Test and Measure of Circularity for Digital Curves

Tristan Roussillon, Isabelle Sivignon, Laure Tougne

Laboratoire d'InfoRmatique en Image et Systèmes d'information
LIRIS UMR 5205 CNRS/INSA de Lyon/Université Claude Bernard Lyon 1/Université Lumière Lyon 2/Ecole Centrale de Lyon
INSA de Lyon, bâtiment J. Verne
20, Avenue Albert Einstein - 69622 Villeurbanne cedex
http://liris.cnrs.fr

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Outline

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Data

Digital object

4-connected set of points of $\mathbb{Z}^2$.

Digital curve

8-connected boundary of a digital object.

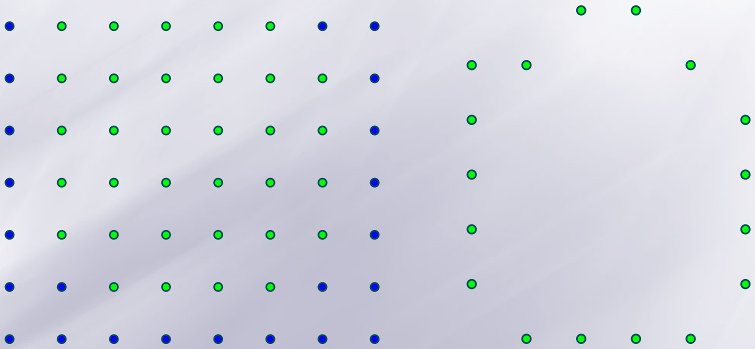
Digital disk

Set of points that can be separated from the other points of $\mathbb{Z}^2$ by a circle.

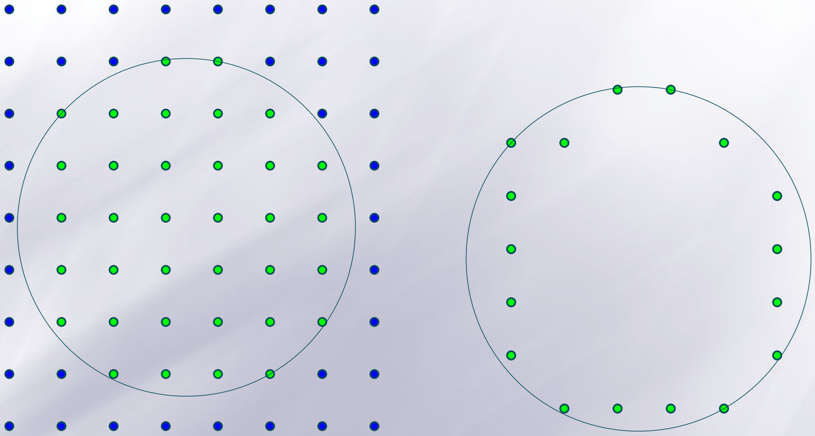
Digital circle

Boundary of a digital disk.

Data



Data



Two goals

Let X be a digital curve

- Test of circularity : Is X a digital circle ?
  - YES. We are done.
  - NO. We know that X is not a digital circle but we would like to know the extent of the discrepancy.

- Measure of circularity : How does X look like a digital circle ?

Outline

The separating circle problem

Let $\mathcal{S}$ and $\mathcal{T}$ be two sets of points of $\mathbb{Z}^2$. If $\mathcal{S}$ is separable from $\mathcal{T}$ by a circle $\mathcal{C}(o, r)$, then $\mathcal{C}$ is such that :

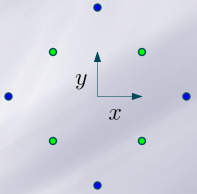
$$\begin{cases} \forall s \in \mathcal{S}, (x_s - x_o)^2 + (y_s - y_o)^2 \leq r^2 \\ \forall t \in \mathcal{T}, (x_t - x_o)^2 + (y_t - y_o)^2 > r^2 \end{cases} \tag{1}$$

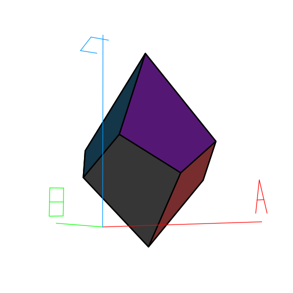
Developing :

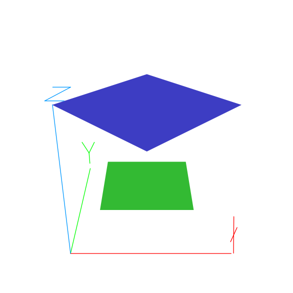
$$\begin{cases} \forall s \in \mathcal{S}, -2ax_s - 2by_s + f(x_s, y_s) + c \leq 0 \\ \forall t \in \mathcal{T}, -2ax_t - 2by_t + f(x_t, y_t) + c > 0 \end{cases}$$

$$\text{where} \quad \begin{cases} a = x_o, \quad b = y_o, \\ c = (a^2 + b^2 - r^2) \\ f(x, y) = x^2 + y^2 \end{cases}$$

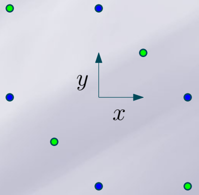
Two sets that are separable by a circle

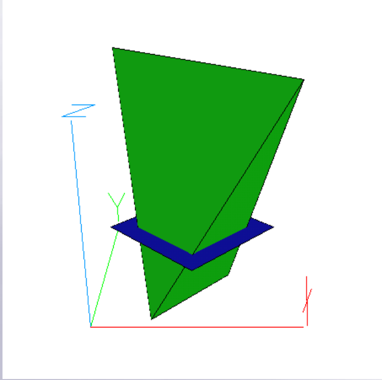






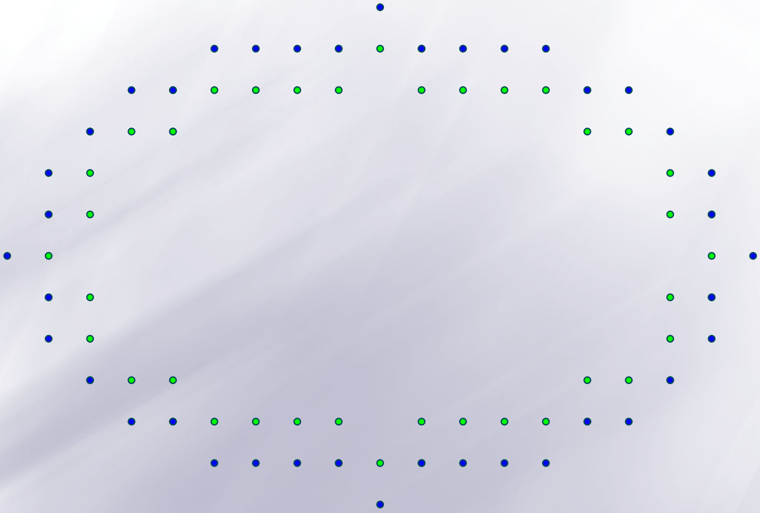
Two sets that are not separable by a circle





Outline

What are $\mathcal{S}$ and $\mathcal{T}$? naïve solution



What are $\mathcal{S}$ and $\mathcal{T}$? second solution

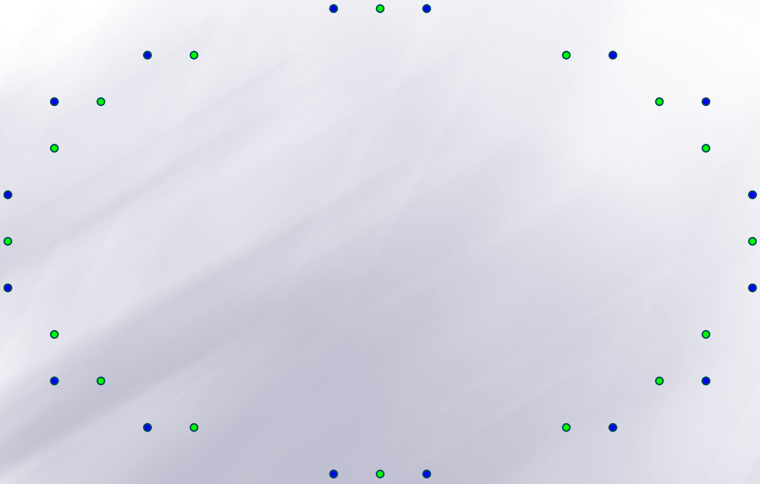
Let X be a digital curve of n points.

- $\mathcal{S}$ is reduced to the vertices of the convex hull of X.
- For each edge of the convex hull of X, $\mathcal{T}$ is reduced to the background point that is closer to the edge (and closer to its middle) than the other background points. [2]
- $|\mathcal{S}| = |\mathcal{T}| = \mathcal{O}(n^{2/3})$ [1]

[AZ95] Dragan M. Acketa and Jovisa D. Zunic.
On the maximal number of edges of convex digital polygons included into a m x m-grid.
Journal of Combinatorial Theory, Series A, 69 :358–368, 1995.

[CGRT04] David Coeurjolly, Yan Gérard, Jean-Pierre Reveillès, and Laure Tougne.
An elementary algorithm for digital arc segmentation.
Discrete Applied Mathematics, 139(1-3) :31–50, 2004.

What are $\mathcal{S}$ and $\mathcal{T}$? second solution



Algorithm

1. compute $\mathcal{S}$ and $\mathcal{T}$ in $\mathcal{O}(n)$.
2. compute $\mathcal{S}' = \{(x_s, y_s, {x_s}^2 + {y_s}^2)\}$ and $\mathcal{T}' = \{(x_t, y_t, {x_t}^2 + {y_t}^2)\}$.
3. compute the 3D convex hull of $\mathcal{S}'$ and $\mathcal{T}'$, denoted by $CH(\mathcal{S}')$ and $CH(\mathcal{T}')$ respectively in $\mathcal{O}(m \log m)$ [3].
4. compute h, the vertical distance between $CH(\mathcal{S}')$ and $CH(\mathcal{T}')$ in $\mathcal{O}(m \log m)$ [3].

[PS85] Franco P. Preparata and Michael I. Shamos.
Computational geometry : an introduction.
Springer, 1985.

Conclude according to the sign of h.

- if $h < 0$, $CH(\mathcal{S}') \cap CH(\mathcal{T}') = \emptyset$, $\mathcal{S}$ and $\mathcal{T}$ are separable by a circle, X is a digital circle.
- if $h \geq 0$, $CH(\mathcal{S}') \cap CH(\mathcal{T}') \neq \emptyset$, $\mathcal{S}$ and $\mathcal{T}$ are not separable by a circle, X is not a digital circle.

Is h a measure of circularity ?

Outline

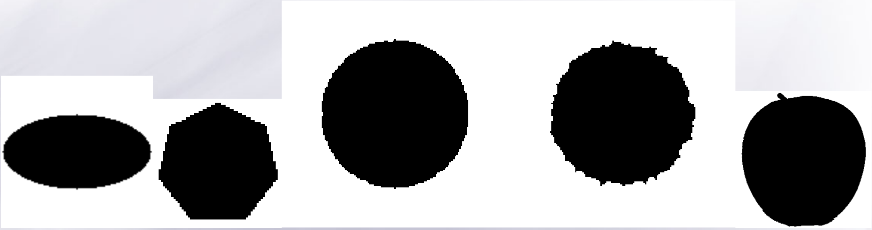
Impact of rigid transformations onto the z-coordinate

- rotation of center the origin and angle θ :
  $f(x_{\hat{p}}, y_{\hat{p}}) = f(x_p, y_p)$.
- scaling by a factor α : $f(x_{\hat{p}}, y_{\hat{p}}) = \alpha^2 f(x_p, y_p)$.

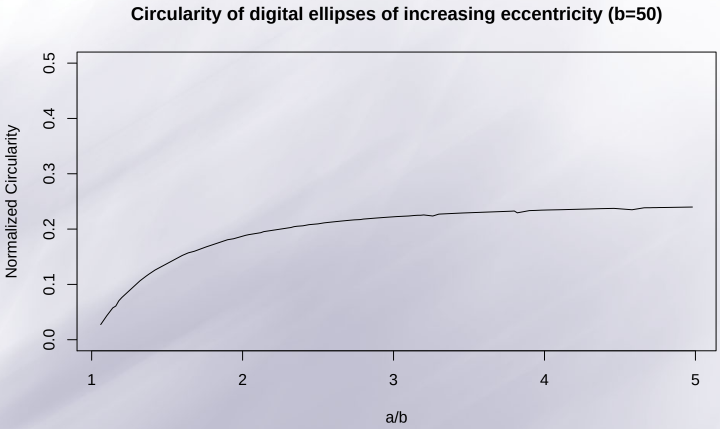
  $h^* = h/d$ where d is the diameter of X.
- translation by a vector $v(x_v, y_v)$:
  $f(x_{\hat{p}}, y_{\hat{p}}) = f(x_p, y_p) - 2x_p x_v - 2y_p y_v - x_v^2 - y_v^2$.

  The origin is set to the barycenter of X.

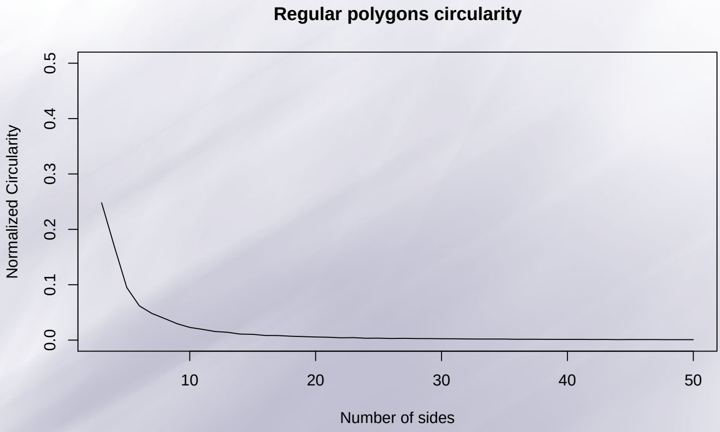
Descriptive behavior



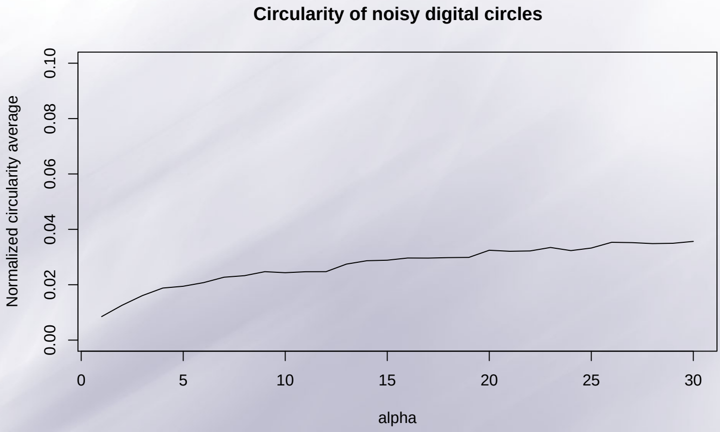
Ellipses of increasing eccentricity



Regular polygons of increasing number of sides



Noisy circles



Outline

1. Data and goals
2. The separating circle problem
3. Test of circularity
4. Measure of circularity
5. Conclusion

Conclusion and perspectives

- Only one geometric algorithm for :
  - Test of circularity in $\mathcal{O}(n)$
  - Measure of circularity in $\mathcal{O}(n)$ for convex digital curves, and $\mathcal{O}(n \log n)$ otherwise.
- A circularity measure that :
  - fulfils basic invariance properties.
  - reflects fairly well the visual dissimilarity between a digital curve and a digital circle.
  - equals 0 for any digital circle and is greater for the digital curves that are not digital circles.
  - may be applied on open digital curves.

C’est fini