A Journey on Configuration Graphs

(Recoloring regular graphs)

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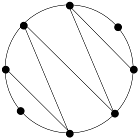
Associahedron

n points in convex position

Triangulation : Non crossing set of edges such that inner faces are triangles.



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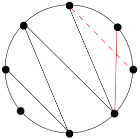

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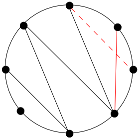
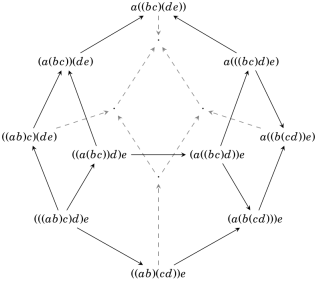


Associahedron $\mathcal{A}(n)$:
Vertices : Triangulations
Edges : Flips.



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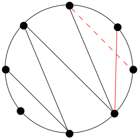

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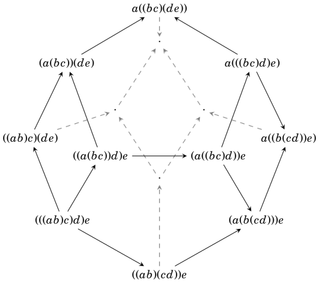

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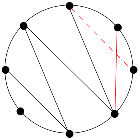

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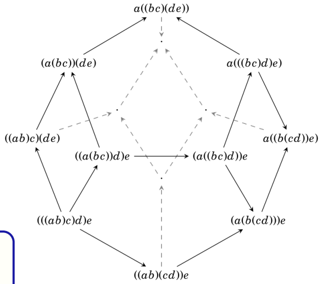
Theorem (Pournin'12)

For every $n \geq 4$, $\mathrm{diam}(\mathcal{A}(n)) = 2n-4$.



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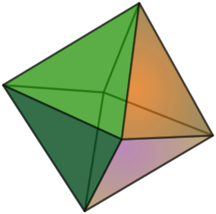
Hirsch conjecture

Polytopes where :
n : number of facets
d : dimension.

Elementary step : Sliding along an edge.



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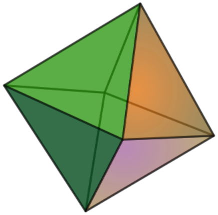

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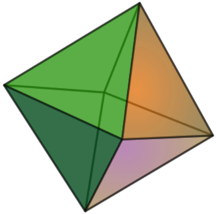

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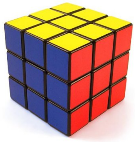
- (Santos'11) Counter-example.
- Relaxations still widely open.

Puzzles & more

One-player games are puzzles : one player makes a series of moves, trying to accomplish some goal.



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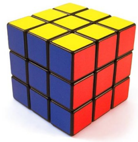

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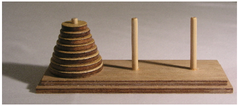

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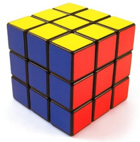

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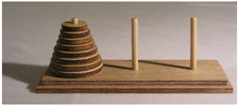
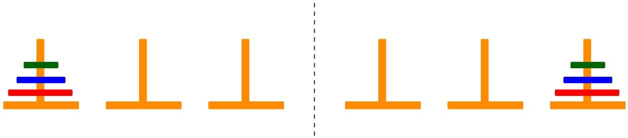


Motivated by problems of random sampling, enumeration, bioinformatics, discrete geometry, games...etc... for decades.

Focus on Hanoi tower



Goal :
Move disks from the first to the last rod moving one disk at every step.

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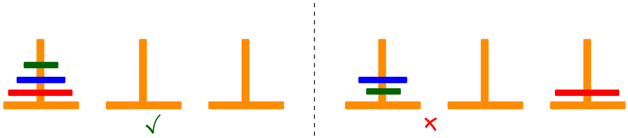

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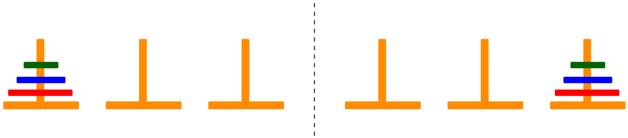

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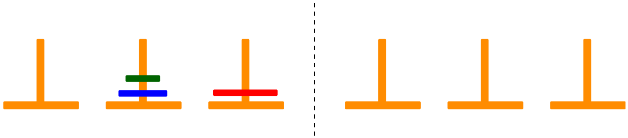

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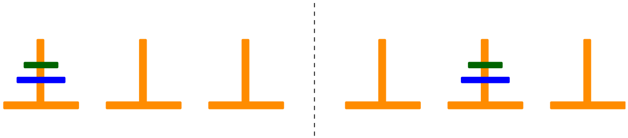

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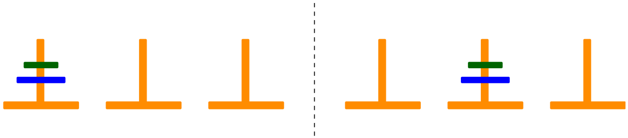

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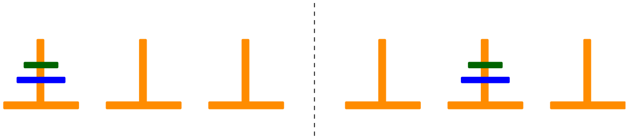

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- Understandable because of symmetry.

  In what follows, symmetry / structure will vanish.

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Definition (Configuration graph $\mathcal{C}(I)$ of I)

- Vertices : Valid solutions of I.
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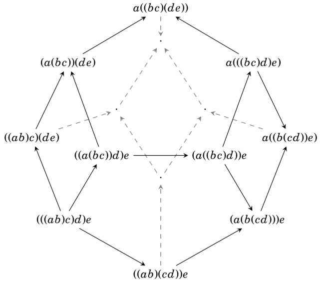

Reconfiguration diameter = Diameter of $\mathcal{C}(I)$ (when connected)

Main questions (in CS)

- **Reachability problem.** Given two configurations, is it possible to transform the first into the other ?
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- **Algorithmics.** Can we efficiently solve these questions ? (In polynomial time, FPT-time...).

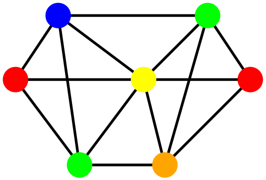
Focus : graph coloring

(Proper) coloring :

Adjacent vertices are colored differently

n : Number of vertices

k : Number of colors



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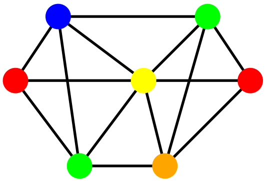

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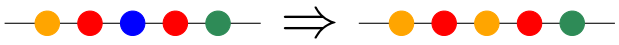
- Single vertex recoloring : 

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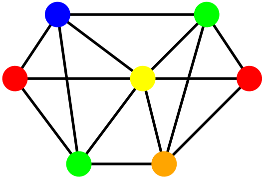

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- Kempe changes :  $\Rightarrow$ 

  Random sampling, physics...

Recoloring paths & grids
(and beyond)

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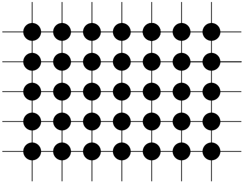

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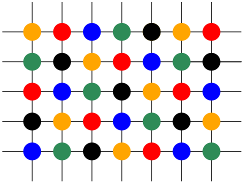

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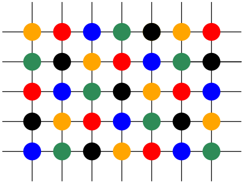

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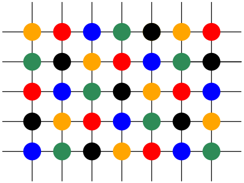

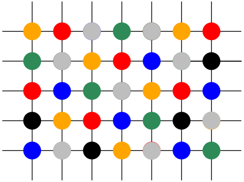

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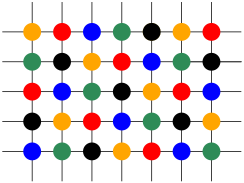

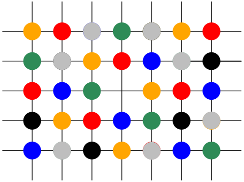

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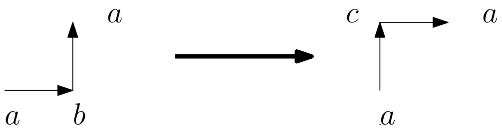

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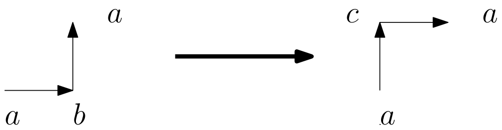

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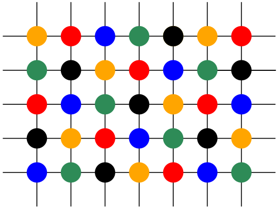
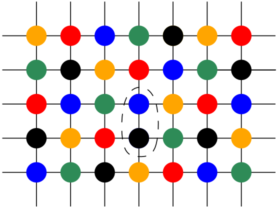
$\Rightarrow$ A transformation from $123123\cdots123$ to $132132\cdots132$ is quadratic.

5-colorings of grids





5-colorings of grids

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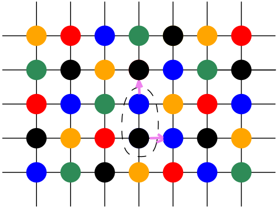

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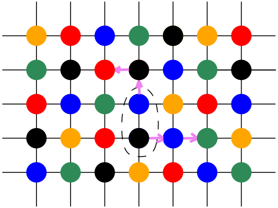

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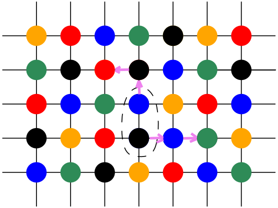

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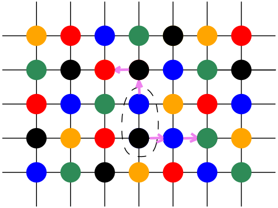

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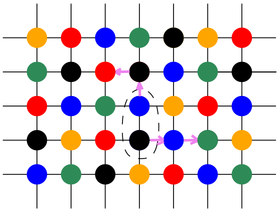

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Question : Is the diameter really quadratic ?

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(Holds for any $(\Delta + 1)$-colorings of any graph as long as $\Delta \geq 3$.)

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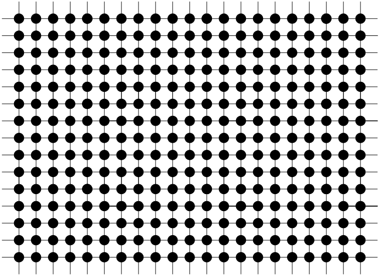
Proof by picture

- Local warming : Duplicate unfrozen vertices locally.



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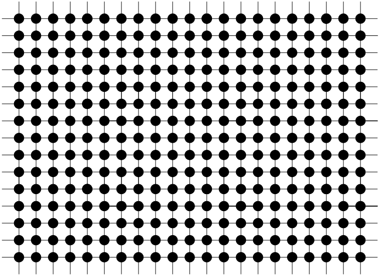

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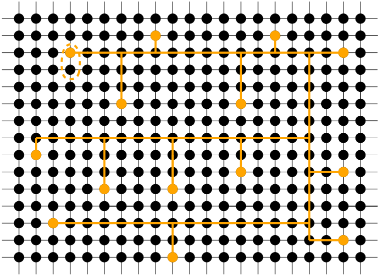

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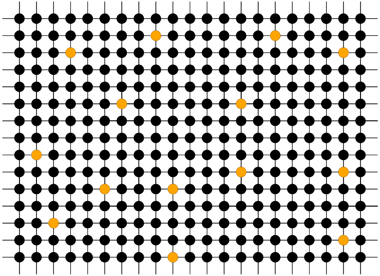

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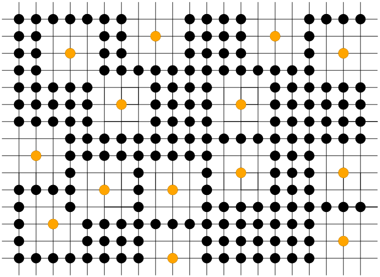

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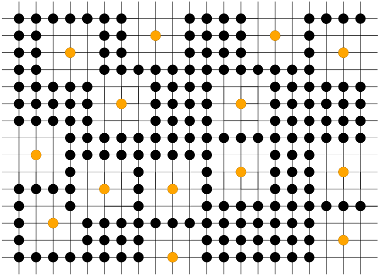

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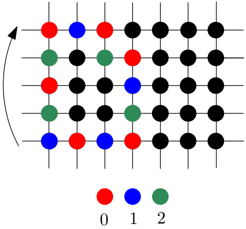

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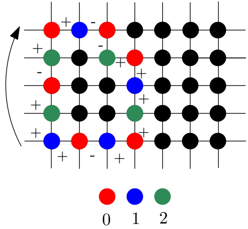

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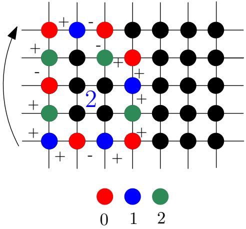

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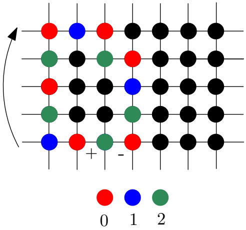

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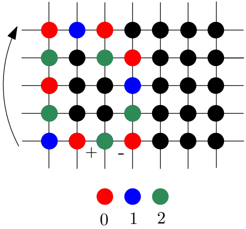

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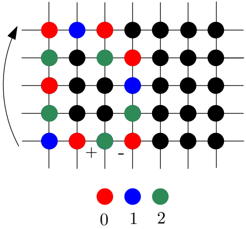

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Remark :
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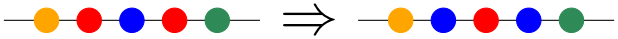

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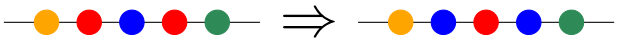

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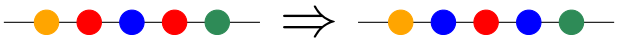

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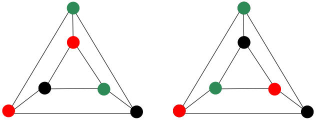

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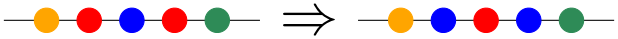

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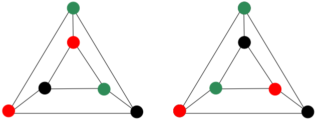

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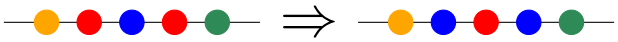

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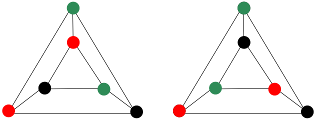

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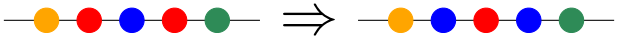

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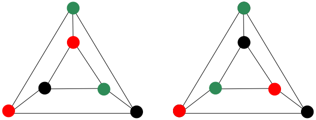

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WSK algorithm : (Wang, Swendsen, Kotecký)

Repeat :

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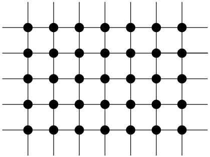

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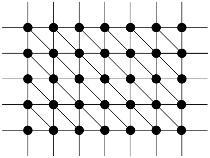

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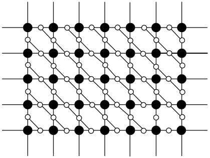

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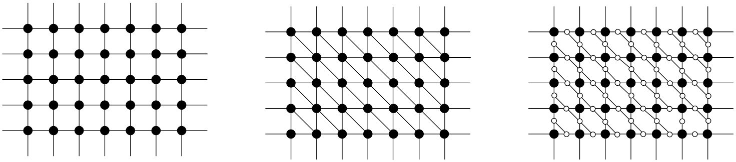

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Missing case :

Triangular grid, 5 colors.

Triangular grid

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Triangular grid $+$ 5 colors $\Rightarrow$ ✓

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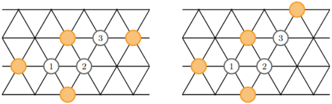

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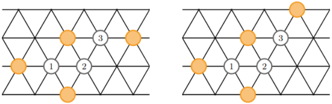

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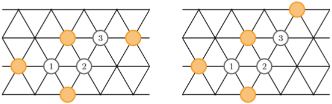

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