the maximum size of a shattered set.

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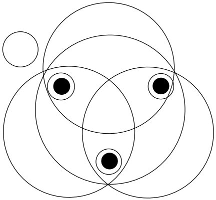

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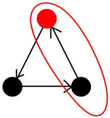
Theorem (Haussler, Welzl '73)

Every hypergraph H of VC-dimension d satisfies

$$\tau \leq 2d\tau^* log(11\tau^*).$$

Transformation into hypergraphs



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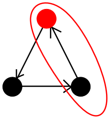

- Consider the hypergraph H where the hyperedges are the closed in-neighborhoods of the vertices of T.

- A hitting set of H is a dominating set of T.

Let X be a hitting set.
For every v, $N^-[v]$ is a hyperedge.
$\Rightarrow$ For every v, X intersects $N^-[v]$.
$\Rightarrow N^+[X] = V$.

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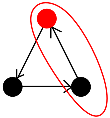

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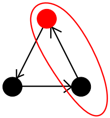

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- A shattered set X is a set of vertices such that for every $Y \subseteq X$ there exists v such that $N^+[v] \cap X = Y$.

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For any directed graph, there exists $w : V \to \mathbb{R}^+$ such that :

- $w(V) = 2$.
- For every $v \in V$, $w(N^-[v]) \geq w(N^+(v))$.

Translation for tournaments :

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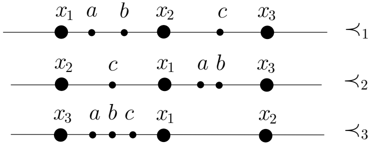
In a tournament :

Bounded VC-dimension $\Rightarrow$ Bounded domination number.

VC-dimension of k-majority tournaments



Shattered set :
$X = \{x_1, x_2, \cdots, x_\ell\}$.

Given X, $\prec_i$ partitions V into $|X| + 1$ classes :

vertices before the 1st vertex of X, between the 1st and the second, ...

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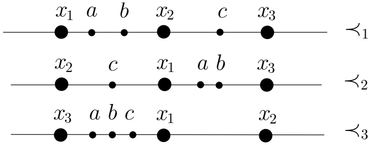

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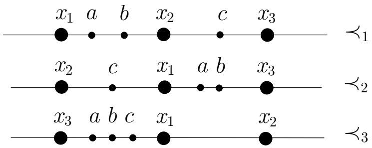

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Claim

Vertices in the same class have the same neighborhood in X.

Proof : u, v in the same class $\Rightarrow \forall x,\ u \prec_i x \Leftrightarrow v \prec_i x$.

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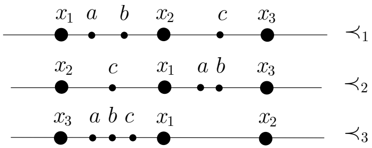

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Proof : u, v in the same class $\Rightarrow \forall x,\ u \prec_i x \Leftrightarrow v \prec_i x$.

$\Rightarrow$ There are at most $(|X| + 1)^{2k-1}$ traces on X in the hypergraph.

Upper bound on the size of X

Let X be a shattered set.

X is shattered $\Rightarrow$ $2^{|X|}$ traces on X.
k-majority tournament $\Rightarrow$ At most $(|X|+1)^{2k-1}$ traces on X.

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VC-dimension : $\mathcal{O}(k \cdot \log k)$.
Fractional transversality : ≤ 2.

$$\Rightarrow \tau \leq \mathcal{O}(k \log k)$$

Generalization

A set of k partial orders $\prec_1, ..., \prec_k$ partition a tournament D if $x_i \to x_j \Leftrightarrow$ there is a unique order ℓ where $x_i \prec_\ell x_j$.

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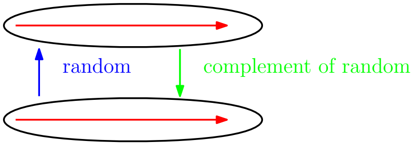

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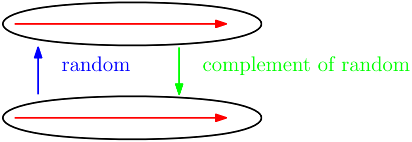

Remark : k-majority tournaments can be partitioned into 2^{2k-2} partial orders.

Related questions

Gyárfás, Pálvölgyi conjecture implies in particular the following :

Theorem (Bárány and Lehel)

Every finite subset X of $\mathbb{R}^d$ can be covered by $f(d)$ X-boxes (i.e. each box has two antipodal points in X).

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GP conjecture is a particular case of :

Conjecture (Erdős, Sands, Sauer, Woodraw '82)

In every k-arc colored tournament, there exist $f(k)$ vertices that can reach any vertex via a monochromatic path.

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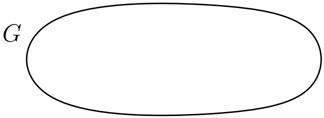
B., Lochet, Thomassé (2017) : ESSW conjecture is true.

Sketch of the proof

Theorem (Fisher)

For any tournament, there exists $w : V \to \mathbb{R}^+$ such that :

- $w(V) = 1$.
- For every $v \in V$, $w(N^-[v]) \geq 1/2$.



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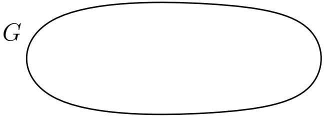

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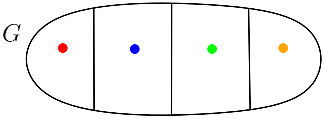

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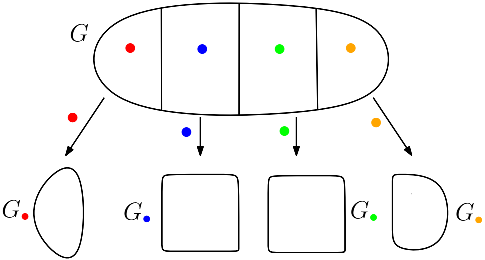

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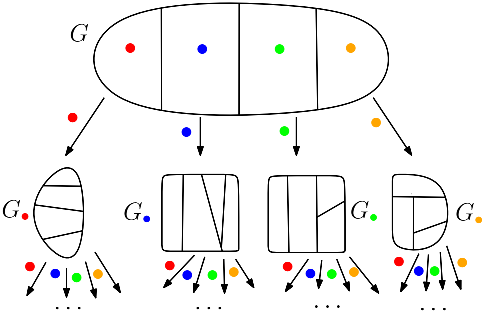

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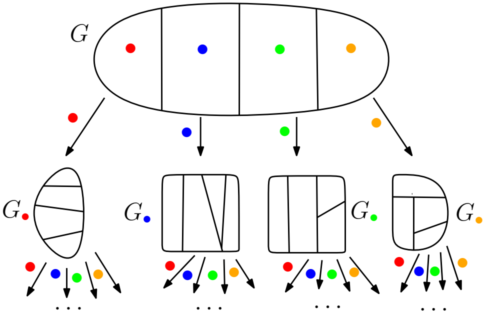
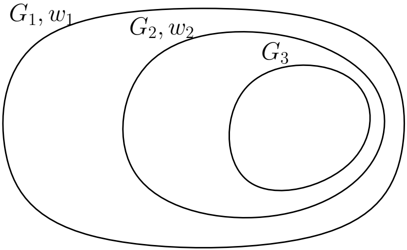
$\Rightarrow$ For any $v \in V$, there exists a color • such that $|N^-_\bullet[v]| \geq 1/2k$.



Repeat $(k+1)$ times : Partition into k^{k+1} parts.

Dominating each part

Goal : Show that each part can be dominated using $f(k)$ vertices.



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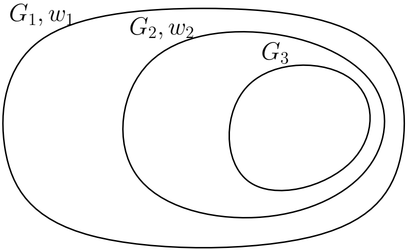
Idea of the proof :

- After $k+1$ extractions, a color • is repeated twice.



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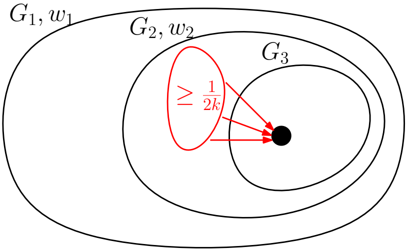
- After $k+1$ extractions, a color • is repeated twice.
- $x \in G_3$ satisfies $w_2(N^-_\bullet[x]) \geq 1/2k$.



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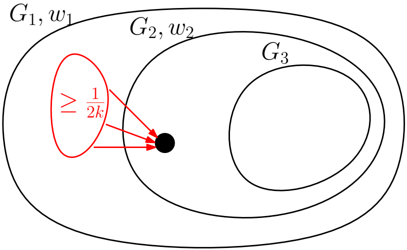

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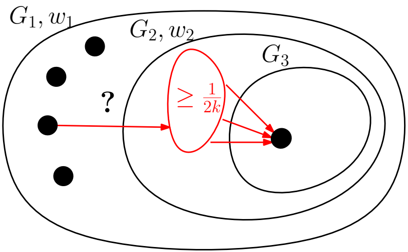

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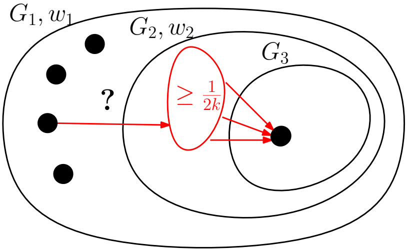
- Sample $f(k)$ vertices X in G_1 with probability rule w_1.



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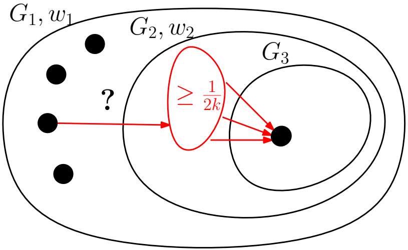

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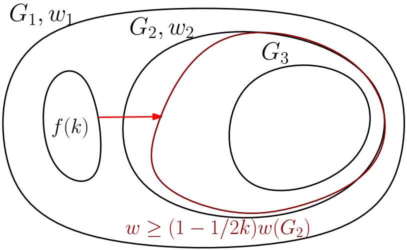

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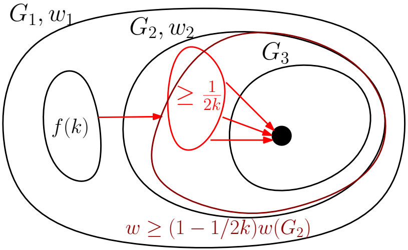

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Conjecture (Erdős, Sands, Sauer, Woodraw)

In every k-arc colored oriented graph, there exist $f(k)$ independent sets $X_1, \ldots, X_{f(k)}$ that "can reach any vertex via a monochromatic path".

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Thanks for your attention !