Reductions in local certification

Louis Esperet, Sébastien Zeitoun

June 13, 2025

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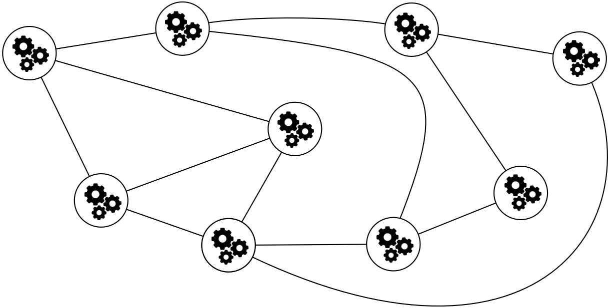
Context: distributed computing

Model: graph, $\begin{cases} \text{vertices} = \text{computation units} \\ \text{edges} = \text{communication channels} \end{cases}$



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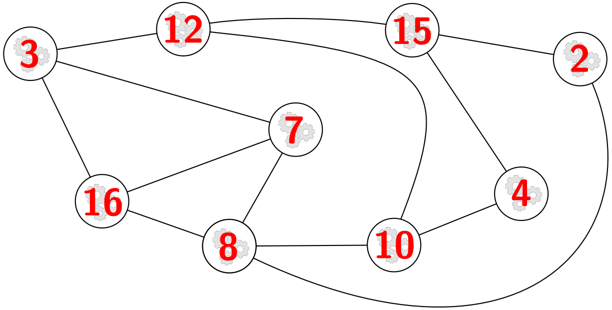

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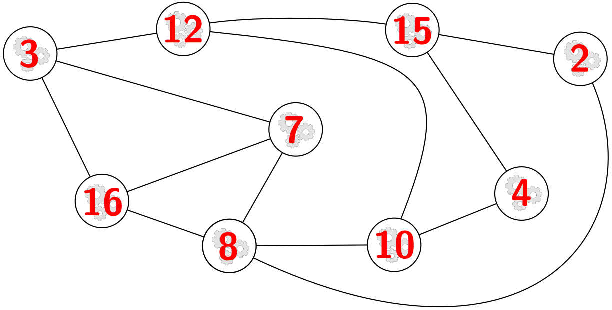

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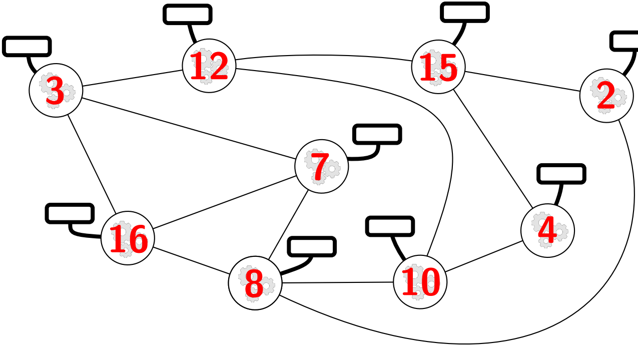

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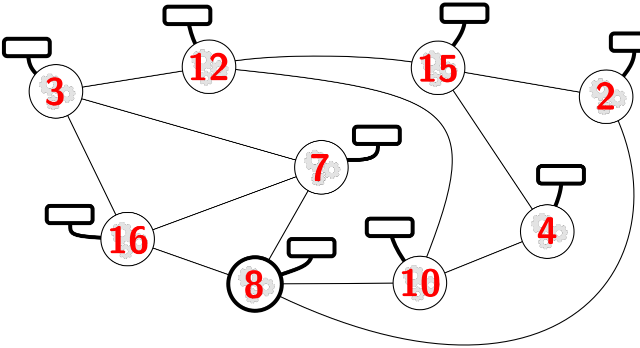

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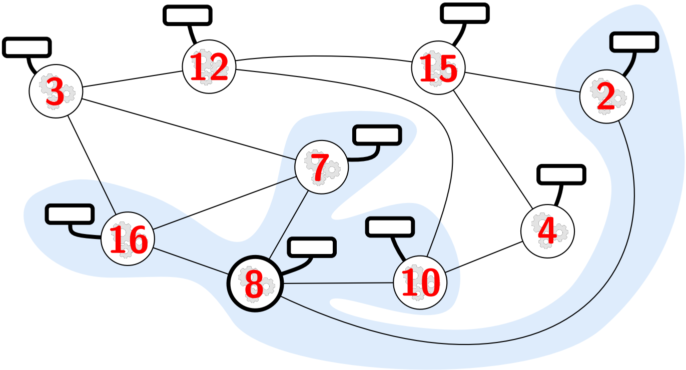

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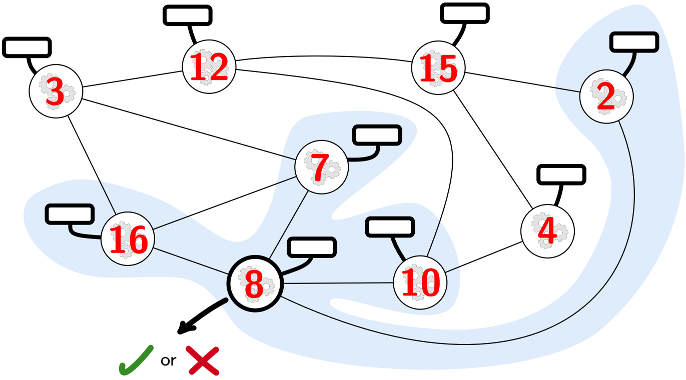

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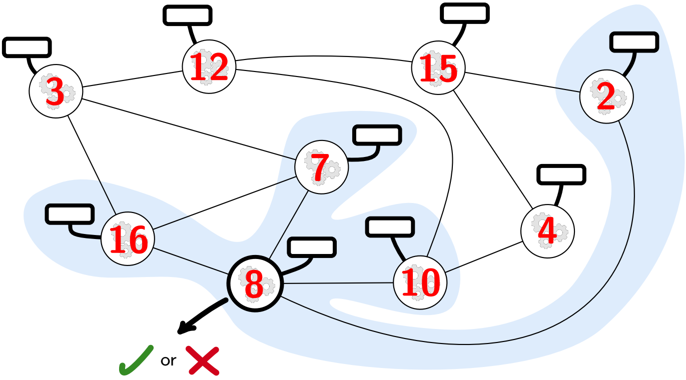

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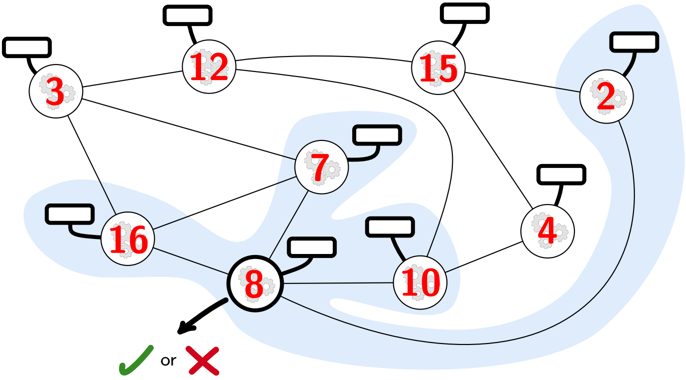

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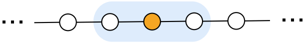

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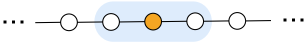

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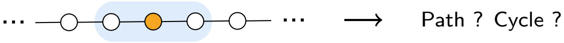

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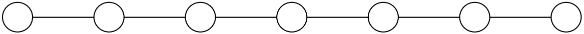

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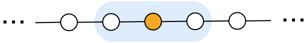

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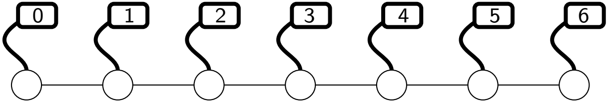

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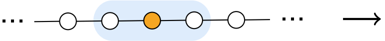

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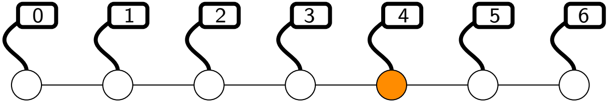

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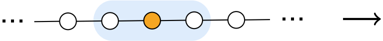

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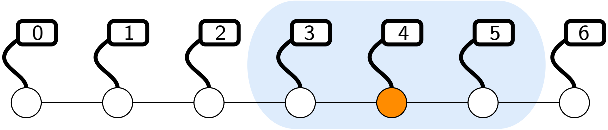

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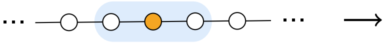

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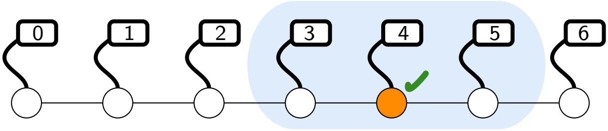

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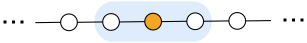

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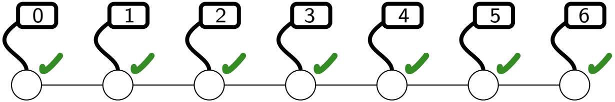

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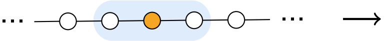

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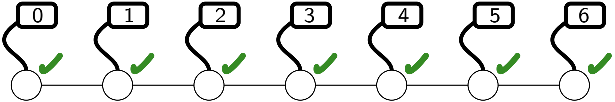

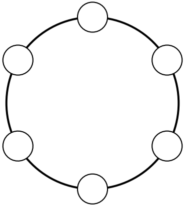

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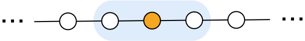

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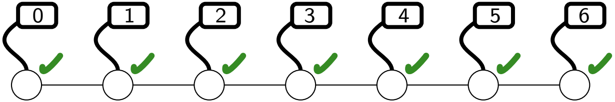

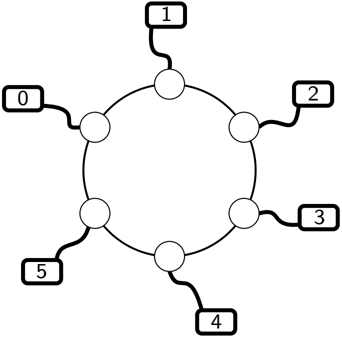

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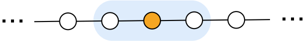

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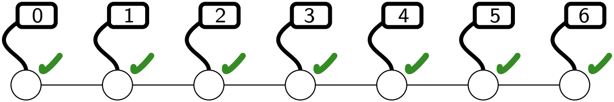

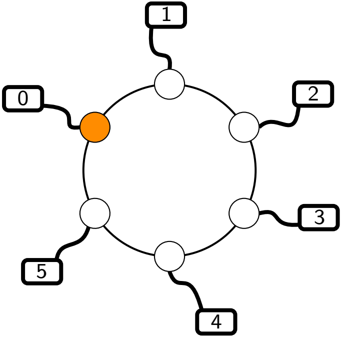

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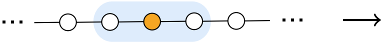

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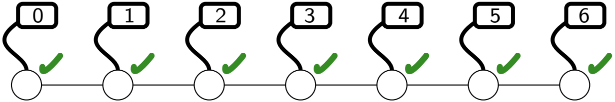

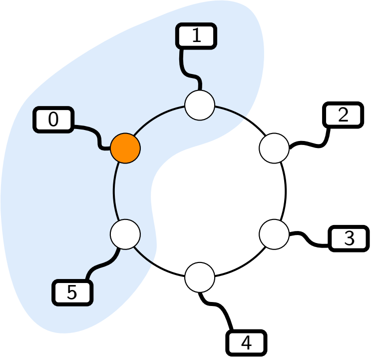

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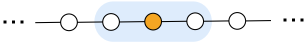

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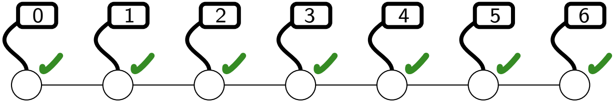

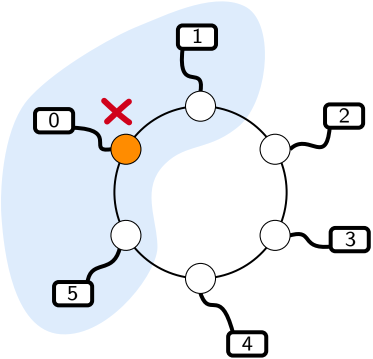

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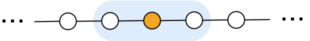

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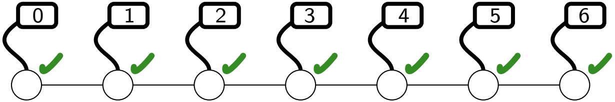

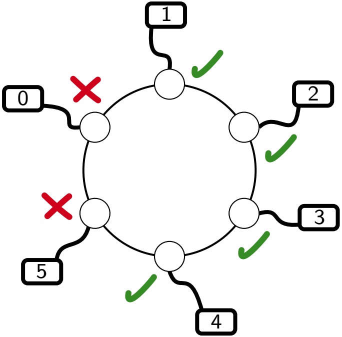

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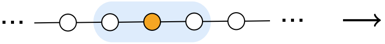

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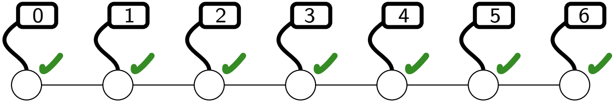

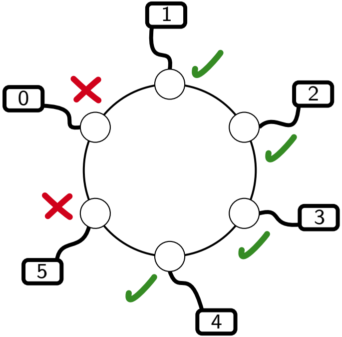

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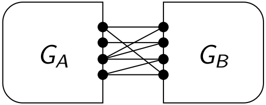

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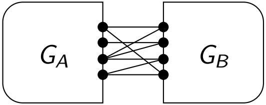

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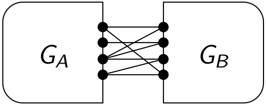

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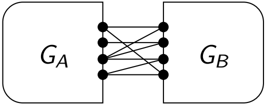

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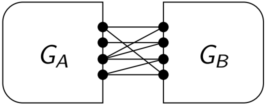

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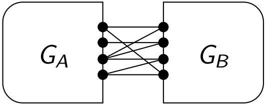

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- Such a reduction has to be **local**: we identified the requirements that it should satisfy.

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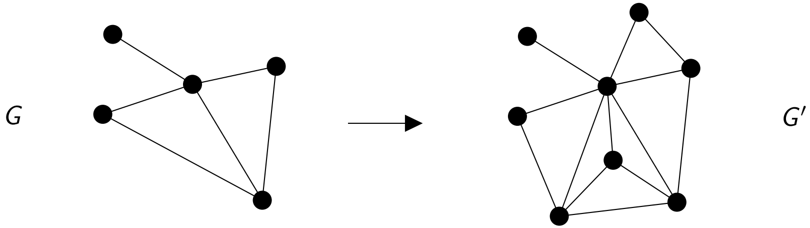

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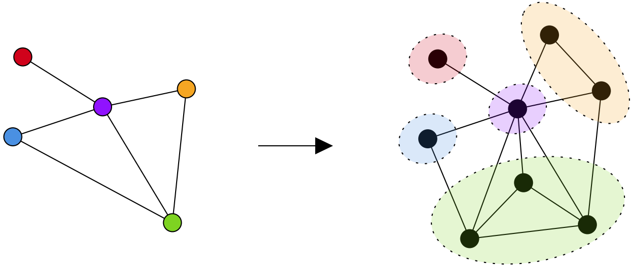

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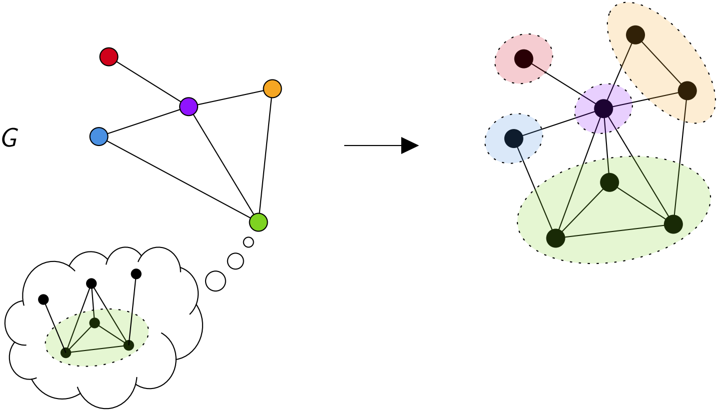

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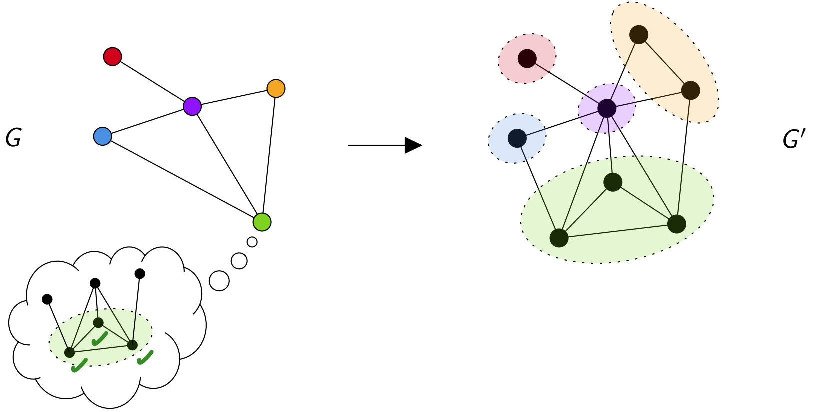

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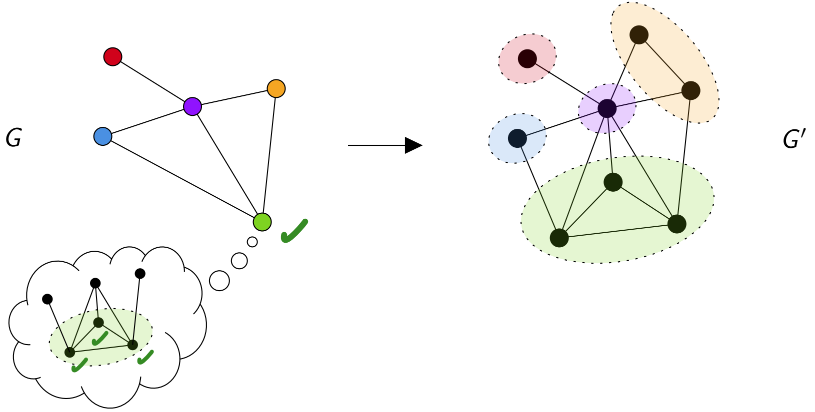

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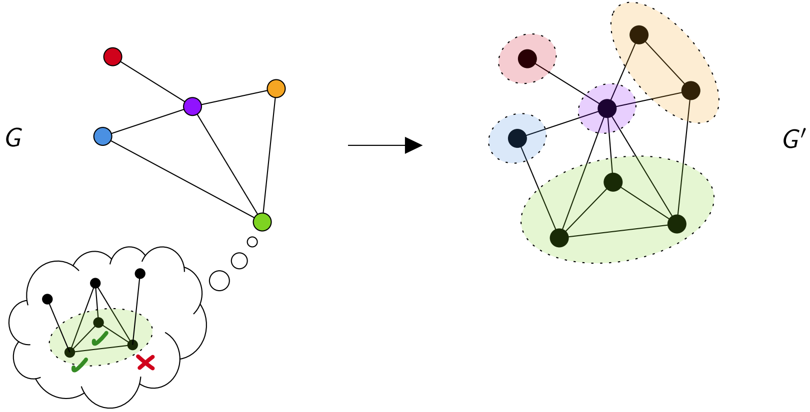

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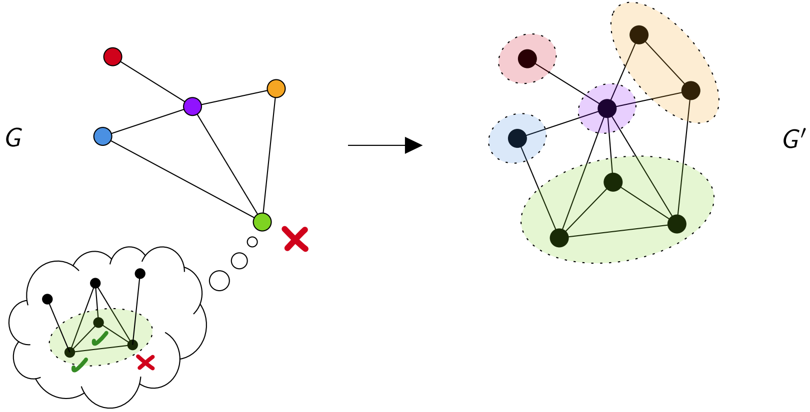

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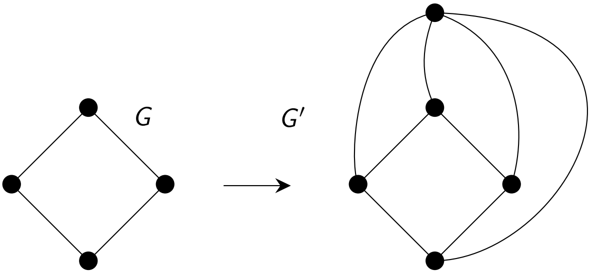

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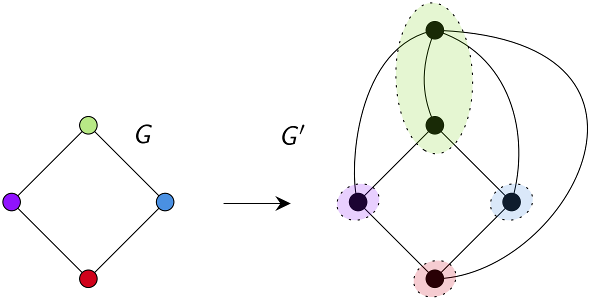

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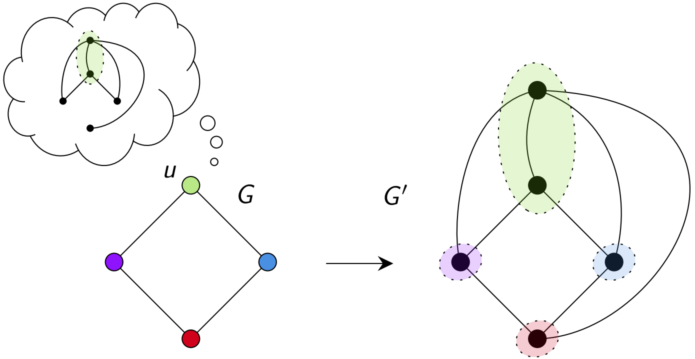

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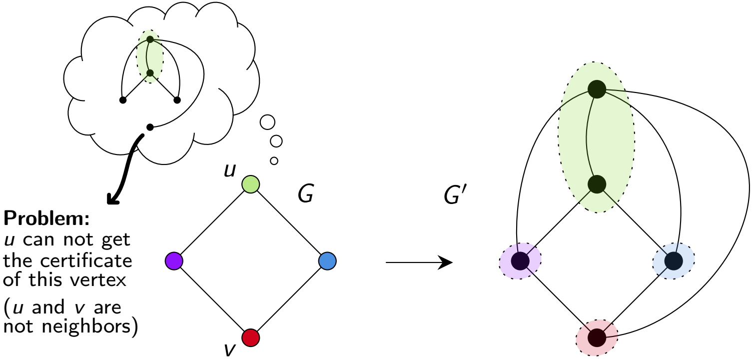

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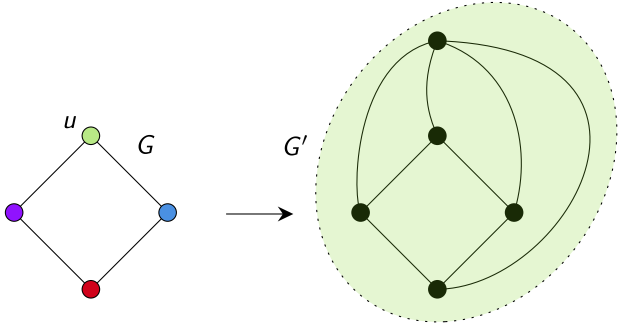

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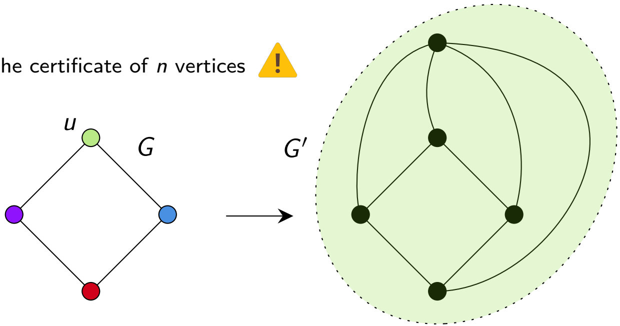

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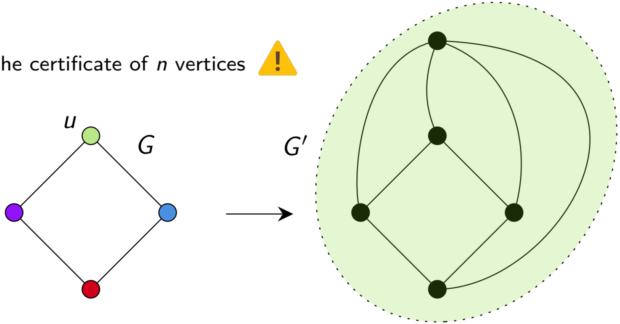

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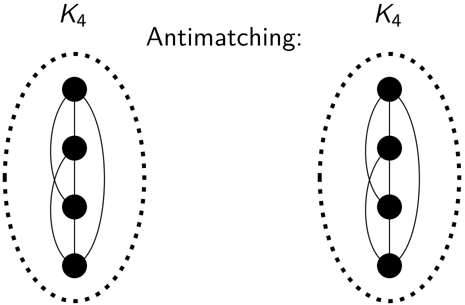

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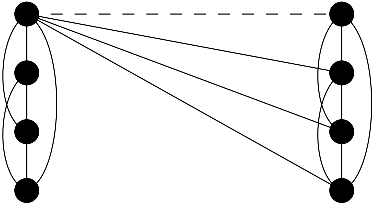

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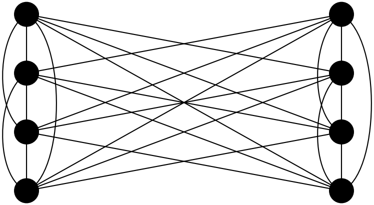

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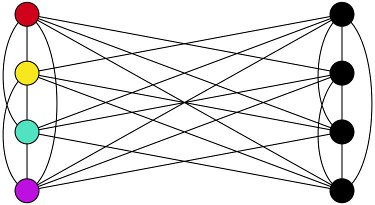

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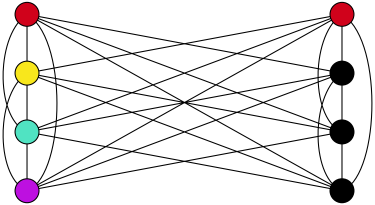

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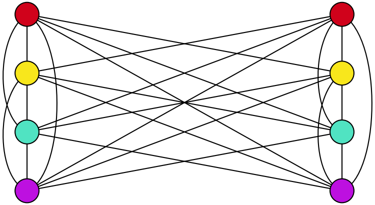

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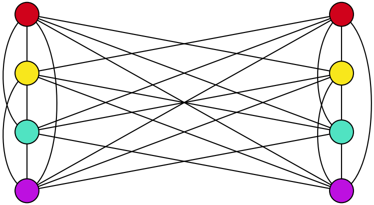

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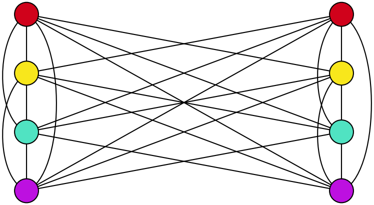

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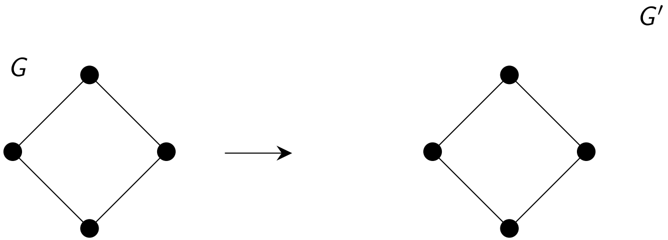

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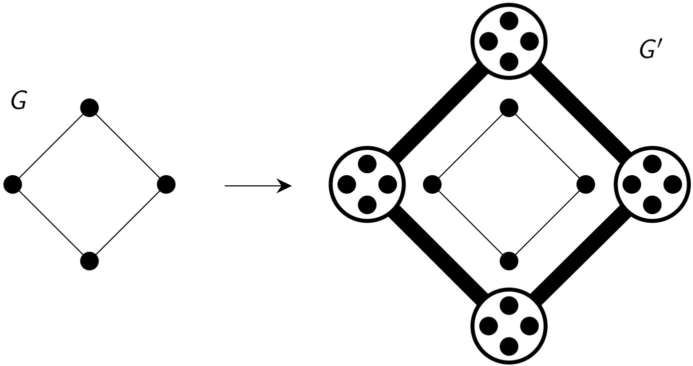

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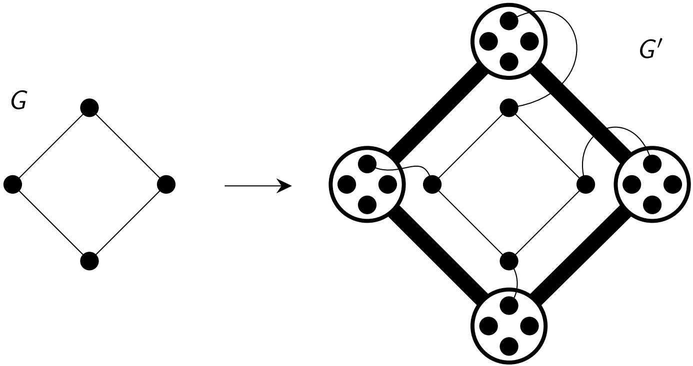

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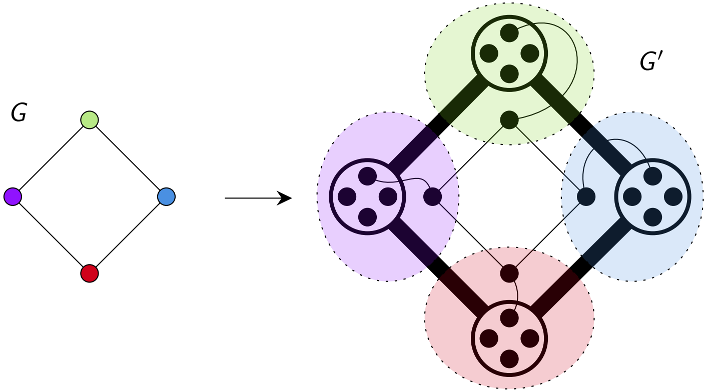

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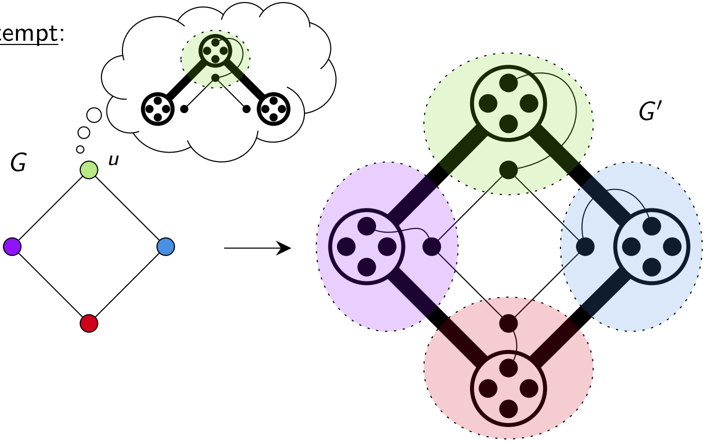

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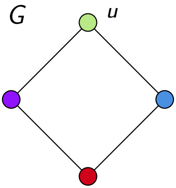

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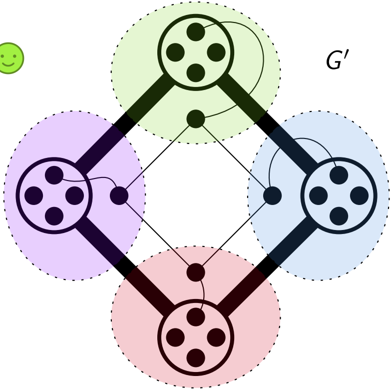

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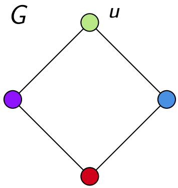

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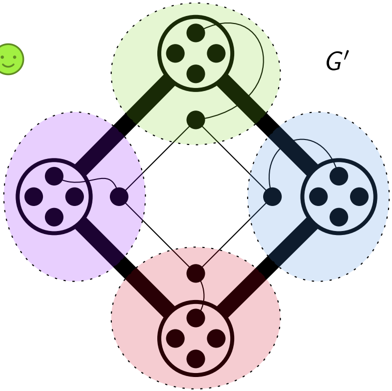

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Theorem (Esperet, Z.)

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Thanks for your attention !