CSc 30400 Introduction to Theory of Computer Science
5th Homework Set

1

Consider the following grammar:

$$E \to E + T | T$$

$$T \to T * F | F$$

$$F \to (E) | D$$

$$D \to 0|1|2|3|4|5|6|7|8|9$$

Give parse trees and derivations for each string

a. 2

b. 5+3

c. (2+5)*6

d. (2+(6+3)+4)

2

Give context-free grammars for the following languages. In all parts the alphabet is $\Sigma = \{0, 1\}$. Which of them are also regular languages (find FA or reg. expr. or just give a reg. grammar)?

a. $\{w | w$ contains at least three 1s$\}$

b. $\{0^n 1^{2n} | n \geq 0\}$

c. $\{w | w$ starts and ends with the same symbol$\}$

d. $\{w |$ the length of w is even$\}$

e. $\{w_1 w_2 |$ the lengths of w_1 and w_2 are equal$\}$

f. $\{w |\ w = w^R$, that is w is palindrome$\}$

3

Convert the following CF grammar into a CF grammar in Chomsky Normal Form

$$S \to ASA|B$$
$$A \to 00|\varepsilon$$
$$B \to 01B|1$$

4

$$S \to aS|Sb|\varepsilon$$

Answer the following questions concerning the above grammar:

a. Which is the language that this grammar produces?

b. Is this grammar ambiguous? Why? If yes try to write an unambiguous one for the same language.

5

An **Extended Right Regular Grammar** is a grammar with productions of the form:

$$A \to \varepsilon$$
$$A \to w$$
$$A \to wB$$

where $A, B \in V$ and $w \in \Sigma^*$. Show that Extended Right Regular Grammars produce the Regular Languages.

Hint: You should work on two directions: Every Extended Right Regular grammar produces a Regular language and every Regular language is produced by an Extended Right Regular grammar. However we know that Right Regular Grammars produce exactly the Regular Languages. So what you should do is to show that Right Regular Grammars and Extended Right Regular Grammars are equivalent. The first direction is immediate: Every Right Regular Grammar is also an Extended one (why?). So the "difficult part" is for every Extended Right Regular Grammar to construct an equivalent Right Regular Grammar.

6

Show that Context Free Languages are closed under concatenation.

Hint: Given two Context Free grammars G_1 and G_2 that produce the Context Free Languages L_1 and L_2 construct a Context Free grammar G that produces $L = L_1 \circ L_2$