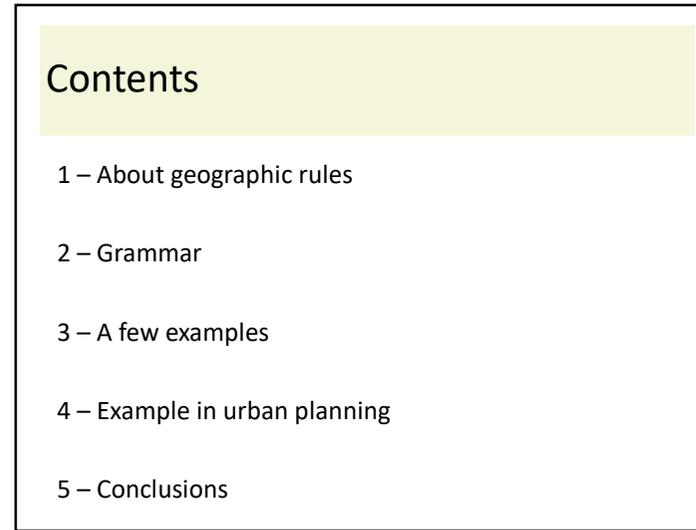
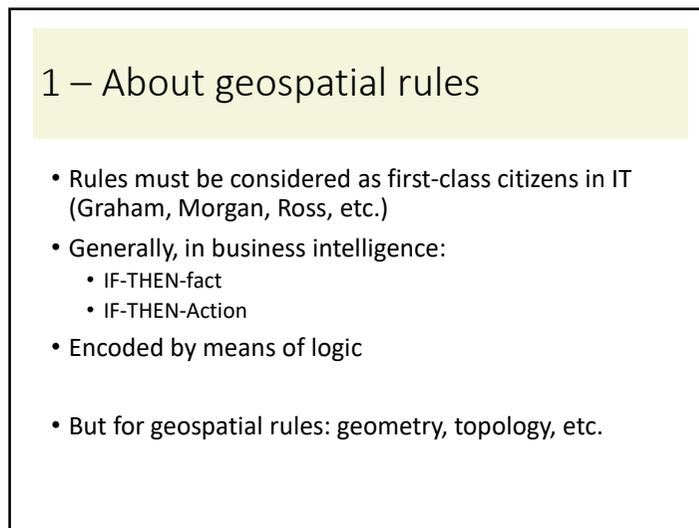


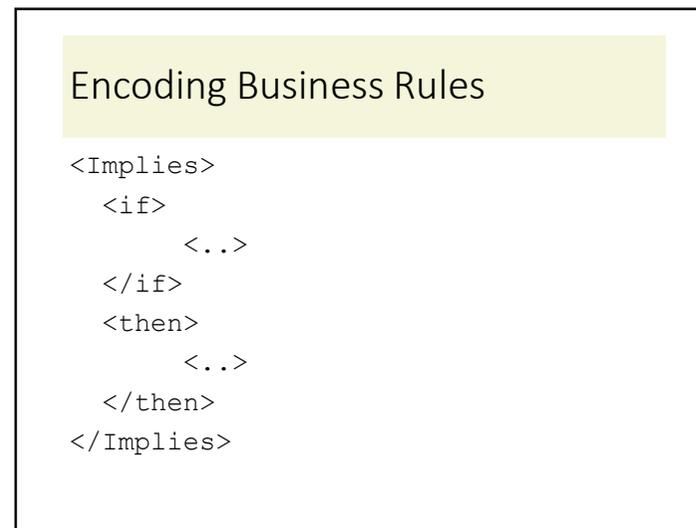
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4

### Examples of geospatial rules (1/2)

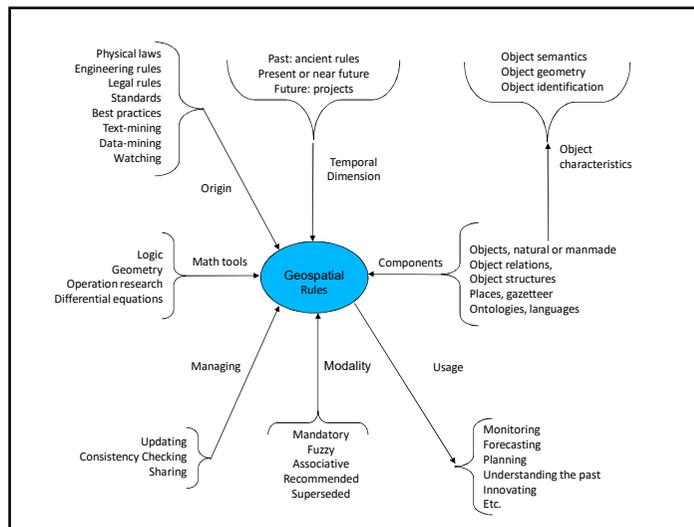
- If a lane is narrow, make it one-way, except if it is a cul-de-sac (dead end);
- When planning a metro, move underground networks;
- No parking, no business;
- Each building must be connected to utility networks (water, electricity, gaz, telephone, internet, etc.);
- Council flats must be connected to urban heating systems;
- If a cross-road is dangerous, install traffic lights;
- In city centers, transform streets into pedestrian precincts;
- When a commercial mall is planned in the neighborhood of a city, shops located in the city center will be in jeopardy;
- If the number of car parking lots is insufficient, encourage using buses or bikes;
- At the vicinity of an airport, limit building heights

5

### Examples of geospatial rules (2/2)

- When a big plant is closing, unemployment will increase;
- At the vicinity of an historic building, no modifications of building are allowed
- Every lamppost can be considered as holder of sensors (temperature, pollution, noise, etc.);
- When defining a new industrial area, unemployment will diminish;
- When a road is wide and buses are running, provide a bus lane;
- If a recreational park is inside a city, provide bike lanes coming to this park;
- In France, it is forbidden to open a new tobacconist shop within 500 meters from an existing one;
- If there is one or several rivers crossing a city, design systems to mitigate floods;
- In a city with many hills, consider cable-cars linking them.

6



7

### From Data Mining

- Malerba et al. (2003) have discovered the following rule:

$$is\_a(X, large\_town) \wedge intersects(X, Y) \wedge is\_a(Y, road) \rightarrow intersects(X, Z) \wedge is\_a(Z, road) \wedge Z \neq Y (91\%, 85\%)$$

- Sallab-Aoussi et al. (2015) have studied geology and mineral deposits in South America.

$$Mines - \exists_{5km}^3 Faults \rightarrow True$$

$$Mines - \exists_{1km}^1 Volcano \rightarrow (active=yes)$$

8

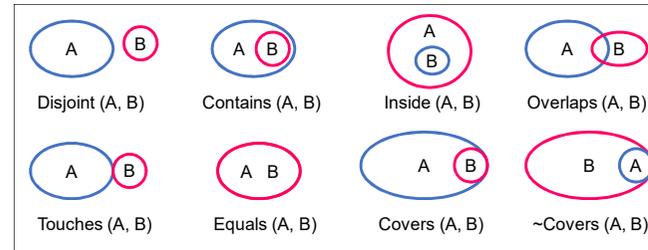
### Expert Rules

- Varadharajulu et al. (2016) have proposed the following rule for checking road length against road type

```
NEWROAD(?R1), ROADSUFFIX(?R1, ?T1),
hasLength(?R1, ?200), SameAs (?T1, ?Close)
-> isAllowed(?R1, true)
```

9

### Egenhofer topological relations



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### Why a Mathematical Language?

- Too early for a computer language
- Analysis of the semantics (to be extended)
- Independence from any software product
- Basis for the design of a computer language
- Links with inference engine

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### Assumptions for a Mathematical Language (1/2)

- a global geographic object called *Earth* includes all existing geographic objects (*GO*) and territories (*Terr*),
- an ontology will describe their types/classes (whatever is the concept) together with some specific attributes and generic relations between them,
- all objects will have valued attributes; in the case of new objects, the attributes are set to null,
- there exists a set named *Projects* which comprises all possible environmental and urban projects,

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## Assumptions for a Mathematical Language (2/2)

- there exists a gazetteer integrating all place names possibly with different variants and in different languages,
- it is assumed that all information is correct and consistent,
- it is assumed that there are no problems neither regarding geometry accuracy, nor multi-representation,
- there are no considerations for storing, implementation, optimization, etc.,
- this language is not a rigid language as in computer sciences, but a sketch in which everybody can add symbols, functions, etc.

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## Characteristics of this language

- First level
  - Set theory ( $\forall, \exists, \cap, \cup, \supset, \subset, \in, \forall, \wedge, \oplus$ , etc.)
  - Logic (First order, fuzzy) ( $p \wedge q \wedge \dots \wedge t \Rightarrow u$ )
  - 2D Computational geometry
  - 2D Topology
- Second level (not yet done !!!)
  - 3D
  - Temporal dimension
  - Operation research
  - Graph theory
  - Differential equations

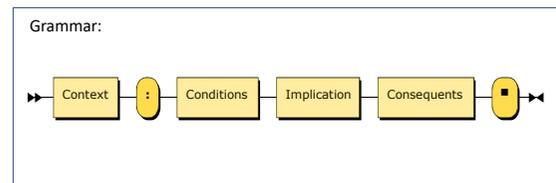
14

## About the symbol =

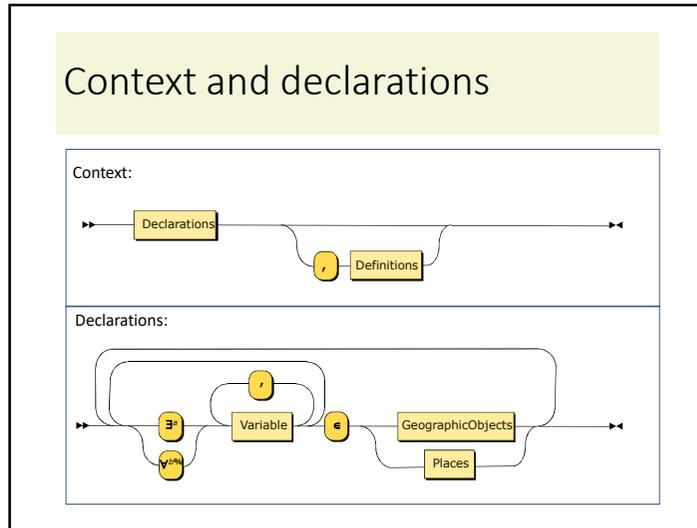
- Three meanings:
  - = as a comparator in Boolean conditions; so that the answer will be either true or false,
  - $\equiv$  for definitions, especially of new variables,
  - := for assigning a new value to a known variable.

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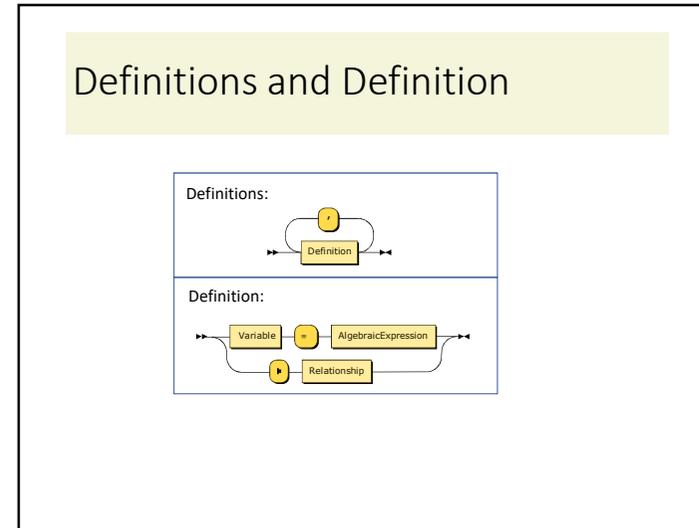
## 2 – Grammar



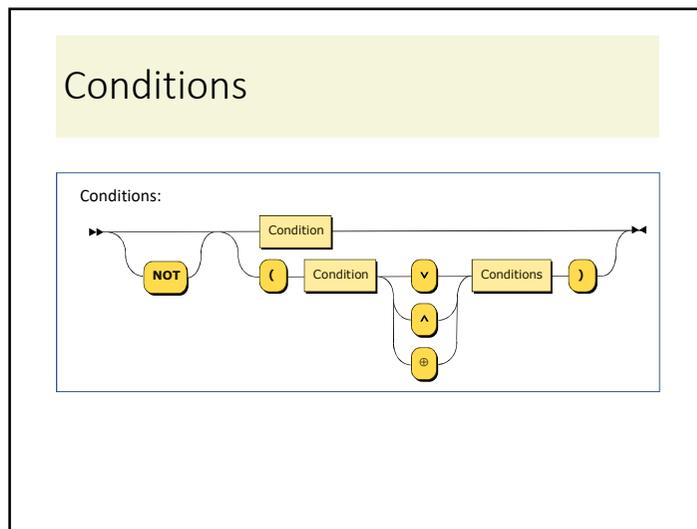
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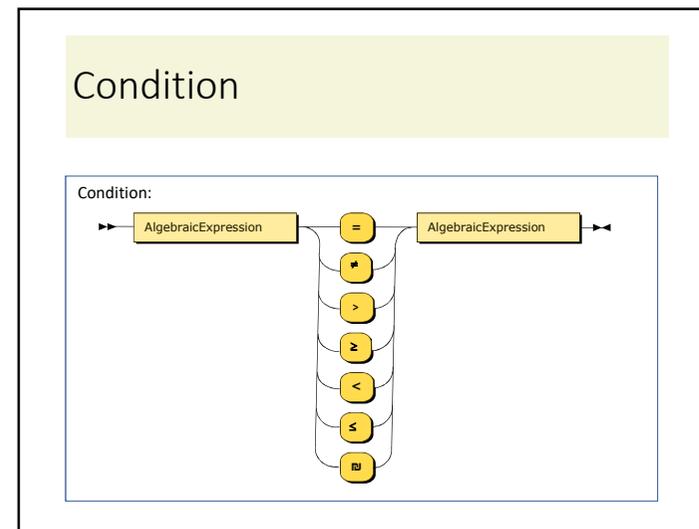
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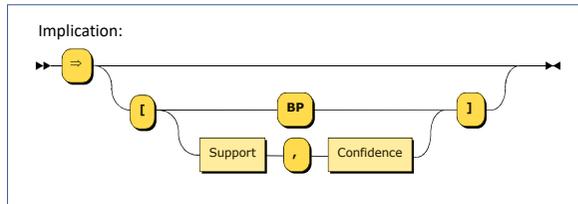


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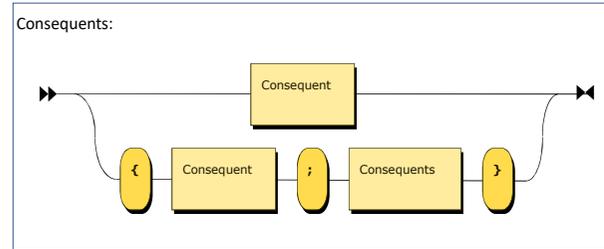
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## Implication



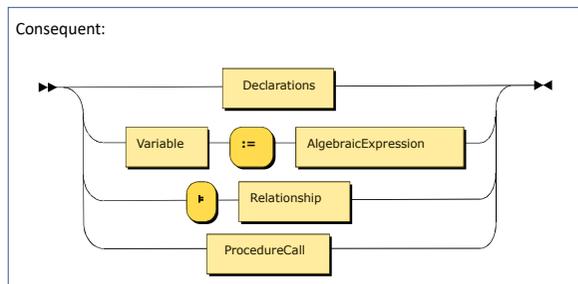
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## Consequents



22

## Consequent



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## 3 – A few Examples

- Creation of a relationship

$\forall p_1, p_2 \in \text{Earth}, \text{GeomType}(p_1) \equiv \text{Point},$ $\text{GeomType}(p_2) \equiv \text{Point}$ $\vdots$ $\text{Latitude}(p_1) > \text{Latitude}(p_2)$ $\Rightarrow$ $\models \text{North}(p_1, p_2) \blacksquare$	Rule 1
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$\forall p_1, p_2 \in \text{Earth},$ $\text{GeomType}(p_1) \equiv \text{Point}, \text{GeomType}(p_2) \equiv \text{Point}$ $\vdots$ $(\text{Longitude}(p_1) > \text{Longitude}(p_2))$ $\wedge (\text{Longitude}(p_1) - \text{Longitude}(p_2) < 180^\circ)$ $\Rightarrow$ $\models \text{West}(p_1, p_2) \blacksquare$	Rule 2
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### Other examples

$\begin{aligned} &\forall Ro \in Road, \forall Ri \in River \\ &: \\ &Area(Intersection(Geom(Ro), Geom(Ri))) \neq 0 \\ &\Rightarrow \\ &\{ \models Cross(Ro, Ri); \models Cross(Ri, Ro) \} \blacksquare \end{aligned}$	Rule 3
$\begin{aligned} &\exists T \in Town, \forall R \in River, Topo(T) \equiv "Smart Town" \\ &: \\ &Overlap(R, T) \vee Covers(T, R) \\ &\Rightarrow \\ &\{ CreateGO(F); Type(F) := "Floodplain"; \\ &Geom(F) := Intersection(Geom(T), Buffer(R, \\ &100)) \} \blacksquare \end{aligned}$	Rule 4

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### In Urban Planning

$\begin{aligned} &\forall P \in Parcel, L \in Plants, B \in Project.Building, \\ &\models contains(P, L), \models contains(P, B) \\ &: \\ &L.Erased \wedge P.Depolluted \\ &\Rightarrow \\ &\models B.Authorized \blacksquare \end{aligned}$	Rule 5
$\begin{aligned} &\exists C \in County, M \in Marsh, \forall B \in Project, \\ &\models Contains(C, M) \\ &: \\ &Contains(M, B) \\ &\Rightarrow \\ &Prohibit(B) \blacksquare \end{aligned}$	Rule 6

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### Best practice

$\begin{aligned} &\exists T, \in Town, \exists R \in Road, \forall L \in Lamppost, \\ &\forall S \in Pollution.Sensor, \\ &Topo(T) \equiv "Smart Town", \\ &Topo(R) \equiv "Churchill Road" \\ &: \\ &Contains(Geom(T), Geom(R)) \wedge Contains(Geom(R), \\ &Geom(L)) \\ &\Rightarrow [BP] \\ &Assign(S, L) \blacksquare \end{aligned}$	Rule 11
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- When studying road incidents in the city of Helsinki, Finland, Karasova et al. have shown by spatial data mining that many incidents occur near bars and restaurants. More exactly, around each incident they have designed a 50 m buffer and see whether there were incidents in those zones (Support 1.7% and confidence 40.0%).

$\begin{aligned} &\exists C \in City, \forall B \in Bar, \forall R \in Restaurant, \\ &\forall I \in Incident, \\ &\exists RiskyZone \in Terr, Topo(C) \equiv "Helsinki", \\ &\models Contains(Geom(C), Geom(B)), \\ &\models Contains(Geom(C), Geom(R)), \\ &\models Contains(Geom(C), Geom(I)), \\ &Geom(RiskyZone) \equiv \\ &Union(Buffer(Centroid(Geom(B), 50), Buffer \\ &(Centroid(Geom(R), 50)) \\ &\Rightarrow [1.7\%, 40.0\%] \\ &\models Contains(Geom(RiskyZone), Geom(I)) \blacksquare \end{aligned}$	Rule 12
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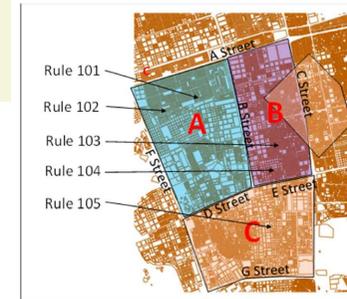
28

- In their study in the city of Antwerp, Belgium, Zhou et al. have a lot of co-location association rules within 600 m buffers.

$\exists C \in \text{City}, \forall K \in \text{Kindergarten}, \exists P \in \text{Playground},$ $\text{Topo}(C) \equiv \text{"Antwerp"}$ $\vdots$ $\text{Contains}(\text{Geom}(C), \text{Geom}(K)),$ $\text{Contains}(\text{Geom}(C), \text{Geom}(P))$ $\Rightarrow [\alpha, \beta]$ $\models \text{Contains}(\text{Geom}(\text{Centroid}(\text{Geom}(K), 600), \text{Geom}(\text{Centroid}(P))), \blacksquare)$	Rule 13
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29

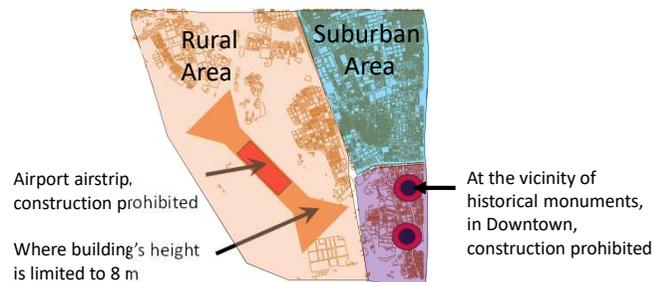
### Located Rules



$\exists C \in \text{City}, \forall B \in \text{Project}, \exists \text{ZoneA} \in \text{Terr},$ $\text{Geom}(\text{ZoneA}) \equiv \text{SurroundedByStreet}(A\_Street,$ $B\_Street, D\_Street, F\_Street)$ $\vdots$ $\text{Contains}(\text{Geom}(\text{ZoneA}), \text{Geom}(B))$ $\Rightarrow$ $\{\text{AppliedRule}(101); \text{AppliedRule}(102)\} \blacksquare$	Rule 17
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### 4 – Examples in Urban Planning

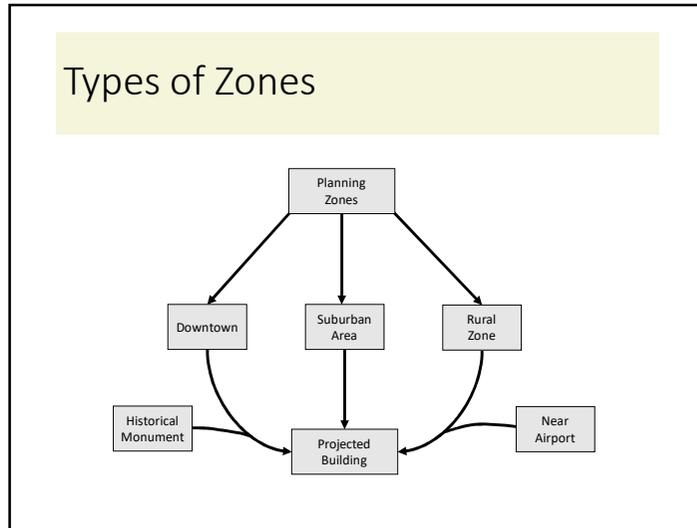


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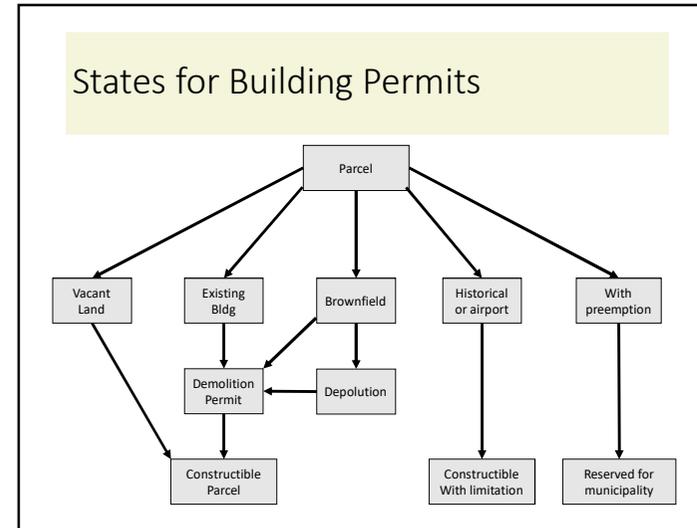
### Creation of Urban Planning Zone

$\exists C \in \text{City}, \exists PZoneA \in \text{Project.Terr},$ $\text{Topo}(C) \equiv \text{"Smart Town"}$ $\text{Geom}(PZoneA) \equiv \text{Polyg}(731, 128; 903, 133; 905,$ $341; 839, 346; 814, 349)$ $\vdots$ $\text{Approved}(PZone)$ $\Rightarrow$ $\text{AffectName}(PZoneA, \text{"Downtown"}) \blacksquare$	Rule 18
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### Parameters

TABLE I. PLANNING ZONES AND THEIR PARAMETERS

Zone ID	Max Height (in m)	Max Floorspace ratio	Max Footprint
Downtown	12	3	80 %
Suburban area	15	4	70 %
Rural area	12	0.5	30 %
Near airport (Bowtie)	8	2	50 %
Airstrip	0	0	0

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### For Suburban Zone

$$\begin{aligned}
 & \exists C \in City, \exists ZoneB \in Terr, \forall B \in Project.BUILDING, \forall \\
 & \quad P \in Parcel, \\
 & \quad Topo(C) \equiv \text{"Smart Town"}, \\
 & \quad Topo(ZoneB) \equiv \text{"Suburban Area"} \\
 & \quad \models Contains(Geom(C), Geom(ZoneB)), \\
 & \quad \models Contains(Geom(ZoneB), Geom(P)), \\
 & \quad \models Contains(Geom(P), Geom(B)) \\
 & \quad \vdots \\
 & \quad B.Height \leq 15 \\
 & \quad \wedge Area(Union(Geom(Floors)))/Area(Geom(P)) \leq 4 \\
 & \quad \wedge Area(B)/Area(Geom(P)) \leq 0.70 \\
 & \quad \Rightarrow \\
 & \quad \models B.ZoneB\_Approved \blacksquare
 \end{aligned}$$

Rule 22

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$  \begin{aligned}  &\exists C \in \text{City}, \exists \text{ZoneA}, \text{ConservA} \in \text{Terr}, \forall B \in \text{Project.Building}, \\  &\forall P \in \text{Parcel}, \forall M \in \text{Monuments}, \\  &\quad \text{Topo}(C) \equiv \text{"Smart Town"}, \\  &\quad \text{Topo}(\text{ZoneA}) \equiv \text{"Downtown"}, \\  &\quad \text{Geom}(\text{ConservA}) \equiv \text{Union}(\text{Buffer}(\text{Centroid}(\text{Geom}(M), 200))), \\  &\quad \models \text{Contains}(\text{Geom}(C), \text{Geom}(\text{ZoneB})), \\  &\quad \models \text{Contains}(\text{Minus}(\text{Geom}(\text{ZoneA}), \text{Geom}(\text{ConservA})), \text{Geom}(P)), \\  &\quad \models \text{Contains}(\text{Geom}(P), \text{Geom}(B)) \\  &\quad \vdots \\  &\quad B.\text{Height} \leq 12 \\  &\quad \wedge \text{Area}(\text{Union}(\text{Geom}(\text{Floors}))) / \text{Area}(\text{Geom}(P)) \leq 3 \\  &\quad \wedge \text{Area}(B) / \text{Area}(\text{Geom}(P)) \leq 0.80 \\  &\quad \Rightarrow \\  &\quad \models B.\text{ZoneA\_Approved} \blacksquare  \end{aligned}  $	<p>Rule 23</p>
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$  \begin{aligned}  &\exists C \in \text{City}, \exists \text{ZoneC}, \text{Bowtie}, \text{Airstrip} \in \text{Terr}, \\  &\quad \forall B \in \text{Project.Building}, \\  &\quad \forall P \in \text{Parcel}, \\  &\quad \text{Topo}(C) \equiv \text{"Smart Town"}, \\  &\quad \text{Topo}(\text{ZoneA}) \equiv \text{"Rural Area"}, \\  &\quad \text{Geom}(\text{Bowtie}) = \text{Polyg}(640, 243; 657, 290; 748, 387; \\  &\quad 796, 405; 743, 459; 729, 406; 636, 316; 580, 297), \\  &\quad \text{Geom}(\text{Airstrip}) = \text{Polyg}(670, 311; 724, 365; 707, 386; \\  &\quad 650, 330), \\  &\quad \models \text{Contains}(\text{Geom}(C), \text{Geom}(\text{ZoneC})), \\  &\quad \models \text{Contains}(\text{Geom}(P), \text{Geom}(B)) \\  &\quad \vdots \\  &\quad (\text{Contains}(\text{Minus}(\text{Geom}(\text{ZoneC}), \text{Geom}(\text{Bowtie})), \text{Geom}(B)), \\  &\quad \wedge B.\text{Height} \leq 12 \wedge \text{Area}(\text{Union}(\text{Geom}(\text{Floors}))) / \text{Area}(\text{Geom}(P)) \leq 0.5 \\  &\quad \wedge \text{Area}(B) / \text{Area}(\text{Geom}(P)) \leq 0.30 \\  &\quad \oplus \text{Disjoint}(\text{Geom}(\text{Airstrip}), \text{Geom}(B)) \\  &\quad \oplus (\text{Contains}(\text{Geom}(\text{Bowtie}), \text{Geom}(B)) \\  &\quad \wedge B.\text{Height} \leq 8 \wedge \text{Area}(\text{Union}(\text{Geom}(B.\text{Floor}))) / \text{Area}(\text{Geom}(P)) \leq 0.5 \\  &\quad \wedge \text{Area}(B) / \text{Area}(\text{Geom}(P)) \leq 0.30 \\  &\quad \Rightarrow \\  &\quad \models B.\text{ZoneC\_Approved} \blacksquare  \end{aligned}  $	<p>Rule 24</p>
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<h2 style="background-color: #e1e5fe; padding: 5px;">Final Approbation</h2>	
$  \begin{aligned}  &\exists C \in \text{City}, \forall B \in \text{Project.Building}, \\  &\quad \text{Topo}(C) \equiv \text{"Smart Town"}, \\  &\quad \models \text{Contains}(\text{Geom}(C), \text{Geom}(B)) \\  &\quad \vdots \\  &\quad (\text{Contains}(\text{Geom}(\text{ZoneA}), \text{Geom}(B)) \wedge B.\text{ZoneA\_Approved}) \\  &\quad \oplus (\text{Contains}(\text{Geom}(\text{ZoneB}), \text{Geom}(B)) \wedge B.\text{ZoneB\_Approved}) \\  &\quad \oplus (\text{Contains}(\text{Geom}(\text{ZoneC}), \text{Geom}(B)) \wedge B.\text{ZoneC\_Approved}) \\  &\quad \Rightarrow \\  &\quad \models B.\text{FullyApproved} \blacksquare  \end{aligned}  $	<p>Rule 25</p>

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<h2 style="background-color: #e1e5fe; padding: 5px;">5 - Conclusions</h2>	
<ul style="list-style-type: none"> <li>• Mathematical language for geographic static rules</li> <li>• Syntax</li> <li>• Examples</li> <li>• Expressive power</li>   <li>• In total 500+ geographic rules encoded</li> </ul>	

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## Perspectives (1/2)

- Integration of 3D, especially for terrain modeling and engineering networks;
- Integration of temporal issues; this will lead to dynamic geospatial rules;
- Integration of rules deriving from continuous fields, especially for dealing with meteorology, pollution, etc. and other aspects in physical geography;
- Integration of additional clauses to extend its expressive power, overall to deal with networks whatsoever, electricity, sewerage, bus lines, etc.;

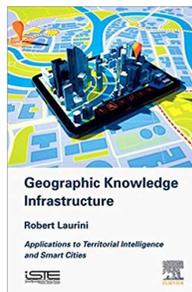
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## Perspectives (2/2)

- Looking for more issues in order to enrich semantics, especially for the automatic adaptation to special contexts; for instance, how to adapt a rule such as "when planning a metro, move underground engineering networks" to various street configurations;
- Transformation of this mathematical language into a computer language;
- Study of metadata relative to geographic rules (origin, etc.);
- Design of an inference engine to reason with those rules;
- Defining the organization of rules together for their access mechanisms taking temporal and spatial superseding mechanisms.

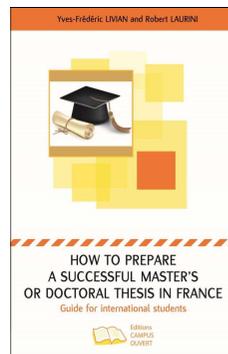
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Thanks for your attention!



• [Roberto.Laurini@gmail.com](mailto:Roberto.Laurini@gmail.com)

• <http://www.laurini.net/robert/>



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