On the economic efficiency of the Combinatorial Clock Auction

Nicolas Bousquet, Yang Cai, Christof Hunkenschröder and Adrian Vetta

SODA'16

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Two main auctions used worldwide:

- The SMRA (Simultaneous Multi-Round Auction).
- The CCA (Combinatorial Clock Auction).

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> While all the bids are not "somehow" disjoint:
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Favorite set: set S maximizing value of S minus price of S (where the price of S is the sum of the prices of items in S).

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Return the "best possible" allocation.

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Amongst all the possible allocations, the best possible allocation is an allocation maximizing the revenue of the auctionneer.

SMRA and CCA

Item vs package bidding:

- Package bidding in the CCA: all or nothing bid at price $p(S)$.
  $\Rightarrow$ The bidder receives either all or none of the items.
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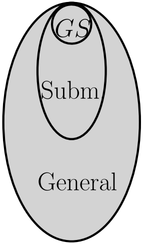
Drawback:

No market clearing $\Rightarrow$ usually market clearing helps for finding guarantees.

Existing results

For the SMRA:

- Gross substitutes and truthful bidding ⇒ Walrasian equilibrium [Milgrom '00].



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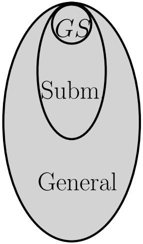
- Submodular valuation functions and truthful bidding ⇒ half of the optimal welfare [Fu, Kleinberg, Lavi '12].



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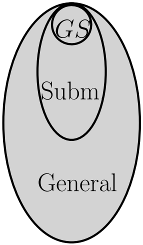

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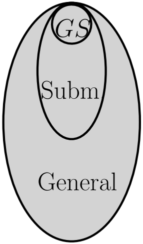

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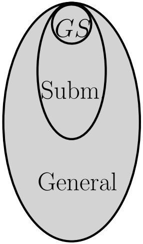

For the CCA?
Nothing is known !

Our result

Theorem (B., Cai, Hunkenschröder, Vetta)

In a k-demand auction with truthful bidding, the welfare allocation of the CCA is at least

$$\Omega\left(\frac{OPT}{k^2 \log n \cdot \log^2 m}\right)$$

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where n is the number of bidders and m the number of items if the stopping rule and price increments are well chosen.

Stopping rule

At $t = 0$, the price of every item is 0.
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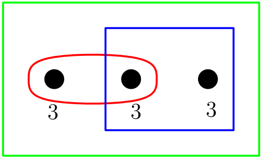
Round 3



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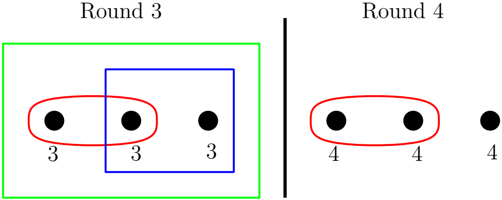

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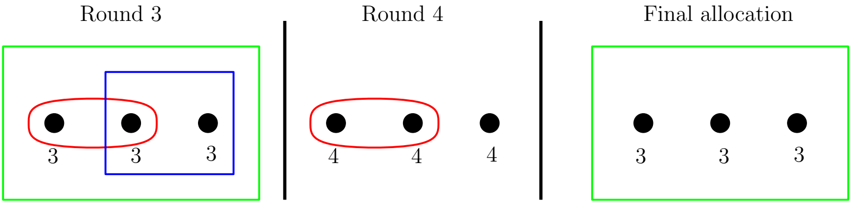

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Consequence: hard to understand why it works and to convince bidders that the auction is strategy-proof.

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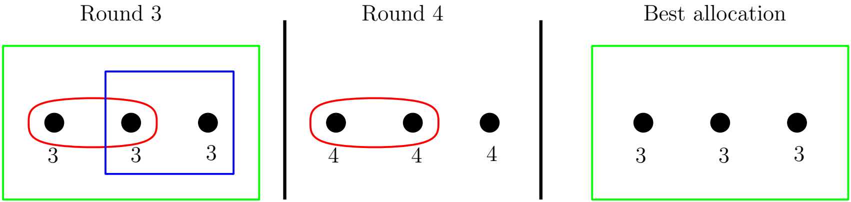

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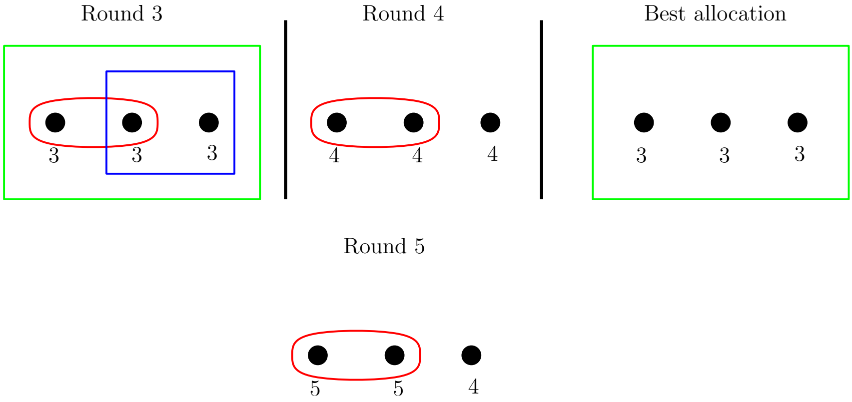

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Ausubel and Baranov (2014)

" Among all design decisions that need to be made prior to the auction [the choice of price increments] is considered relatively unimportant and is often overlooked by the design team."

Proof sketch

We take all the bids made by all the bidders during the auction.

Let v be a (well-chosen) threshold. We consider the following greedy allocation $\bullet$:

> As long as there remains a bid of price $\geq v$
> Let (i, S_i) of maximum price p_i.
> Add i to $\bullet$ and allocate S_i to her.
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The • bidders are (i) not in • and (ii) have welfare $\gg v$ in the optimal allocation.

Lemma

The welfare of the • allocation satisfies the conclusion
Or number of • bidders $\gg$ number of • bidders.

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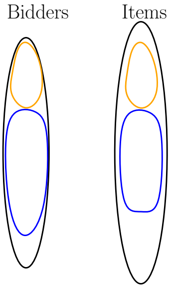
Assume by contradiction that the first point does not hold.

Decrease the utility of • bidders



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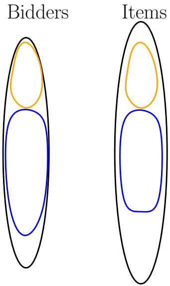

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We can show that $u(\bullet)$ is $\leq v$ at the end of the auction.

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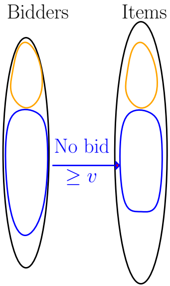

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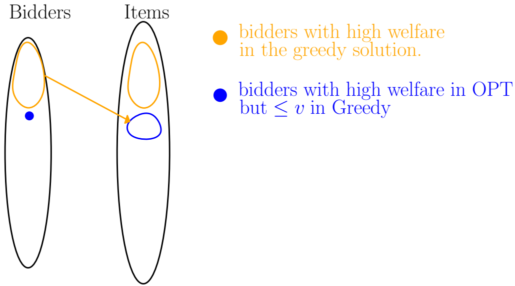

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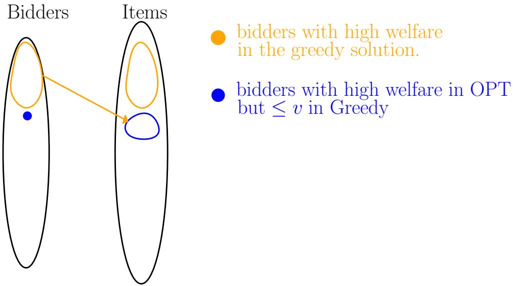

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$\Rightarrow$ Price increments above v are due to • bidders.

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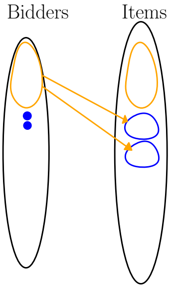

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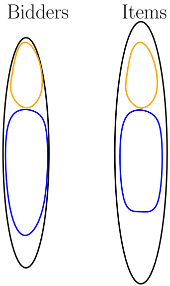

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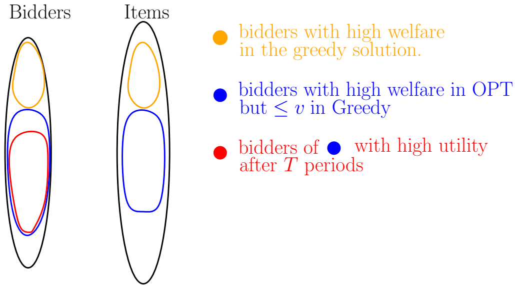

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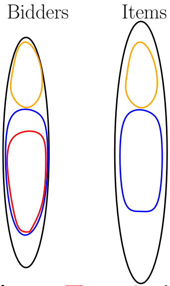
... and it holds for *all* the • bidders.
$\Rightarrow$ Many, many periods (say T+1) are needed to decrease by an ϵ-fraction the utility of an ϵ'-fraction of the • bidders.
$\Rightarrow$ After T-periods, a $(1-\epsilon')$ fraction of • bidders still have utility $\geq (1-\epsilon)\cdot w(\bullet)$: the • bidders.

On what sets • bidders are bidding on?



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- bidders with high welfare in OPT but $\leq v$ in Greedy
- bidders of • with high utility after T periods

So during at least T periods, bids of • have utility $\geq (1-\epsilon)w(\bullet)$.
And the total number of bids of • bidders is $\gg OPT$.

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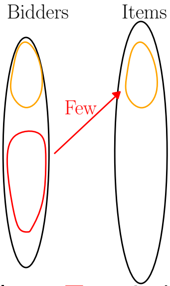

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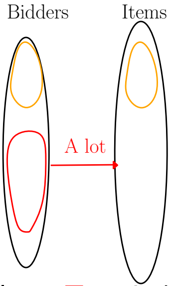

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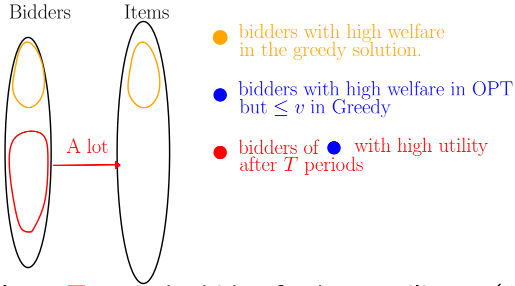

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No • bid contains an item of price at least v:
$\Rightarrow$ Each • bidder bids on a lot of disjoint bundles of utility at least $(1-\epsilon)\cdot u(\bullet)$ before period T.

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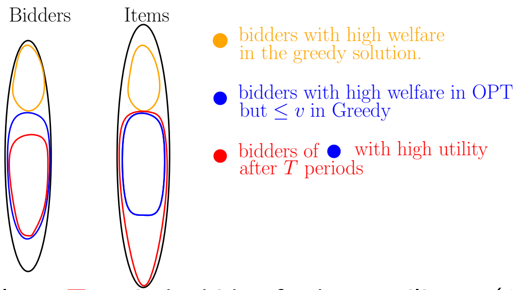

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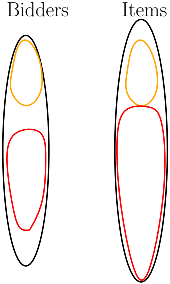

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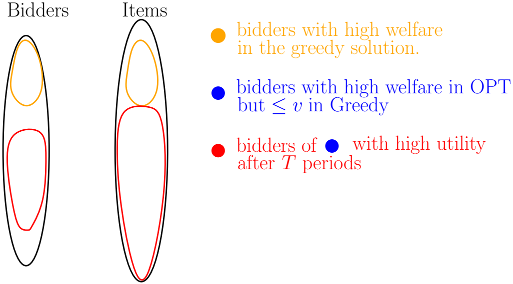

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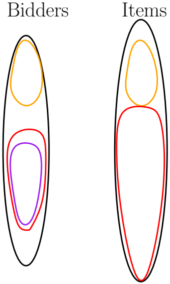

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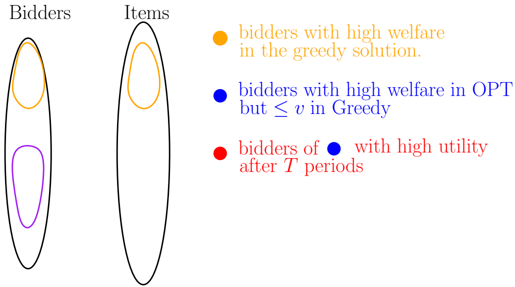

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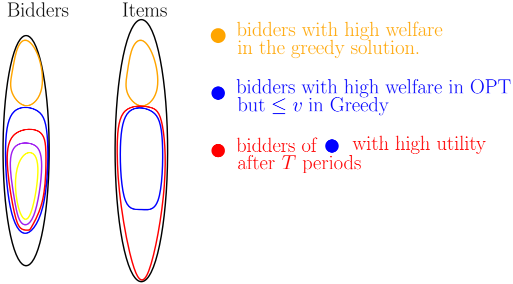
And we repeat the process

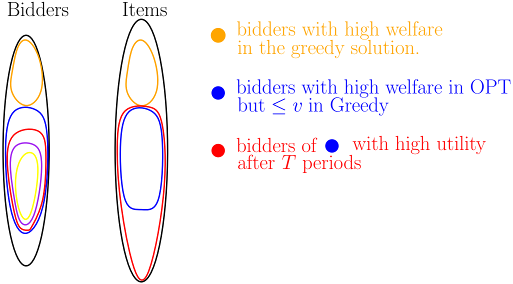
Decrease the utility of • bidders



We essentially repeat the same process...
⇒ Price increments between v and $\epsilon \cdot w(\bullet)$ on ON ANY SET on which • bids on before period T are due to • bidders.
We need $T' \gg T$ periods to decrease by $2\epsilon \cdot u(\bullet)$ the utility of an ϵ'-fraction of • bidders.
And we repeat the process

...

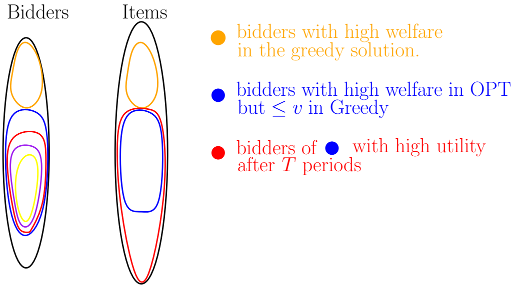
Conclusion step



We repeat this operation. At each step:

- The number of bidders decreases.
- The number of items increases.

Conclusion step



We repeat this operation. At each step:

- The number of bidders decreases.
- The number of items increases.

We can show that if ϵ, ϵ', v...etc.. are well-chosen, the number of items **must** be larger than m before the number of bidders reach 0: a contradiction.

Questions

- Understand the impact of activity rules for the welfare of the CCA.
- Price of Anarchy of the SMRA / the CCA ?

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Thanks for your attention !