PhD defense:
Vertex covering under constraints

Valentin Gledel

under the supervision of Éric Duchêne and Aline Parreau

24/09/2019
Why you are all here
Why you are all here
Why you are all here
Why you are all here
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Pictures are non-contractual. Actual buffet might differ.
A mathematical problem

How can I satisfy everybody?
A mathematical problem

How can I satisfy everybody?

What is the minimum number of meals that can be selected so that everyone has something to eat?
A mathematical problem

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What is the minimum number of meals that can be selected so that everyone has something to eat?
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How can I satisfy everybody?

What is the minimum number of sets that can be selected so that every points is inside one set?
Set cover

Given a hypergraph $\mathcal{H} = (X, \mathcal{F})$, the goal is to find a minimal subset $\mathcal{F}'$ of $\mathcal{F}$ such that every vertex of $X$ is in one hyperedge of $\mathcal{F}'$. 

Set cover

This problem is NP-complete (Karp, 1972).
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Set cover is a very general problem, we can often restrict ourselves to a more constraint structure.
Structure

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Graph structure

![Graph structure diagram]
Structure

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Geometric structure

- hyperedges are sets of points that can be covered by the same disk
My PhD

Set cover

Structure:
- Graphs
- Geometric

Variation:
- Propagation
- Game
- Identification

Strong geodetic number
Maker-Breaker domination game
Power Domination
Identification of points using disks
My PhD

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Maker-Breaker Domination Game
Domination in graphs (Ore, 1961)

Let $G = (V, E)$ be a graph and $S$ be a subset of $V$. $S$ dominates $G$ if all vertices of $G$ are in $S$ or adjacent to a vertex of $S$. 

![Graph Diagram]

8/49
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![Graph diagram with vertex subset highlighted in red]
# Domination games

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- **Game domination number**
  - Two players: Dominator and Staller
  - Alternate select a vertex of the graph that dominates at least one new vertex
  - Dominator wants the dominating set to be small
  - Staller wants it to be large

Determining the number of moves in an optimal game of the domination game is **PSPACE-complete** (Brešar et al., 2016).
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- **Domination game**: Two players: Dominator and Staller. They alternately select a vertex of the graph that dominates at least one new vertex. The Dominator wants the dominating set to be small, and the Staller wants it to be large.

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![Diagram of a graph with vertex D dominating other vertices]
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Maker-Breaker domination game
(Duchêne, G., Parreau and Renault, 2018+)

- Played on a graph $G = (V, E)$
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```
G = (V, E)

V = {D, S, D}
E = {{D, S}, {S, D}}
```
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![Diagram of Maker-Breaker domination game]
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The problem

The goal is to decide which player has a winning strategy.
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The possible outcomes are the following:

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Maker-Breaker games

- Played on an hypergraph \((X, \mathcal{F})\).
- Two players: Maker and Breaker.
- They alternately select vertices of \(X\).
- Maker wins if and only if he selected all the vertices of a hyperedge \(A \in \mathcal{F}\).

- Hex
  - Maker has a winning strategy (Nash, 1952)
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- \(\mathcal{F} = \{\text{the dominating sets}\}\),
  - Dominator = Maker.
Maker-Breaker games

- Played on an hypergraph \((X, F)\).
- Two players: **Maker** and **Breaker**.
- They alternately select vertices of \(X\).
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The Maker-Breaker Domination game is a Maker Breaker game.

- \(F = \{\text{the dominating sets}\}\),
  - **Dominator** = **Maker**.
- \(F = \{\text{the closed neighborhoods}\}\),
  - **Staller** = **Maker**.
Maker-Breaker games

**Theorem (Folklore)**

If **Maker** wins the Maker-Breaker game on \((X, F)\) as the second player, then he also wins as first player.
Maker-Breaker games

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→ It is never interesting to pass
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**Theorem (Schaefer, 1978)**

Deciding the outcome of Maker-Breaker is a PSPACE-complete problem.
There exist graphs for the three possible outcomes.

- Dominator starts
- Staller starts
Outcomes

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\[ N \quad D \quad S \]
A winning condition for Dominator

Lemma

If a graph can be partitioned into cliques of size at least 2 then its outcome is $\mathcal{D}$. 
A winning condition for Dominator

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If a graph can be partitioned into cliques of size at least 2 then its outcome is $D$. 

$K_2 \times K_2 \times K_4 \times 15/49$
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[Diagram showing a graph partitioned into cliques $K_4$ and $K_2$.]
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![Diagram](image-url)
Pairing dominating sets

Definition (Duchêne, G., Parreau, Renault, 2018+)

A set of pairs of vertices \( \{(u_1, v_1), \ldots, (u_k, v_k)\} \) is a pairing dominating set if:

- all vertices are distinct,
- \( V = \bigcup_{i=1}^{k} N[u_i] \cap N[v_i] \).

\( G \) has a pairing dominating set \( \iff \) \( G \) has outcome \( \mathcal{D} \).
Pairing dominating sets

Theorem
Deciding if a graph admits a pairing dominating set is an NP-complete problem.

The proof uses a reduction from SAT.

Gadget for a variable
Pairing dominating sets

There exist graphs of outcome $D$ that do not admit pairing dominating sets.
Deciding the outcome of a Maker-Breaker domination game position is a PSPACE-complete problem.

This result is proved by reduction from Maker-Breaker games which are PSPACE-complete (Schaeffer, 1978).
Complexity

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Dominator follows Breaker’s strategy
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![Diagram of Maker-Breaker game with nodes and edges illustrating the strategy of Dominator following Breaker's strategy.](image)

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Staller follows Maker's strategy
Complexity

- Graphs
- Bipartite graphs
- Chordal graphs
- Split graphs
- Paths

PSPACE-c
Complexity

Graphs

Bipartite graphs

Chordal graphs

Split graphs

Trees

Paths

PSPACE-c

P
For paths, removing $P_2$’s preserves the outcome.
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\[ P_5 \rightarrow P_3 \]
For paths, removing $P_2$’s preserves the outcome.

Is it still true for other graphs?
Glue operator

We "glue" two graphs on a vertex.
Glue operator

We "glue" two graphs on a vertex.
Glue operator

We want to find the couples \((G, u)\) such that for all \(H\),
\[ G_u \cong H \equiv H: \]

\[ G \quad u \]

\[ G \quad u \equiv H \equiv H: \]
A graph \((G, u)\) is neutral for the glue operator if and only if

- \(G\) has outcome \(\mathcal{N}\)
- \(G \setminus \{u\}\) has outcome \(\mathcal{D}\)
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Theorem

Maker-Breaker Domination Game is polynomial on trees.

Removing pendant $P_2$’s can be done in polynomial time.
Theorem

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Removing pendant $P_2$'s can be done in polynomial time.
Trees

**Theorem**

Maker-Breaker Domination Game is polynomial on trees.

Any tree can be reduced to one of the following configurations:

- $\emptyset$ (empty graph)
- $K_{1,0}$
- $K_{1,n}$
- $T$
Trees

Theorem

Maker-Breaker Domination Game is polynomial on trees.

Any tree can be reduced to one of the following configurations:

- \( \emptyset \) empty graph
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- \( S \)
Complexity

- Graphs
  - Bipartite graphs
  - Trees
  - Paths
  - Chordal graphs
  - Split graphs

- PSPACE-c

- P

PDS property: Having outcome $D \iff$ having a pairing dominating set
Complexity

Graphs

- Cographs: P
- Bipartite graphs: PSPACE-c
- Trees: P
- Paths: P

Chordal graphs: PSPACE-c

Split graphs: PSPACE-c
Complexity

- Cographs
- Bipartite graphs
- Trees
- Paths
- Chordal graphs
- Split graphs
- Interval graphs
- k-Trees

PSPACE-c

P

P

P

?
PDS property(*): Having outcome $\mathcal{D}$ $\iff$ having a pairing dominating set
PDS property(*): Having outcome $\mathcal{D} \iff$ having a pairing dominating set
Other works and perspectives

Other works

- The Maker-Breaker domination numbers
  (G., Iršič and Klavžar, 2019)
  - The difference between the "Dominator starts" and the "Staller starts" values are unbounded
  - PSPACE-complete
  - Solved for cycles and trees

- The Maker-Breaker total domination game
  (Henning, G., Iršič and Klavžar, 2019)
  - Solved on cacti

- The Avoider-Enforcer domination game
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Other works and perspectives

Other works

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Perspectives

• Maker-Breaker domination numbers of cographs
• Study of the pairing dominating sets
Identification of points using disks
Set cover

Structure:
- Graphs
- Geometric

Variation:
- Propagation
- Game
- Identification

Strong geodetic number
Maker-Breaker domination game
Power Domination
Identification of points using disks
Back to the post-defense buffet

I found a plate. Who does it belong to?
Back to the post-defense buffet

I found a plate. Who does it belong to?
I found a plate. Who does it belong to?

We can **identify** the right guest.
Identification in hypergraphs

Two goals:

- Covering
- Separation
Identification in hypergraphs

Two goals:

- Covering
- Separation
Linked problems

- Test cover (Moret and Shapiro, 1985)

- Identifying codes in graphs (Karpovsky, Chakrabarty and Levitin, 1998)
  - Unit disk graphs (Müller and Sereni, 2009)
  - Unit interval graphs (Foucaud, Mertzios, Naserasr, Parreau and Valicov, 2015).
Identification of points with disks
(G. and Parreau, 2019)

**Input of the problem**

A set $\mathcal{P}$ of points in the plane

**Output**

A set $\mathcal{D}$ of closed disks verifying:
- Every point of $\mathcal{P}$ must belong to at least one disk of $\mathcal{D}$. (Covering)
- Two points of $\mathcal{P}$ must belong to two different subsets of $\mathcal{D}$. (Separation)

$\gamma_D^{ID}(\mathcal{P})$: Minimal number of disks necessary to identify $\mathcal{P}$. 

Separation of points using convex sets (Gerbner and Toth, 2012)
Separation of points using lines parallels to the axis (Calinescu, Dumitrescu, Karlo and Wan, 2005)
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Lower bound

**Theorem (Folklore)**

Putting $k$ disks in the plane defines at most $k^2 - k + 1$ intersection areas.
**Lower bound**

**Theorem (Folklore)**

Putting $k$ disks in the plane defines at most $k^2 - k + 1$ intersection areas.

**Corollary**

Let $\mathcal{P}$ be a set of $n$ points in the plane

$$\gamma^ID_P(\mathcal{P}) \geq \left\lceil \frac{1 + \sqrt{1 + 4(n-1)}}{2} \right\rceil \sim \sqrt{n}.$$
Upper bound

**Theorem (Adapted from Gerbner and Toth, 2012)**

Let $\mathcal{P}$ be a set of $n$ points in the plane, $\gamma_D^\text{ID}(\mathcal{P}) \leq \lceil \frac{n+1}{2} \rceil$. 
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![Diagram](image-url)
Let $\mathcal{P}$ be a set of $n$ points in the plane, $\gamma_D^*(\mathcal{P}) \leq \lceil \frac{n+1}{2} \rceil$. 

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**Theorem (Adapted from Gerbner and Toth, 2012)**

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Let $\mathcal{P}$ be a set of $n$ points in the plane, $\gamma_D^D(\mathcal{P}) \leq \left\lceil \frac{n+1}{2} \right\rceil$. 

Theorem (Adapted from Gerbner and Toth, 2012)
Let $\mathcal{P}$ be a set of $n$ points in the plane, $\gamma_D^L(\mathcal{P}) \leq \lceil \frac{n+1}{2} \rceil$. 

Theorem (Adapted from Gerbner and Toth, 2012)
The points are colinear

Let $\mathcal{P}$ be a set of $n$ collinear points, $\gamma_{D}^{ID}(\mathcal{P}) = \lceil \frac{n+1}{2} \rceil$.

The disks go through $n+1$ areas on the same line to cover each point and separate each pair of points.
The points are colinear

**Theorem**

Let \( \mathcal{P} \) be a set of \( n \) colinear points, \( \gamma_D^I(\mathcal{P}) = \lceil \frac{n+1}{2} \rceil \).

The disks go through \( n + 1 \) areas on the same line to cover each point and separate each pair of points.
Upper bound in general configuration

The previous upper bound is tight for colinear and cocyclic sets of points.

**Theorem**

Let $\mathcal{P}$ be a set of $n$ points such that no three points are colinear and no four points are cocyclic, $\gamma^\text{ID}_D(\mathcal{P}) \leq 2\lceil \frac{n}{6} \rceil + 1$. 
Principle

Same principle as the previous proof:

• Separating the points into equal size areas using lines
• Iteratively separating points from each area with disks
Principle

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- Separating the points into equal size areas using lines
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  - Use of Delaunay’s triangulation
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Upper bound in the general case

**Theorem (J. G. Ceder, 1964)**

Let $\mathcal{P}$ be a set of $n$ points in the plane such that no three points are colinear. Using three concurrent lines, it is possible to divide the plane into six areas containing between $\lceil \frac{n}{6} \rceil - 1$ and $\lceil \frac{n}{6} \rceil$ points.
Upper bound in the general case

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<thead>
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</thead>
<tbody>
<tr>
<td>Radii</td>
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- Unit disk graph: NP-complete
- Unit interval graph: ?

- Müller and Sereni, 2009
- Foucaud, Mertzios, Naserasr, Parreau and Valicov, 2015
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<td>?</td>
<td>$O(1)$</td>
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<td>Unit disk graph (NP-complete)</td>
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- Müller and Sereni, 2009
- Foucaud, Mertzios, Naserasr, Parreau and Valicov, 2015
The following problem is NP-complete:

**Instance:** A set $\mathcal{P}$ of points in the plane and a number $k \in \mathbb{N}$.

**Question:** Is it possible to identify the points of $\mathcal{P}$ using $k$ disks of radius 1?

The proof uses a reduction from $P_3$-partition in grid graphs, a NP-complete problem. The $P_3$’s become the following structure:
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Colinear points and fixed radius

Theorem

The following problem can be solved in linear time:

**Instance:** A set \( \mathcal{P} \) of colinear points and a number \( k \in \mathbb{N} \).

**Question:** Is it possible to identify the points of \( \mathcal{P} \) using \( k \) disks of radius 1?
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The proof has two steps:

- Showing that there always exists a minimum identifying set of disks in normal form,
- Using a greedy algorithm to find such a set.
Colinear points and fixed radius

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The proof has two steps:

- Showing that there always exists a minimum identifying set of disks in **normal form**,
- Using a greedy algorithm to find such a set.
A set of disk is in **normal form** if each connected component of $\mathcal{P}$ is of odd size $k$ and:

- the first and last disks contain exactly two points,
- all the other disks contain exactly three points.
Colinear points and fixed radius

Each minimum identifying set of disks can be transformed:
Colinear points and fixed radius

Each minimum identifying set of disks can be transformed:

- By showing that we can divide the connected component so that they are of odd size and identified by $\frac{k+1}{2}$ disks,
Colinear points and fixed radius

Each minimum identifying set of disks can be transformed:

- By showing that we can divide the connected component so that they are of odd size and identified by $\frac{k+1}{2}$ disks,
- Then showing that each of these connected component can be in normal form.
### Complexity

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<tr>
<td>points**</td>
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Perspectives

- Random disposition of points
- Validity of the results for other shapes or for higher dimensions
General conclusion

<table>
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- G., Iršič and Klavžar,
  *Strong geodetic cores and Cartesian product graphs*,
  Applied Mathematics and Computation, 2019
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Strong geodetic number
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- Duchêne, G., Parreau and Renault,
  *Maker-Breaker domination game*,
- G., Iršič and Klavžar,
  *Maker-Breaker domination number*,
  Bulletin of the Malaysian Mathematical Sciences Society, 2019
- Henning, G., Iršič and Klavžar,
  *Maker-Breaker total domination game*,
  To be published, 2019
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Strong geodetic number
Maker-Breaker domination game
Power Domination
Identification of points using disks

- G. and Iršič,
  *Strong geodetic number of complete bipartite graphs, crown graphs and hypercubes,*
  To be published, 2018.

- G., Iršič and Klavžar,
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- Bose, G., Pennarun and Verdonschot,
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- Duchêne, G., Parreau and Renault,
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- Henning, G., Iršič and Klavžar,
  *Maker-Breaker total domination game,*
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General conclusion

**Strong geodetic number**

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Thank you