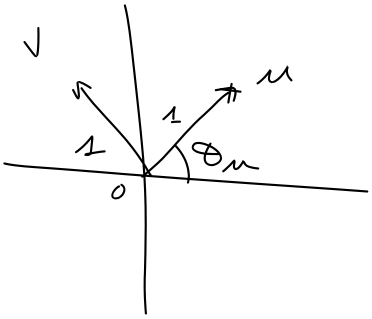
Geometry basics

Dot product

$$u \begin{pmatrix} u_0 \\ u_1 \\ u_2 \end{pmatrix} \qquad v \begin{pmatrix} v_0 \\ v_1 \\ v_2 \end{pmatrix}$$

$$u \circ v = u_0 \times v_0 + u_1 \times u_1 + u_2 v_2 = u^t \times v$$

Projection

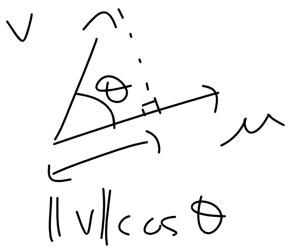


$$u \begin{pmatrix} \cos\theta_u \\ \sin\theta_u \end{pmatrix} \qquad v \begin{pmatrix} \cos\theta_v \\ \sin\theta_v \end{pmatrix}$$

$$\begin{aligned} u \circ v &= \cos\theta_u \cos\theta_v + \sin\theta_u \sin\theta_v \\ &= \cos(\theta_v - \theta_u) \end{aligned}$$

General

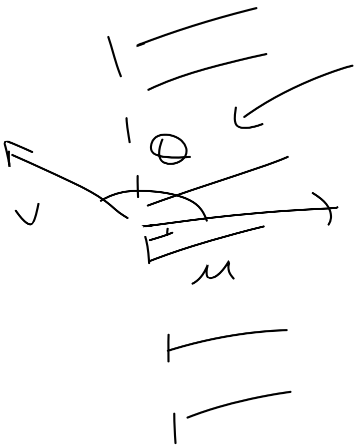
$$u \circ v = \|u\| \|v\| \cos(\langle u, v \rangle)$$



u normalized.

$$v \circ u = \|v\| \circ \cos\theta$$

Alignment



aligned with u

$$u \circ v \begin{cases} \text{positive} \rightarrow \text{aligned} \\ \text{negative} \rightarrow \text{not aligned} \end{cases}$$

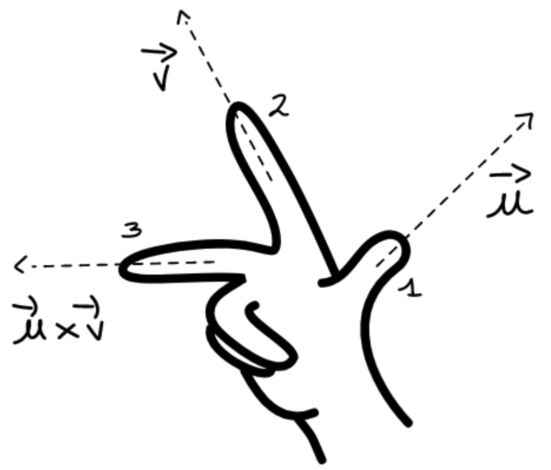
Cross product

$$u \begin{pmatrix} u_0 \\ u_1 \\ u_2 \end{pmatrix} \qquad v \begin{pmatrix} v_0 \\ v_1 \\ v_2 \end{pmatrix}$$

$$\rightarrow \begin{matrix} u \times v \\ u \wedge v \end{matrix} = \begin{pmatrix} u_1 v_2 - v_1 u_2 \\ u_2 v_0 - v_2 u_0 \\ u_0 v_1 - v_0 u_1 \end{pmatrix}$$

$$u \times u \rightarrow 0$$

Right hand rule

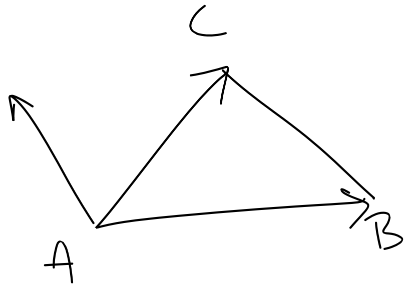


$\vec{u} \times \vec{v}$ orthogonal to $\vec{u}$ and $\vec{v}$

direction

$(u, v, u \times v) \rightarrow$ form a direct frame.

Normal



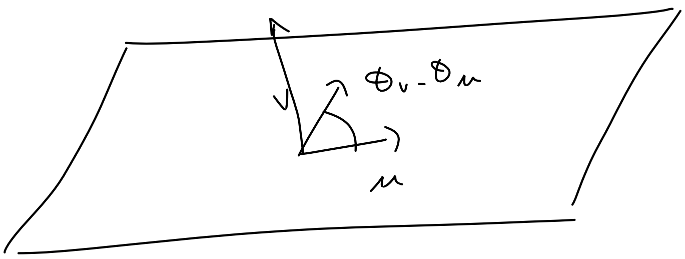
$(\overrightarrow{B - A}) \times (\overrightarrow{C - A}) \rightarrow$ normal

Norm of the cross product

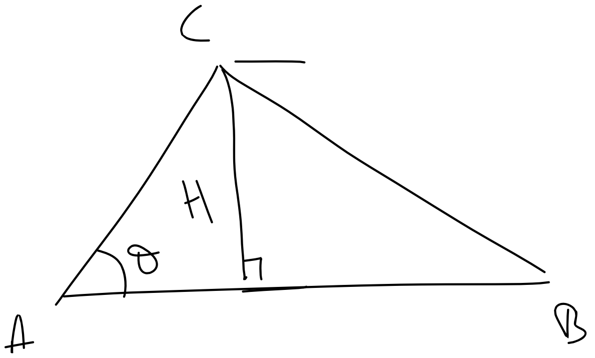
$$u \begin{pmatrix} \cos\theta_u \\ \sin\theta_u \\ 0 \end{pmatrix} \qquad v \begin{pmatrix} \cos\theta_v \\ \sin\theta_v \\ 0 \end{pmatrix}$$

$$u \times v = \begin{pmatrix} 0 \\ 0 \\ \underbrace{\cos\theta_u \sin\theta_v - \cos\theta_v \sin\theta_v}_{\text{norm}} \end{pmatrix}$$

$$\cos\theta_u \sin\theta_v - \cos\theta_v \sin\theta_v = \sin(\theta_v - \theta_u)$$



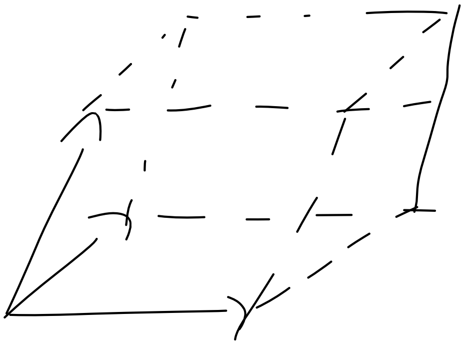
$$\|u \times v\| = \|u\|\|v\||\sin(\theta_v - \theta_u)|$$



$$\frac{\|B-A\| \times \|H\|}{2}$$

$$\|H\| = \|C-A\||\sin\theta|$$

$$\text{Area}(A,B,C) = \frac{\|(B-A)\times(C-A)\|}{2}$$



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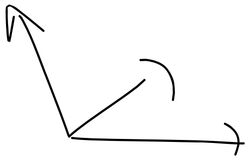

$$(u, v, w) = \left(u, v, w \right) = (u \times v)\cdot w$$