Local certification of local properties : tight bounds, trade-offs and new parameters

Nicolas Bousquet, Laurent Feuilloley, Sébastien Zeitoun

March 13, 2024

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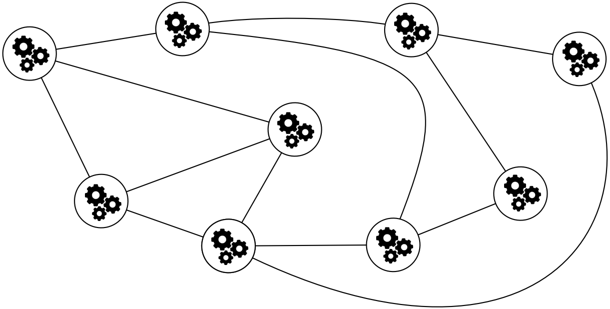

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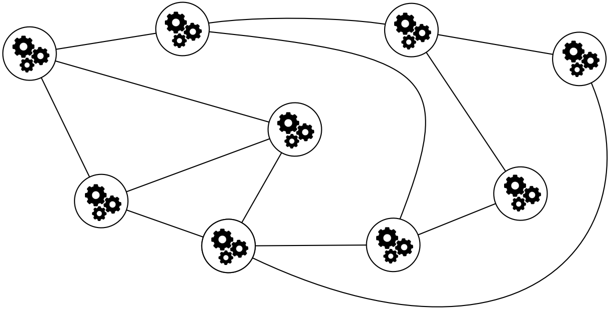

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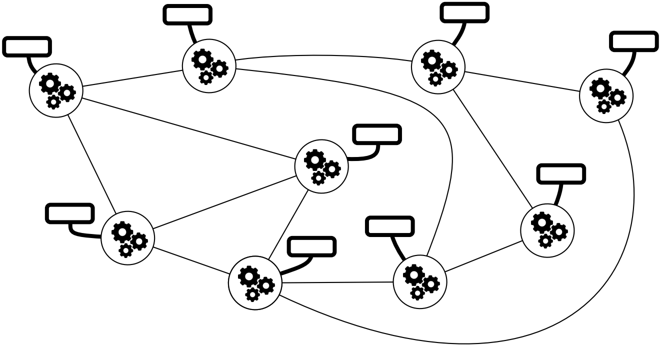

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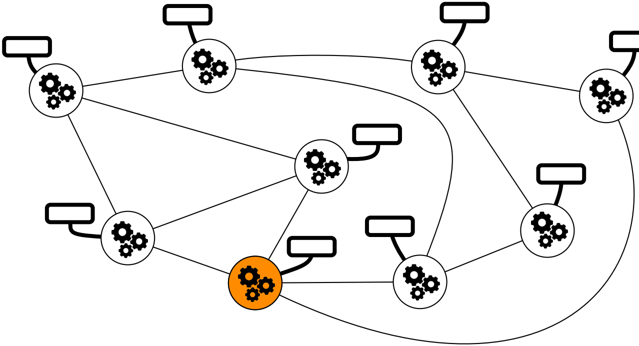

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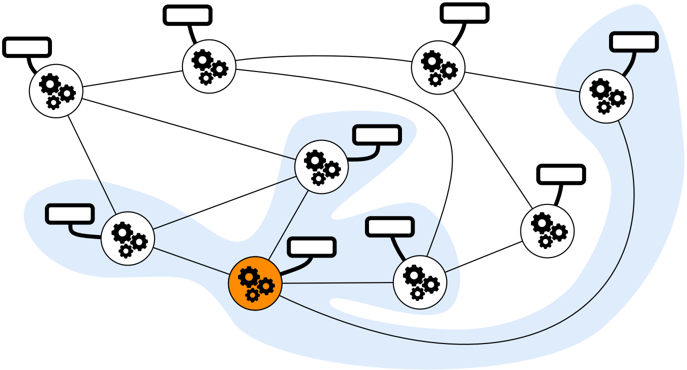

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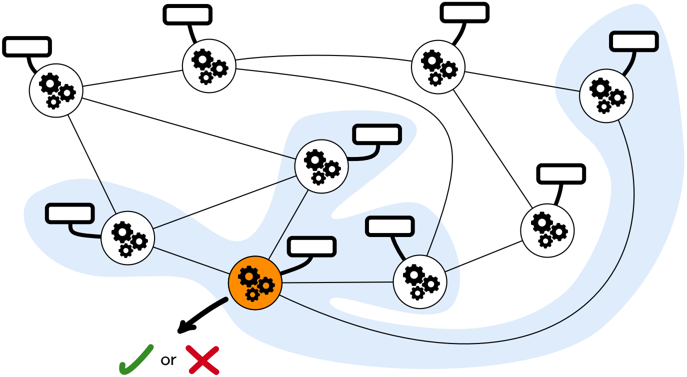

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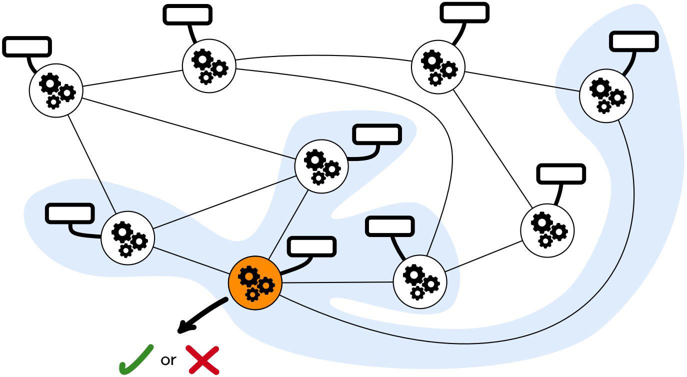

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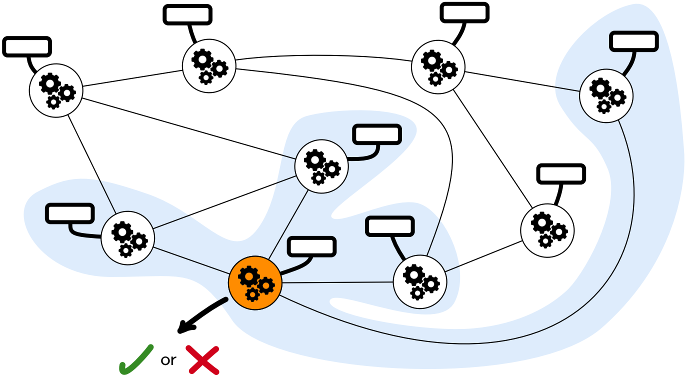

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G satisfies $\mathcal{P} \iff$ there exists an assignment of the certificates such that G is accepted

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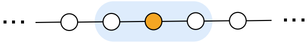

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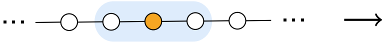

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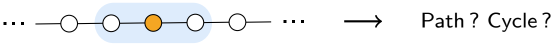

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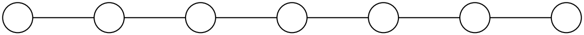

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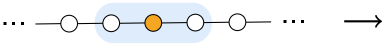

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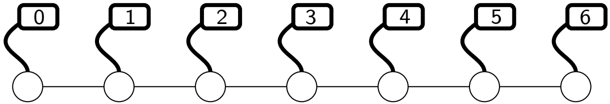

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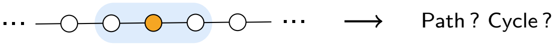

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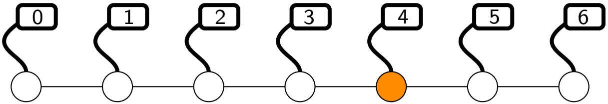

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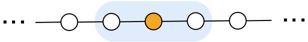

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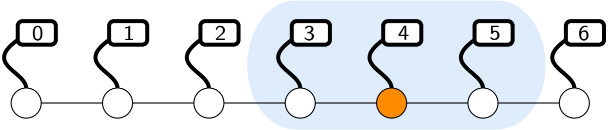

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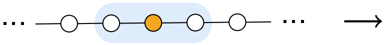

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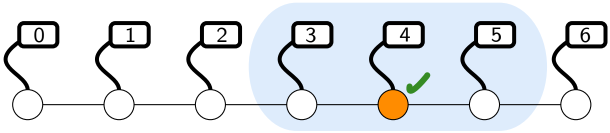

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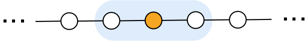

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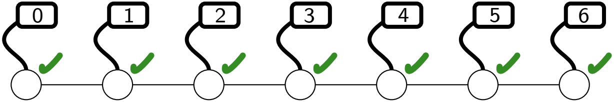

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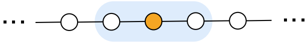

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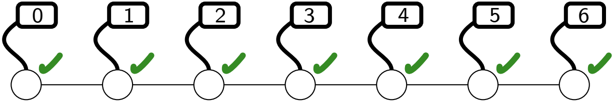

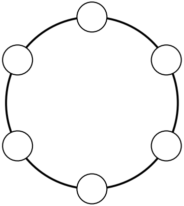

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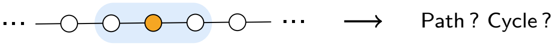

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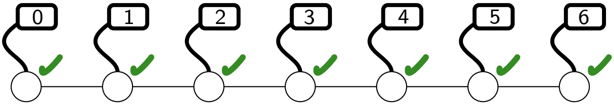

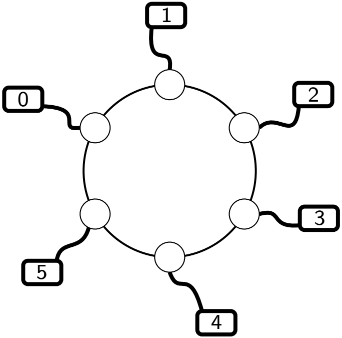

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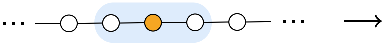

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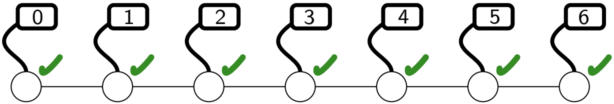

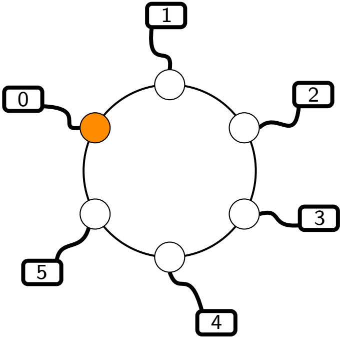

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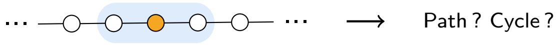

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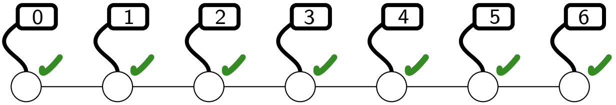

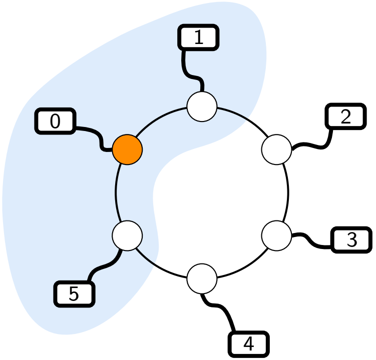

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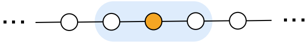

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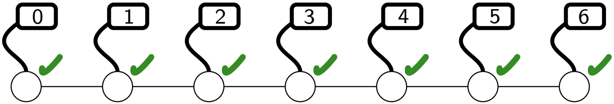

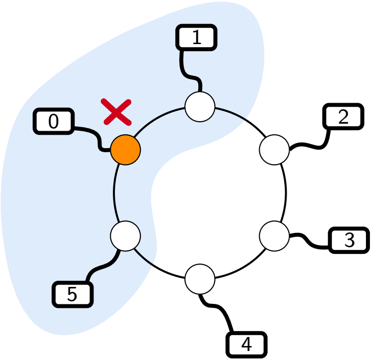

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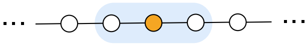

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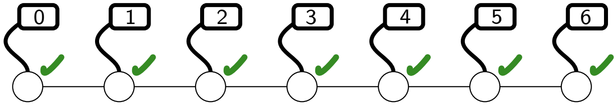

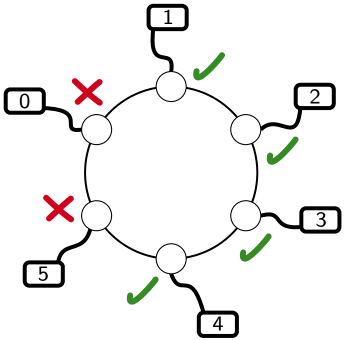

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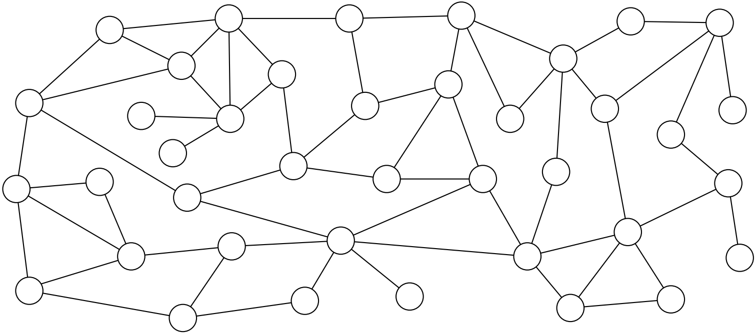

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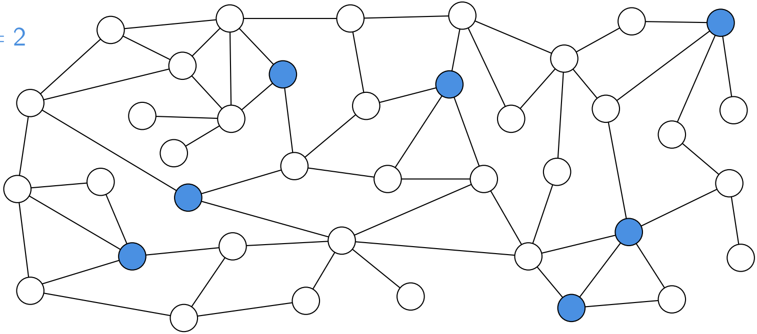

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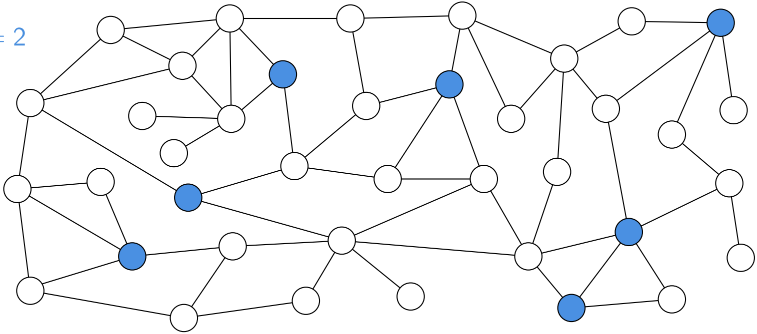

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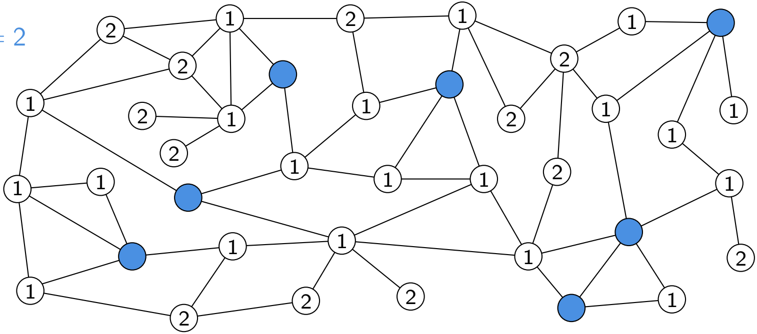

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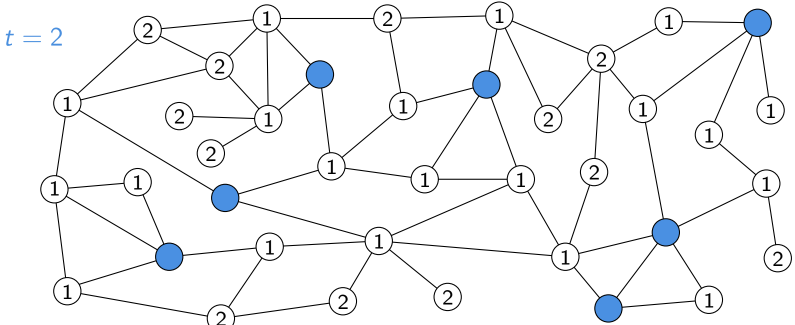

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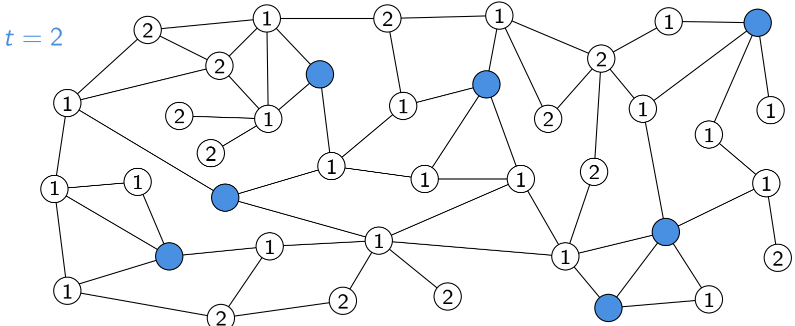

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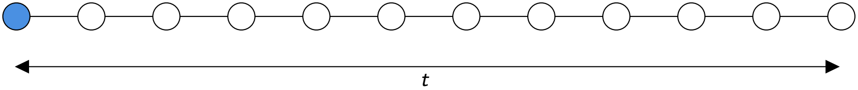

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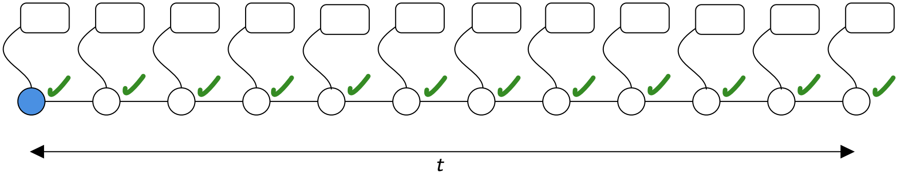

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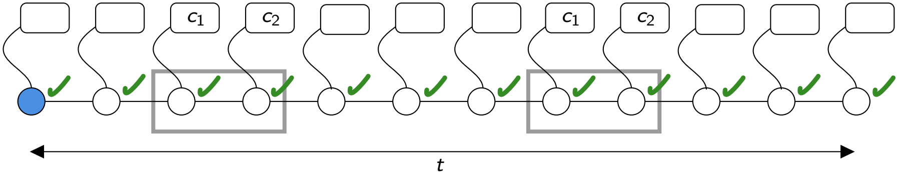

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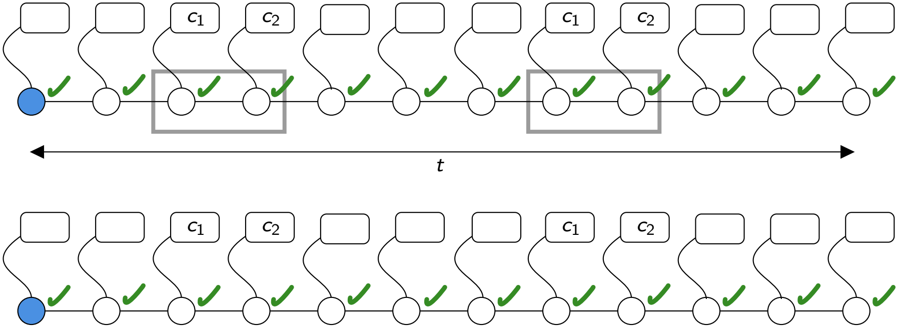

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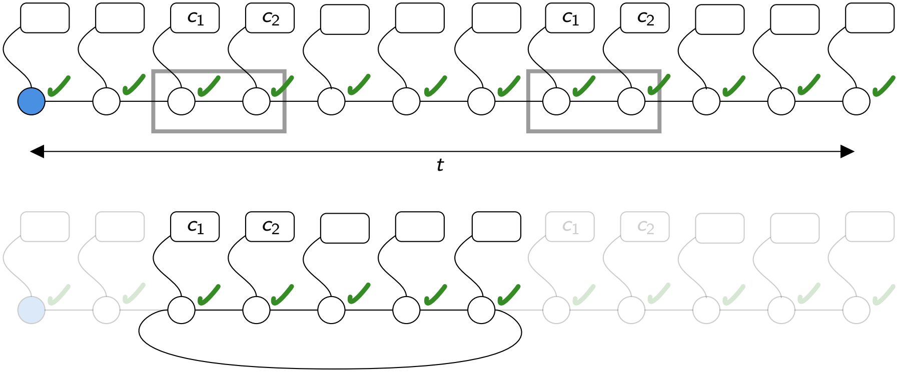

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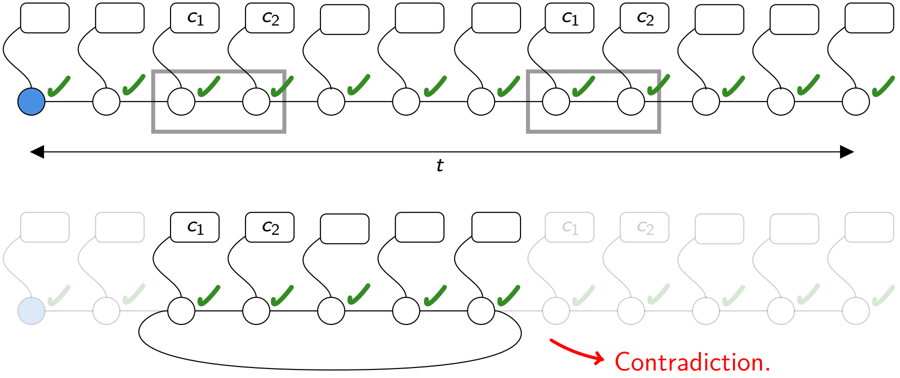

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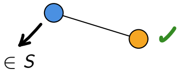

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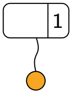

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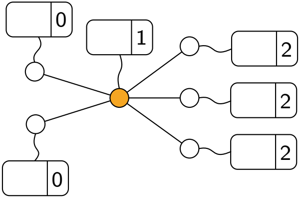

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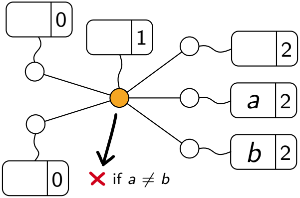

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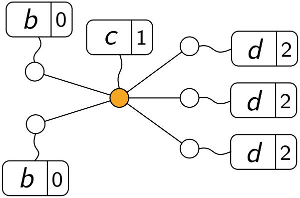

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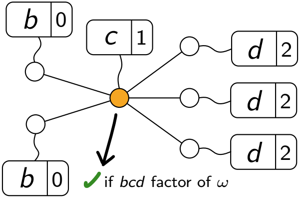

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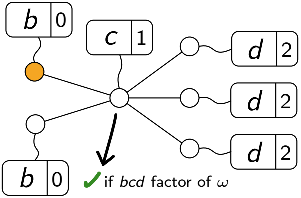

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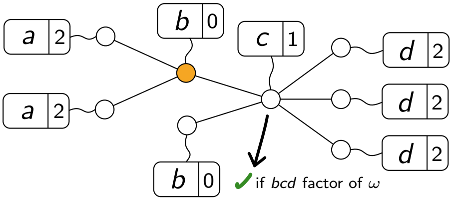

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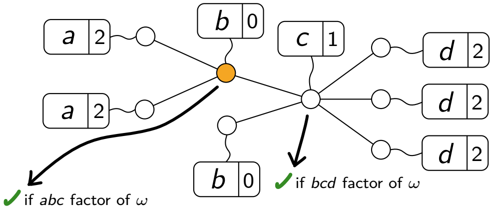

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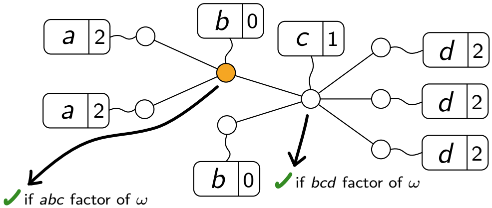

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$\Downarrow$

$\omega = ...abcd.....$

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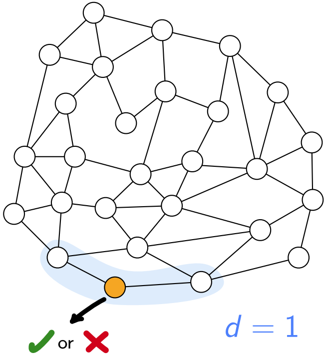

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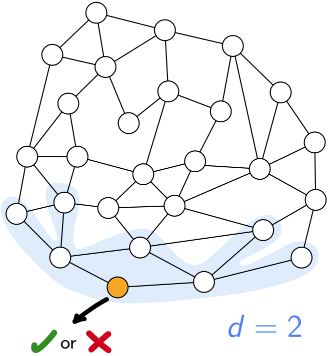

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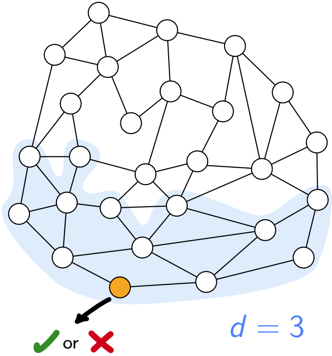

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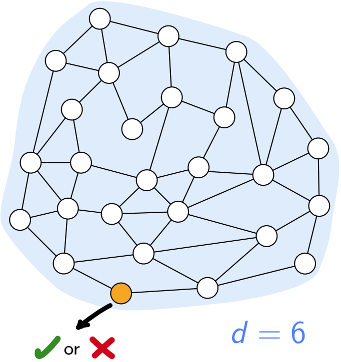

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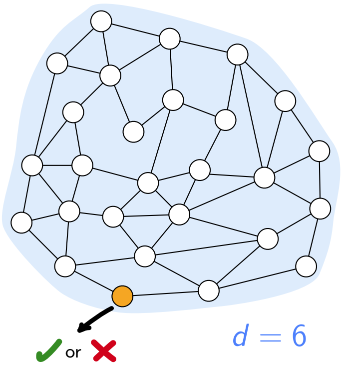

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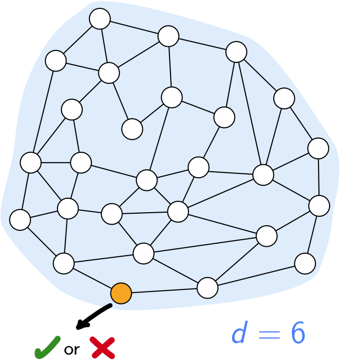

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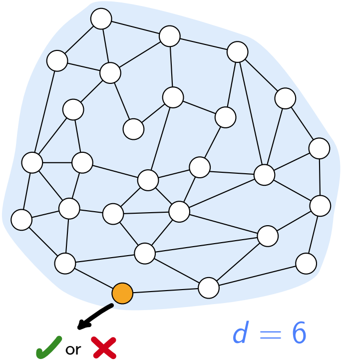

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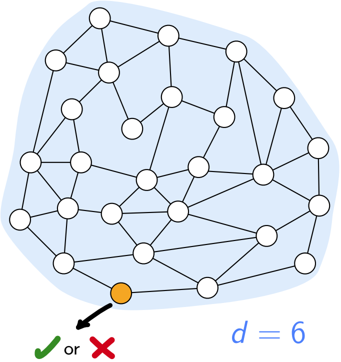

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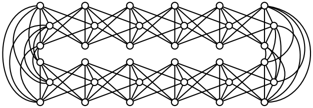

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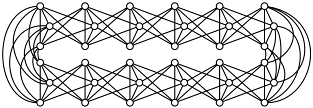

($k = 3$)

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Question : lower bound at distance $d > 1$?

Thanks for your attention !