CSc 30400 Introduction to Theory of Computer Science
7th Homework Set

Chapter 2

1. Prove that the following language is context free (in order to do so you can find either a non deterministic PDA that recognizes the language or a context free grammar that produces it).The alphabet is $\Sigma = \{0, 1\}$.

   $L_1 = \{w|w \text{ contains twice as many 1s as 0s}\}$

2. As we said, there are several languages that are not context free. Some of them were discussed in class (like $L_2 = \{0^{2^n}, n \geq 0\}$ or $L_3 = \{w\#w|w \in \{0, 1\}^*\}$). One other language that it is not context free is the language $L_4 = \{a^n b^n c^n, n \geq 0\}$. Show that if you had two stacks (instead of just one) then you would be able to recognize all those languages. (This automaton is named a 2-PDA)

   Hints:

   - In order to define formally a 2-PDA you should take care of both stacks, so in the transition you need to add an extra $a \rightarrow b$ to denote the change in the second stack.
   - Try to think the "high level" approach to the problem. First describe in words what the machine should do in order to accept or to reject.
   - For L_2 (probably) you need to save your input first in the stack during the time you are reading it and then work up the saved input (under just ε moves) in order to decide whether the string is accepted or not. You should (probably) consider the halving method that we described for Turing Machines.
   - For L_3 you could think how the first w can be transformed into a w^R. Then it will be easy to compare it with the second w by using a stack.
   - For L_4 take advantage of the second stack to count both as and bs and then compare them with the cs

3. (a) Show that $L_5 = \{a^n b^n c^k, n, k \geq 0\}$ and $L_6 = \{a^k b^n c^n, n, k \geq 0\}$ are both context free.

(b) Show (by using the fact that $L_4 = \{a^n b^n c^n, n \geq 0\}$ is not context free and part a) that context free languages are not closed under intersection.

(c) Show that the context free languages are not closed under complement

Hint: Use the following facts:

- De Morgan Law,
- Context free languages are closed under the union operation and
- part b.

Suppose that CF-languages were closed under complement. Then find a language that should be context free (because it only contains operations between cf-languages) but it is not and obtain a contradiction.

Chapter 3

4. In many cases as we saw in class, it is useful when designing a Turing machine to have a special symbol (for example a \$) to denote the start (left end) of the tape (just like the \$ symbol we used in PDAs to denote the end of the stack). By using this symbol you can easily move between states (if you are in state q_i and see the \$ symbol then move to state q_j). Design a transition diagram that shifts the input by one block to the right and places a \$ in the leftmost position and accepts when the process is done. Assume that when starting the computation the head of the machine is in the leftmost position

   Example: $\boxed{0}\boxed{1}\boxed{0}\boxed{0}\boxed{1}\boxed{}\cdots$, should produce $\boxed{\$}\boxed{0}\boxed{1}\boxed{0}\boxed{0}\boxed{1}\boxed{}\cdots$

5. Design Turing Machines that accept the languages $L_1 - L_4$ of questions 1 and 2. Make sure that you also give a "high level program" for ever case and not only the precise transition diagram of the machine. In all parts the alphabet Σ contains only the symbols that are used each time in the language (for example, for L_3: $\Sigma = \{0, 1, \#\}$). You are free to define the alphabet Γ of the tape to be whatever you want.