PhD defense:
Vertex covering under constraints

Valentin Gledel

under the supervision of Éric Duchêne and Aline Parreau

24/09/2019

Why you are all here

Why you are all here

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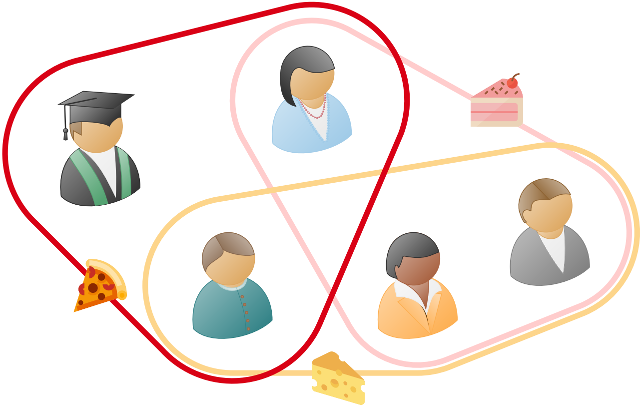
Pictures are non-contractual. Actual buffet might differ.

A mathematical problem

How can I satisfy everybody?



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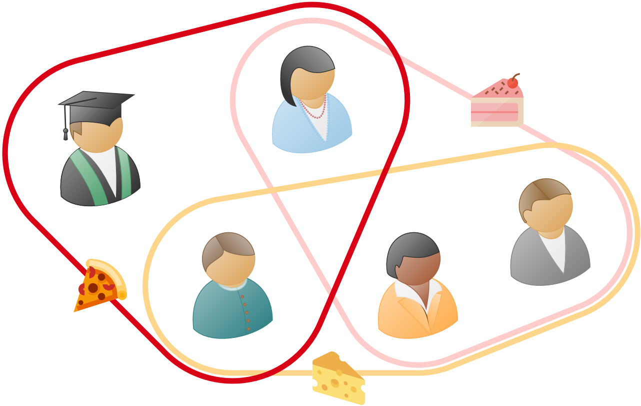

What is the minimum number of meals that can be selected so that everyone has something to eat?

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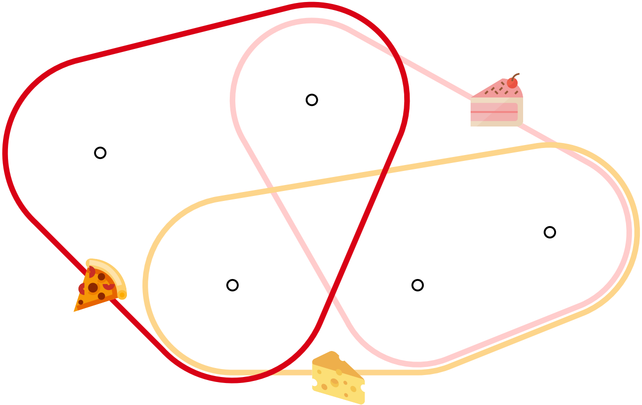

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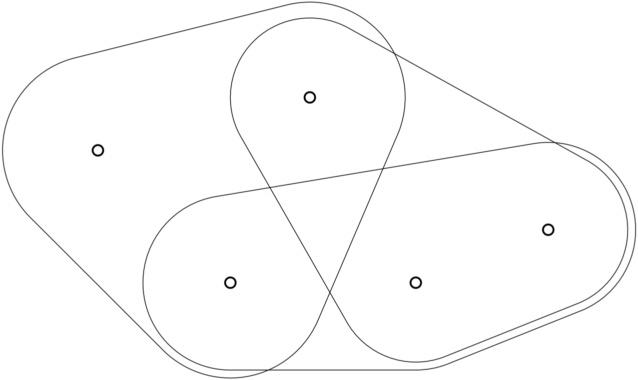

What is the minimum number of **sets** that can be selected so that every **points** is inside one set?

Set cover

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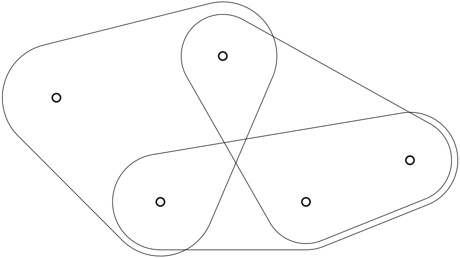
Given a hypergraph $\mathcal{H} = (X, \mathcal{F})$, the goal is to find a minimal subset $\mathcal{F}'$ of $\mathcal{F}$ such that every vertex of X is in one hyperedge of $\mathcal{F}'$.



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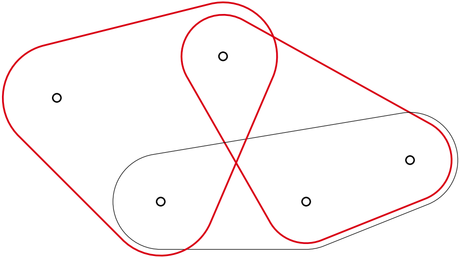

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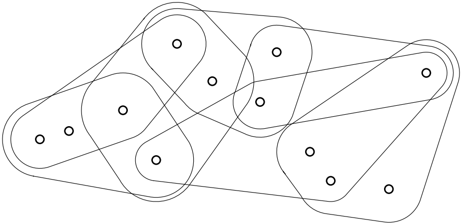

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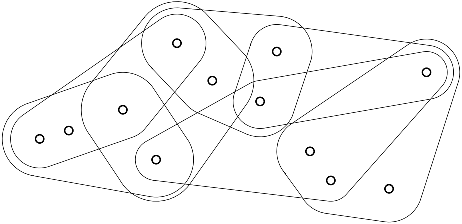

- This problem is NP-complete (Karp, 1972)

Structure

Set cover is a very general problem, we can often restrict ourselves to a more constraint structure.

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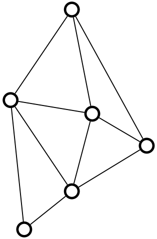
Graph structure



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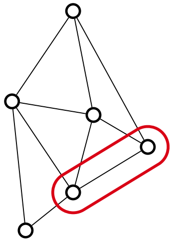

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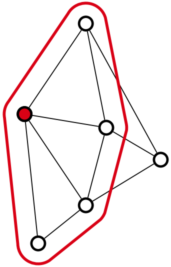

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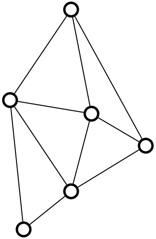

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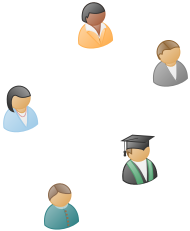

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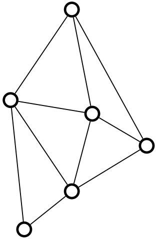

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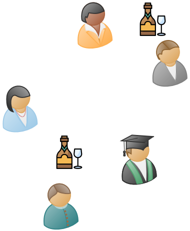

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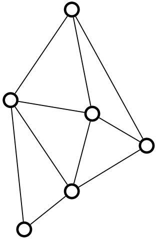

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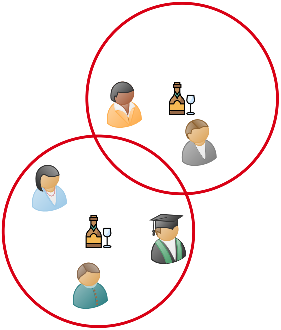
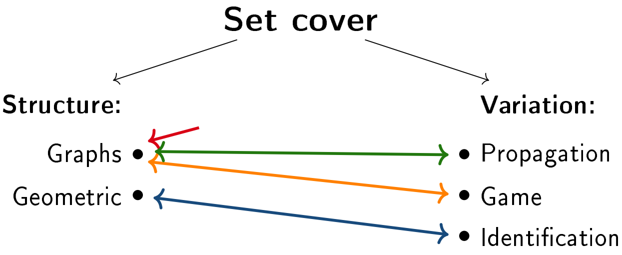


- hyperedges are sets of points that can be covered by the same disk

My PhD



Strong geodetic number

Maker-Breaker domination game

Power Domination

Identification of points using disks

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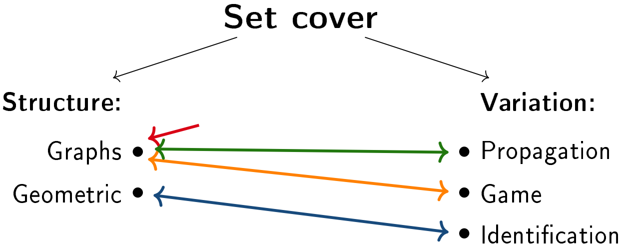

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→ **Maker-Breaker domination game**

Power Domination

→ **Identification of points using disks**

Maker-Breaker Domination Game

Domination in graphs (Ore, 1961)

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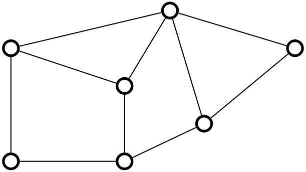
S dominates G if all vertices of G are in S or adjacent to a vertex of S.



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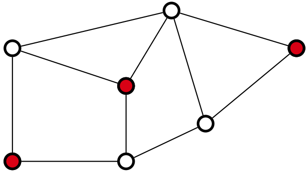

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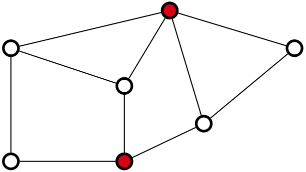

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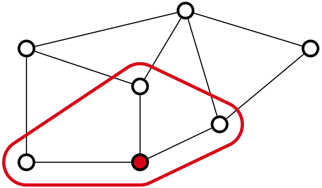

Domination games

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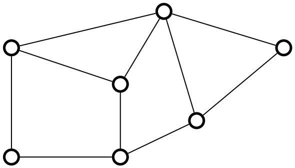

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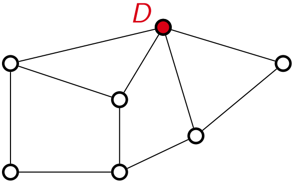

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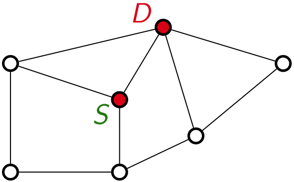

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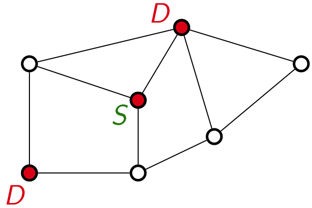

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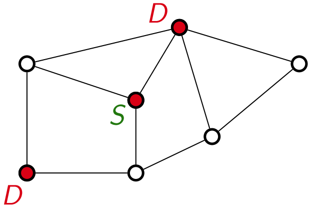

Determining the number of moves in an optimal game of the domination game is *PSPACE*-complete (Brešar et al., 2016).

Maker-Breaker domination game

(Duchêne, G., Parreau and Renault, 2018+)

Maker-Breaker domination game

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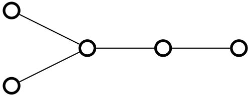

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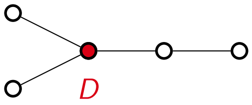

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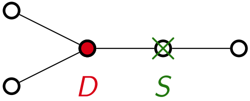

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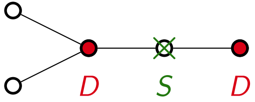

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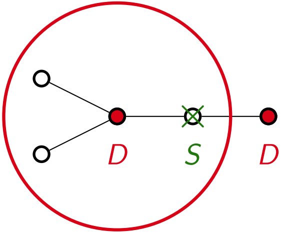

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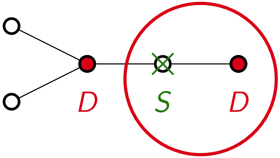

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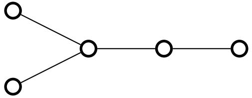

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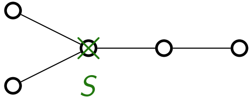

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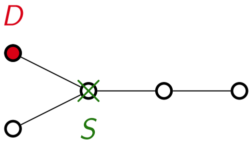

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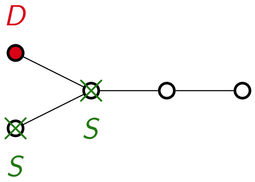

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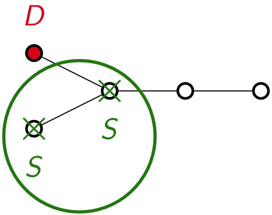

The problem

The goal is to decide which player has a winning strategy.

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The possible **outcomes** are the following:

Staller starts / Dominator starts	Dominator wins	Staller wins
Dominator wins	$\mathcal{D}$	$\mathcal{N}$
Staller wins	$\mathcal{P}$	$\mathcal{S}$

Maker-Breaker games

Maker-Breaker games

- Played on an hypergraph $(X, \mathcal{F})$.
- Two players: Maker and Breaker.
- They alternately select vertices of X.
- Maker wins if and only if he selected all the vertices of a hyperedge $A \in \mathcal{F}$.

- Hex
  - Maker has a winning strategy (Nash, 1952)

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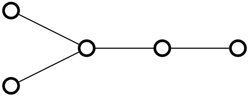

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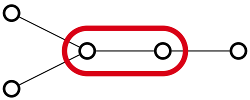

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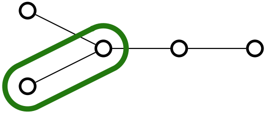

- $\mathcal{F}$ = {the dominating sets},
Dominator = Maker.
- $\mathcal{F}$ = {the closed neighborhoods},
Staller = Maker.

Maker-Breaker games

Theorem (Folklore)

If Maker wins the Maker-Breaker game on $(X, \mathcal{F})$ as the second player, then he also wins as first player.

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$\rightarrow$ There is no game of outcome $\mathcal{P}$

Theorem (Schaefer, 1978)

Deciding the outcome of Maker-Breaker is a PSPACE-complete problem.

Outcomes

There exist graphs for the three possibles outcomes.

			
Dominator starts			
Staller starts			

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Outcomes

There exist graphs for the three possibles outcomes.

			
Dominator starts	Dominator		
Staller starts			

Outcomes

There exist graphs for the three possibles outcomes.

			
Dominator starts	Dominator		
Staller starts	Staller		
	$\mathcal{N}$		

Outcomes

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Dominator starts	Dominator		
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Dominator starts	Dominator	Dominator	
Staller starts	Staller		

$\mathcal{N}$

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Dominator starts	Dominator	Dominator	
Staller starts	Staller	Dominator	
	$\mathcal{N}$	$\mathcal{D}$	

Outcomes

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			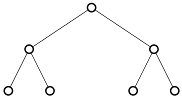
Dominator starts	Dominator	Dominator	
Staller starts	Staller	Dominator	
	$\mathcal{N}$	$\mathcal{D}$	

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			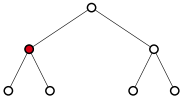
Dominator starts	Dominator	Dominator	
Staller starts	Staller	Dominator	

$\mathcal{N}$ $\mathcal{D}$

Outcomes

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			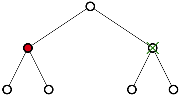
Dominator starts	Dominator	Dominator	
Staller starts	Staller	Dominator	

$\mathcal{N}$ $\mathcal{D}$

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			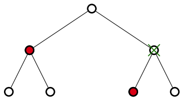
Dominator starts	Dominator	Dominator	
Staller starts	Staller	Dominator	
	$\mathcal{N}$	$\mathcal{D}$	

Outcomes

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			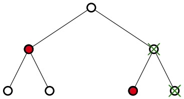
Dominator starts	Dominator	Dominator	Staller
Staller starts	Staller	Dominator	

$\mathcal{N}$ $\mathcal{D}$

Outcomes

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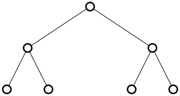
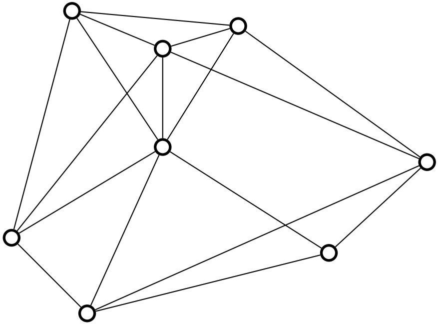
			
Dominator starts	Dominator	Dominator	Staller
Staller starts	Staller	Dominator	Staller
	$\mathcal{N}$	$\mathcal{D}$	$\mathcal{S}$

A winning condition for Dominator

Lemma

If a graph can be partitioned into cliques of size at least 2 then its outcome is $\mathcal{D}$.



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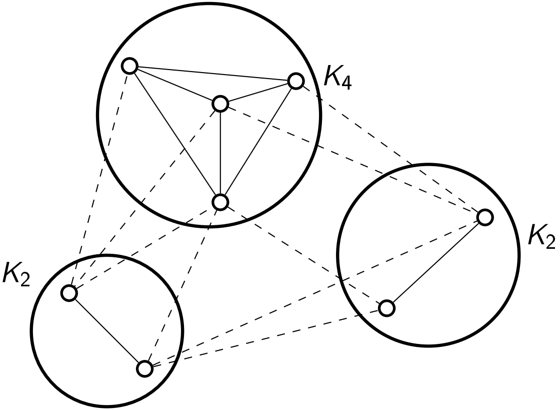

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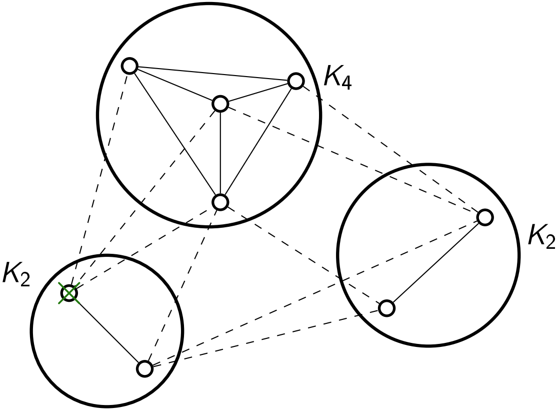

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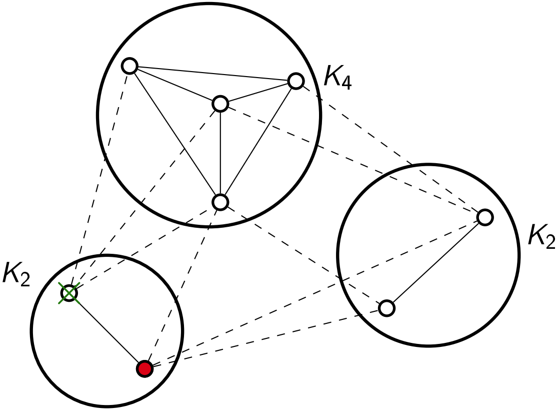

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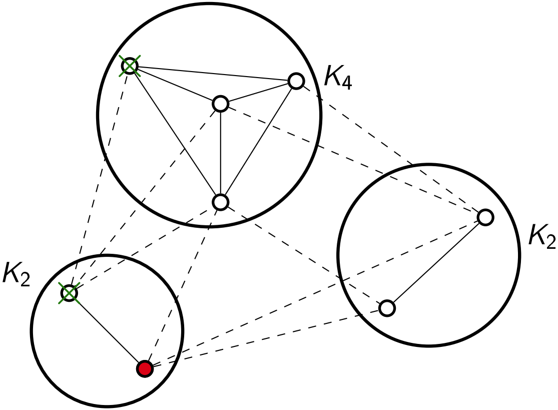

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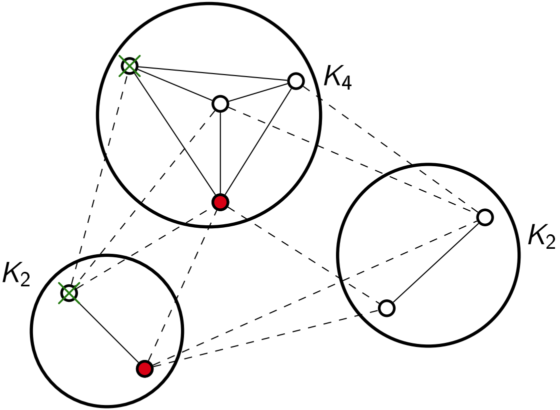

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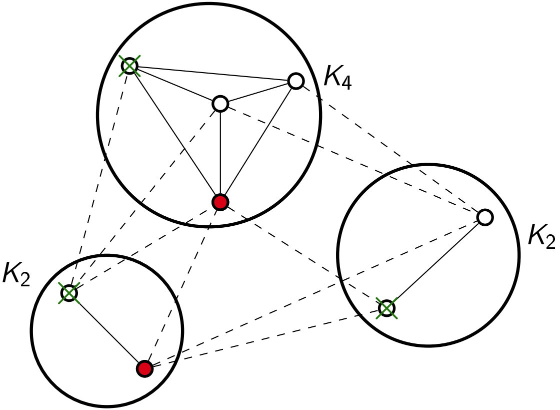

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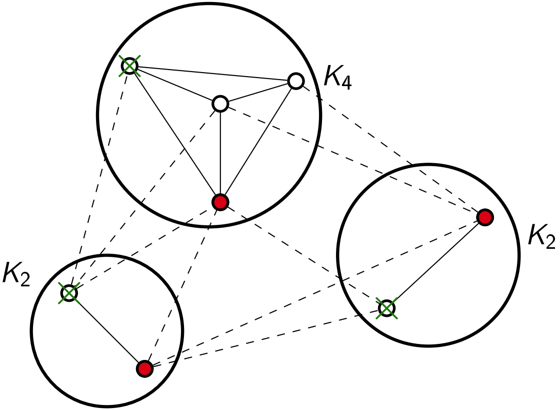

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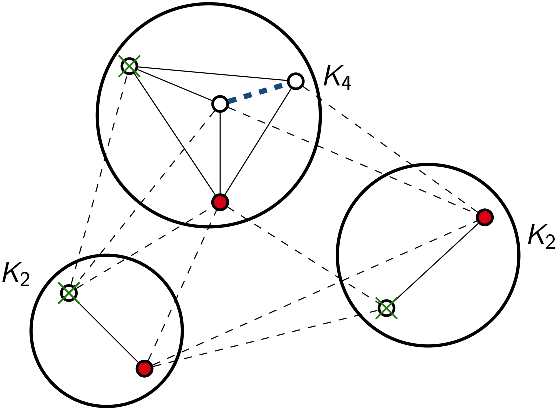
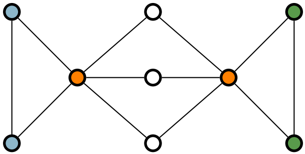


Pairing dominating sets

Definition (Duchêne, G., Parreau, Renault, 2018+)

A set of pairs of vertices $\{(u_1, v_1), \ldots, (u_k, v_k)\}$ is a pairing dominating set if:

- all vertices are distinct,
- $V = \bigcup_{i=1}^{k} N[u_i] \cap N[v_i]$.



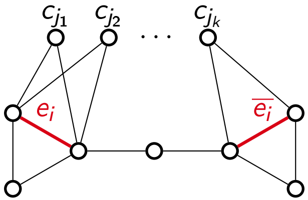
G has a pairing dominating set $\implies$ G has outcome $\mathcal{D}$.

Pairing dominating sets

Theorem

Deciding if a graph admits a pairing dominating set is an NP-complete problem.

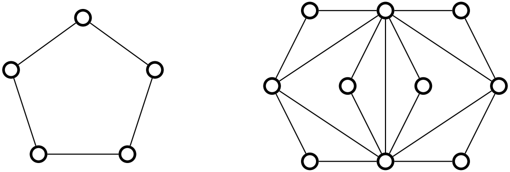
The proof uses a reduction from SAT.



Gadget for a variable

Pairing dominating sets

There exist graphs of outcome $\mathcal{D}$ that do not admit pairing dominating sets.

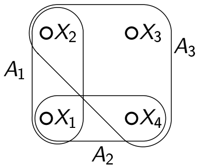


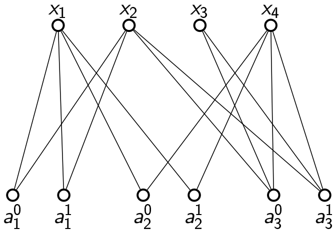
Complexity

Theorem

Deciding the outcome of a Maker-Breaker domination game position is a PSPACE-complete problem.

This result is proved by reduction from Maker-Breaker games which are PSPACE-complete (Schaeffer, 1978).





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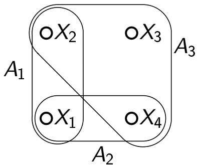

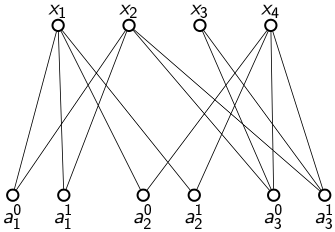

Dominator follows Breaker's strategy

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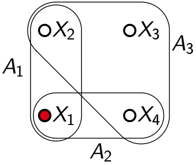

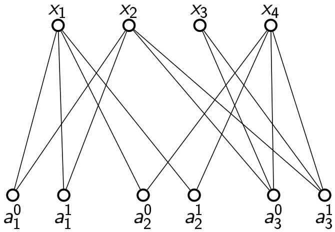

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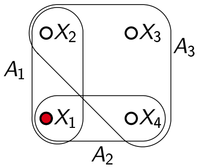

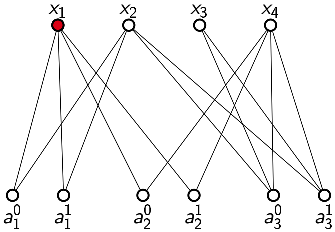

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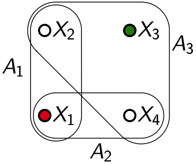

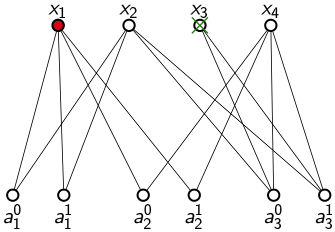

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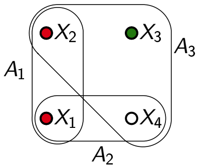

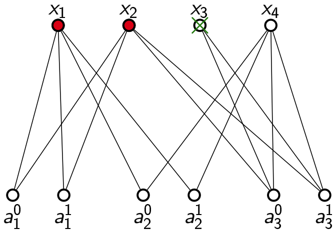

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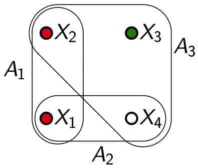

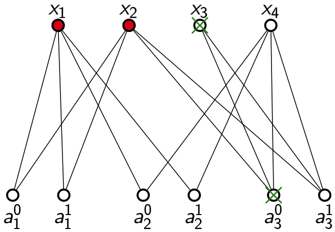

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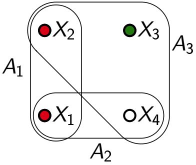

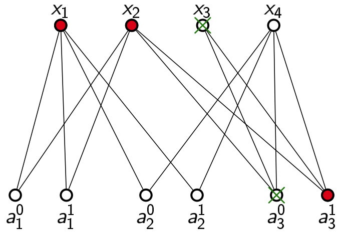

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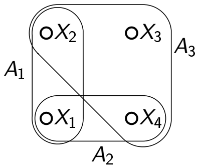
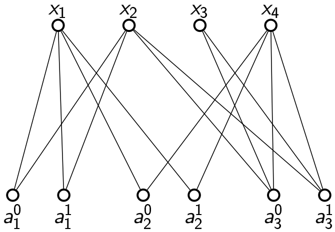
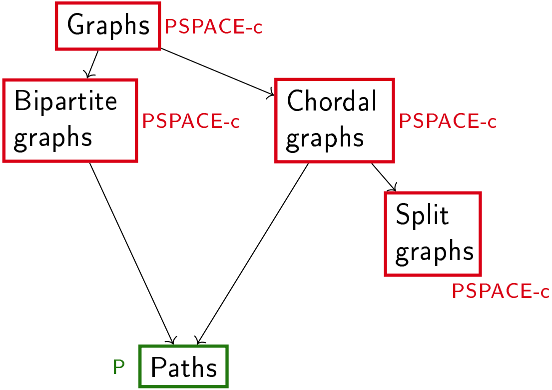




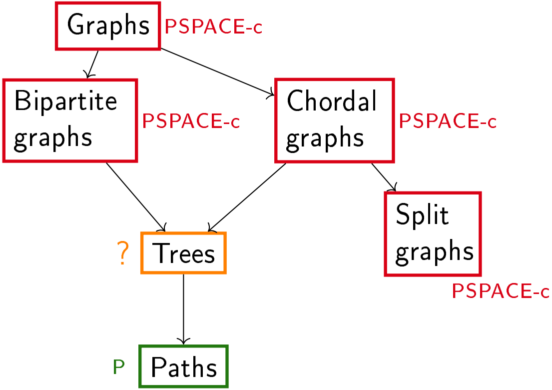
Staller follows Maker's strategy

Complexity



Graphs
PSPACE-c
Bipartite graphs
PSPACE-c
Chordal graphs
PSPACE-c
Split graphs
PSPACE-c
P
Paths

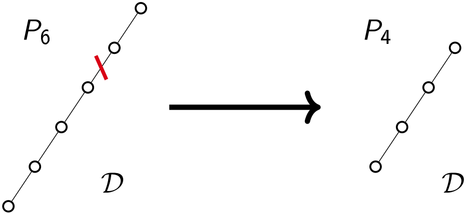
Complexity



Graphs
PSPACE-c
Bipartite graphs
PSPACE-c
Chordal graphs
PSPACE-c
?
Trees
Split graphs
PSPACE-c
P
Paths

Maker-Breaker domination game on trees

For paths, removing P_2's preserves the outcome.



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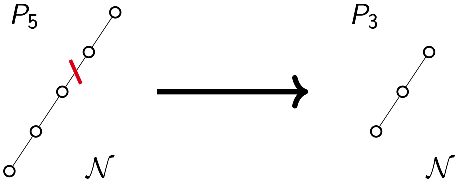

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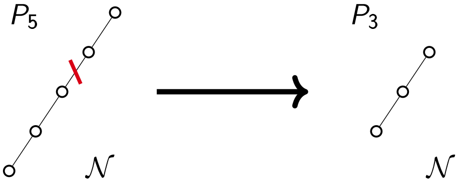
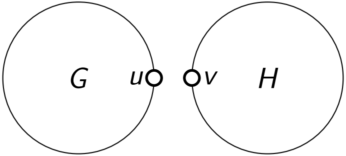


Is it still true for other graphs?

Glue operator

We "glue" two graphs on a vertex.



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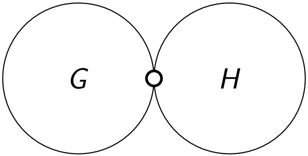
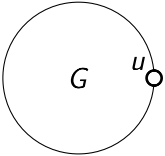
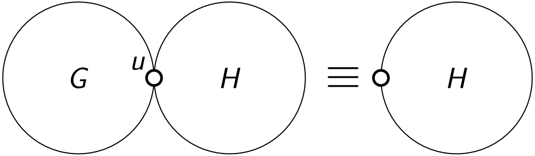


$G \underset{u}{\asymp}\underset{v}{} H$

Glue operator

We want to find the couples (G, u) such that for all H,
$G \underset{u}{\bowtie} H \equiv H$:



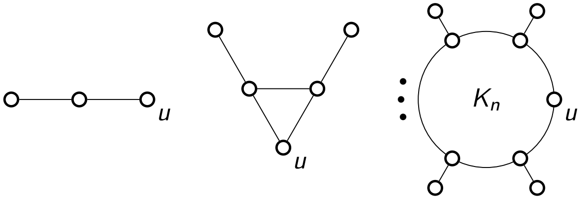


Glue operator

Theorem

A graph (G, u) is neutral for the glue operator if and only if

- G has outcome $\mathcal{N}$
- $G \setminus \{u\}$ has outcome $\mathcal{D}$



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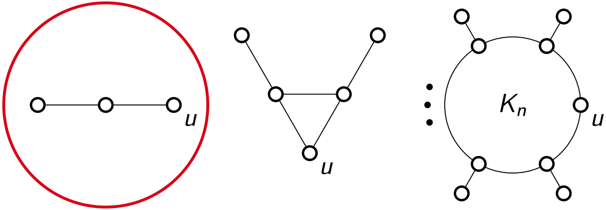
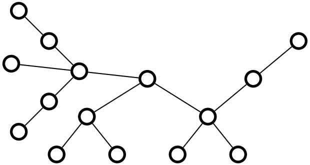


Trees

Theorem

Maker-Breaker Domination Game is polynomial on trees.

Removing pendant P_2's can be done in polynomial time.



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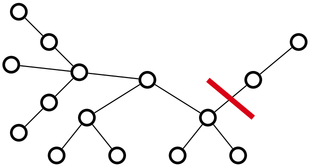

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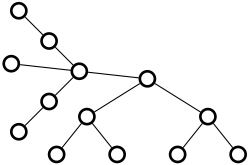

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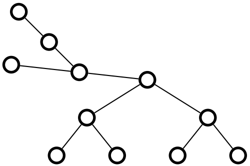

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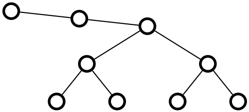

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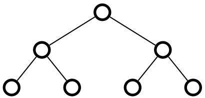


Trees

Theorem

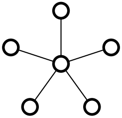
Maker-Breaker Domination Game is polynomial on trees.

Any tree can be reduced to one of the following configurations:

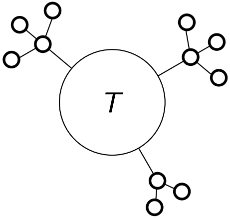
$\emptyset$

empty graph

$K_{1,0}$



$K_{1,n}$

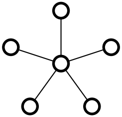
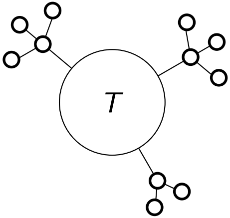


Trees

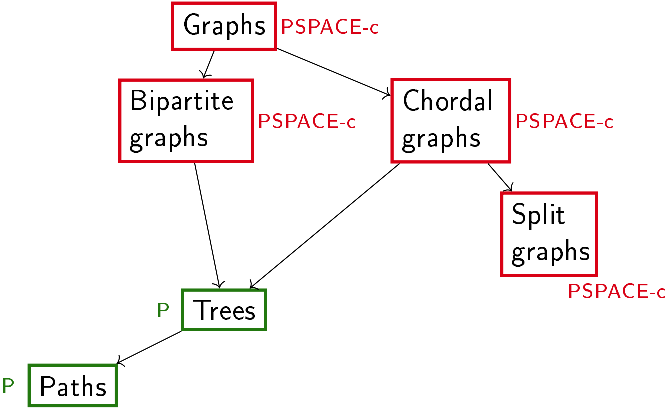
Theorem

Maker-Breaker Domination Game is polynomial on trees.

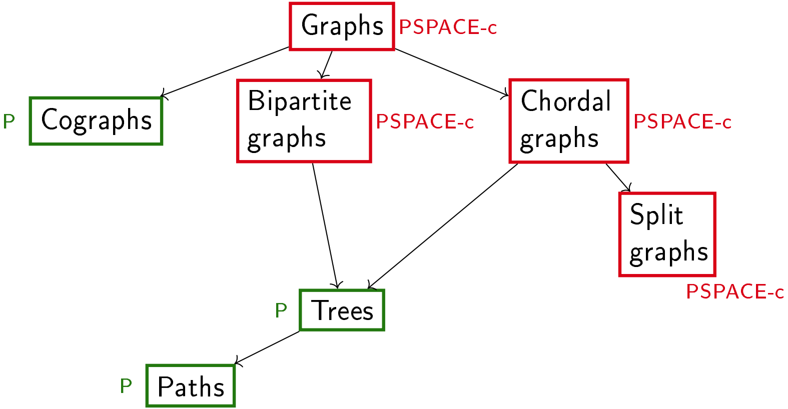
Any tree can be reduced to one of the following configurations:

$\emptyset$			
empty graph	$K_{1,0}$	$K_{1,n}$	
$\mathcal{D}$	$\mathcal{N}$	$\mathcal{N}$	$\mathcal{S}$

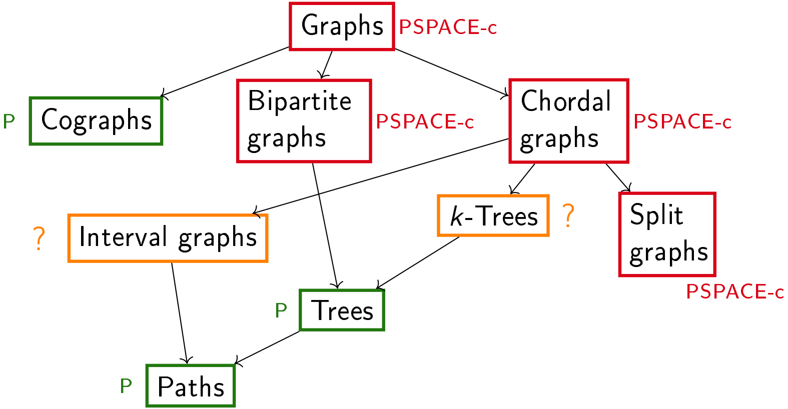
Complexity


Graphs
PSPACE-c
Bipartite graphs
PSPACE-c
Chordal graphs
PSPACE-c
Split graphs
PSPACE-c
P
Trees
P
Paths

Complexity


Graphs
PSPACE-c
P
Cographs
Bipartite graphs
PSPACE-c
Chordal graphs
PSPACE-c
Split graphs
PSPACE-c
P
Trees
P
Paths

Complexity


Graphs
PSPACE-c
P
Cographs
Bipartite graphs
PSPACE-c
Chordal graphs
PSPACE-c
?
Interval graphs
k-Trees
?
Split graphs
PSPACE-c
P
Trees
P
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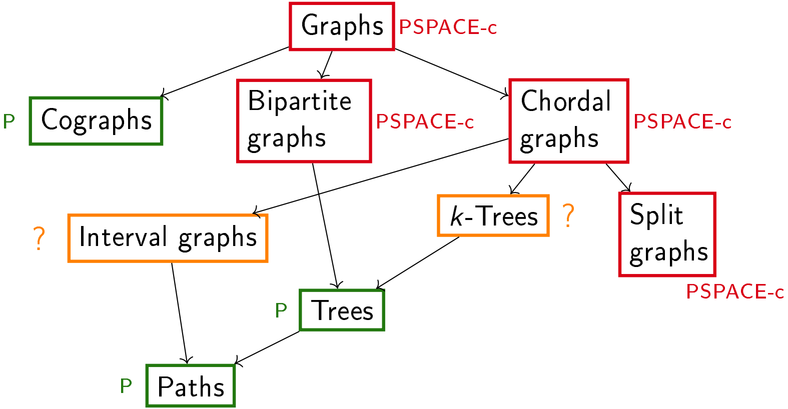
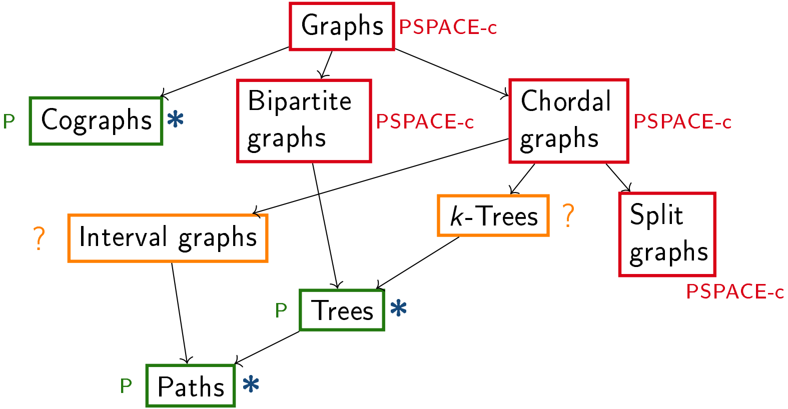
Complexity



PDS property(*): Having outcome $\mathcal{D} \iff$ having a pairing dominating set

Complexity



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Other works and perspectives

Other works

- The Maker-Breaker domination numbers
  (G., Iršič and Klavžar, 2019)
  - The difference between the "Dominator starts" and the "Staller starts" values are unbounded
  - PSPACE-complete
  - Solved for cycles and trees
- The Maker-Breaker total domination game
  (Henning, G., Iršič and Klavžar, 2019)
  - Solved on cacti
- The Avoider-Enforcer domination game
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Perspectives

- Maker-Breaker domination numbers of cographs
- Study of the pairing dominating sets

Identification of points using disks

My PhD

Set cover

Structure:

- Graphs
- Geometric

Variation:

- Propagation
- Game
- Identification

Strong geodetic number

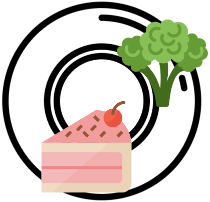
Maker-Breaker domination game

Power Domination

→ **Identification of points using disks**

Back to the post-defense buffet

I found a plate. Who does it belong to?



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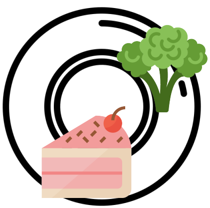

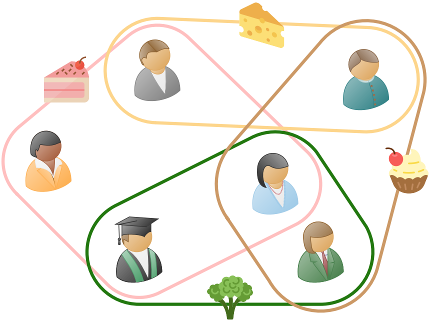

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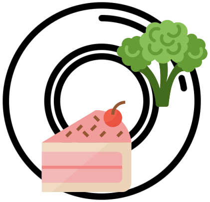
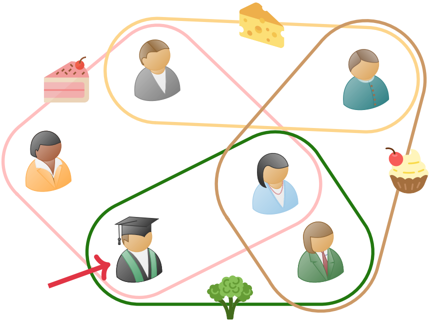


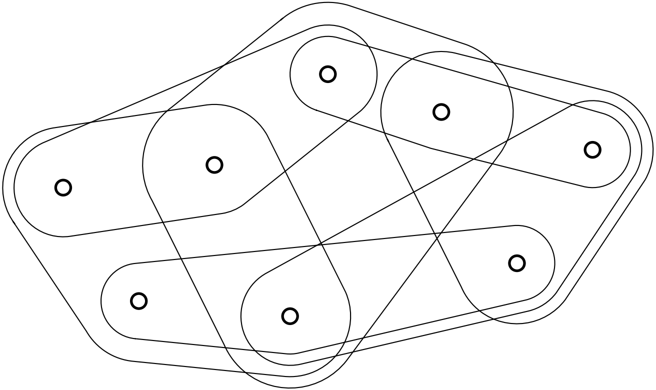


We can **identify** the right guest.

Identification in hypergraphs

Two goals:

- Covering
- Separation



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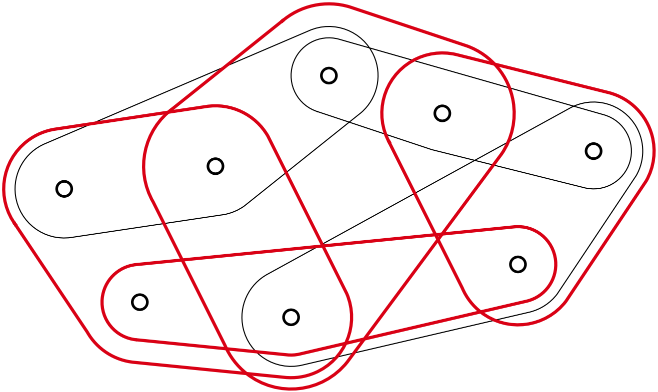


Linked problems

- Test cover (Moret and Shapiro, 1985)
- Identifying codes in graphs (Karpovsky, Chakrabarty and Levitin, 1998)
  - Unit disk graphs (Müller and Sereni, 2009)
  - Unit interval graphs (Foucaud, Mertzios, Naserasr, Parreau and Valicov, 2015).

Identification of points with disks

(G. and Parreau, 2019)

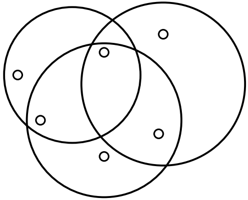
Input of the problem

A set $\mathcal{P}$ of points in the plane

Output

A set $\mathcal{D}$ of closed disks verifying:

- Every point of $\mathcal{P}$ must belong to at least one disk of $\mathcal{D}$. (Covering)
- Two points of $\mathcal{P}$ must belong to two different subsets of $\mathcal{D}$. (Separation)



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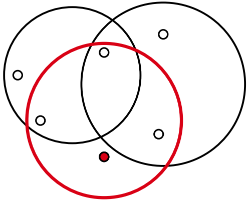

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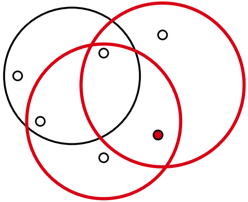

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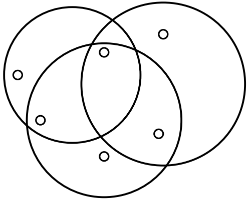


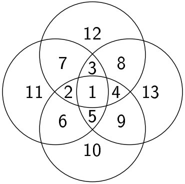
$\gamma_D^{ID}(\mathcal{P})$: Minimal number of disks necessary to identify $\mathcal{P}$.

- Separation of points using convex sets (Gerbner and Toth, 2012)
- Separation of points using lines parallels to the axis (Calinescu, Dumitrescu, Karloff and Wan, 2005)

Lower bound

Theorem (Folklore)

Putting k disks in the plane defines at most $k^2 - k + 1$ intersection areas.



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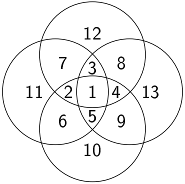


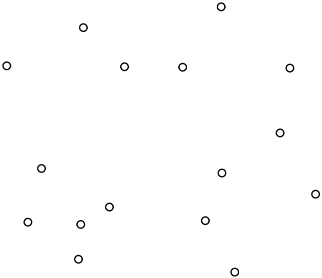
Corollary

Let $\mathcal{P}$ be a set of n points in the plane
$\gamma_D^{ID}(\mathcal{P}) \geq \left\lceil \frac{1+\sqrt{1+4(n-1)}}{2} \right\rceil \sim \sqrt{n}$.

Upper bound

Theorem (Adapted from Gerbner and Toth, 2012)

Let $\mathcal{P}$ be a set of n points in the plane, $\gamma_D^{ID}(\mathcal{P}) \leq \lceil \frac{n+1}{2} \rceil$.



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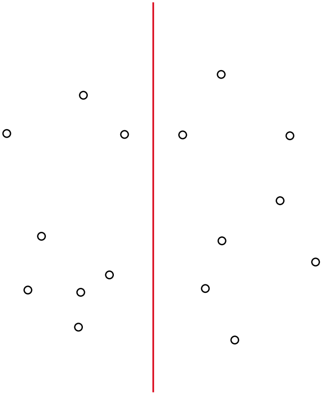

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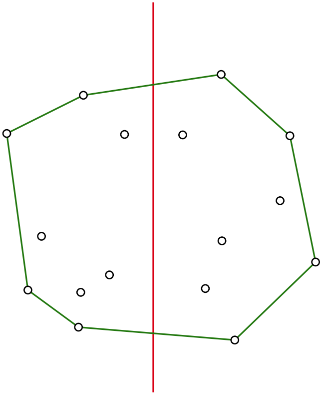

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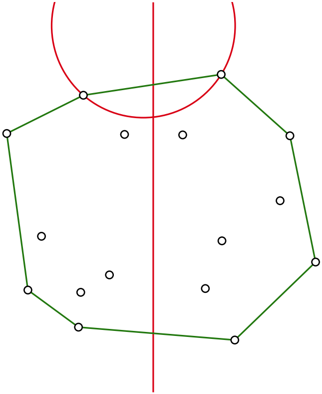

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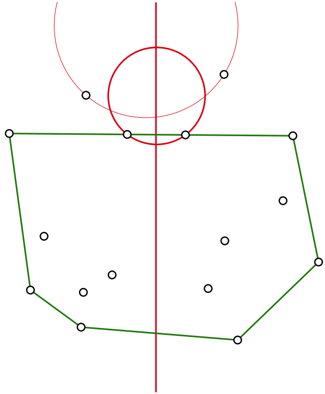

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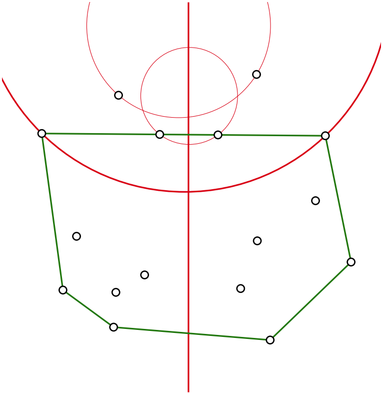

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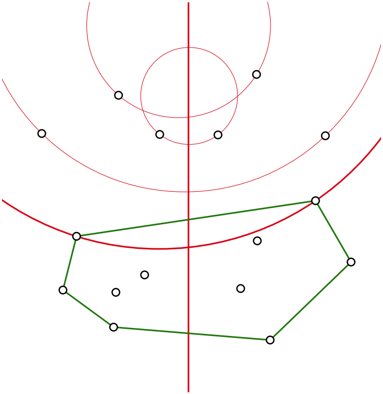

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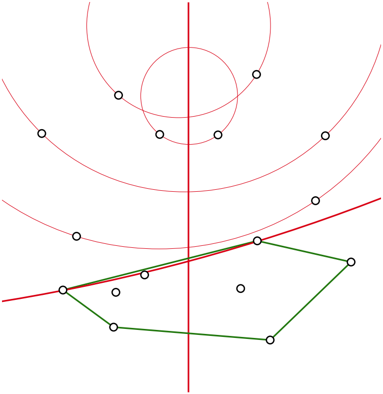

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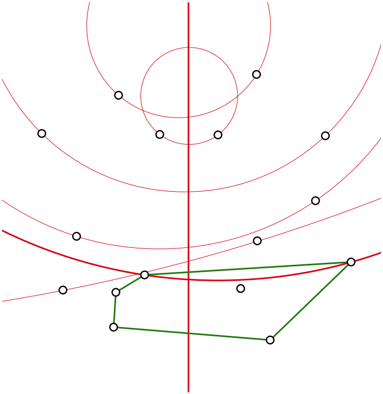

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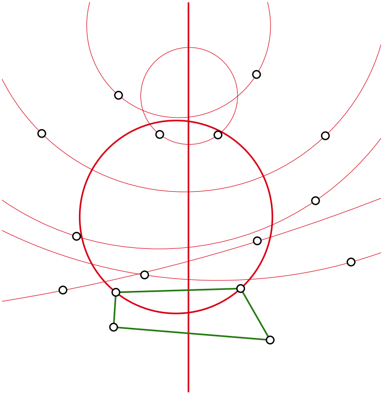

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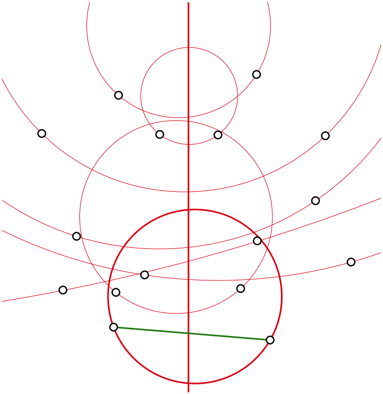

The points are colinear

Theorem

Let $\mathcal{P}$ be a set of n colinear points, $\gamma_D^{ID}(\mathcal{P}) = \lceil \frac{n+1}{2} \rceil$.

The disks go through $n+1$ areas on the same line to cover each point and separate each pair of points.

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Upper bound in general configuration

The previous upper bound is tight for colinear and cocyclic sets of points.

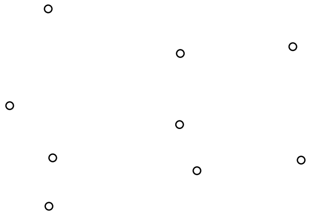
Theorem

Let $\mathcal{P}$ be a set of n points such that no three points are colinear and no four points are cocyclic, $\gamma_D^{ID}(\mathcal{P}) \leq 2\lceil \frac{n}{6} \rceil + 1$.

Principle

Same principle as the previous proof:

- Separating the points into equal size areas using lines
- Iteratively separating points from each area with disks



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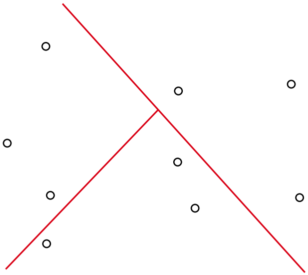

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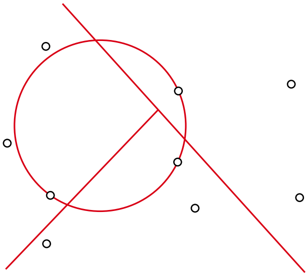

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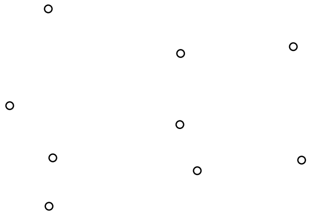

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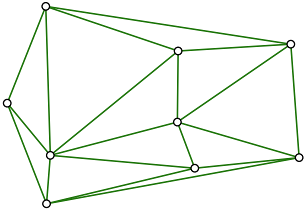
  - Use of Delaunay's triangulation



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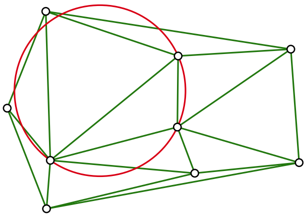

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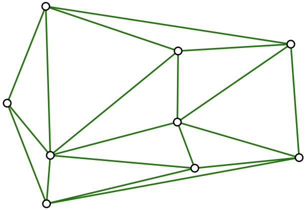

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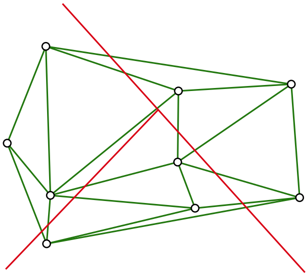

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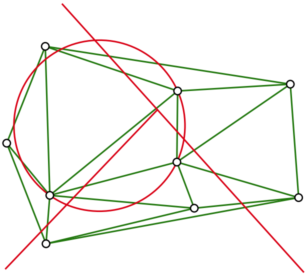

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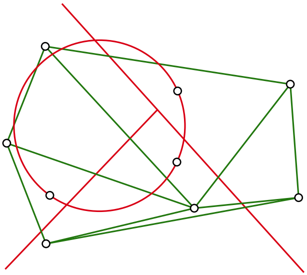

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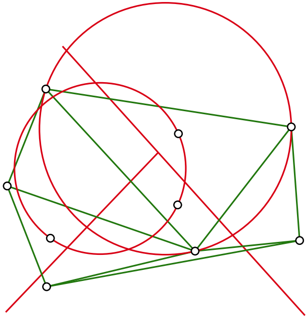
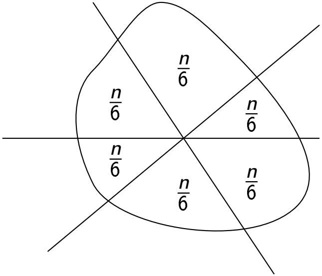


Upper bound in the general case

Theorem (J. G. Ceder, 1964)

Let $\mathcal{P}$ be a set of n points in the plane such that no three points are colinear. Using three concurrent lines, it is possible to divide the plane into six areas containing between $\lceil \frac{n}{6} \rceil - 1$ and $\lceil \frac{n}{6} \rceil$ points.



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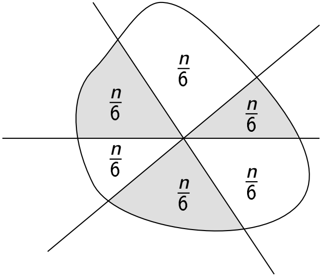

Upper bound in the general case

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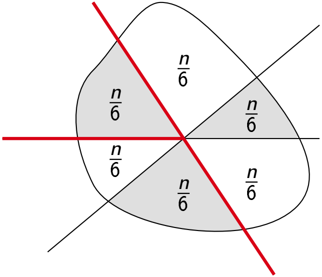

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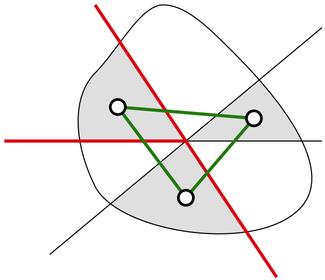

Complexity

	Points in the plane		Points on a line	
Radii / Centers	Any values	Fixed to the same value	Any values	Fixed to the same value
Anywhere				
Fixed on the points				

Complexity

	Points in the plane		Points on a line	
Radii / Centers	Any values	Fixed to the same value	Any values	Fixed to the same value
Anywhere				
Fixed on the points		Unit disk graph NP-complete		Unit interval graph ?

- Müller and Sereni, 2009
- Foucaud, Mertzios, Naserasr, Parreau and Valicov, 2015

Complexity

	Points in the plane		**Points on a line**	
Radii / **Centers**	Any values	Fixed to the same value	Any values	Fixed to the same value
Anywhere			$O(1)$	
Fixed on the points		Unit disk graph NP-complete		Unit interval graph ?

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Complexity

	Points in the plane		Points on a line	
Radii / **Centers**	Any values	Fixed to the same value	Any values	Fixed to the same value
Anywhere	?	?	$O(1)$	?
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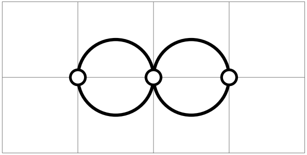
Fixing the radius

Theorem

The following problem is NP-complete:
Instance: A set $\mathcal{P}$ of points in the plane and a number $k \in \mathbb{N}$.
Question: Is it possible to identify the points of $\mathcal{P}$ using k disks of radius 1?

The proof uses a reduction from P_3-partition in grid graphs, a NP-complete problem. The P_3's become the following structure:



Complexity

	Points in the plane		**Points on a line**	
Radii / **Centers**	Any values	Fixed to the same value	Any values	Fixed to the same value
Anywhere	?	**NP-complete**	$O(1)$	?
Fixed on the points	?	Unit disk graph NP-complete	?	Unit interval graph ?

Complexity

Radii / Centers	Points in the plane: Any values	Points in the plane: Fixed to the same value	Points on a line: Any values	Points on a line: Fixed to the same value
Anywhere	?	**NP-complete**	$O(1)$	?
Fixed on the points	?	Unit disk graph NP-complete	?	Unit interval graph ?

Colinear points and fixed radius

Theorem

The following problem can be solved in linear time:
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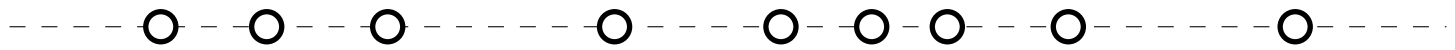
Question: Is it possible to identify the points of $\mathcal{P}$ using k disks of radius 1?



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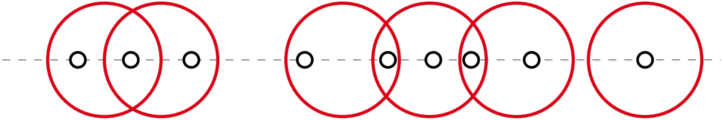

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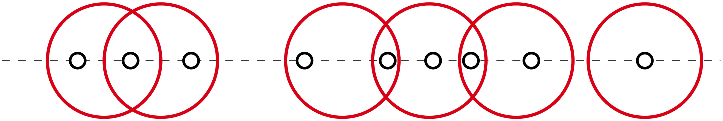

The proof has two steps:

- Showing that there always exists a minimum identifying set of disks in **normal form**,
- Using a greedy algorithm to find such a set.

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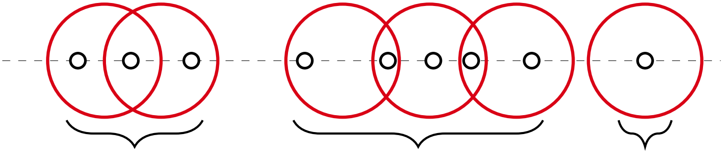


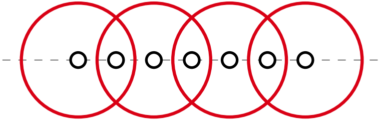
The proof has two steps:

- Showing that there always exists a minimum identifying set of disks in **normal form**,
- Using a greedy algorithm to find such a set.

Colinear points and fixed radius

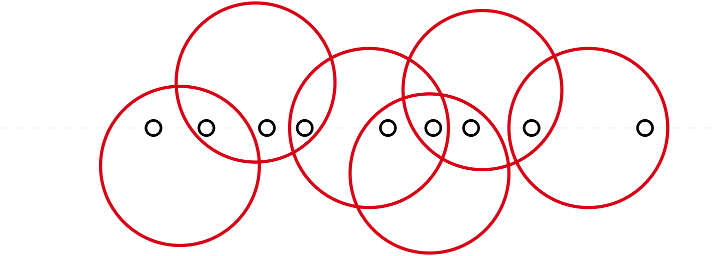
A set of disk is in **normal form** if each connected component of $\mathcal{P}$ is of odd size k and:

- the first and last disks contain exactly two points,
- all the other disks contain exactly three points.



Colinear points and fixed radius

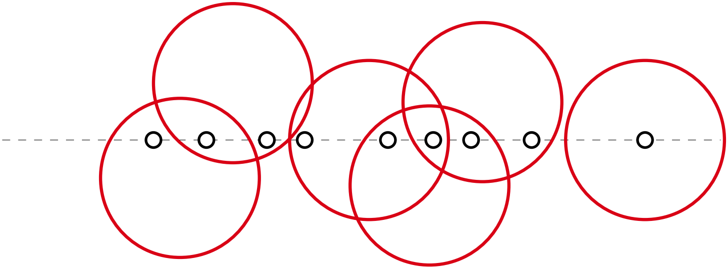
Each minimum identifying set of disks can be transformed:



Colinear points and fixed radius

Each minimum identifying set of disks can be transformed:

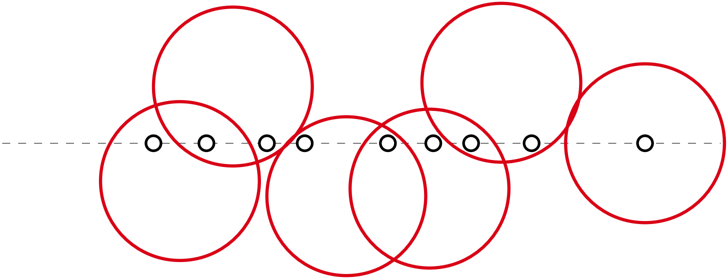
- By showing that we can divide the connected component so that they are of odd size and identified by $\frac{k+1}{2}$ disks,



Colinear points and fixed radius

Each minimum identifying set of disks can be transformed:

- By showing that we can divide the connected component so that they are of odd size and identified by $\frac{k+1}{2}$ disks,
- Then showing that each of these connected component can be in normal form.



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Radii / **Centers**	Any values	Fixed to the same value	Any values	Fixed to the same value
Anywhere	?	NP-complete	$O(1)$	**Linear**
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Perspectives

- Random disposition of points
- Validity of the results for other shapes or for higher dimensions

General conclusion

Strong geodetic number

Maker-Breaker domination game

Power Domination

Identification of points using disks

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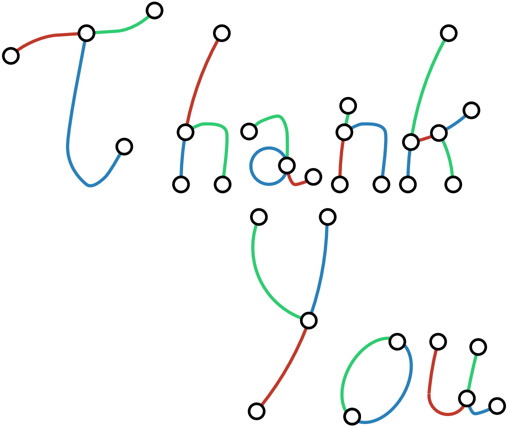
Thank
you