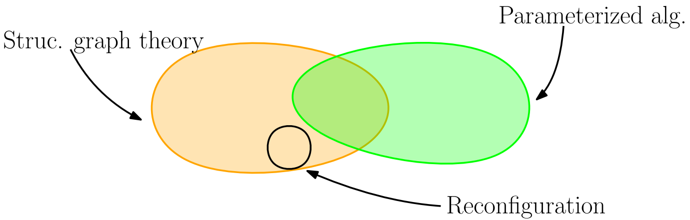
A Journey on Configuration Graphs

Coloring and Independent Set Reconfiguration

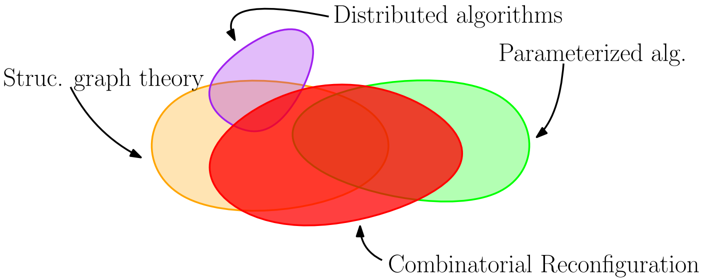
Nicolas Bousquet

July, 3rd, 2024

My research - 11 years ago



My research - Now



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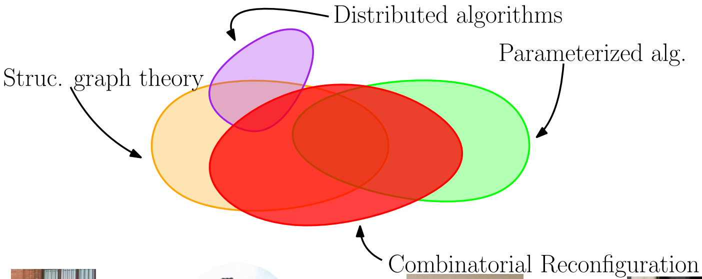


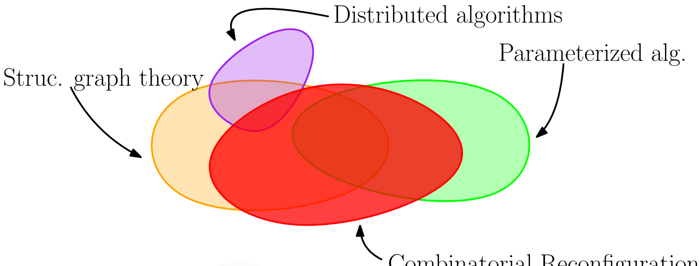
Reconf. & games

Dom. Set Rec.

Col. & IS Reconf.

Recol. & Metric dim.

My research - Now



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Geometric & Col. Rec.

Local certification

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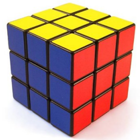

Question :
Giving my current position, can I reach a fixed target position ?

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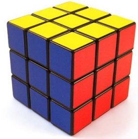

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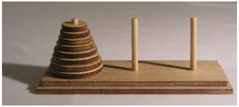

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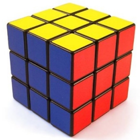

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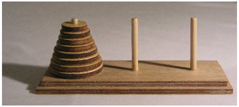

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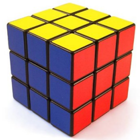

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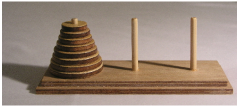

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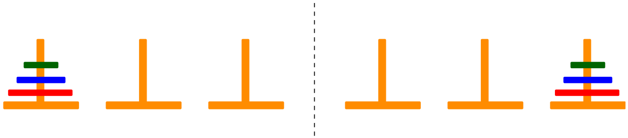
  Colorings, independent sets, dominating sets, cliques, list colorings, bases of matroids, CSP and boolean formulas...
- Important problems in random sampling, bioinformatics, discrete geometry, games...etc... for decades.

Focus on Hanoi tower



Goal :

Move disks from the first to the last rod moving one disk at every step.

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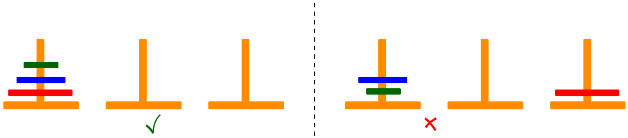

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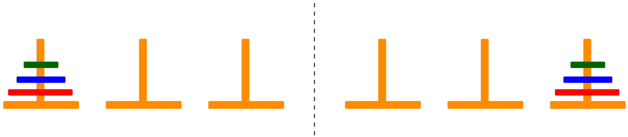

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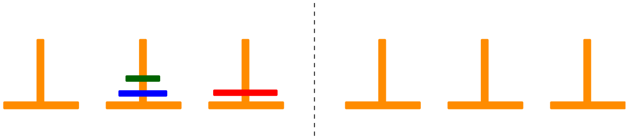

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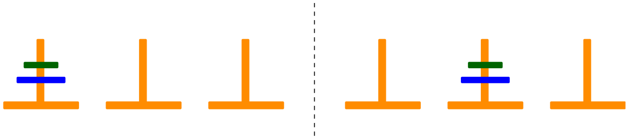

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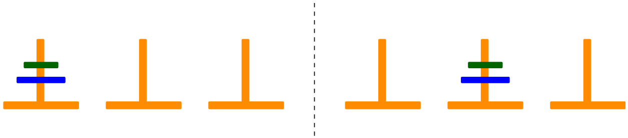

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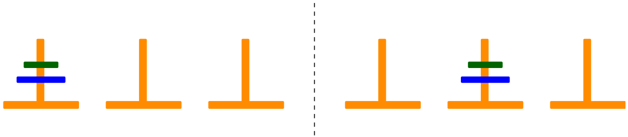

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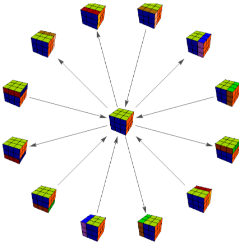
  Looks simple but computationally hard.
- Understandable because of symmetry.
  In what follows, symmetry / structure will vanish.

Configuration graph

Definition (Configuration graph $\mathcal{C}(I)$ of I)

- Vertices : Valid solutions of I.
- Create an edge between any two solutions if we can transform one into the other in one elementary step.



Reconfiguration diameter = Diameter of $\mathcal{C}(I)$ (when connected)

Main questions

- **Reachability problem.** Given two configurations, is it possible to transform the first into the other ?

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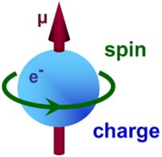
Outline of the presentation :
Focus on Graph Recoloring an Independent Set Reconfiguration.

PART I

Graph Recoloring

Genesis of graph recoloring



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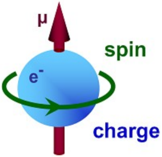

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Spin of fermions $\in \{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots\}$.
Spin of bozons $\in \{0, 1, 2, \ldots\}$.

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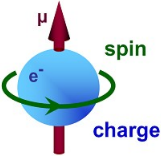

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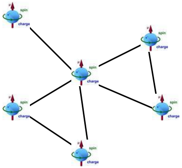
A spin system is a set of spins given with :



- An integer k being the number of states.
- An interaction $\{0, 1\}$ (symmetric) matrix modelizing the interaction between spins.
  - $0 =$ no interaction $=$ no link.
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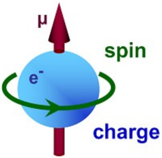

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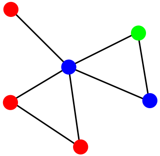

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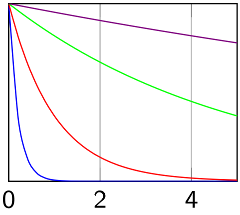
A spin configuration is a function $f : S \to \{1, \ldots, k\}^n$.
$\Leftrightarrow$ A (non necessarily proper) graph coloring.

Antiferromagnetic Potts model



$T = 5, 1, 0.2, 0.05$

$H(\sigma)$: number of monochromatic edges.

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Edges with both endpoints of the same color.

Gibbs measure at fixed temperature T :

$$\nu_T(\sigma) = e^{-\frac{H(\sigma)}{T}}$$

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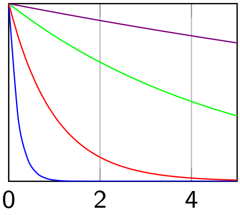

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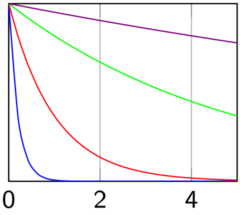

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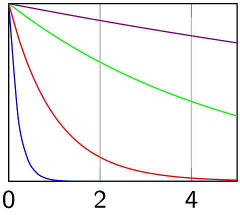

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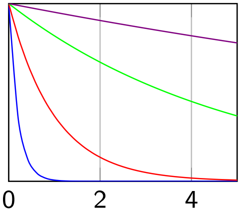

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Limit of an antiferromagnetic Potts model when $T \to 0$.
$\Rightarrow$ Only **proper** colorings have positive measure.

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- Start with an initial coloring c ;

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Questions :

- Can we generate every solution ?

  Is the configuration graph connected ?
- How long shall we wait to "*sample a solution almost at random*" ?

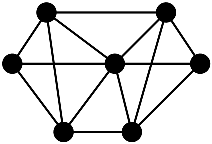
  What is the mixing time ? (Related with the diameter of the configuration graph).

Cereceda's conjecture

Conjecture (Cereceda '08)

The $(d+2)$-recoloring diameter of any d-degenerate graph is $\mathcal{O}(n^2)$.

A graph is d-degenerate if there exists an ordering $v_1, \ldots, v_n$ such that for every i, $|N(v_i) \cap \{v_{i+1}, \ldots, v_n\}| \leq d$.



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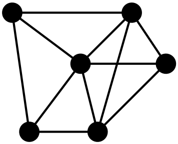

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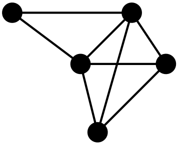

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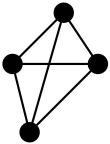

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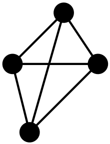

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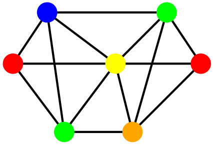

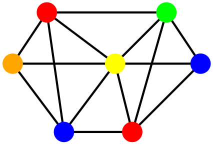

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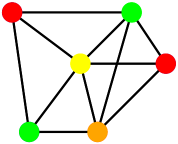

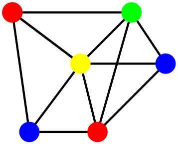

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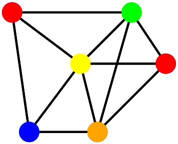

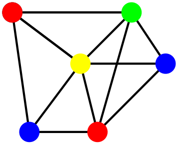

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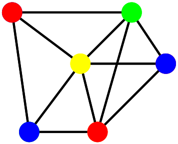

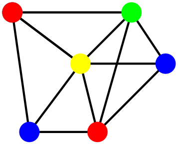

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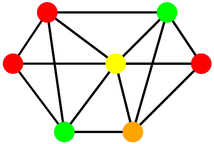

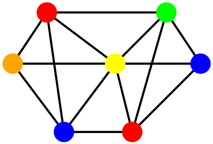

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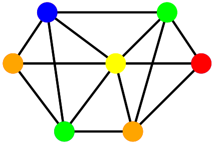

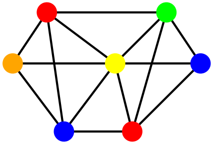

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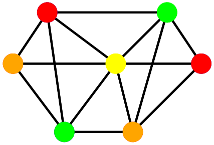

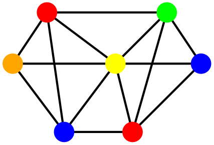

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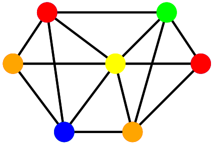

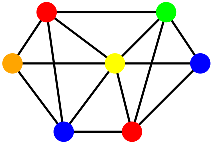

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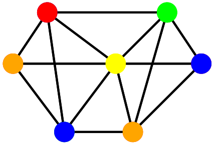

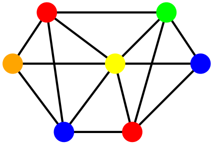

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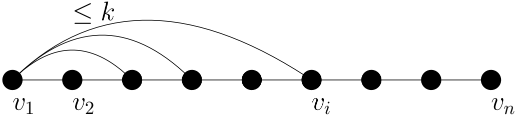

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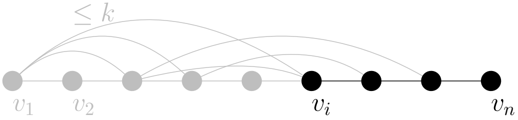

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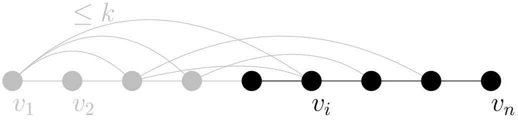

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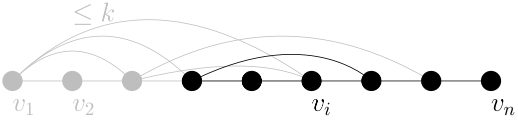

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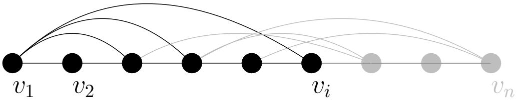

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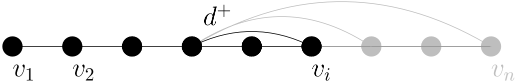

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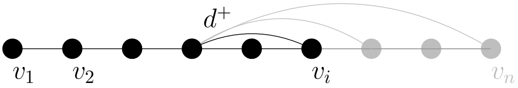

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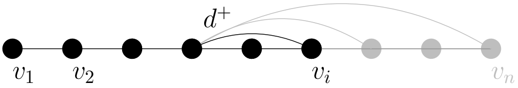

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Cerededa's conjecture for $d = 2$

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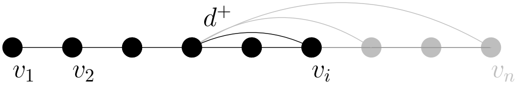

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Cerededa's conjecture for $d = 2$... and $\Delta = 4$!

[Feghali, Johnson, Paulusma '17] $d = 2$ and $\Delta = 3$ is true.

A recent trend - Linear diameter

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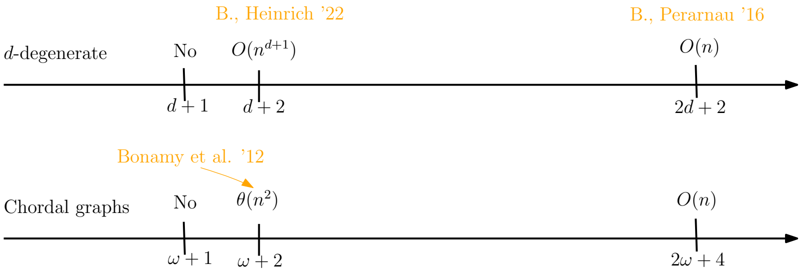

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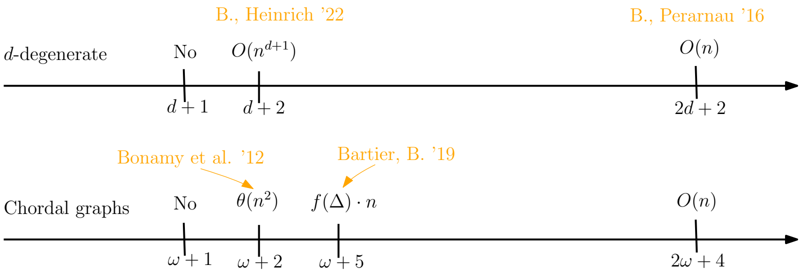

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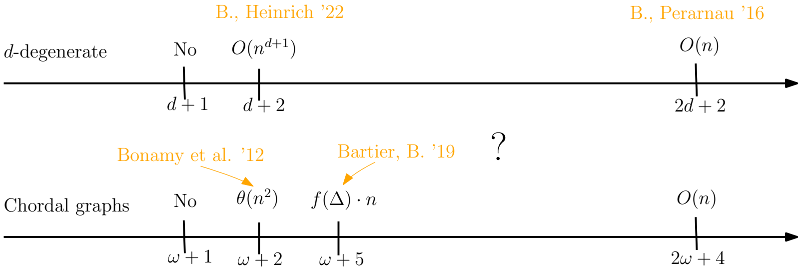

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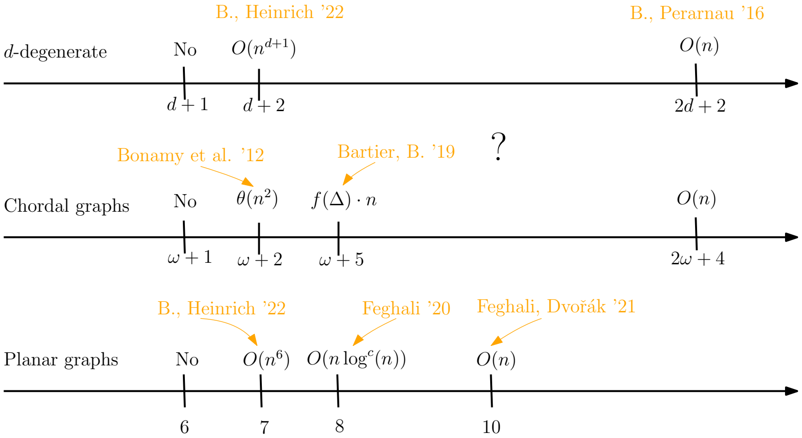

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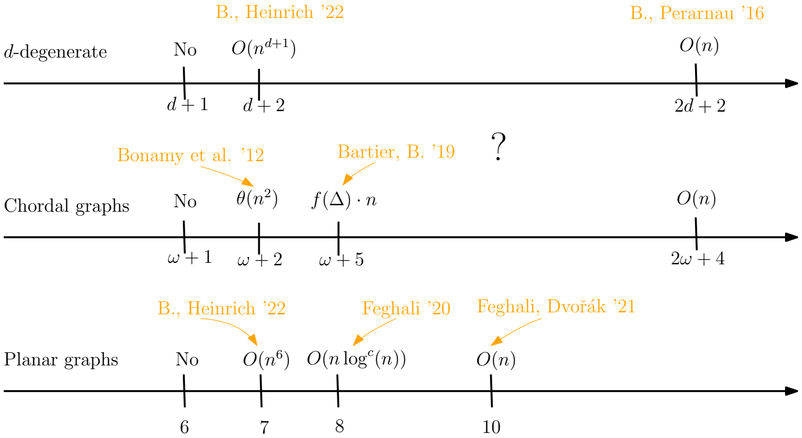

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$(2-\epsilon)tw(G)$ colors ? $(2-\epsilon)d$?

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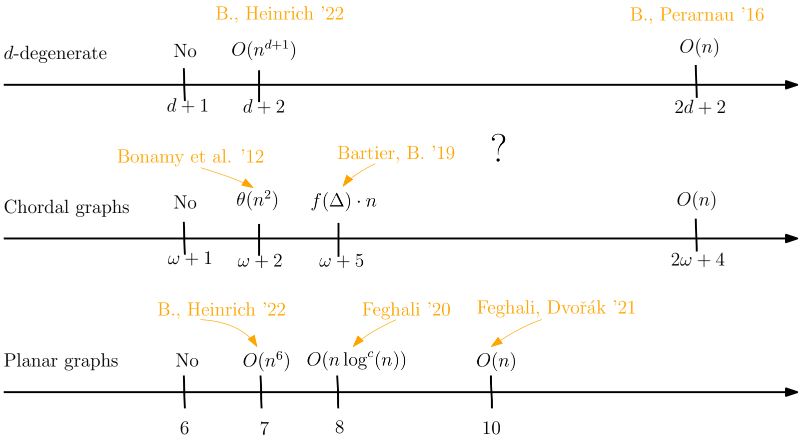

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7 colors for planar graphs ?

Part II

Independent Set Reconfiguration

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  ⇒ PSPACE-complete (but they need large robots).

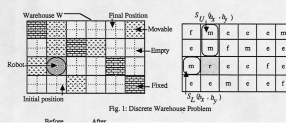

Fig. 1: Discrete Warehouse Problem

Genesis of ISR

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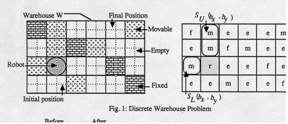

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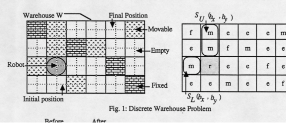

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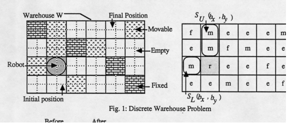

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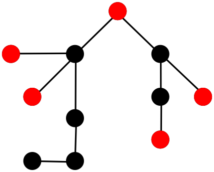
Theorem [Hearn, Demaine '04]

The problem is PSPACE-complete.

Token Jumping vs Token Sliding



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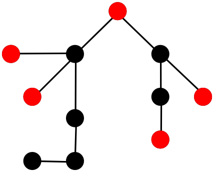

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Select one vertex of I and move it anywhere else.
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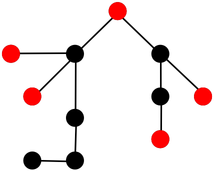

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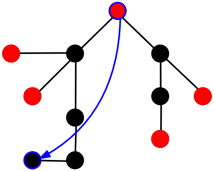

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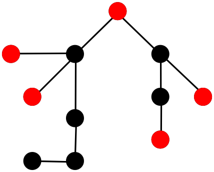

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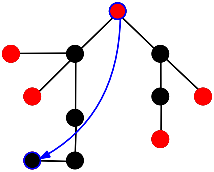

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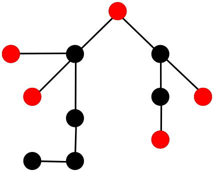

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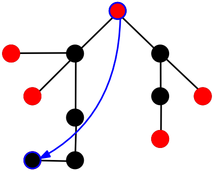

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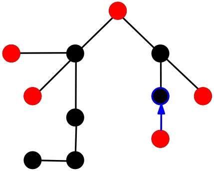

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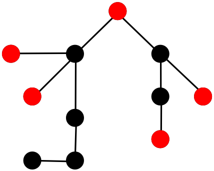

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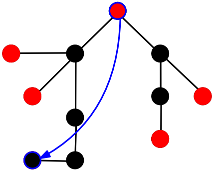

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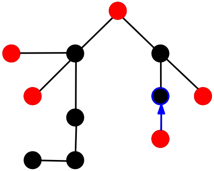

Question : What is the complexity of TS / TJ-REACHABILITY ?

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Input : A graph G, two independent sets I, J.
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Today :

Focus on parameterized algorithms.

Parameterized complexity

A problem Π parameterized by k is FPT if it can be decided in $f(k) \cdot Poly(n)$.

In this talk :
Parameter = size of the IS.

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Theorem (Bodlaender, Groenland, Swennenhuis '21)

TS and TJ-Reachability are XL-complete.

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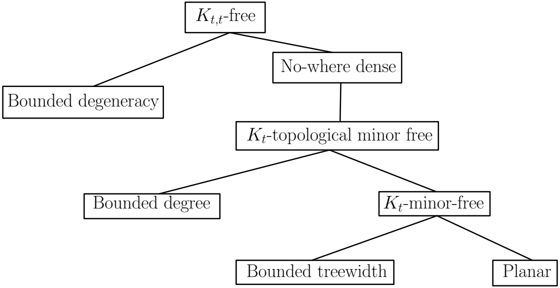

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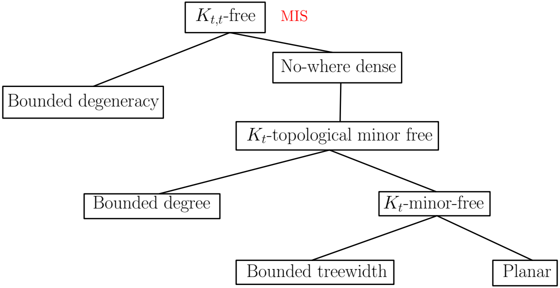

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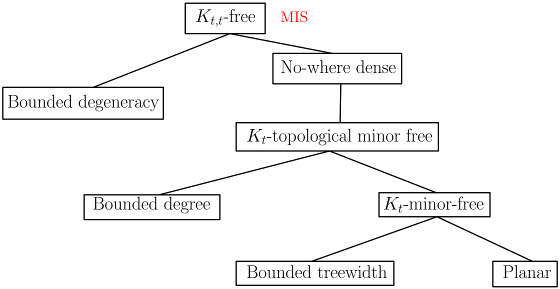

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- [Ito et al. '14] Planar graphs.

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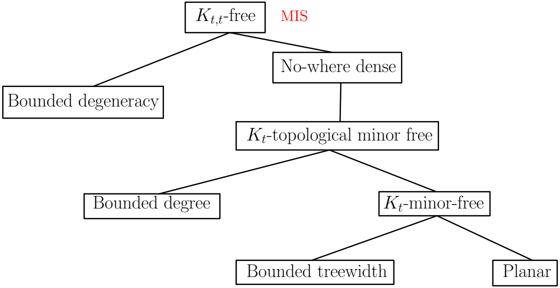

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- [Ito et al. '14] Planar graphs.
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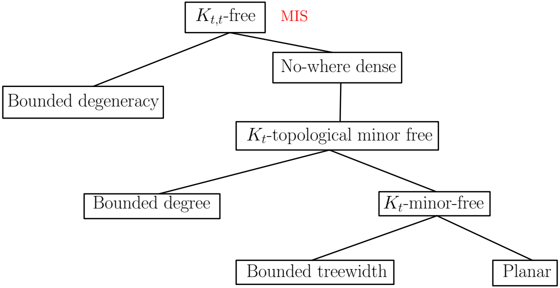

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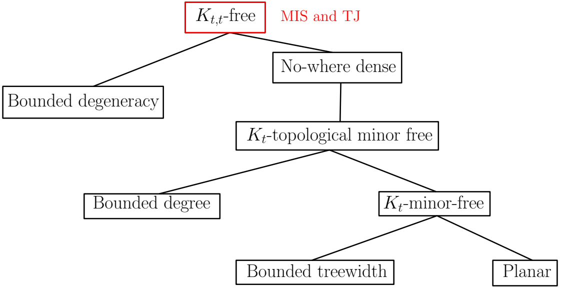

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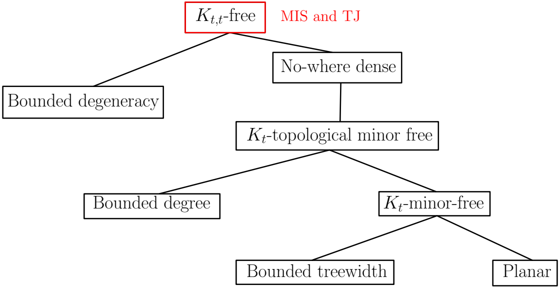

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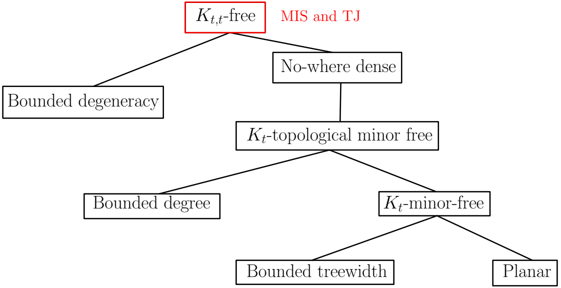

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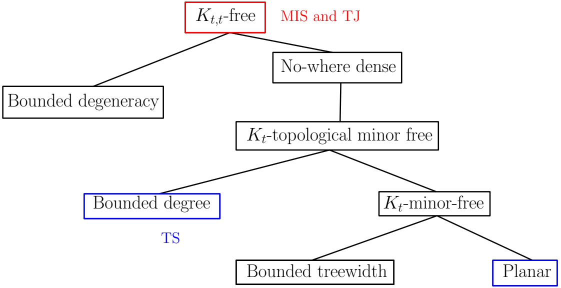

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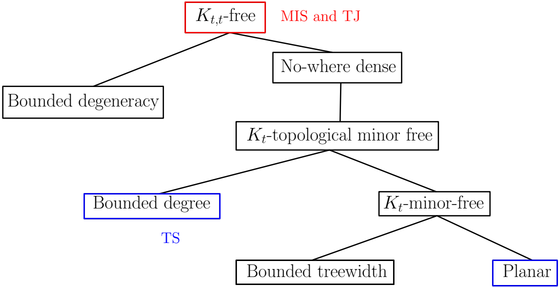

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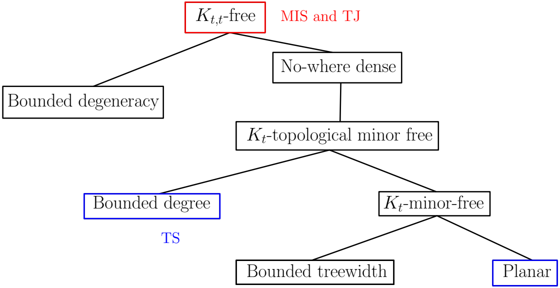

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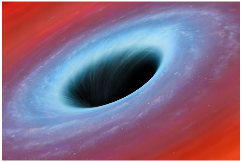

A galactic graph is a graph with special vertices called black holes that :

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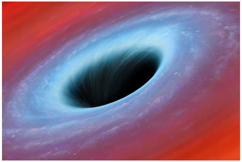

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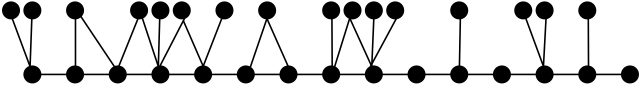

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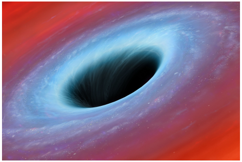

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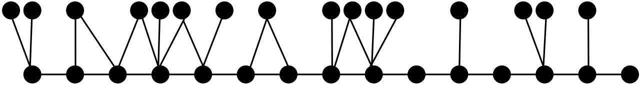

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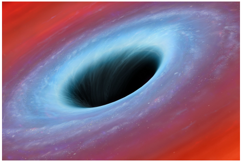

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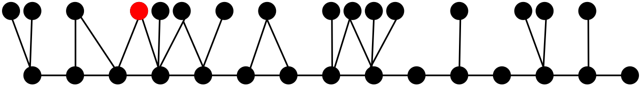

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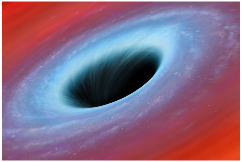

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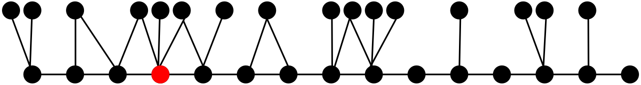

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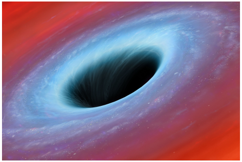

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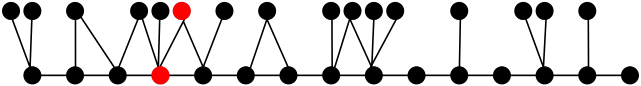

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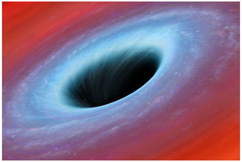

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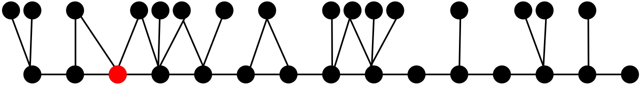

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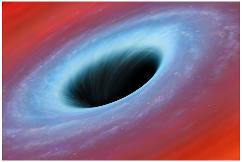

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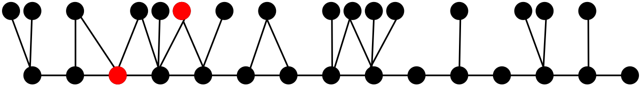

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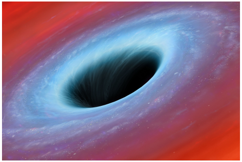

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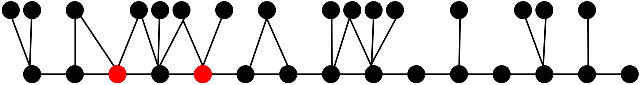

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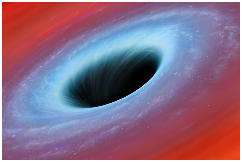

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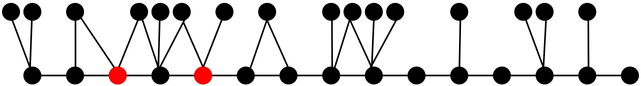

Consequences :

- FPT on bounded degree graphs.
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  On going work : bounded treewidth.

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Thanks for your attention !