

Sobol' Sequences with Guaranteed-Quality 2D Projections

Supplementary Materials: Rendering results

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1 Rendering results

Figures 1, 2 and 3 provide rendered images for small per-pixel sample counts (4 to 256). MSE convergence graphs are presented in the main document. First, the Cornell-box scene presents path traced results with direct lighting and 1 bounce of indirect lighting thus requiring $5 + 6 + 6$ dimensions: 5 dimensions are used to sample pixel, lens and time, 3 dimensions then sample the direct lighting and 3 more dimensions are sampled to find the next vertex of the path. Extending the path one more time requires another 6 dimensions. The Spaceship scene samples longer refractive paths thus requiring 4 indirect bounces, resulting in a total of 35 dimensions. The last scene, the Dining-Room, is lit both directly and indirectly, we use simple 2 bounces paths and 17 dimensions.

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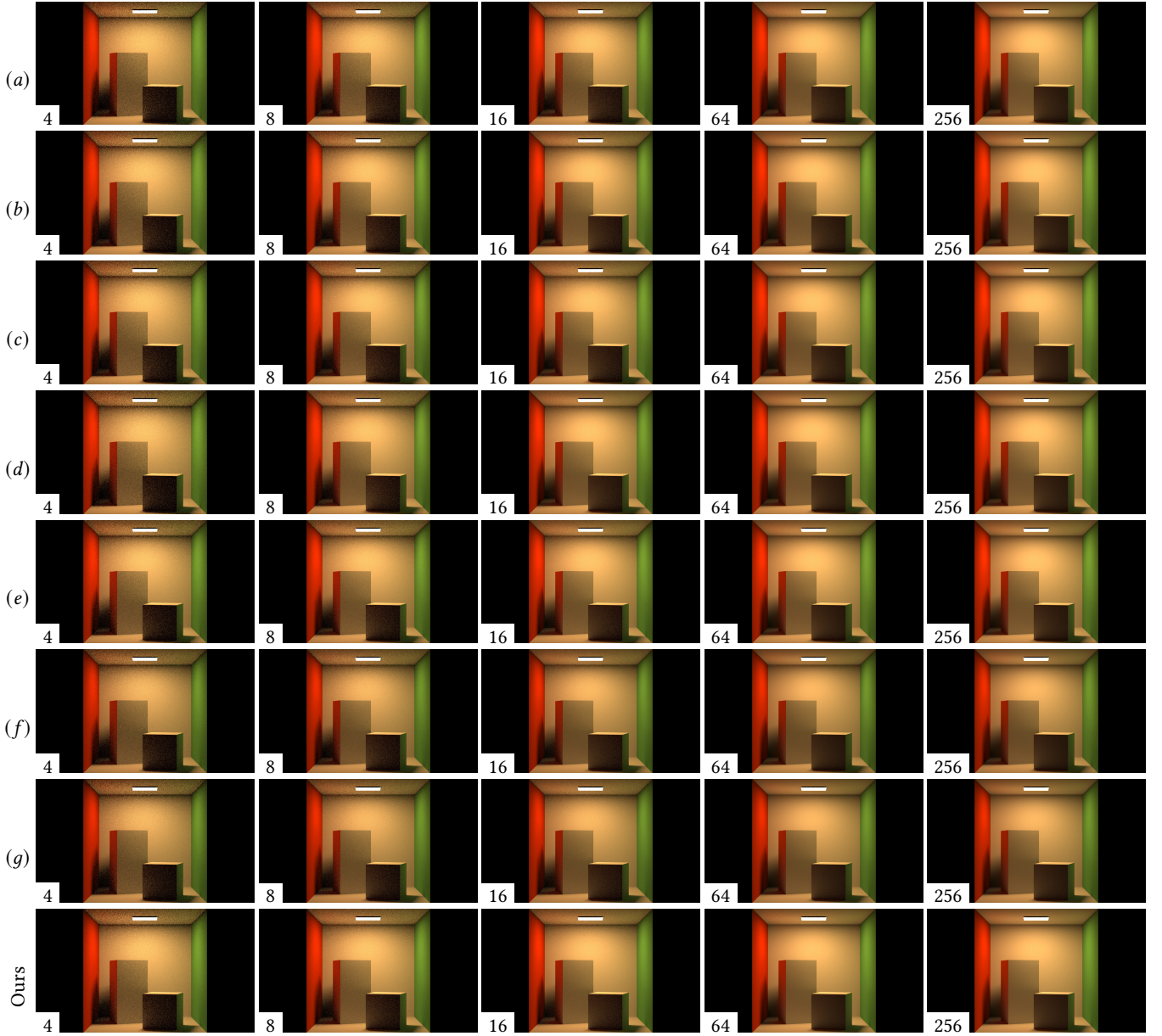


Fig. 1. Rendering 2 bounces indirect lighting in 17D. We compare our results with the ones obtained from White Noise (a), Sobol' with Joe and Kuo [2008] (b), Faure and Lemieux [2016] (c), Quad Optimized in GF(3) [Ostromoukhov et al. 2024] (d), Cascaded Sobol' [Paulin et al. 2021] (e), ZeroTwo [Pharr et al. 2023] (f) and Padded Sobol' 4D [Burley 2020] (g).



Fig. 2. Rendering 5 bounces indirect lighting in 35D. We compare our results with the ones obtained from White Noise (a), Sobol' with Joe and Kuo [2008] (b), Faure and Lemieux [2016] (c), Quad Optimized in GF(3) [Ostromoukhov et al. 2024] (d), Cascaded Sobol' [Paulin et al. 2021] (e), ZeroTwo [Pharr et al. 2023] (f) and Padded Sobol' 4D [Burley 2020] (g).



Fig. 3. Rendering 2 bounces indirect lighting in 17D. We compare our results with the ones obtained from White Noise (a), Sobol' with Joe and Kuo [2008] (b), Faure and Lemieux [2016] (c), Quad Optimized in GF(3) [Ostromoukhov et al. 2024] (d), Cascaded Sobol' [Paulin et al. 2021] (e), ZeroTwo [Pharr et al. 2023] (f) and Padded Sobol' 4D [Burley 2020] (g).