Maker-Breaker Domination Number

Valentin Gledel

CGTC 3, Lisboa

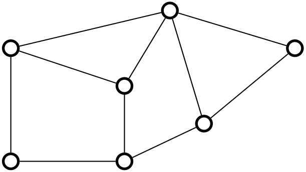
January 24, 2019

Joint work with Vesna Iršič and Sandi Klavžar

Domination in graphs

Let $G = (V, E)$ be a graph and $S \subseteq V$.

S dominates G if all vertices of G are in S or adjacent to a vertex of S.



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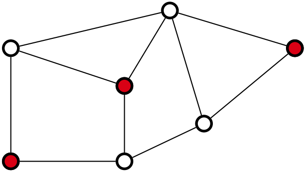

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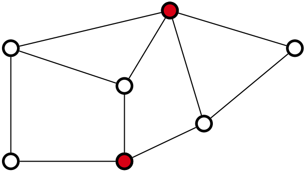

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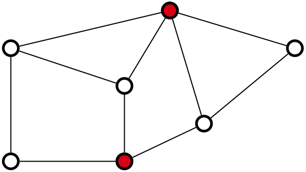

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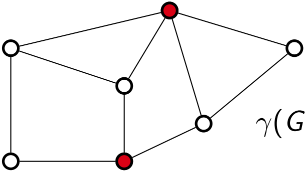

$\gamma(G) = 2$

The objective is to find $\gamma(G)$, the size of a minimum dominating set in G

Domination Game

(Brešar, Klavžar and Rall, 2010)

- Two players: Dominator and Staller
- Alternately select a vertex of the graph that dominates at least one new vertex.
- Dominator wants the dominating set to be small.
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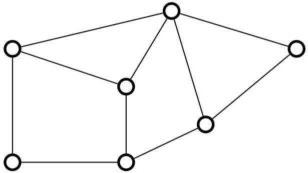
γ_g : Size of the obtained dominating set



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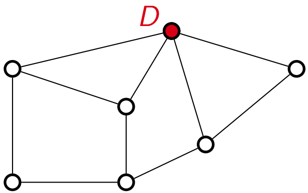

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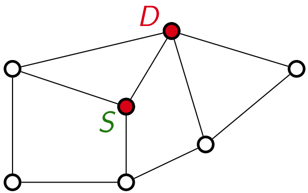

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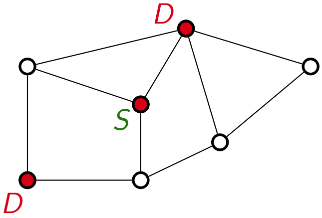

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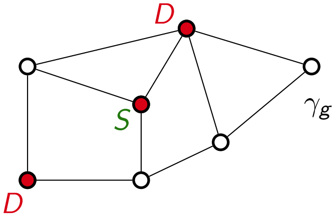

$\gamma_g(G) = 3$

Maker-Breaker Domination Game

(Duchêne, G, Parreau and Renault, 2018+)

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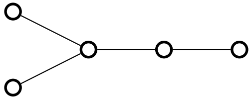

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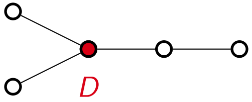

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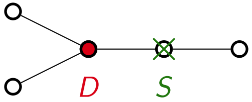

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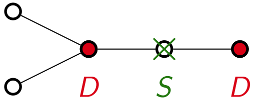

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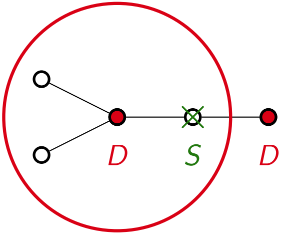

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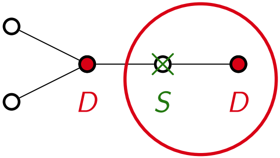

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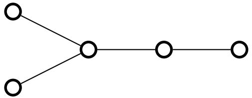

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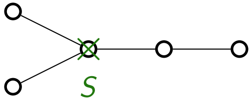

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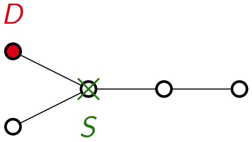

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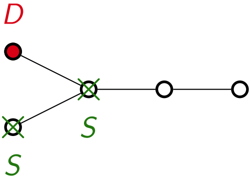

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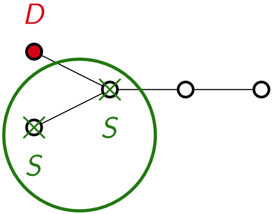

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- A variant of the general Maker-Breaker games
  (see J. Beck 2008 for a survey)
- Solved for the union and the join
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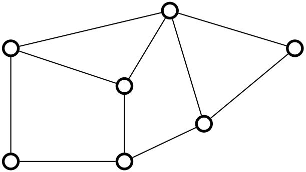
One open question that we will cover today : How many moves are needed to win ?

Maker-Breaker Domination Number

Can Dominator win on this graph ? If yes in how many moves ?



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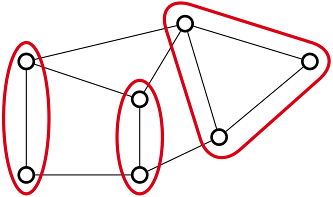

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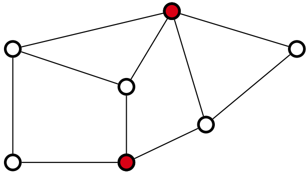

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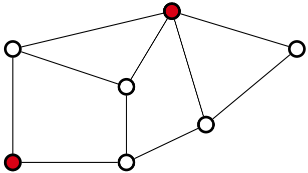

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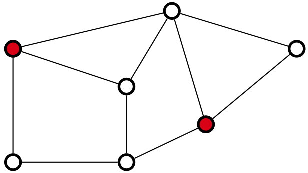

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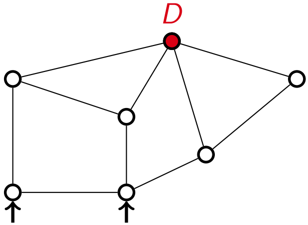

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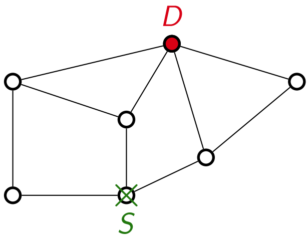

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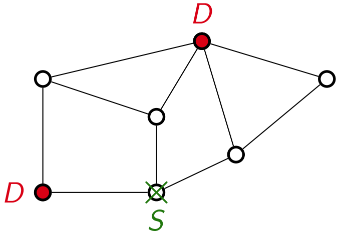

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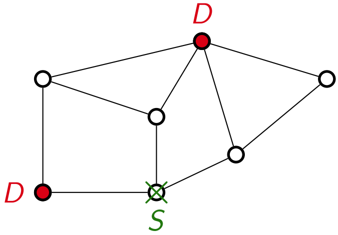

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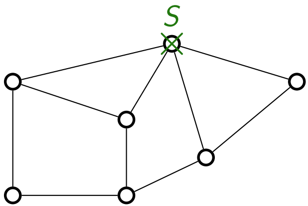

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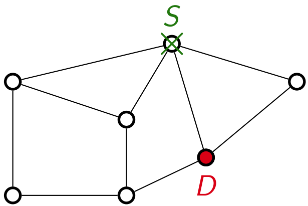

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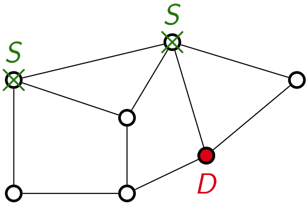

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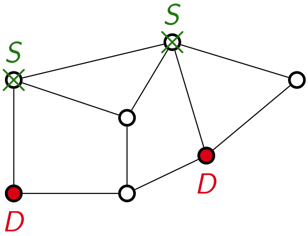

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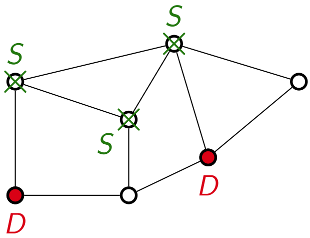

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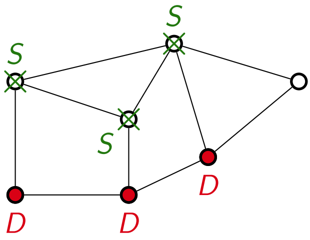

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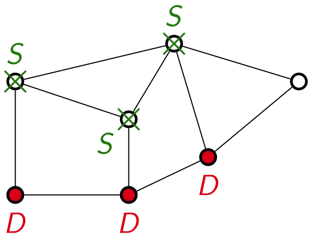


$$\gamma_{MB}(G) = 2$$

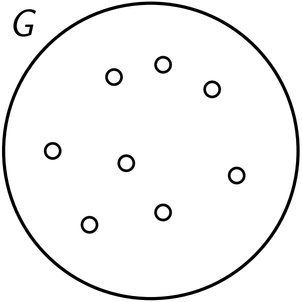
$$\gamma'_{MB}(G) = 3$$

Possible outcomes

Theorem

Let G be a graph, $\gamma_{\mathrm{MB}}(G) \leq \gamma'_{\mathrm{MB}}(G)$

Assume that Dominator has a strategy to achieve $\gamma'_{\mathrm{MB}}(G) = a$



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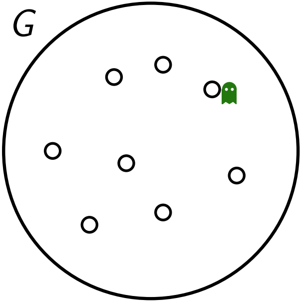

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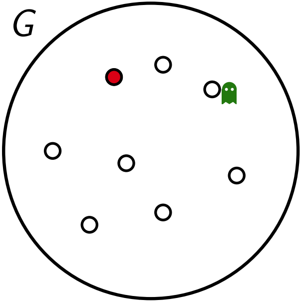

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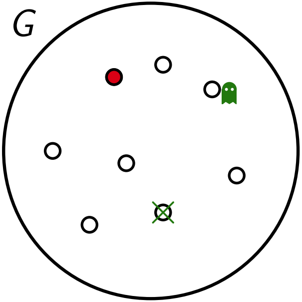

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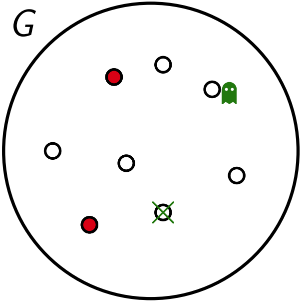

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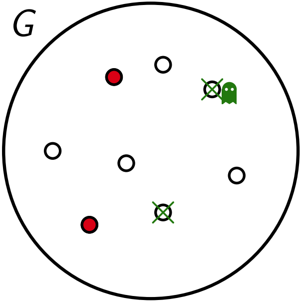

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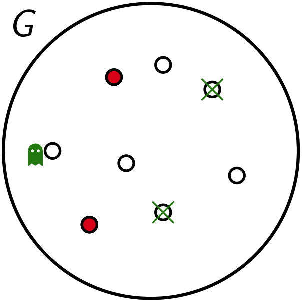

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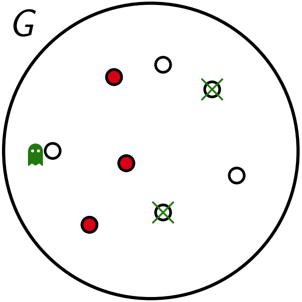
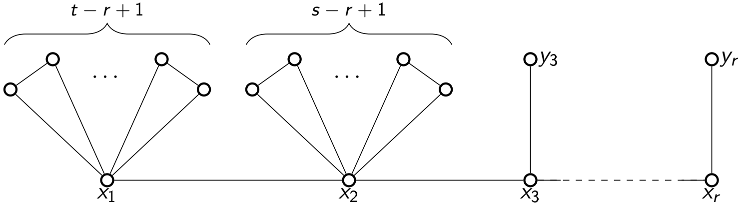


What are the possible values ?

Theorem

For any $2 \leq r \leq s \leq t$, there exists a graph G such that $\gamma(G) = r$, $\gamma_{\mathrm{MB}}(G) = s$, and $\gamma'_{\mathrm{MB}}(G) = t$.



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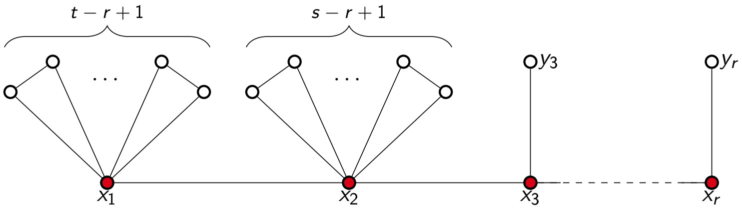

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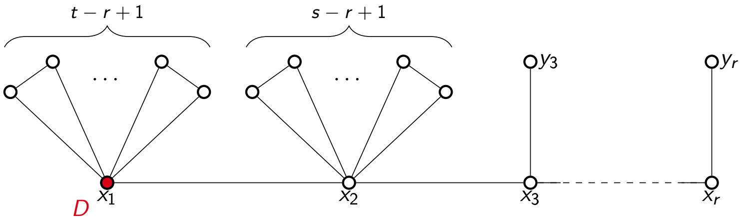
For any $2 \leq r \leq s \leq t$, there exists a graph G such that $\gamma(G) = r$, $\gamma_{\mathrm{MB}}(G) = s$, and $\gamma'_{\mathrm{MB}}(G) = t$.



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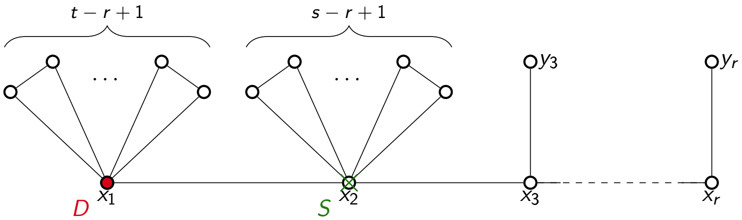

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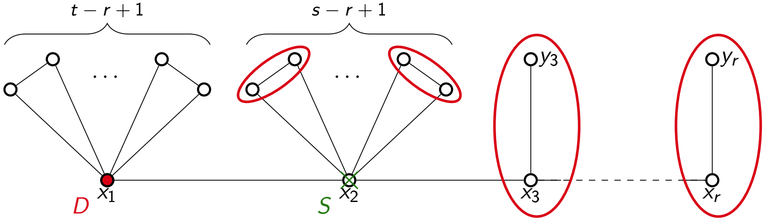

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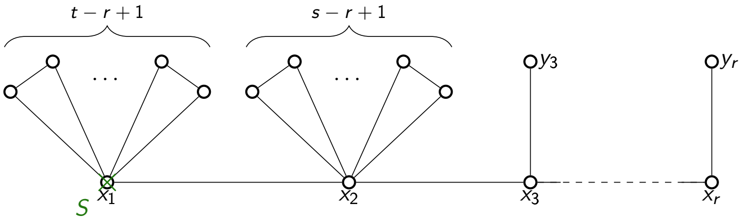

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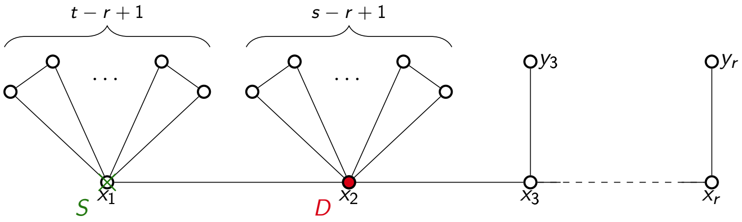

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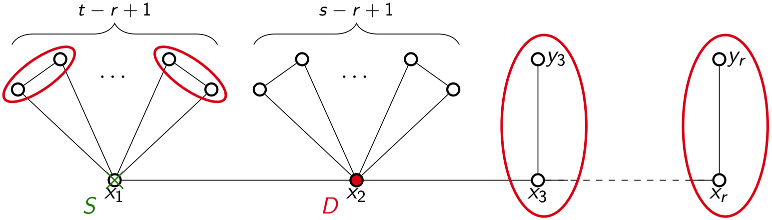

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Maker-Breaker domination number on cycles

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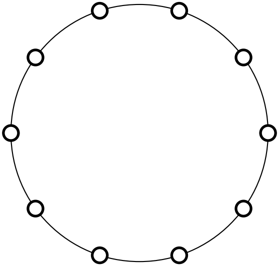

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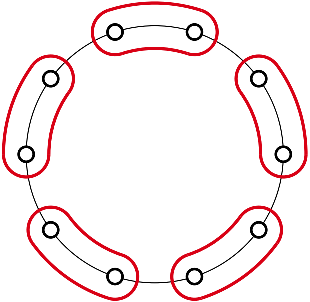

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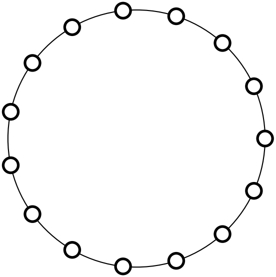

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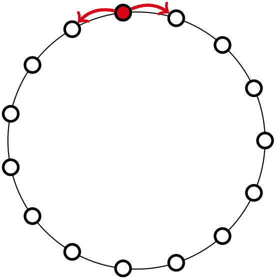

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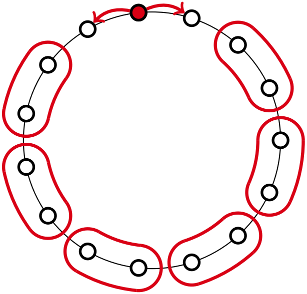

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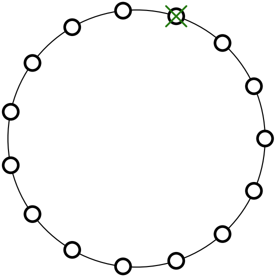

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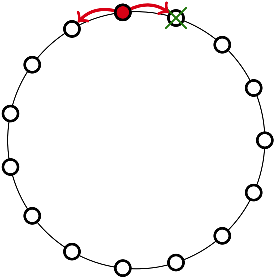

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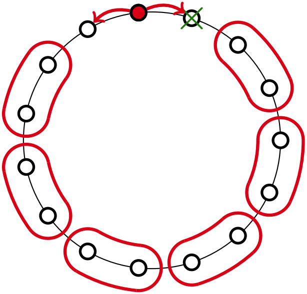

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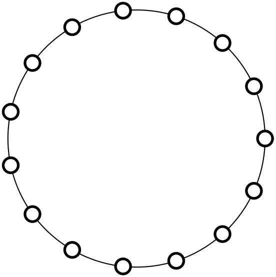

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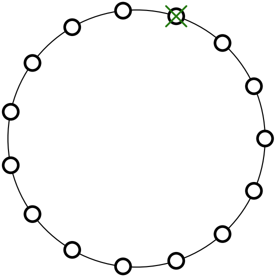

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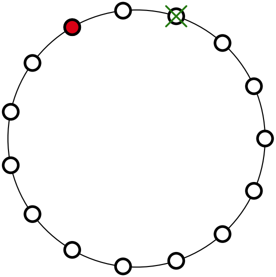

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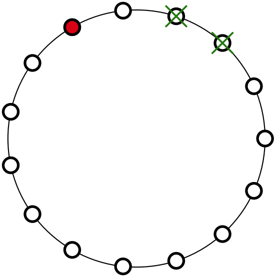

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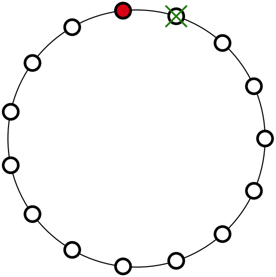

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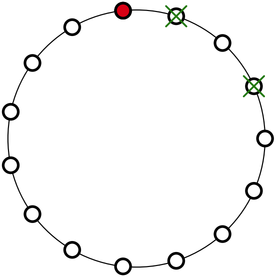

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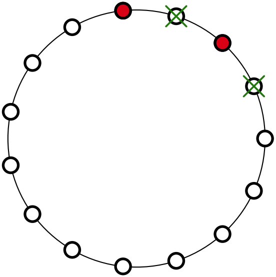

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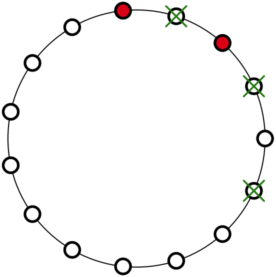

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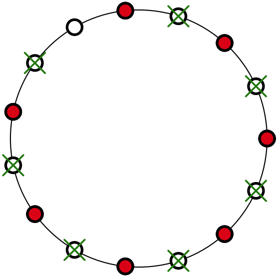

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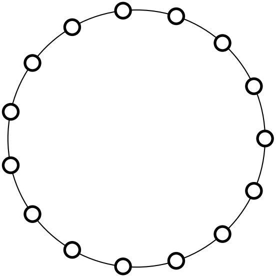

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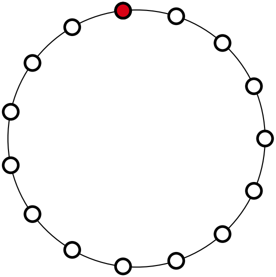

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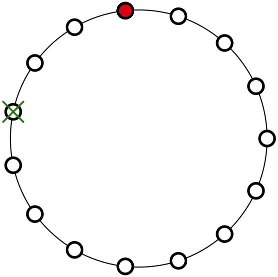

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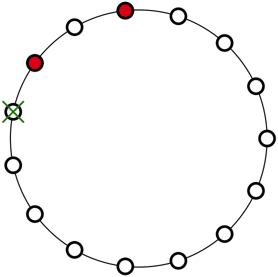

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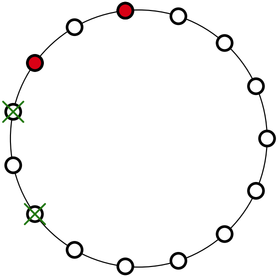

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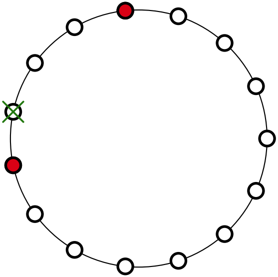

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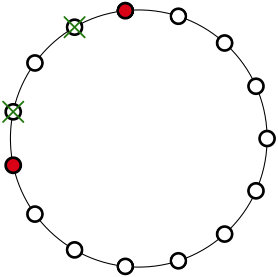

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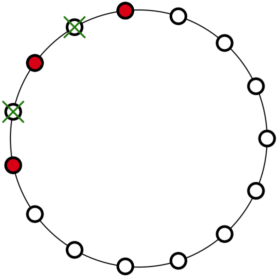

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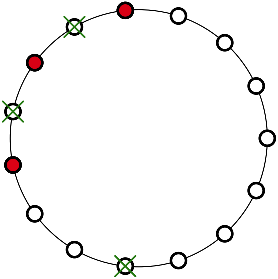

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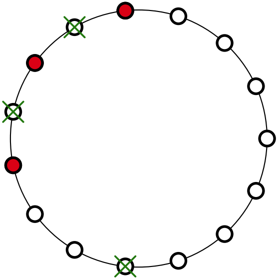

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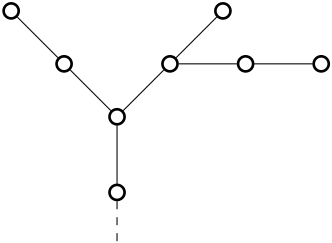
Residual graphs

Idea : Reducing the graph by removing P_2's.



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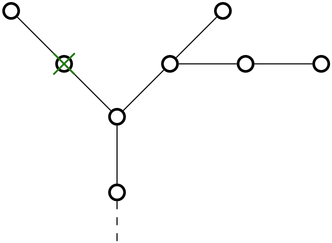

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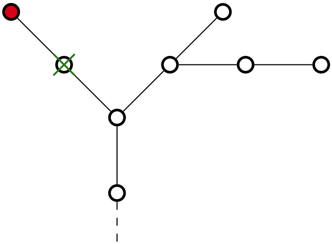

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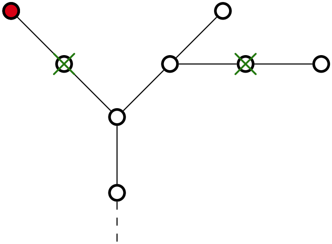

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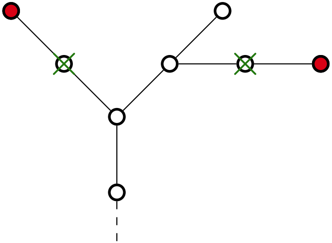

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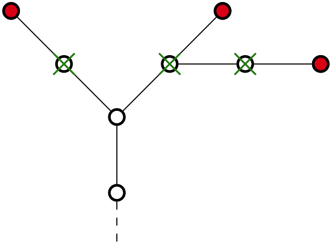

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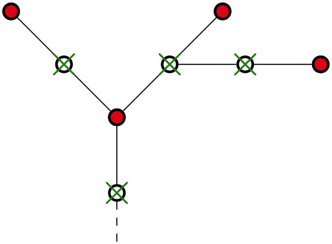

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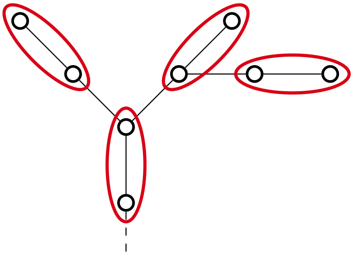
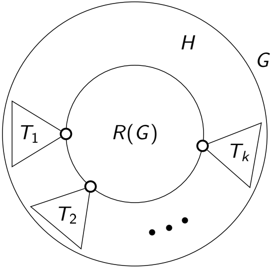


Residual graphs

Definition

Let G be a graph. The **residual graph** of G, $R(G)$, is the graph obtained by iteraltely removing pendant P_2's from G.



Residual graphs

Theorem

Let G be a graph, $\gamma'_{\mathrm{MB}}(G) = \gamma'_{\mathrm{MB}}(R(G)) + \frac{|V(H)|}{2}$,

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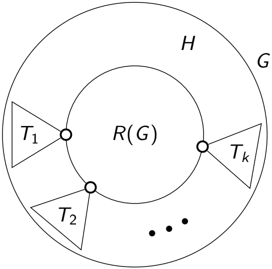

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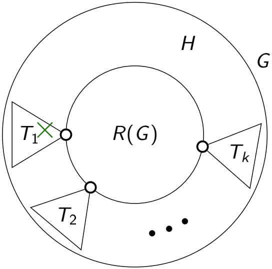

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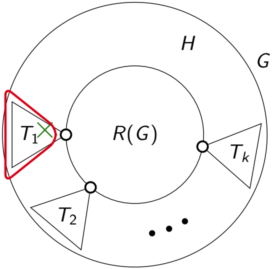

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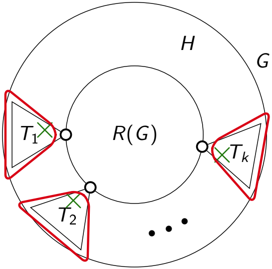

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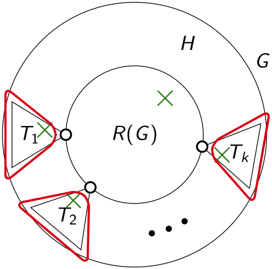

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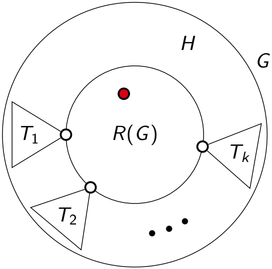

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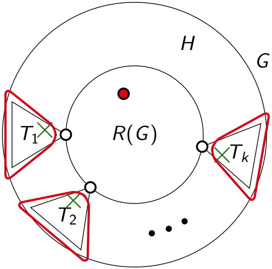

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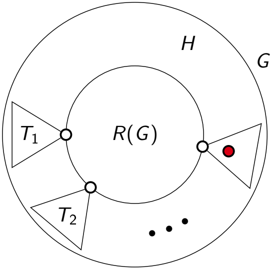

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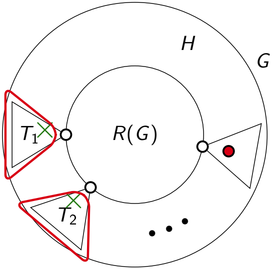

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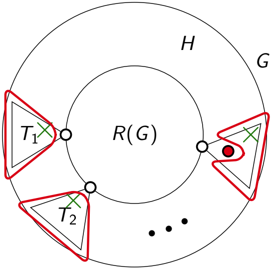

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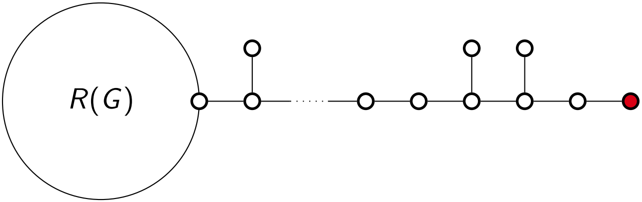

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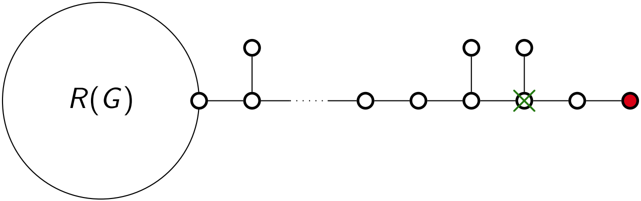

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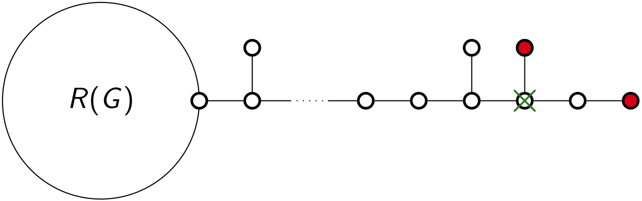

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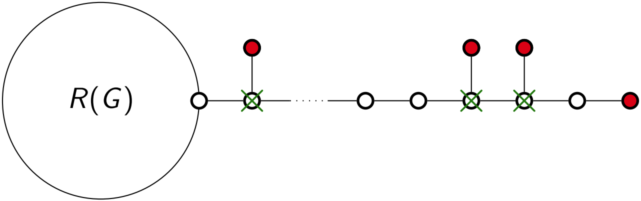

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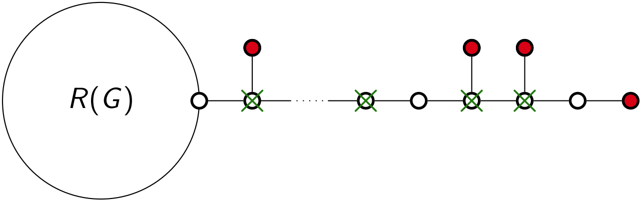

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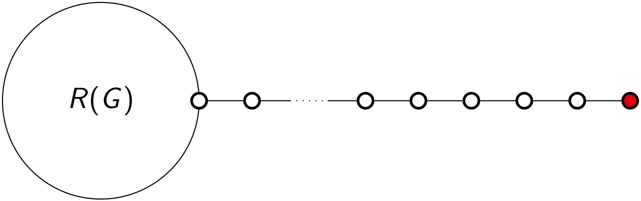

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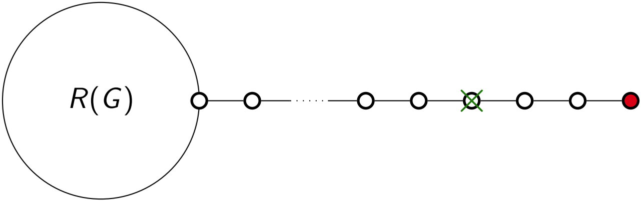

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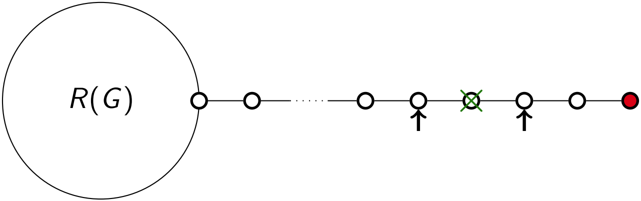

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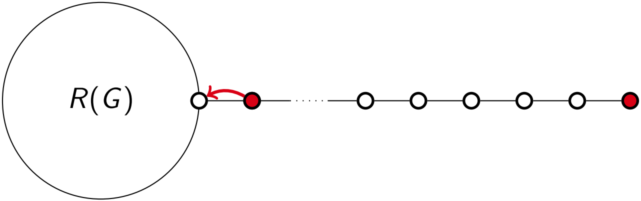

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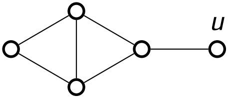
The lower bound is reached when $\gamma'_{\mathrm{MB}}(G|u) = \gamma_{\mathrm{MB}}(G) - 1$.



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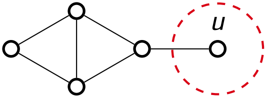

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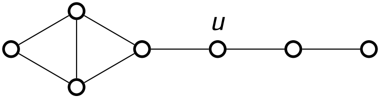

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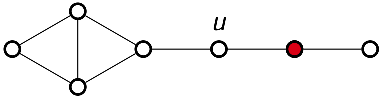
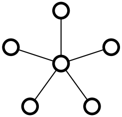
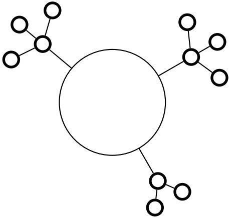


Trees

Maker-Breaker domination number for trees:

$R(T)$	$\emptyset$	 $K_{1,0}$	 $K_{1,n}$	
$\gamma_{\mathrm{MB}}(T)$				
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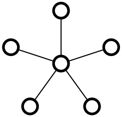
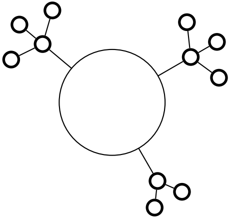
$R(T)$	$\emptyset$	 $K_{1,0}$	 $K_{1,n}$	
$\gamma_{\mathrm{MB}}(T)$	$\frac{\lvert V(T)\rvert}{2}$			
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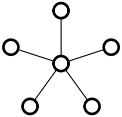
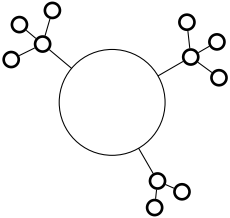
$R(T)$	$\emptyset$	 $K_{1,0}$	 $K_{1,n}$	
$\gamma_{\mathrm{MB}}(T)$	$\frac{\lvert V(T)\rvert}{2}$	$\left\lfloor \frac{\lvert V(T)\rvert}{2} \right\rfloor$		
$\gamma'_{\mathrm{MB}}(T)$	$\frac{\lvert V(T)\rvert}{2}$	∞		

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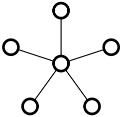
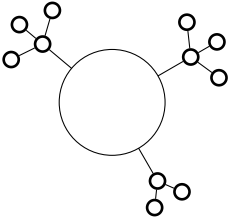
$R(T)$	$\emptyset$	 $K_{1,0}$	 $K_{1,n}$	
$\gamma_{\rm MB}(T)$	$\frac{\lvert V(T)\rvert}{2}$	$\left\lfloor \frac{\lvert V(T)\rvert}{2} \right\rfloor$	$\frac{\lvert V(T)\rvert - n + 1}{2}$	
$\gamma'_{\rm MB}(T)$	$\frac{\lvert V(T)\rvert}{2}$	∞	∞	

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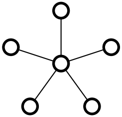
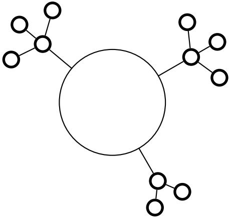
$R(T)$	$\emptyset$	 $K_{1,0}$	 $K_{1,n}$	
$\gamma_{\mathrm{MB}}(T)$	$\frac{\lvert V(T)\rvert}{2}$	$\left\lfloor \frac{\lvert V(T)\rvert}{2} \right\rfloor$	$\frac{\lvert V(T)\rvert - n + 1}{2}$	∞
$\gamma'_{\mathrm{MB}}(T)$	$\frac{\lvert V(T)\rvert}{2}$	∞	∞	∞

Perspectives

- Maker-Breaker domination numbers of cographs ?
- Maker-Breaker domination numbers of cartesian products or other operations on graphs ?
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