Analytical Description of Digital Circles

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Introduction

In this work we propose an analytical description of different kinds of digital circles that appear in the literature and especially in digital circle recognition algorithms. The digitization of a discrete object E is morphological in nature :

$$D_d(E) = (E \oplus B_d(1)) \bigcap \mathbb{Z}^2$$

where $A \oplus B = \{a+b, a \in A, b \in B\}$ is the Minkowski sum.

With d_∞ : Supercover model

(x,y) belongs to $\mathbb{C}_\infty(x_o,y_o,R) = \left((\mathcal{C} \oplus B_\infty(1)) \cap \mathbb{Z}^2\right)$ iff :

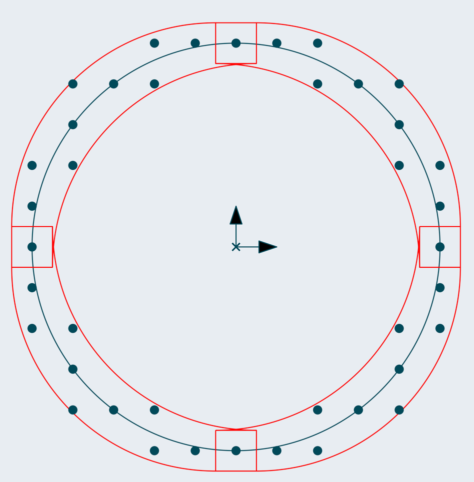
$$|y-y_o| \le \tfrac{1}{2} \text{ and } \big||x-x_o|-R\big| \le \tfrac{1}{2}$$

or

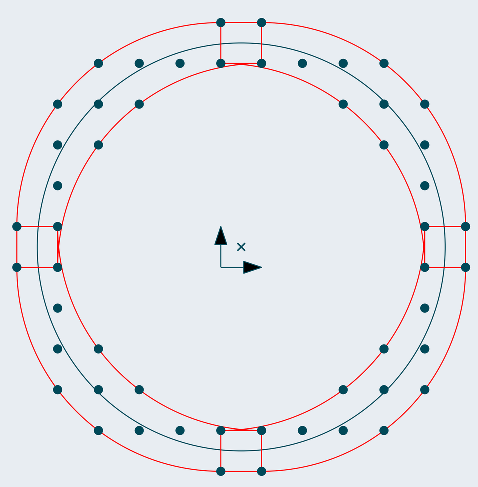
$$|x-x_o| \le \tfrac{1}{2} \text{ and } \big||y-y_o|-R\big| \le \tfrac{1}{2}$$

or

$$R^2 - \frac{1}{2} - (|x-x_o|+|y-y_o|) \le (x-x_o)^2+(y-y_o)^2 \le R^2 - \frac{1}{2} + (|x-x_o|+|y-y_o|)$$



(a) $\mathbb{C}_\infty(0,0,5)$



(b) $\mathbb{C}_\infty(\frac{1}{2},\frac{1}{2},5)$

With d_1 : Closed naive model

(x,y) belongs to $\mathbb{C}_1(x_o,y_o,R) = \left((\mathcal{C} \oplus B_1(1)) \cap \mathbb{Z}^2\right)$ iff :

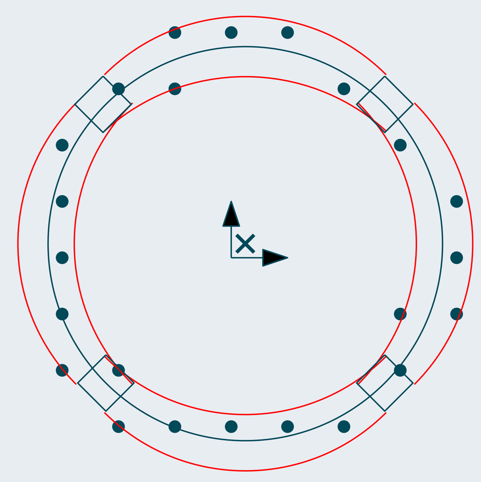
$$|(x-y)-(x_o-y_o)| \le \tfrac{1}{2} \text{ and } \left||x+y-(x_o+y_o)|-R\sqrt{2}\right| \le \tfrac{1}{2}$$

or

$$|(x+y)-(x_o+y_o)| \le \tfrac{1}{2} \text{ and } \left||x-y-(x_o-y_o)|-R\sqrt{2}\right| \le \tfrac{1}{2}$$

or

$$R^2 - \frac{1}{4} - \max(|x-x_o|,|y-y_o|) \le (x-x_o)^2+(y-y_o)^2 \le R^2 - \frac{1}{4} + \max(|x-x_o|,|y-y_o|)$$



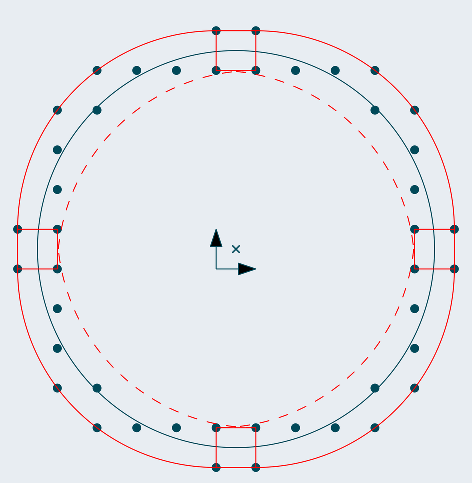
(c) $\mathbb{C}_1(\frac{1}{4},\frac{1}{4},\frac{7}{2})$

A Bresenham circle is a closed naive circle but also an inner and outer naive circle.

Naive circles are thus an extension of Bresenham circles for arbitrary centers and radii.

Standard

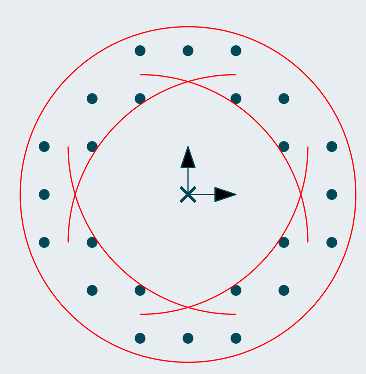
Example of an inner standard circle



Gauss

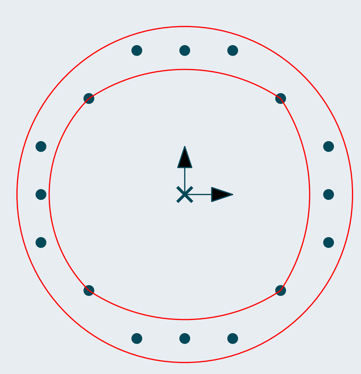
(x,y) belongs to $\mathbb{G}_\infty(x_o,y_o,R)$ iff :

$$R^2 - 2(|x-x_o|+|y-y_o|) - 1 < (x-x_o)^2+(y-y_o)^2 \le R^2$$



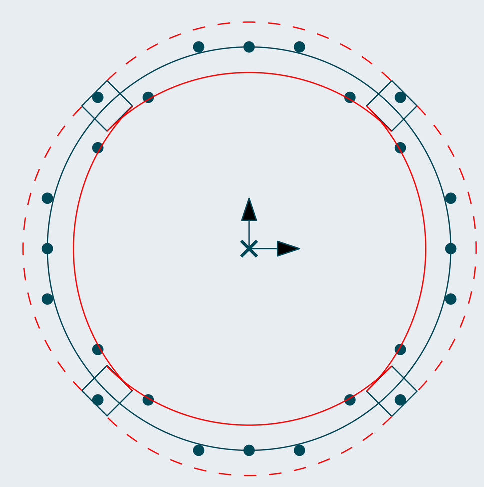
(x,y) belongs to $\mathbb{G}_1(x_o,y_o,R)$ iff :

$$R^2 - 2\max(|x-x_o|,|y-y_o|) - 1 < (x-x_o)^2+(y-y_o)^2 \le R^2$$



Naive

Example of an outer naive circle



Conclusion

We proposed analytical inequalities describing the supercover, inner and outer standard, closed naive, inner and outer naive, d_∞ and d_1-Gauss circles.

Having an analytical characterization has many advantages :

- verifying if a set of points belongs to a digital circle ;
- verifying the correctiveness of digital circle generation algorithm ;
- leads to a unified framework for digital circle generation and recognition algorithms ;
- recognition of subclasses of digital circles by linear programming techniques.