CSc 30400 Introduction to Theory of Computer Science

Useful Tips

1 Proofs.

1. Make sure that you know what you want to prove. Express all the English sentences in mathematics.
2. If you started with one direction in your proof and you are stuck then try the other direction.
3. Always look where you are, where you want to go and what is given and compare them.
4. In most of the cases:
   - you should try a proof by *Contradiction* when you want to prove negative sentences such as "there is no..." (assume that there is).
   - for statements like "$\forall n \in \mathbb{N}$" use *Induction*.
   - use *Construction* when you want to show that something exists (since you were able to construct it, this means that it exists).

2 Induction.

1. Make sure that you didn't forget to prove the base case (remember the example where we "proved" that any even number is equal to an odd number).
2. Make sure that the inductive step holds for every n (remember the example with the horses where the inductive step wasn't true from 1 to 2 horses).
3. When you want to prove that a statement is true for a structure (for example a graph) and the statement says *for every structure* make sure that in the inductive step you are considering the general case of size n+1 (**do not** start from a general structure of size n and simply enlarge by 1 the size of this structure, creating a specific instance of size n+1. Doing so you are only proving the statement for the specific instance and not for the general case! Always start from the general case of a structure of size n+1 and try to remove some items and then use the inductive hypothesis on the smaller structure).

4. When you want to prove that a statement is true for a structure make sure that when you remove something from the structure this was there, and when you add something in the structure this was not there (for example if you want to remove a leaf you have to make sure that there is a leaf on the graph, or when you want to add an edge between two vertices first make sure that this edge doesn't already exist).

5. When you remove or add something to a structure and you want to apply the inductive hypothesis, make sure that the new structure has the same form with the initial structure and you can apply the inductive hypothesis on it. For example, if you have a tree and you remove a node, you have to make sure that the graph you have now is again a tree.

3 Expressions.

- "There are at least k" (or just "there are k") means that there are k (maybe more). To prove a statement beginning with this sentence find k items (probably by construction) or just show that there are k items (probably by contradiction).

- "There are at most k" means that there are from 0 to k. To prove a statement beginning with this sentence prove that there are no k+1 such items (probably by contradiction).

- "There are exactly k" means that there are at least k and at most k and you have to prove them both. First find k and then show that there are no k+1.

- "$\exists x$ such that" means that there is at least one x such that...

- "$\exists! x$ such that" means that there is exactly one x such that...

- "$\forall x$" means for every x (prove it probably by induction).

- "$\nexists x$ such that" means that there are no xs such that... (very similar to the above $\forall x$, prove that for every x the property doesn't hold).