Distributed Computing Models & Algorithms

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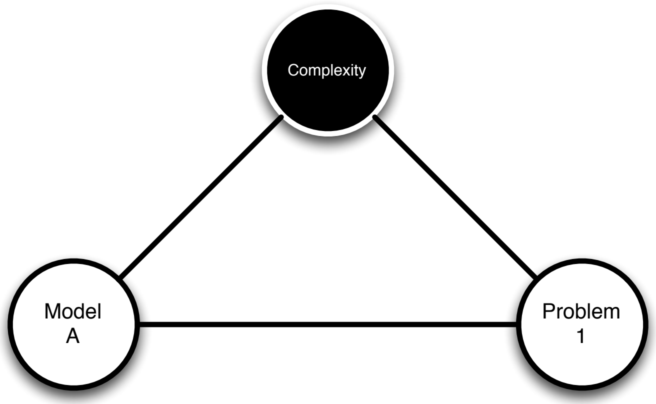
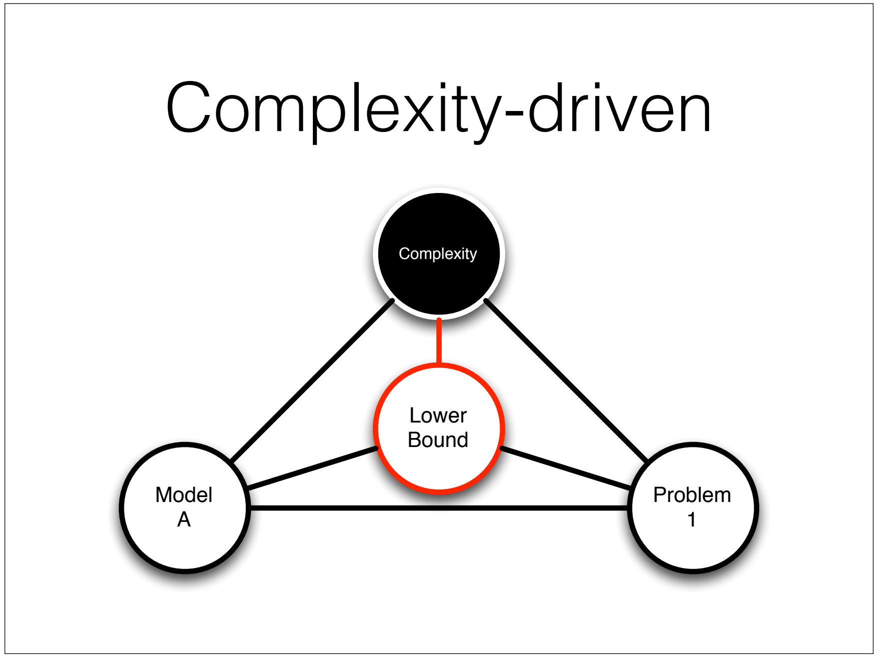
sebastien.tixeuil@lip6.fr

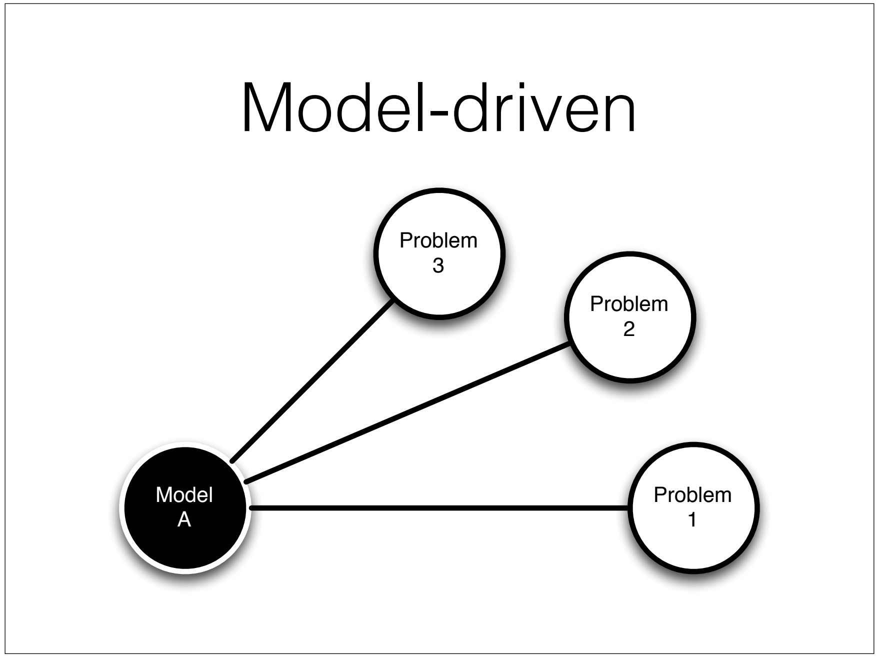
Three Domains

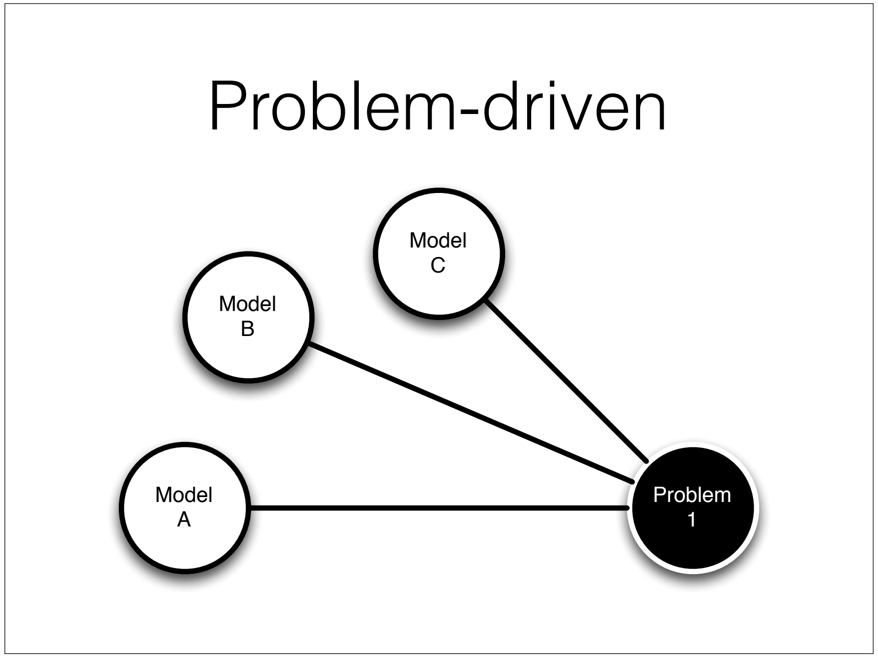
Three Approaches

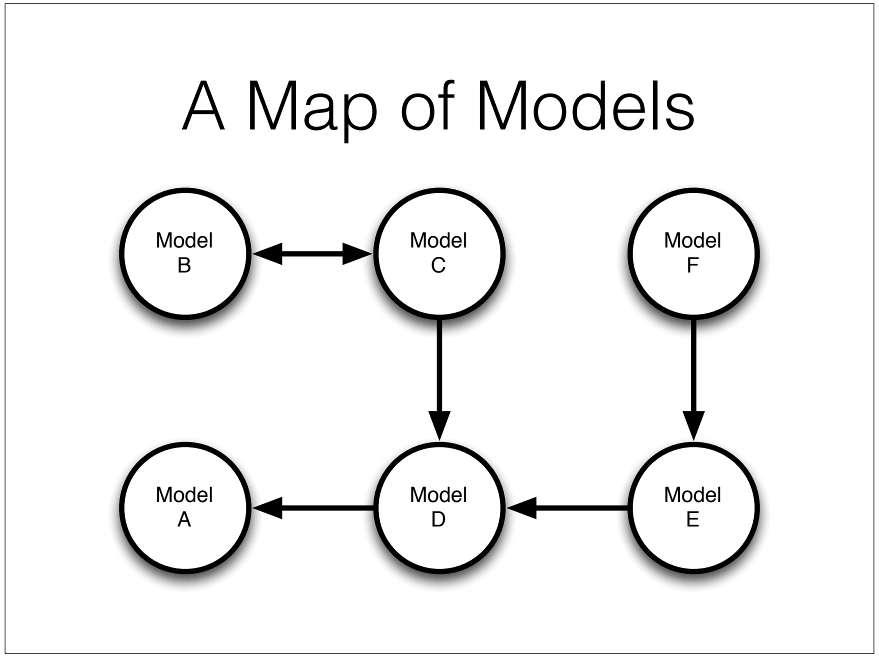
Complexity-driven

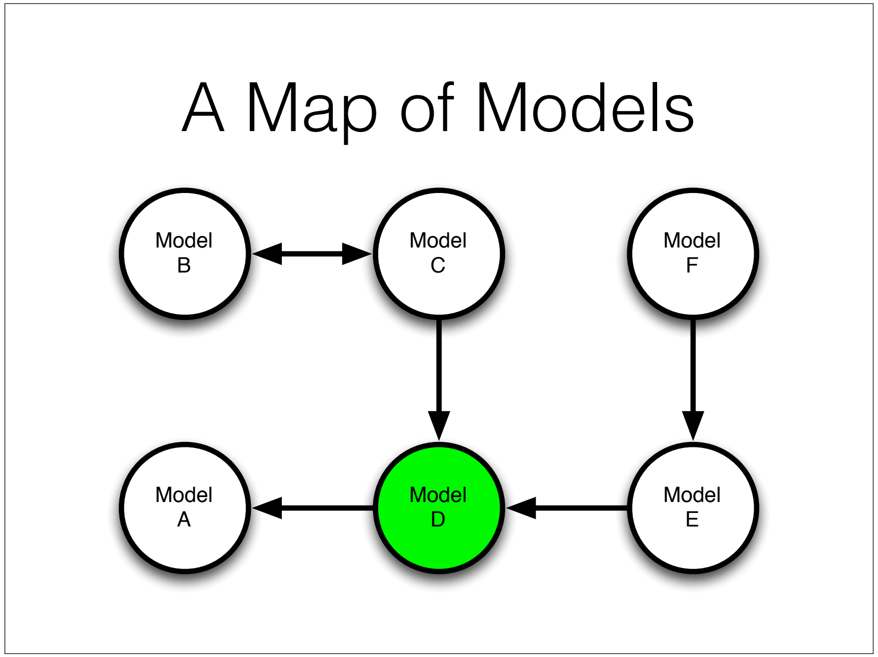



Complexity-driven
Complexity
Lower Bound
Model A
Problem 1


Model-driven
Problem 3
Problem 2
Model A
Problem 1


Problem-driven
Model C
Model B
Model A
Problem 1


A Map of Models
Model B
Model C
Model F
Model A
Model D
Model E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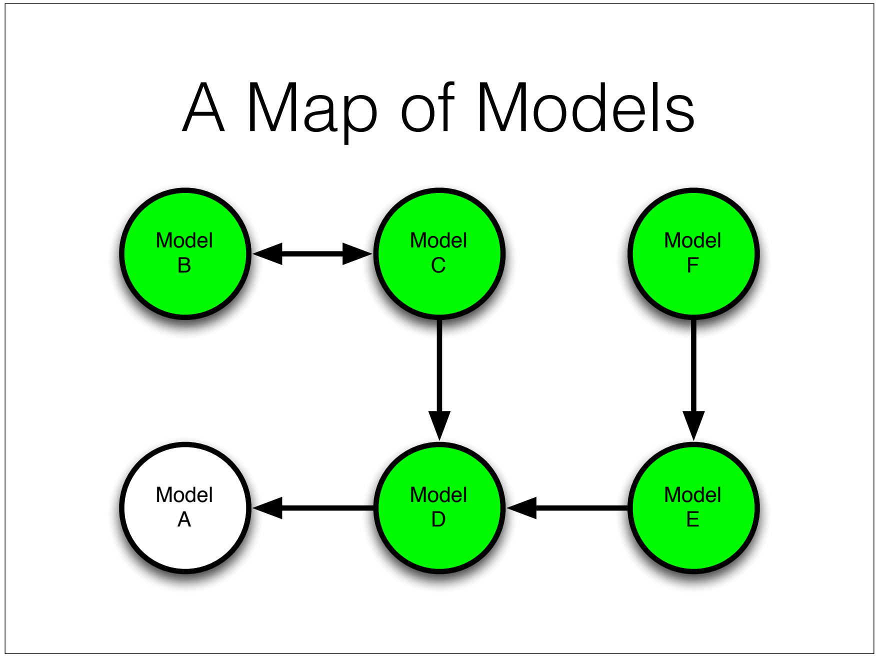
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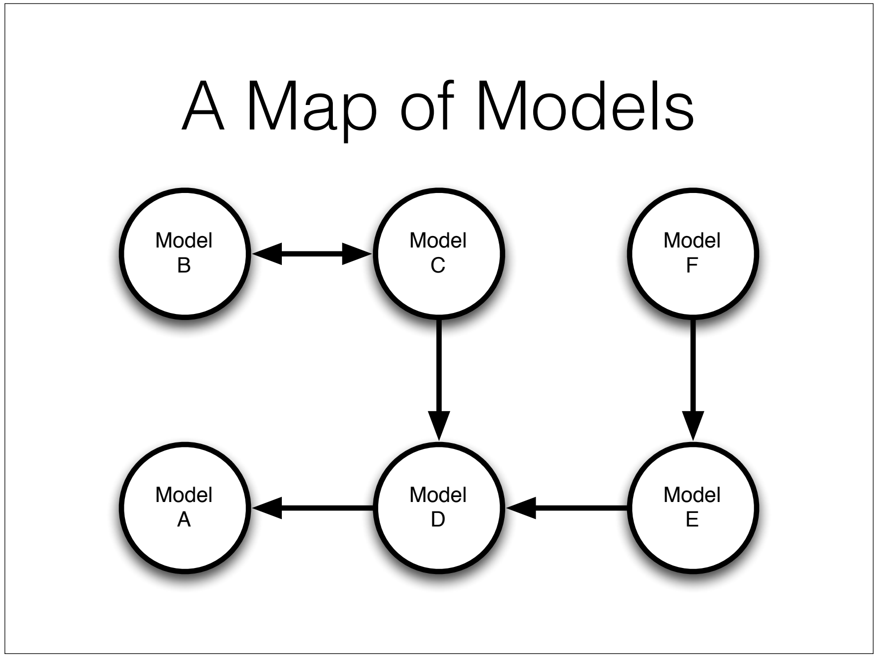
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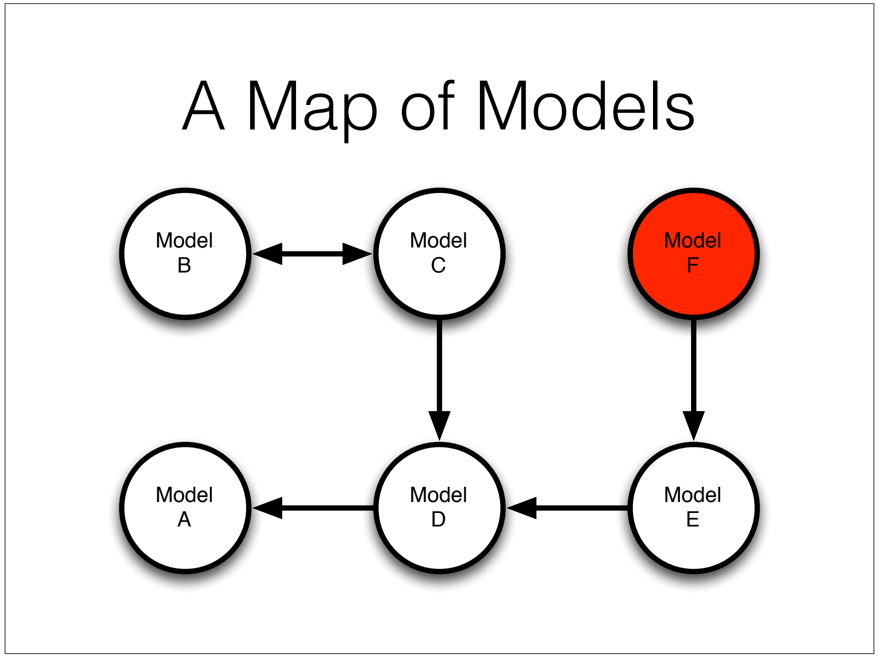
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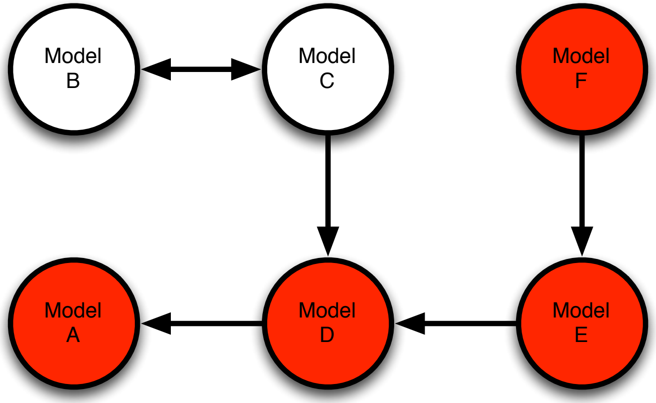
A Map of Models



Static Networks

Parallel Computing

Parallel Computing

Parallel Computing

Parallel Computing

- Tractable Sequential Problems
- Homogeneity
- Synchrony
- Reliable
- Focus on **Efficiency**

Distributed Computing

Distributed Computing

Distributed Computing

Distributed Computing

Distributed Computing

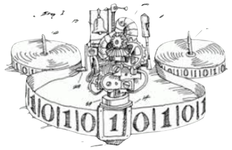
- Intrinsically distributed problems
- Heterogeneity
- Asynchrony
- Unreliable
- Focus on **Computability and Complexity**

Distributed Computing

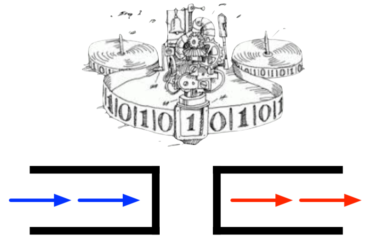
A distributed system is one in which the failure of a computer you didn't even know existed can render your own computer unusable.

Leslie Lamport

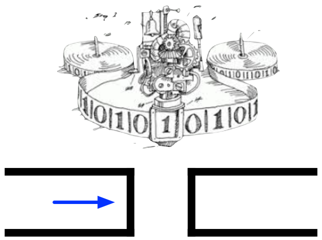
Distributed Computing Elements



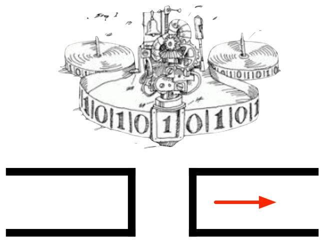
Distributed Computing Elements



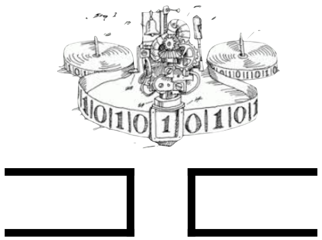
Distributed Computing Processor Actions



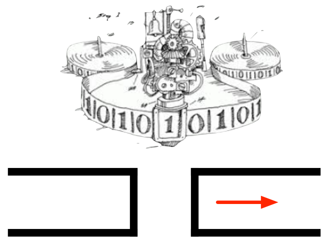
Distributed Computing Processor Actions



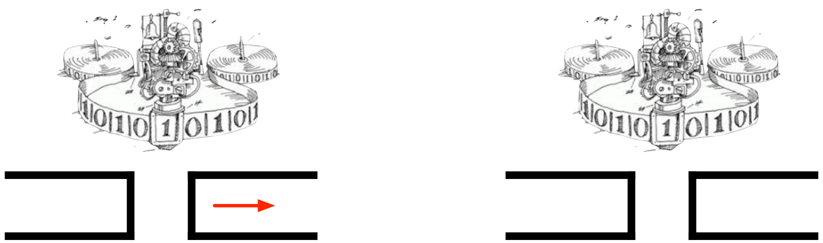
Distributed Computing
Processor Actions



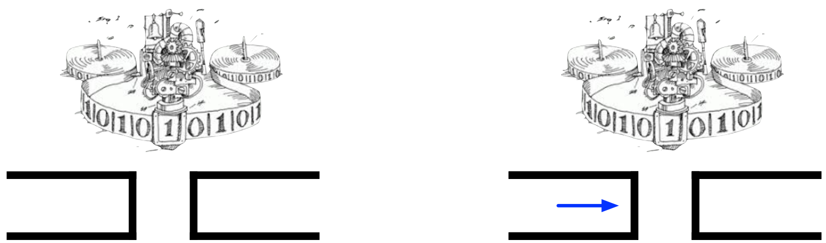
Distributed Computing
Processor Actions



Distributed Computing
Link Actions



Distributed Computing
Link Actions



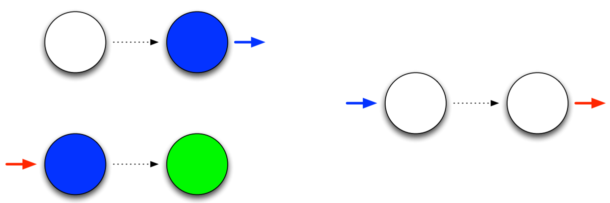
Asynchronous Distributed Execution

- **Sequence** of « processor or link » **actions**
- (***liveness***) Each processor executes an infinite number of actions (or terminates)
- (***liveness***) Each enabled link action eventually occurs

Client-Server

- *Initially*:
  - **Send** Request **to** Server
- *Upon receipt of Response from Server*:
  - **Terminate**
- *Upon receipt of Request from Client*:
  - **Send** Response **to** Client

Client-Server



Client-Server

Client-Server

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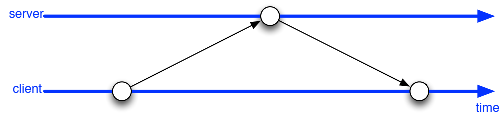
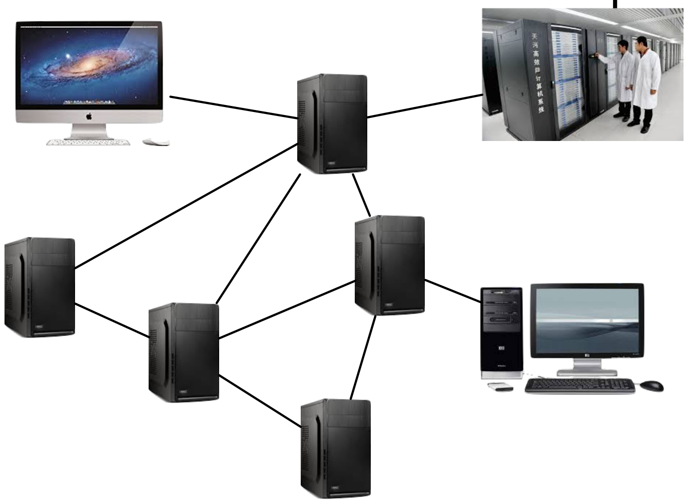
Client-Server

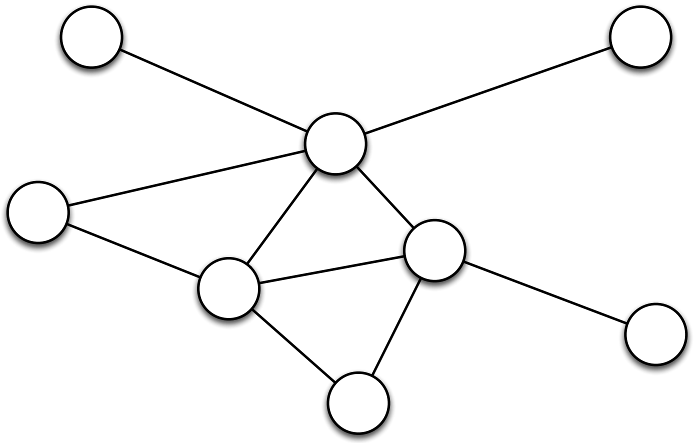
Space-Time Diagram



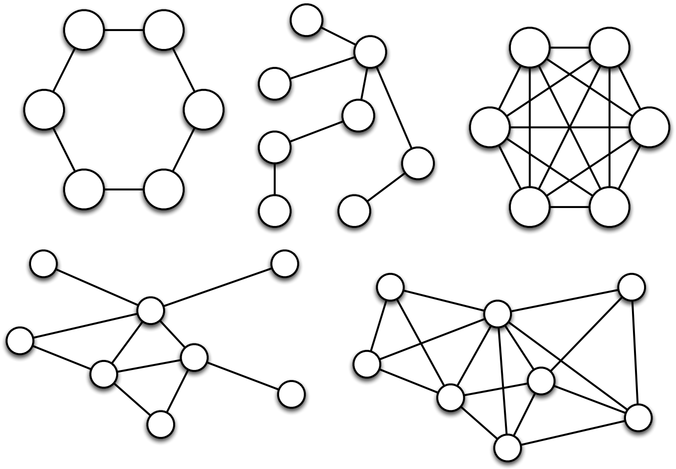
Communication Graph



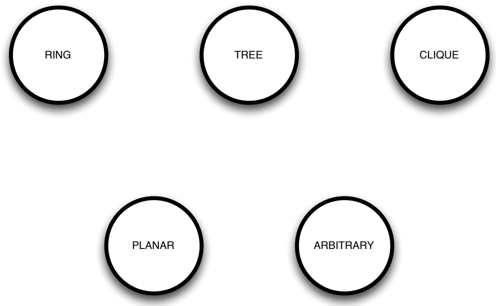
Communication Graph



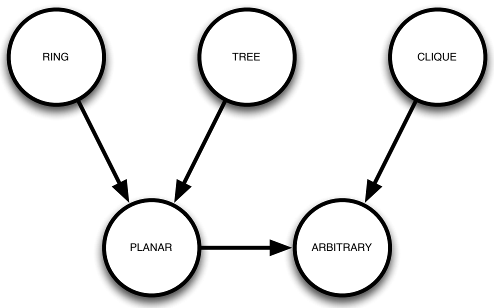
Communication Graphs



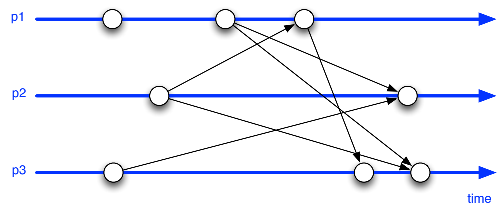
Communication Graph Models



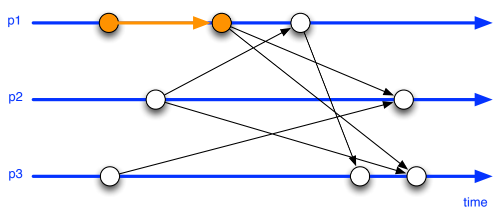
Communication Graph Models



Space-Time Diagram



Happens Before



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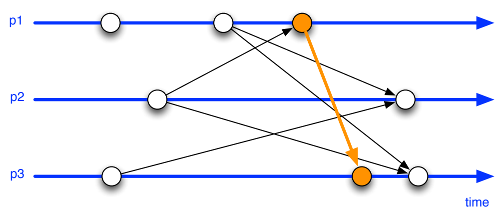

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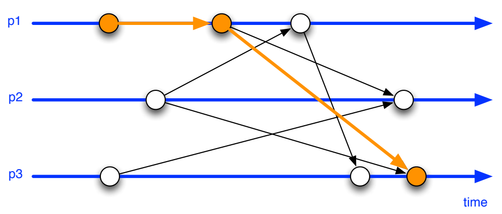
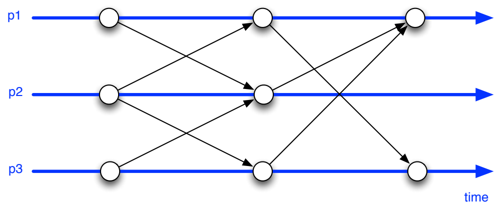


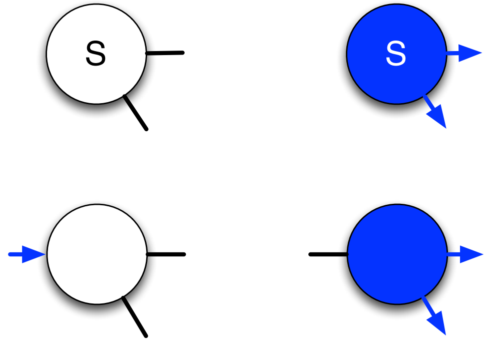
Synchronous Distributed Execution

- **Alternating sequence** of processor and link **phases**
  - In the **processor phase**, all processors that have not terminated execute their actions
  - In the **link phase**, all links execute their actions

Space-Time Diagram



Flooding



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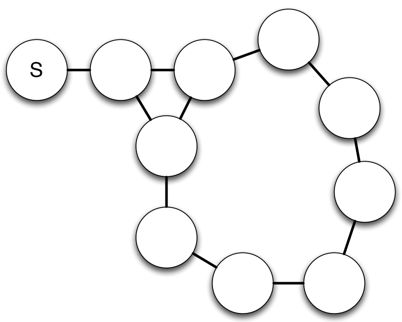

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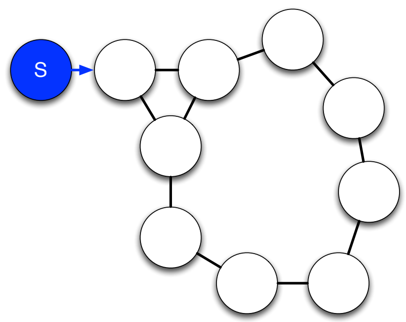

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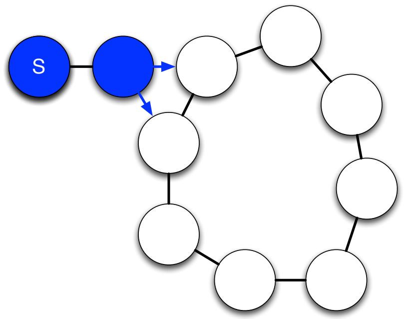

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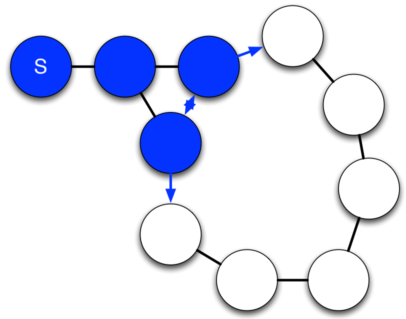

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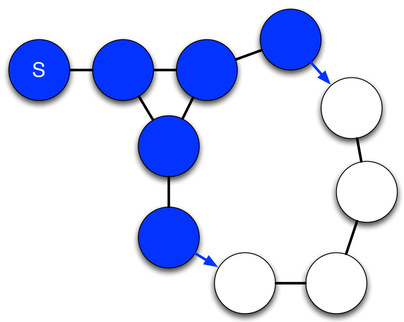

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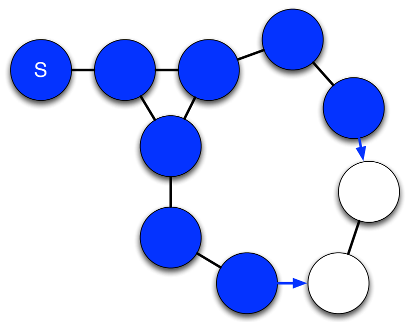

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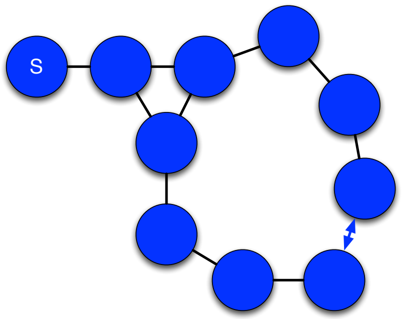
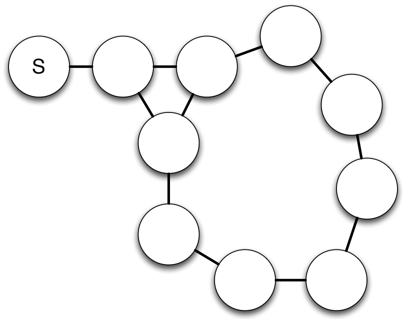


Asynchronous Flooding



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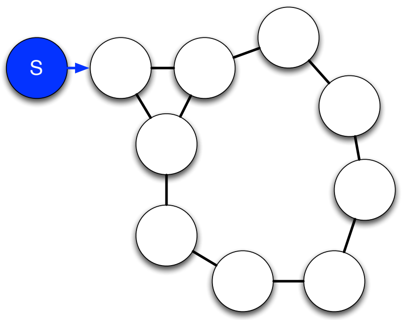

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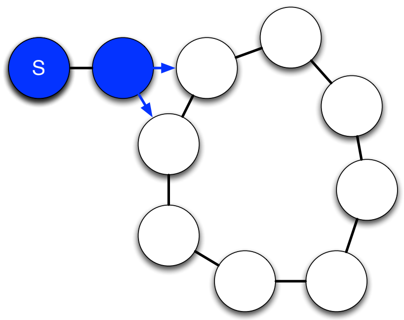

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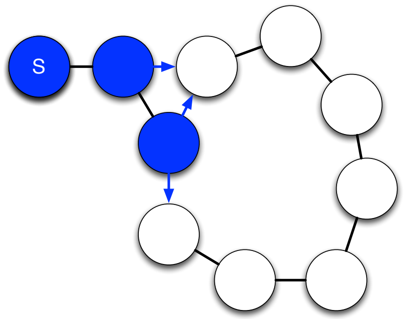

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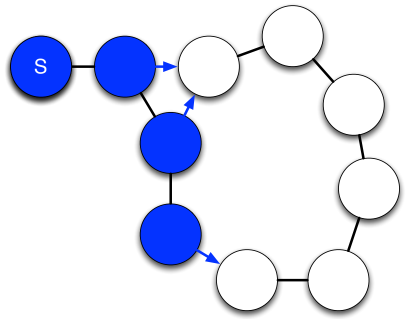

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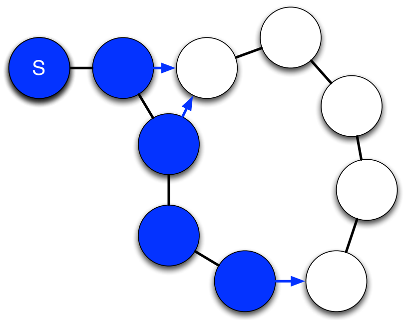

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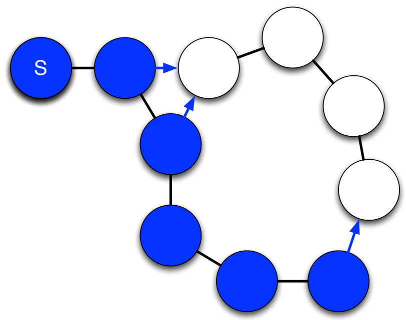

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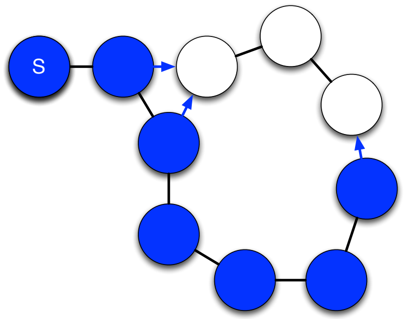

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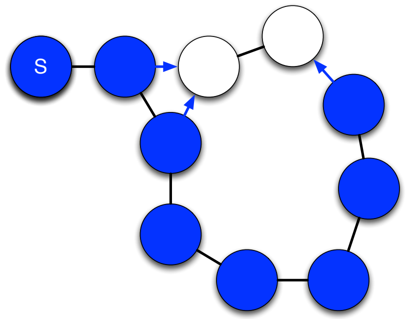

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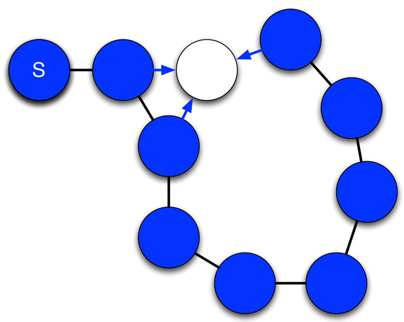

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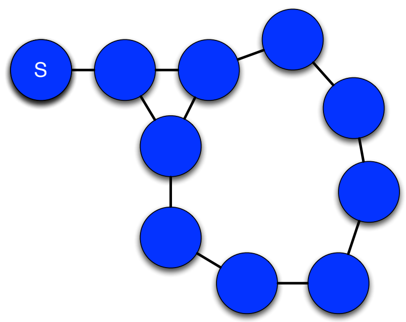
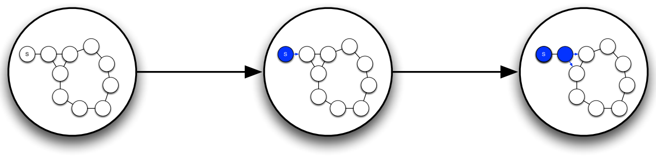


Configurations



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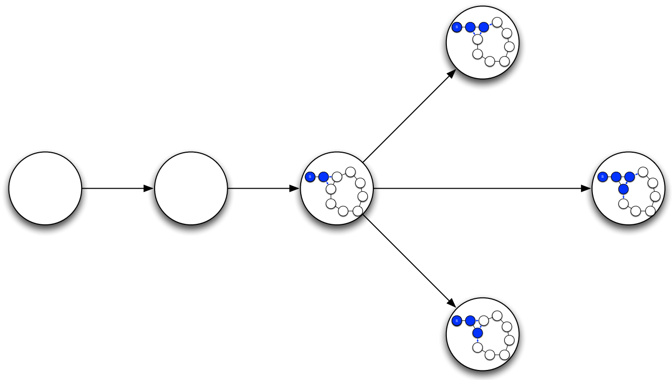

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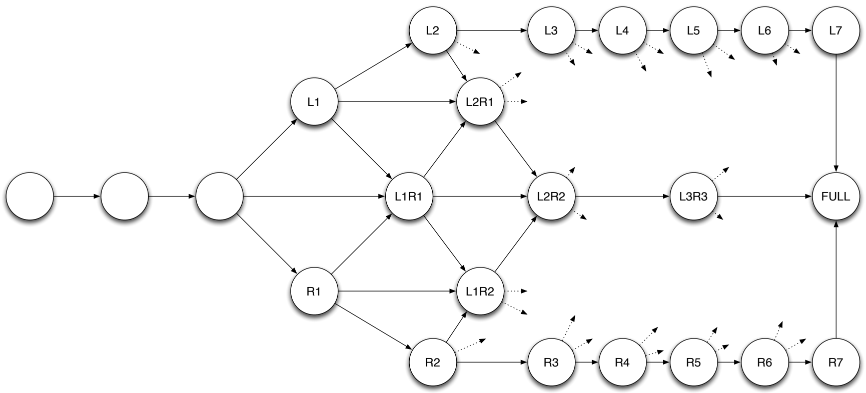
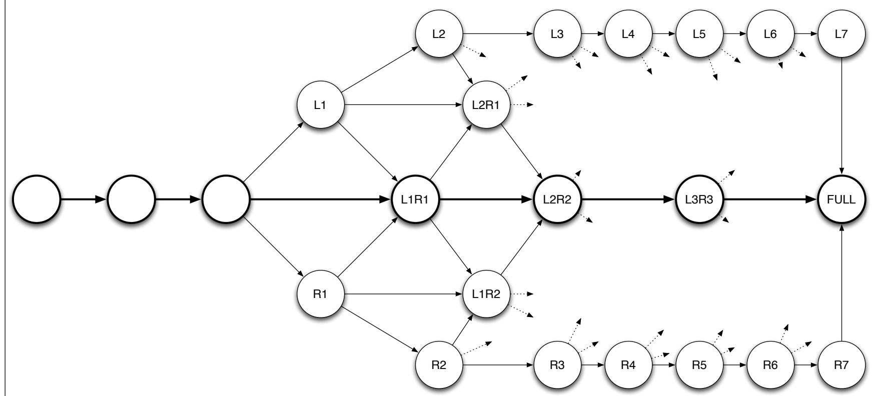


Executions



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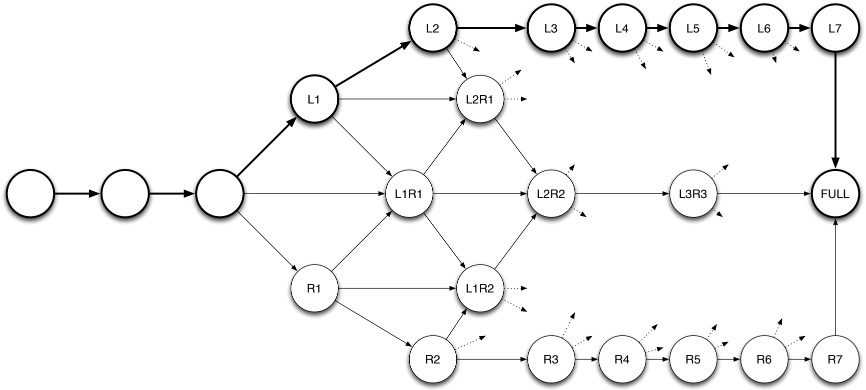
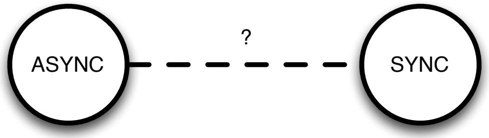


Synchronous *vs.* Asynchronous



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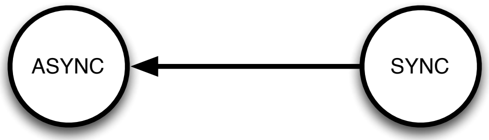
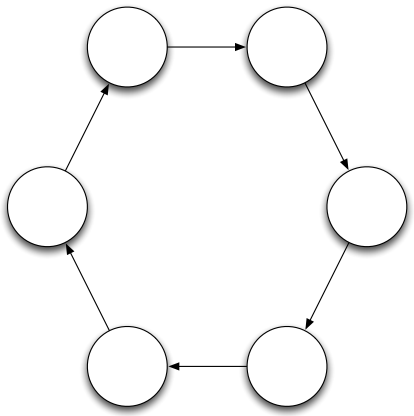


Leader Election



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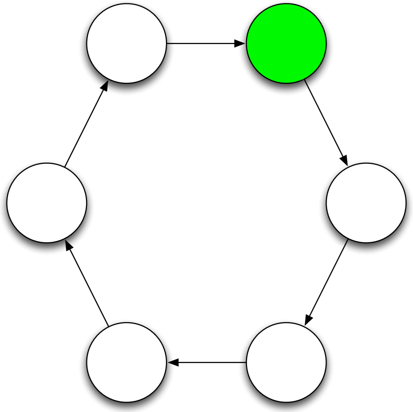

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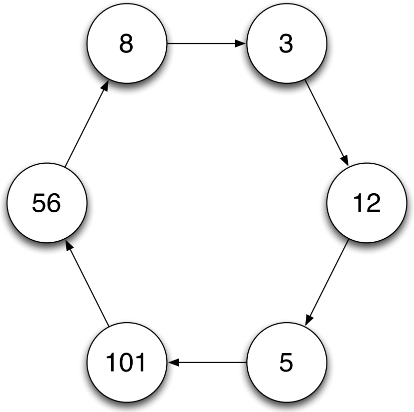

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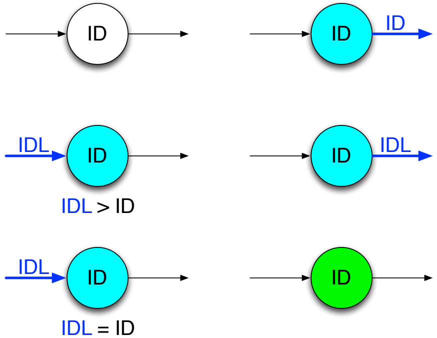

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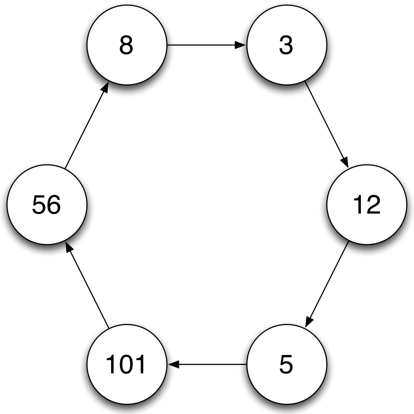

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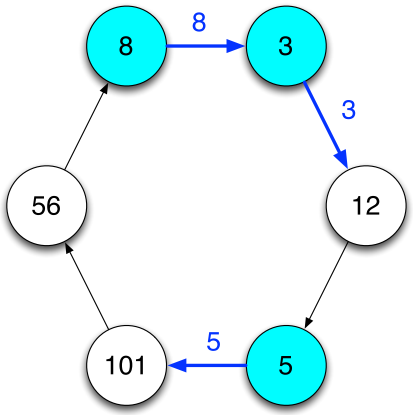
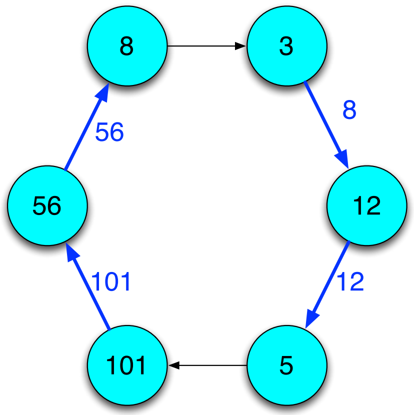


Leader Election



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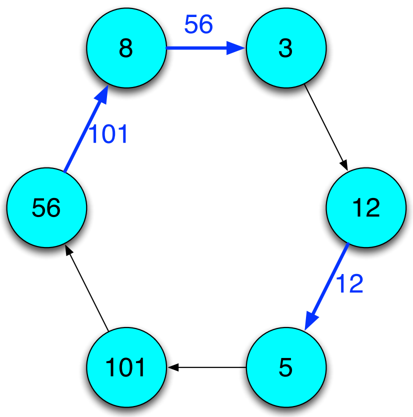

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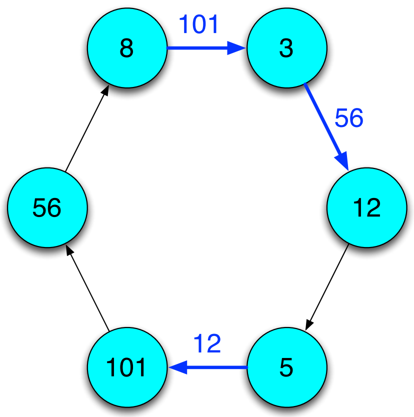

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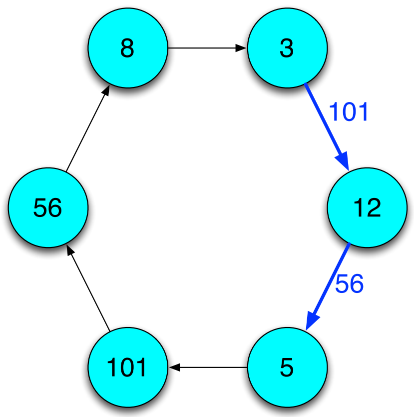
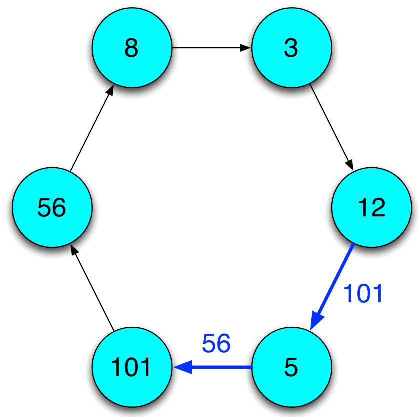


Leader Election



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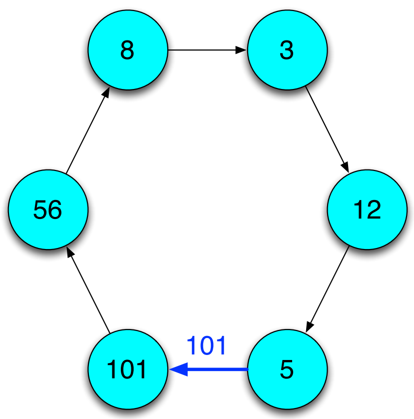

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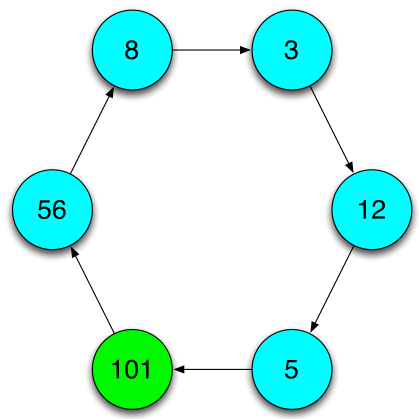
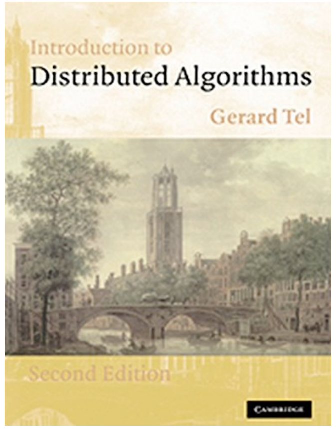
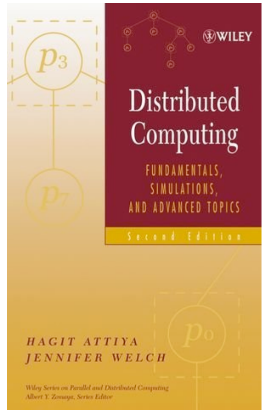


Leader Election

- **Message complexity ?**
  - Lower bound ?
- **Weaker model ?**
  - No IDs ?
  - No Orientation ?
  - General communication graph ?

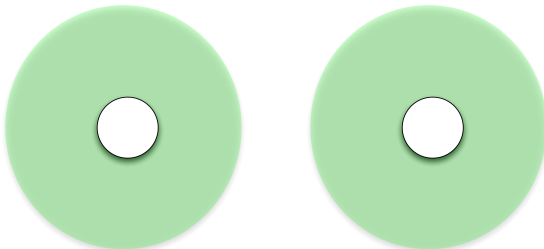
Static Networks



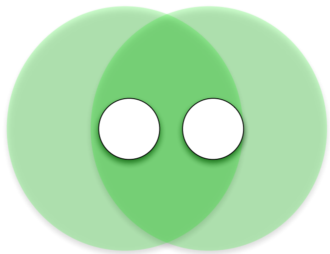


Passively Mobile Networks

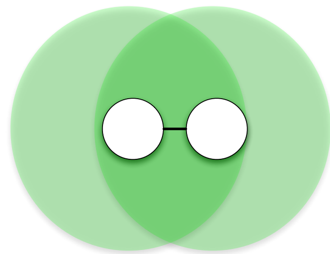
Mobility-induced Dynamic Networks



Mobility-induced
Dynamic Networks



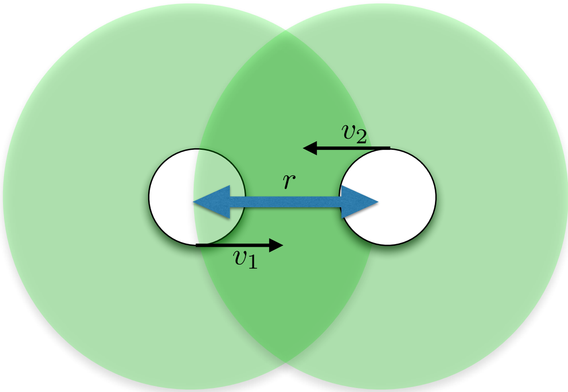
Mobility-induced
Dynamic Networks



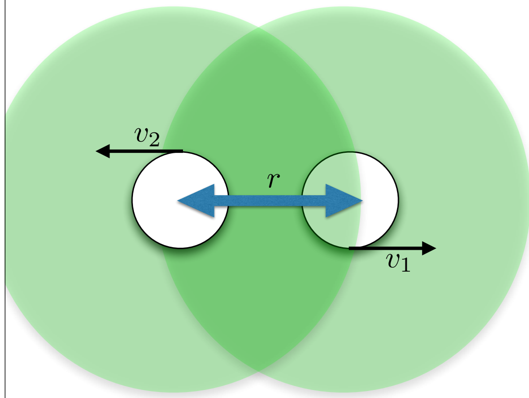
Static Algorithms for
Mobile Networks

Link Lifetime

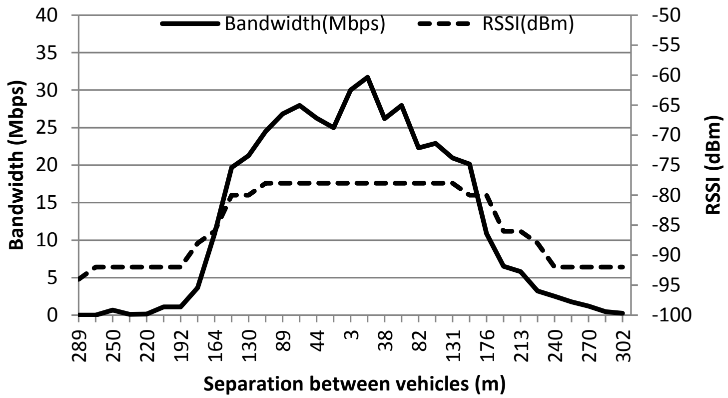
Link Lifetime



Link Lifetime

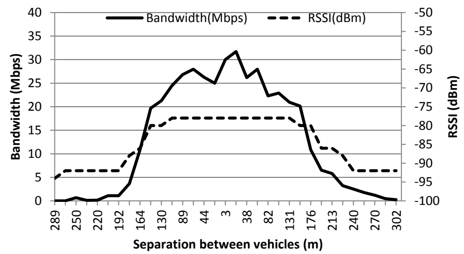


Link Lifetime



On the use of WiMAX and Wi-Fi to provide in-vehicle connectivity and media distribution. Lerotholi S. Mojela ; Marthinus J. Booysen, Industrial Technology (ICIT), 2013 IEEE International Conference on

Link Lifetime



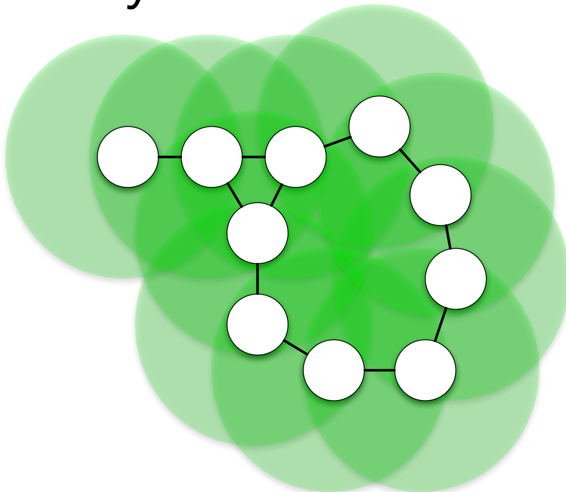
On the use of WiMAX and Wi-Fi to provide in-vehicle connectivity and media distribution. Lerotholi S. Mojela ; Marthinus J. Booysen, Industrial Technology (ICIT), 2013 IEEE International Conference on

Fig. 4b shows a graph of the two vehicles traveling in opposite directions at an average relative speed of 64 km/h in an urban area. The average contact time recorded was 33s and the average maximum communication range was found to be 302 m with an average bandwidth of 13.7 Mbps per test run taken over the period of established contact, average jitter of 1.88 ms and an average of 51.7 MB data transferred per contact period. The maximum peak bandwidth of 31.7 Mbps

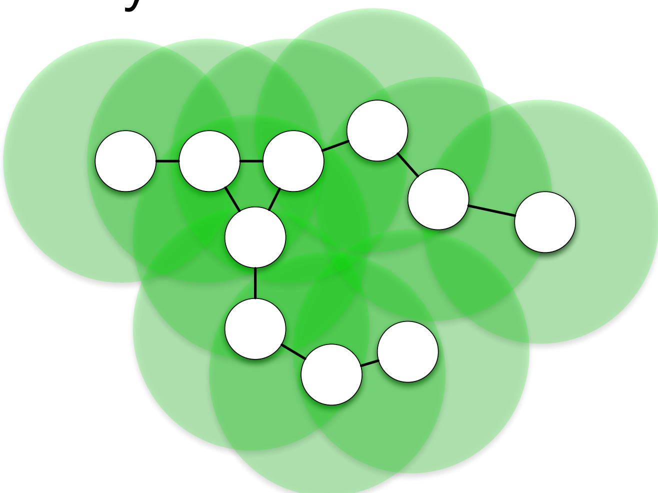
Round-trip Time

	Barcelona	Paris	Tokyo	Toronto	Washington
Amsterdam	44.173ms	11.876ms	242.819ms	96.619ms	93.342ms
Auckland	295.323ms	280.111ms	241.541ms	199.854ms	211.48ms
Berlin	49.71ms	20.041ms	275.013ms	103.027ms	113.909ms
Copenhagen	43.745ms	25.574ms	272.459ms	117.751ms	100.623ms
Dallas	143.543ms	114.941ms	145.957ms	76.173ms	34.194ms
London	35.431ms	4.542ms	240.519ms	85.102ms	75.484ms
Los Angeles	157.289ms	141.484ms	112.265ms	58.689ms	71.881ms
Moscow	74.806ms	55.67ms	299.067ms	137.808ms	144.68ms
New York	130.6ms	77.551ms	171.641ms	17.78ms	47.161ms
Paris	31.085ms	—	260.569ms	103.015ms	107.643ms
Stockholm	59.785ms	27.894ms	285.462ms	123.358ms	115.657ms
Tokyo	267.753ms	268.811ms	—	172.841ms	186.818ms

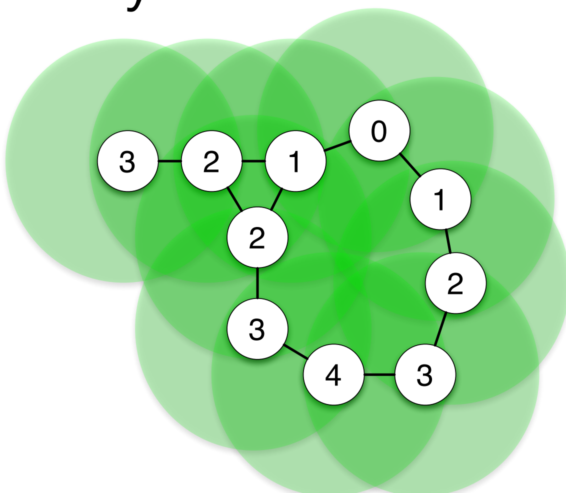
Mobility *vs.* Global State



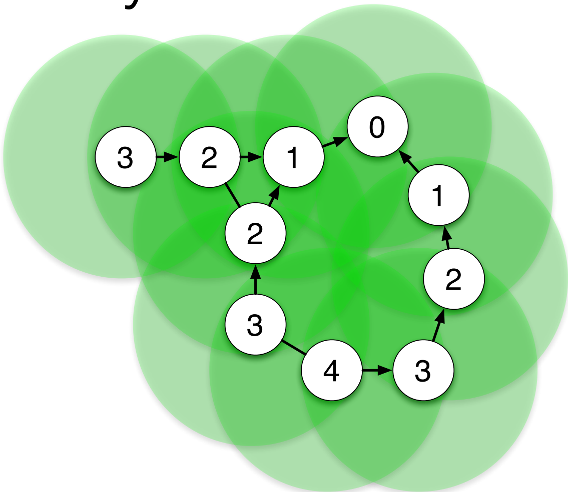
Mobility *vs.* Global State



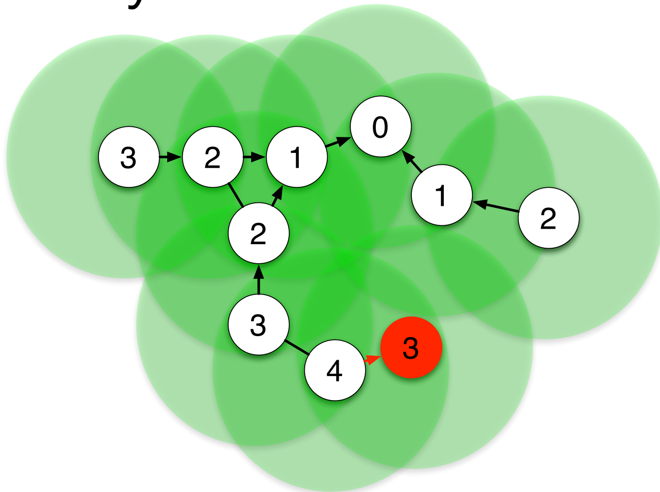
Mobility *vs.* Global State



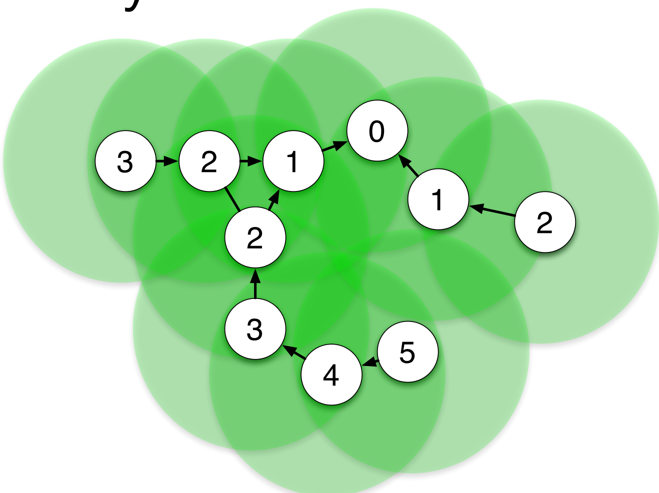
Mobility *vs.* Global State



Mobility *vs.* Global State

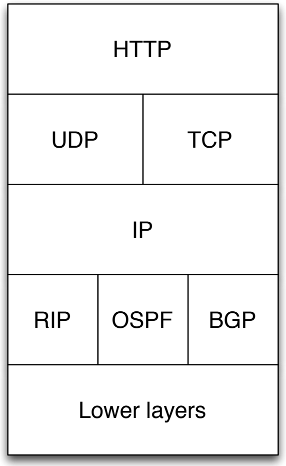


Mobility *vs.* Global State

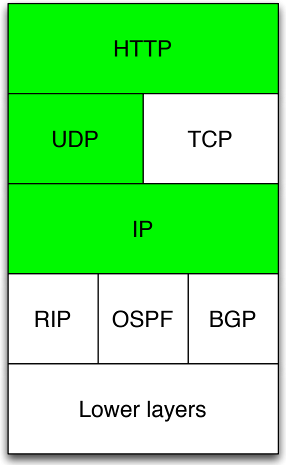


Stateless Algorithms

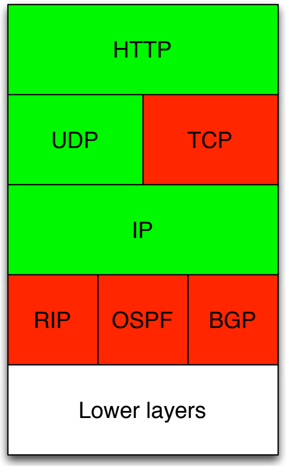
Statelessness



Statelessness



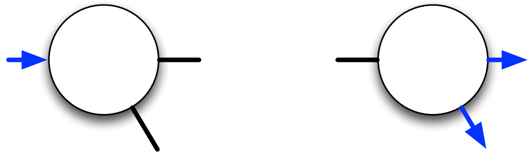
Statelessness



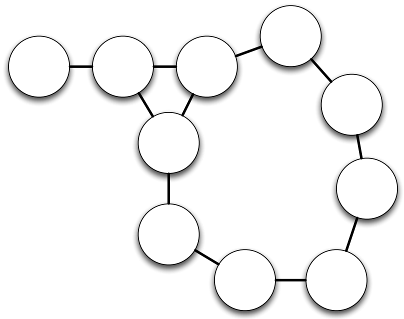
Stateless Routing

A routing algorithm is **stateless** if it is designed such that devices store *no information* about messages *between transmissions*. It is **stateful** otherwise.

Flooding



Stateless Flooding



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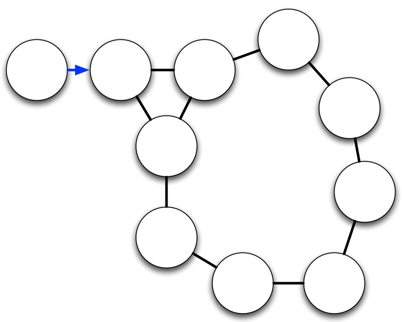

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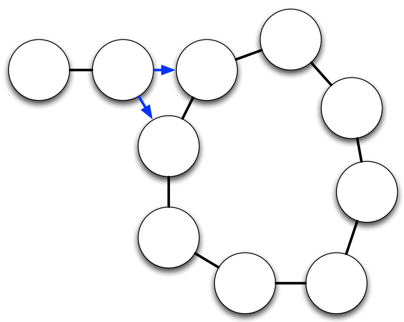

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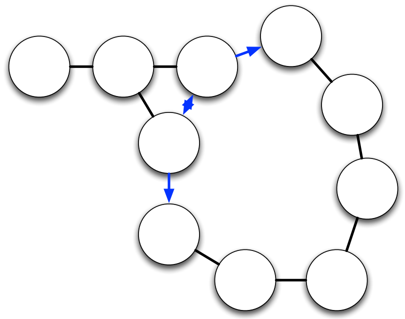
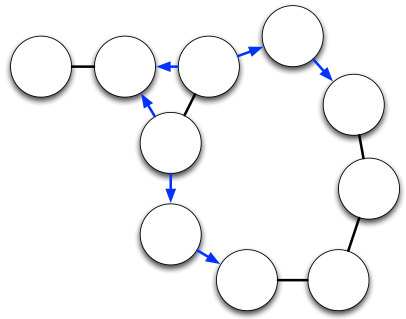


Stateless Flooding



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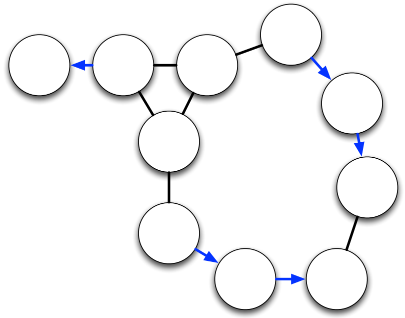

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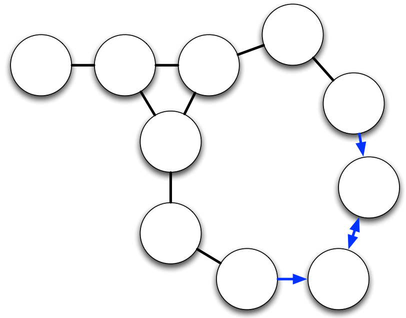

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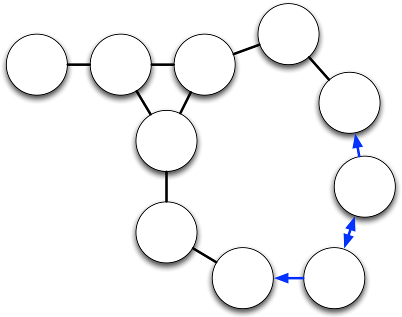

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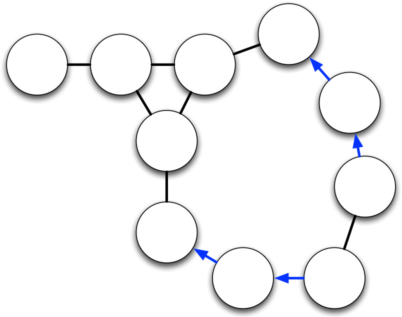

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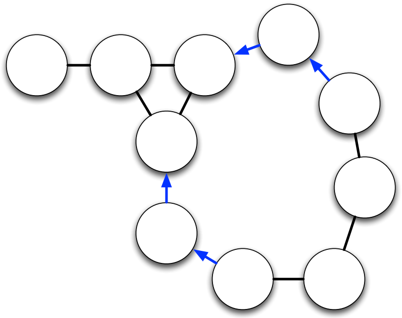

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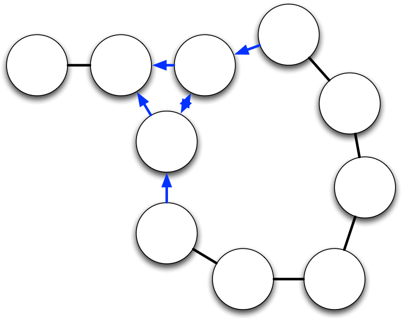

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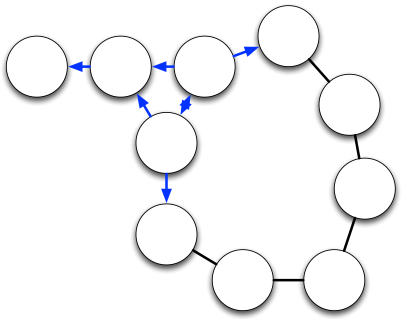

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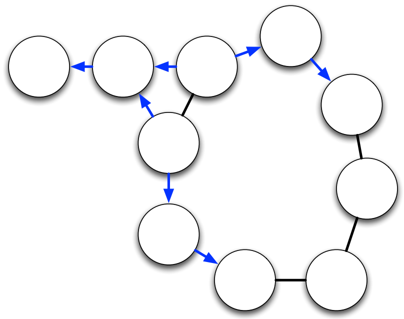

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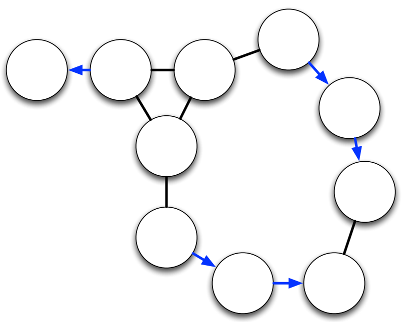
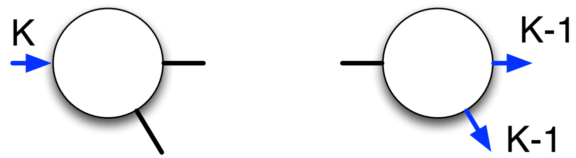
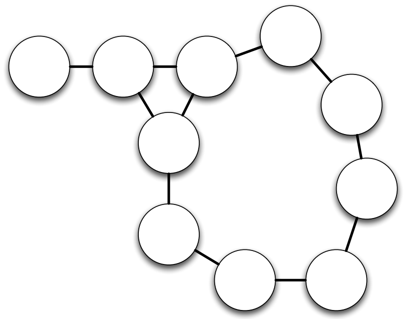


Flooding v2



TTL Flooding



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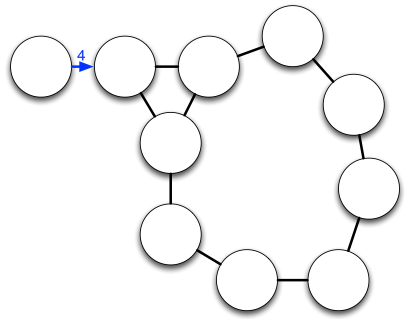

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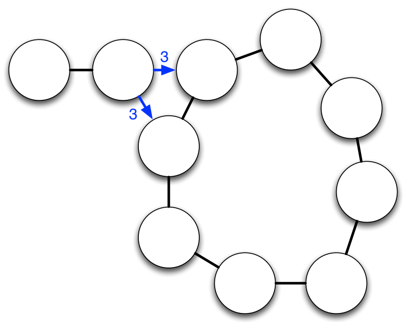
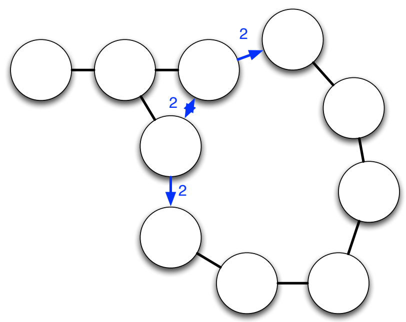


TTL Flooding



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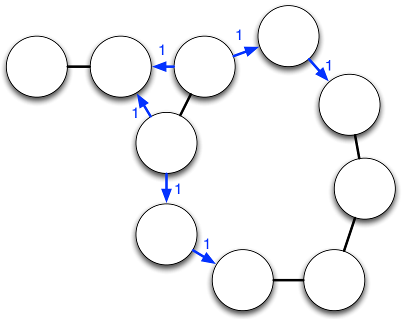

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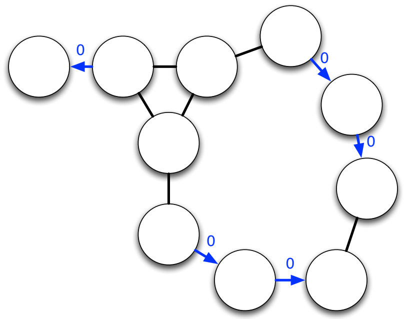

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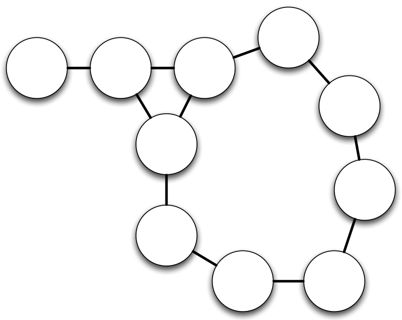
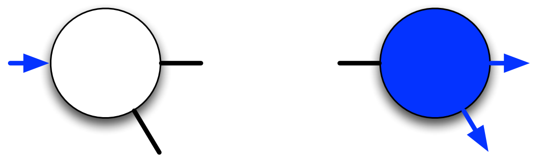
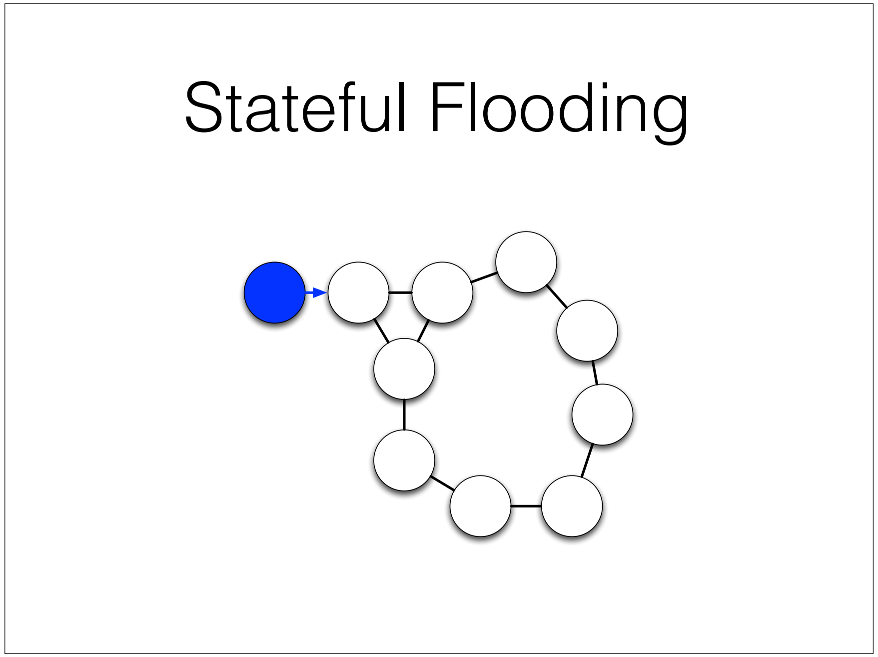
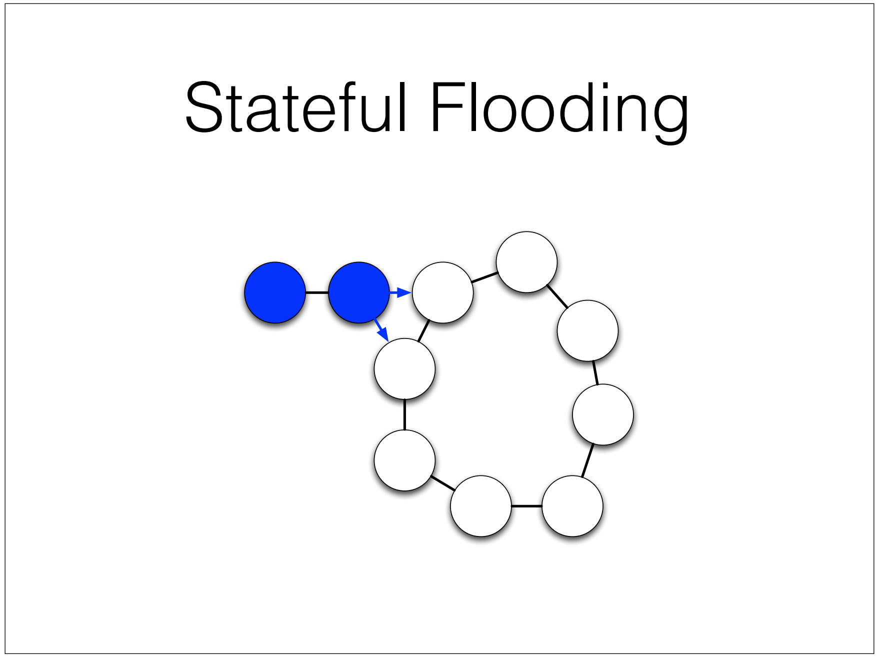
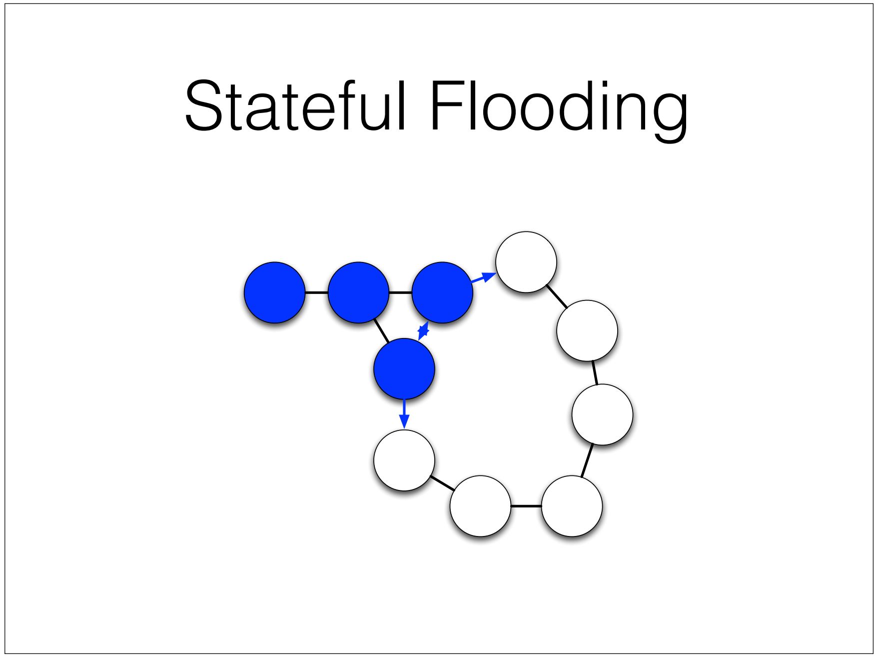


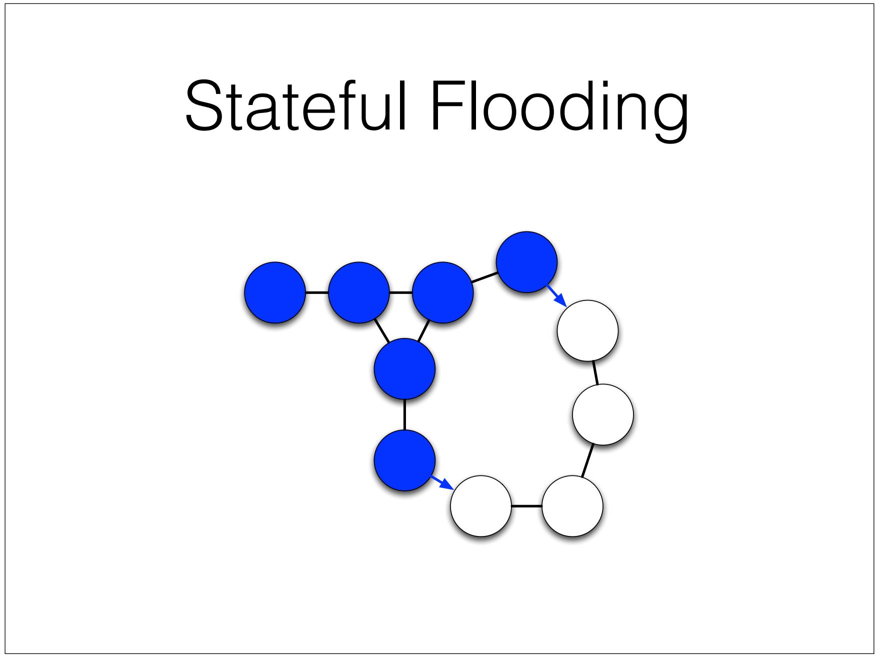
Flooding v3



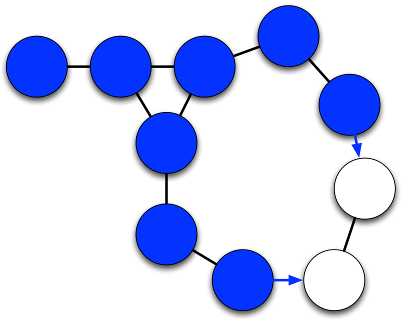

Stateful Flooding


Stateful Flooding


Stateful Flooding


Stateful Flooding

Stateful Flooding



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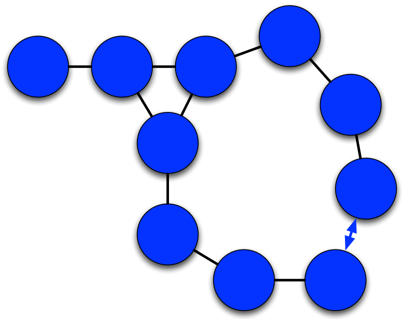

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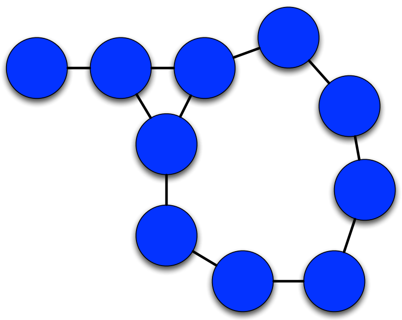
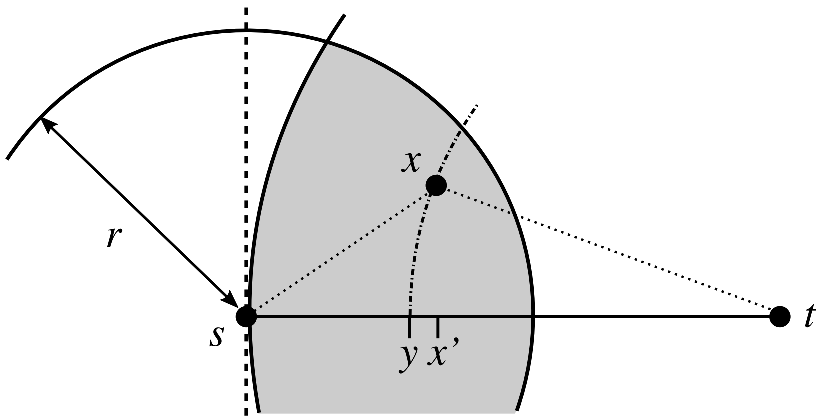


Geometric Routing

- Each node is aware of its *coordinates* (and those of its neighbors)
- The message contains the coordinates of the destination
- **Goal**: deliver the message to the destination *without routing tables*

Stojmenovic, Ivan (2002). "Position based routing in ad hoc networks". IEEE Communications Magazine. 40 (7): 128–134.

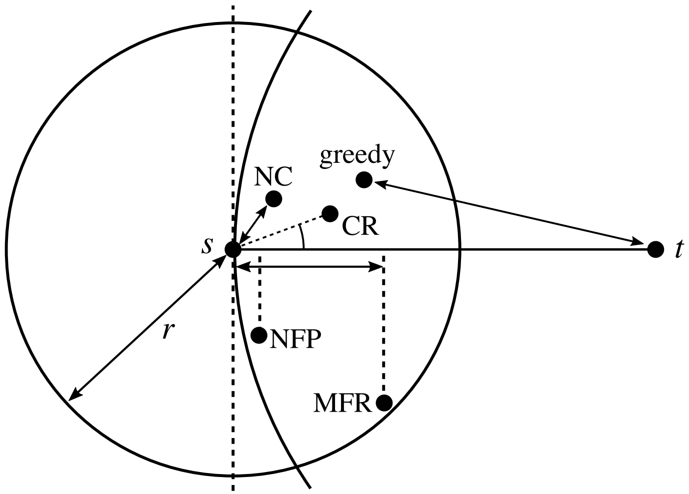
Progress *vs.* Distance



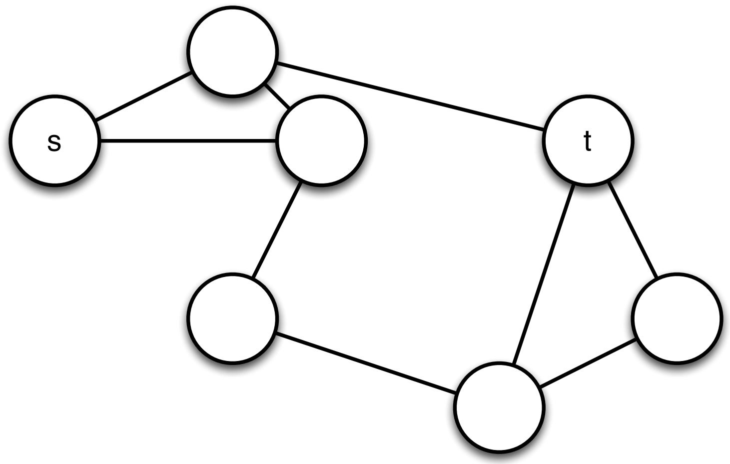
Which Criterion?

- **MFR**: most forwarding progress
- **CR**: minimize angular criterion
- **Greedy**: minimize distance to destination
- **NC**: nearest closer
- **NFP**: nearest with forwarding progress

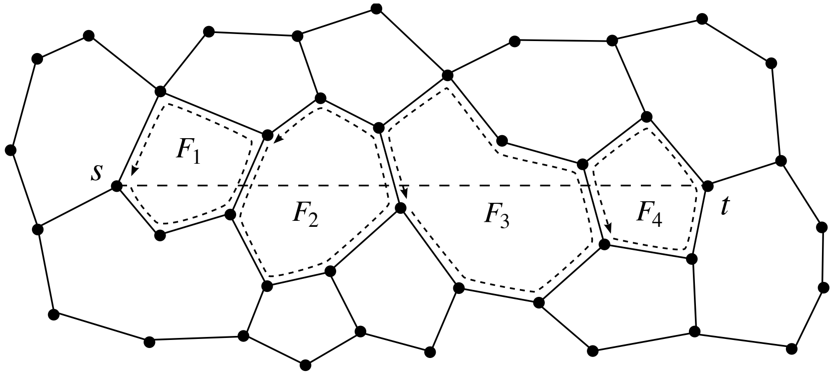
Which Criterion?



Delivery Guarantee?

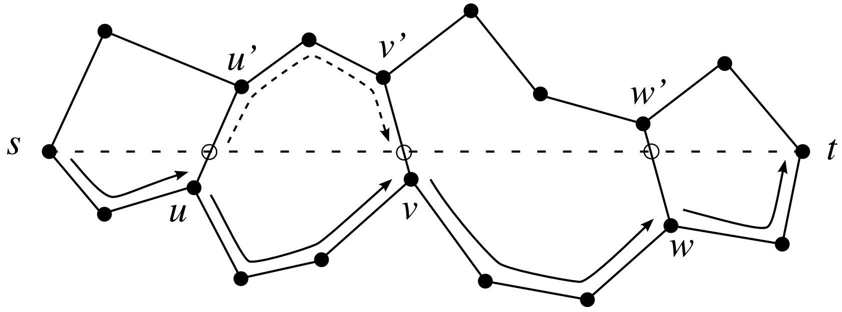


Planar Graph Routing

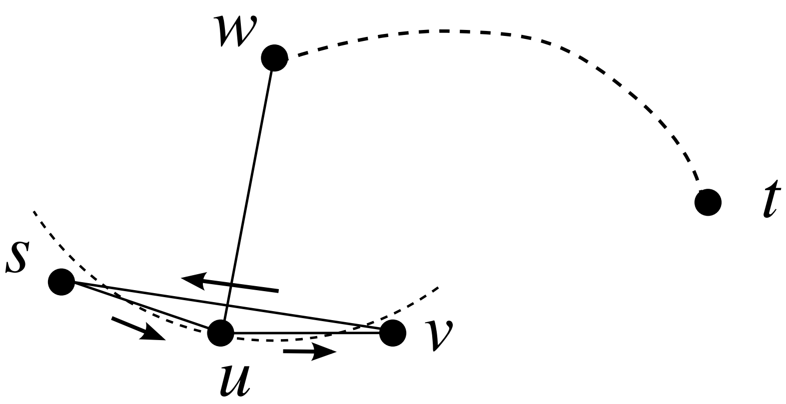


Bose, P.; Morin, P.; Stojmenovic, I.; Urrutia, J. (1999). "Routing with guaranteed delivery in ad hoc wireless networks". Proc. of the 3rd international workshop on discrete algorithms and methods for mobile computing and communications (DIALM '99). pp. 48–55.

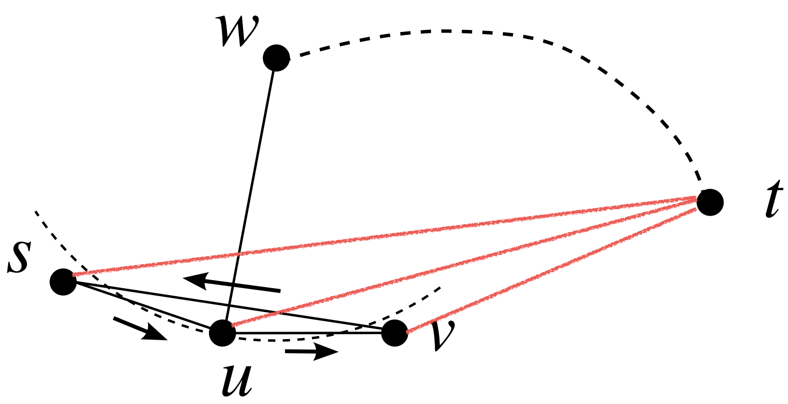
Face Routing



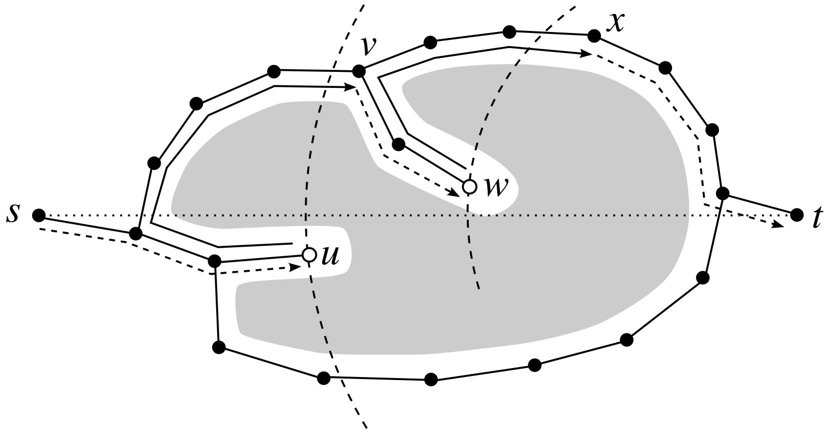
Planar Graphs!



Planar Graphs!



Greedy / Face / Greedy



Self-stabilization

Example

$U_0 = a$

$U_{n+1} = \frac{U_n}{2}$ if U_n is even

$U_{n+1} = \frac{3U_n+1}{2}$ if U_n is odd

Example

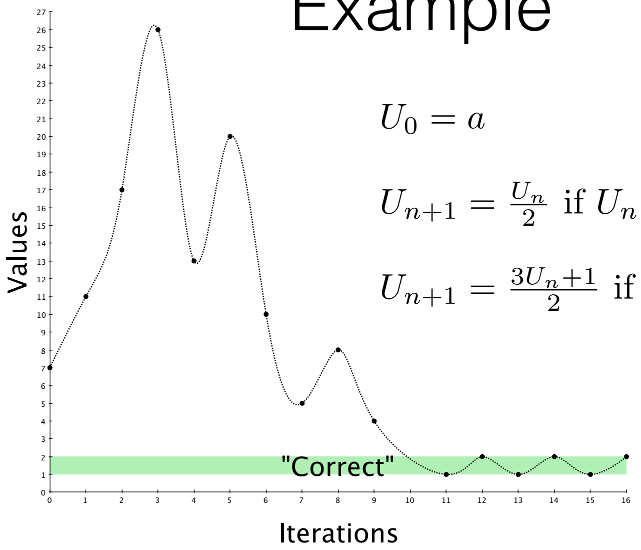
$U_0 = a$

$U_{n+1} = \frac{U_n}{2}$ if U_n is even

$U_{n+1} = \frac{3U_n+1}{2}$ if U_n is odd

n	0	1	2	3	4	5	6	7	8	9	10	11	12
U_n	7	11	17	26	13	20	10	5	8	4	2	1	2

Example

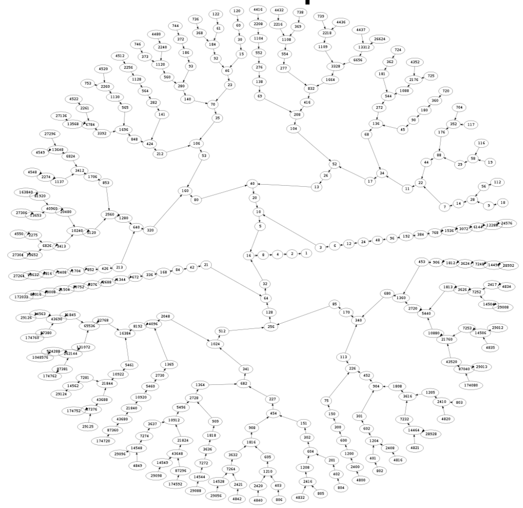


$$U_0 = a$$

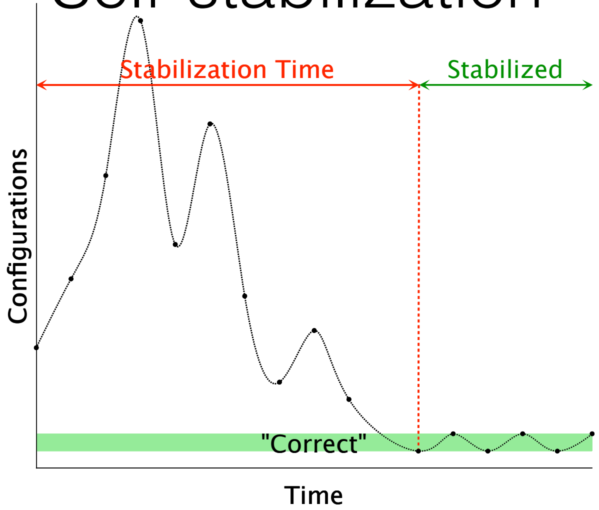
$$U_{n+1} = \frac{U_n}{2} \text{ if } U_n \text{ is even}$$

$$U_{n+1} = \frac{3U_n+1}{2} \text{ if } U_n \text{ is odd}$$

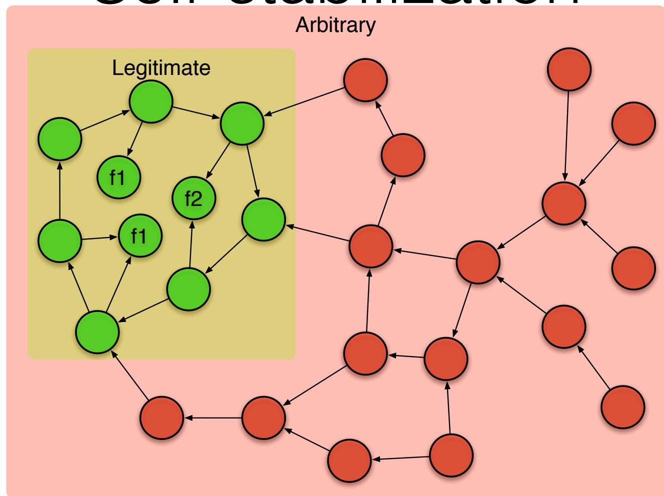
Example



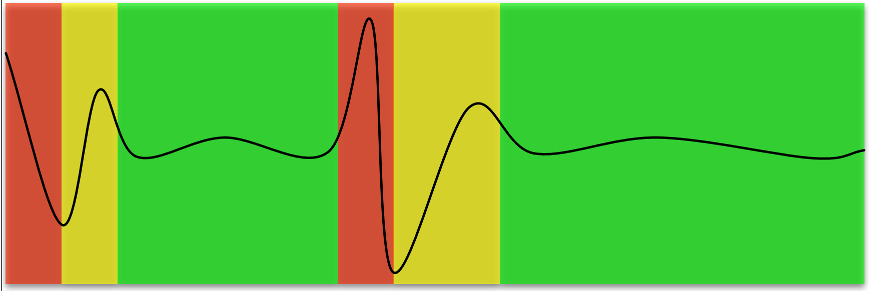
Self-stabilization



Self-stabilization



Self-stabilization



Distributed Systems

- **Configuration**: product of the local states of system components
- **Execution**: interleaving of the local executions of the system components

Distributed Systems

- **Classical**: Starting from a particular initial configuration, the system immediately exhibits correct behavior
- **Self-stabilizing**: Starting from any initial configuration, the system eventually reaches a configuration from which its behavior is correct

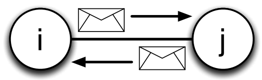
Distributed Systems

- **Self-stabilizing**: Starting from any initial configuration, the system eventually reaches a configuration from which its behavior is correct
  - Defined by *Dijkstra* in 1974
  - Advocated by *Lamport* in 1984 to address fault-tolerant issues
  - Stale states due to **mobility** can be recovered!

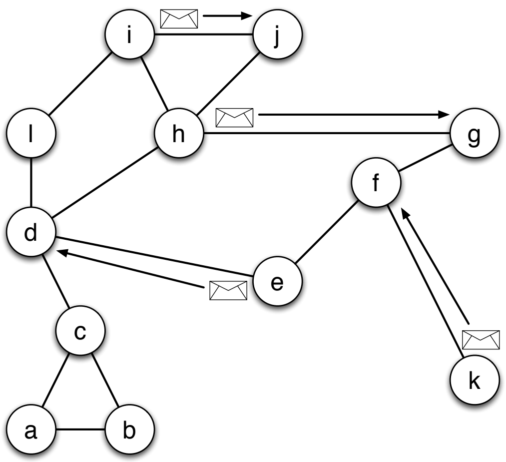
Configurations

```
int x = 0;
...
if( x == 0 ) {
  // code assuming x equals 0
}
else {
  // code assuming x does not equal 0
}
```

Configurations



Configurations



Hypotheses

Atomicity

- A «*stabilizing*» sequential program

```
int x = 0;
...
while( x == x ) {
  x = 0;
  // code assuming x equals 0
}
```

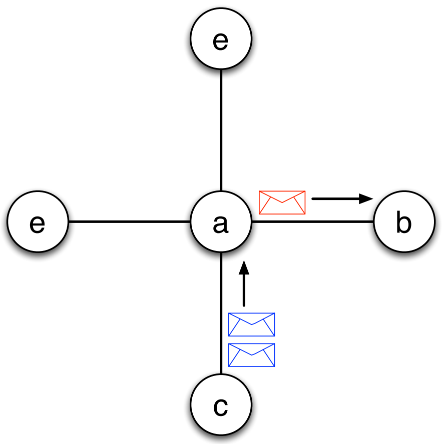
Atomicity

- A «*stabilizing*» sequential program

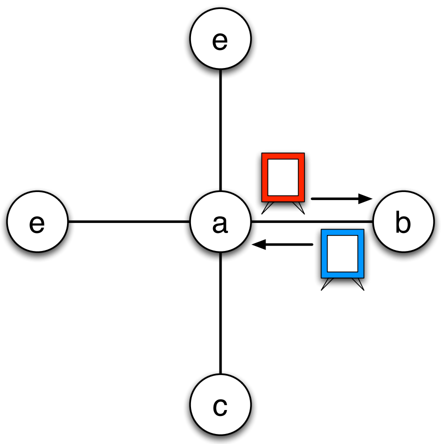
```
0 iconst_0
1 istore_1
2 goto 7
5 iconst_0
6 istore_1
7 iload_1
8 iload_1
9 if_icmpeq 5
```

Problem

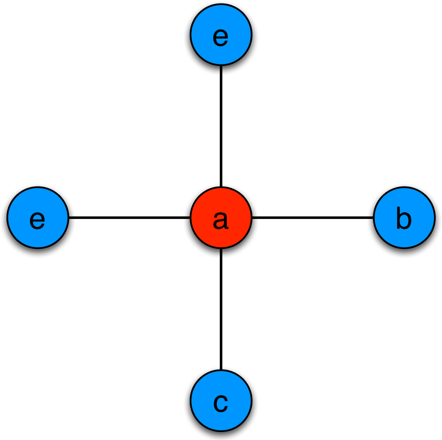
Communications



Communications



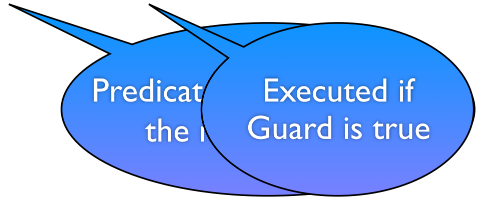
Communications



Example

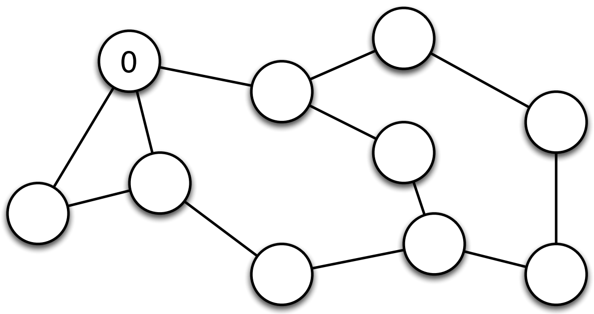
- **Shared memory**: in one atomic step, read the state of all neighbors and write own state
- **Guarded command**

$$Guard \rightarrow Action$$



Example

$$true \rightarrow Distance_i := Min_{j \in Neighbors_i}\{Distance_j + 1\}$$



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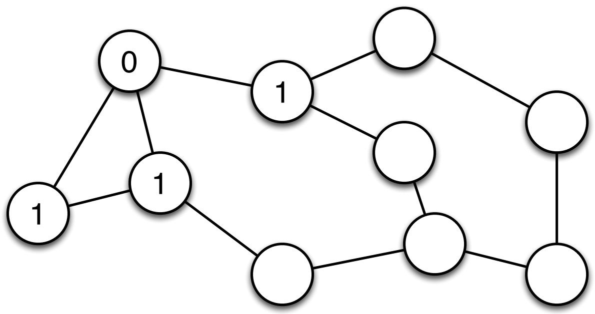

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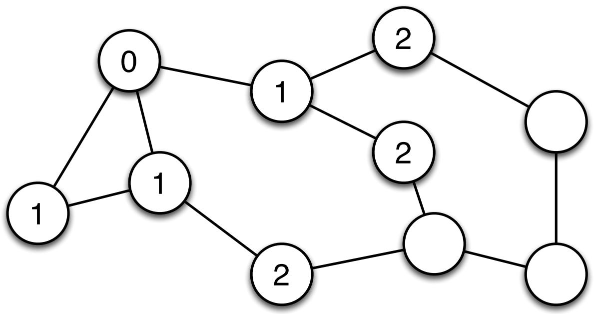

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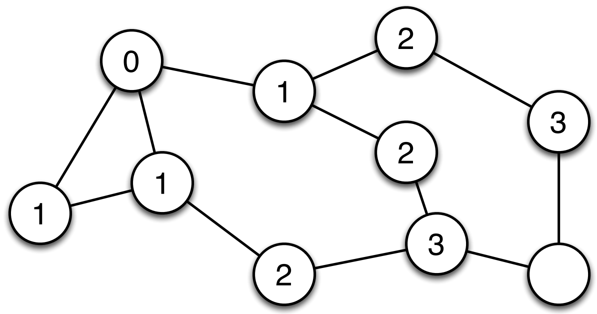

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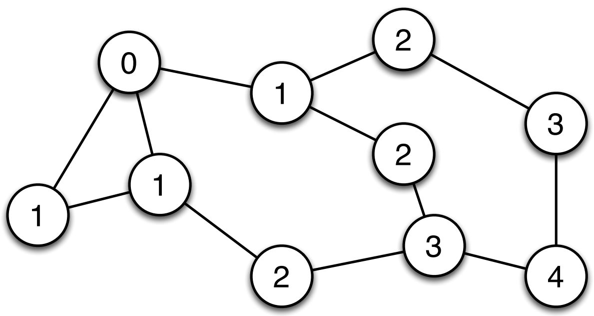

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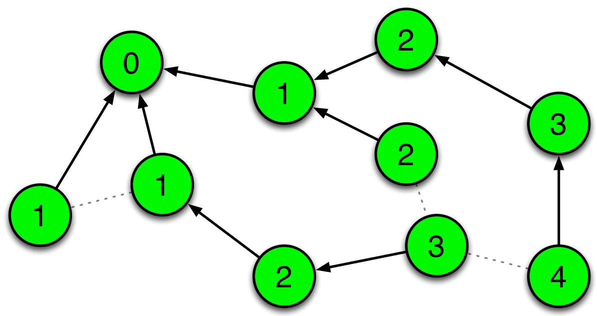

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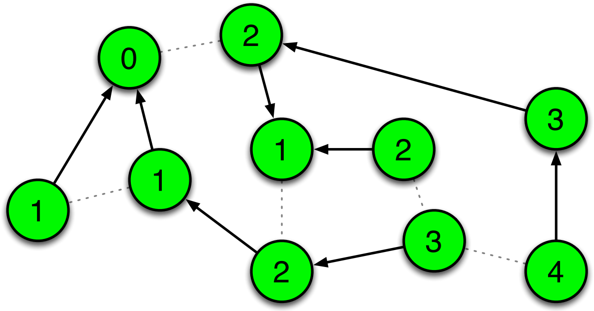

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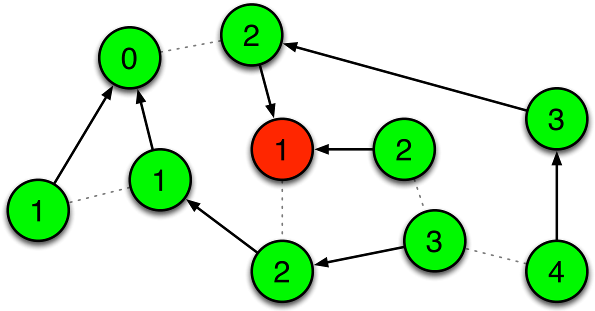

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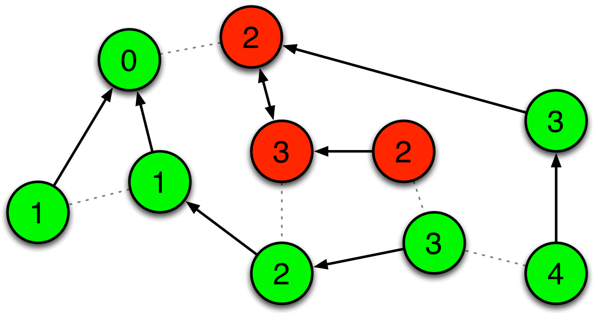

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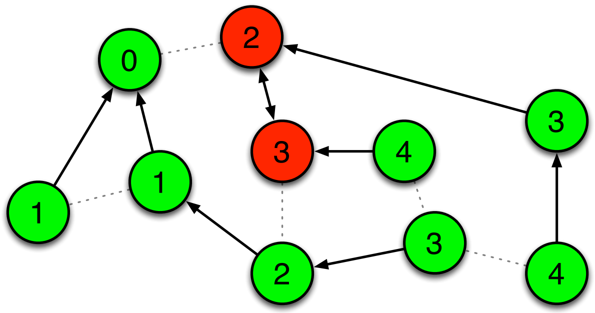

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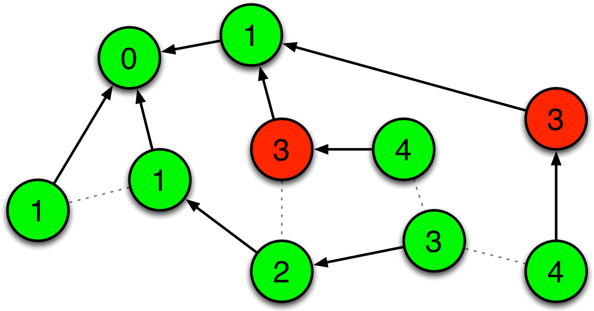

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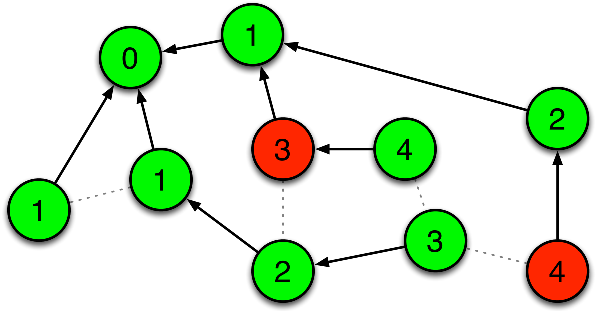

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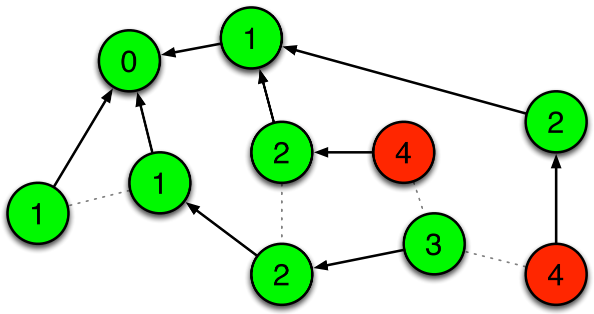

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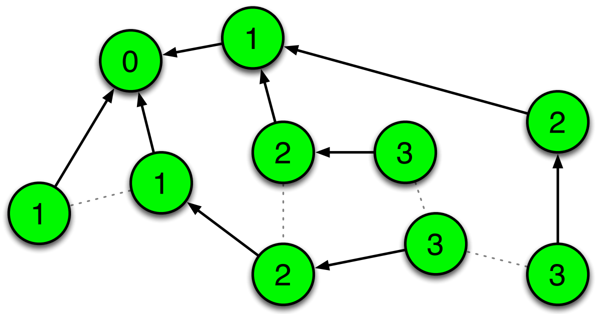
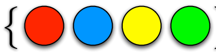


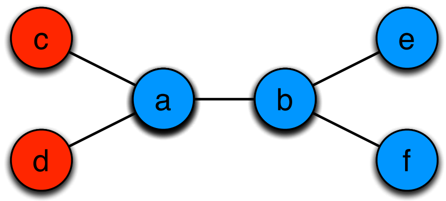
Scheduling

- **Scheduler** (a.k.a. **Daemon**): the daemon chooses among activatable processes those that will execute their actions
  - can be seen as an *adversary* whose role is to prevent stabilization

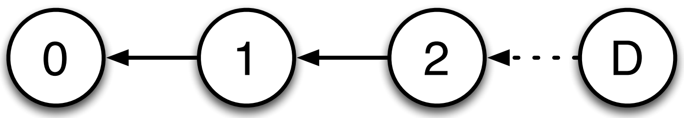
Spatial Scheduling

$$true \rightarrow color_i := Min\{\Delta \setminus \{color_j | j \in Neighbors_i\}\}$$

$\Delta = \{$  $\}$



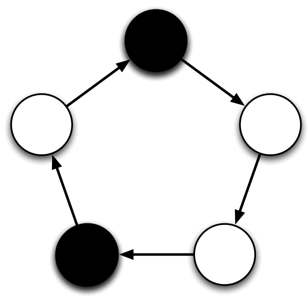
Spatial Scheduling



Temporal Scheduling

$$token \rightarrow pass\ token\ to\ left\ neighbor\ with\ probability\ \frac{1}{2}$$

$token = \bullet \quad no\ token = \circ$



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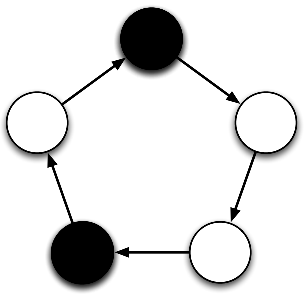
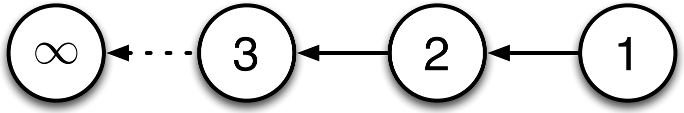
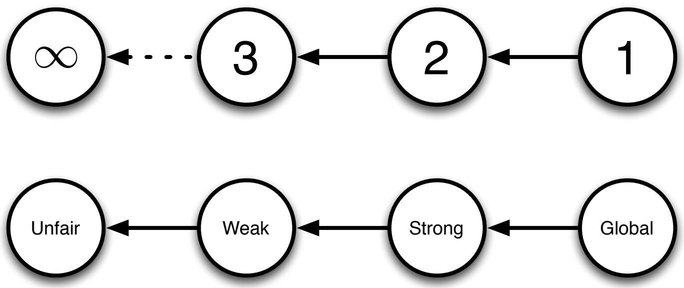


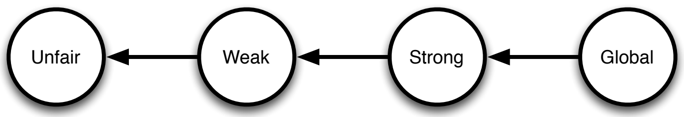
Temporal Scheduling



Temporal Scheduling



A Map of Daemons



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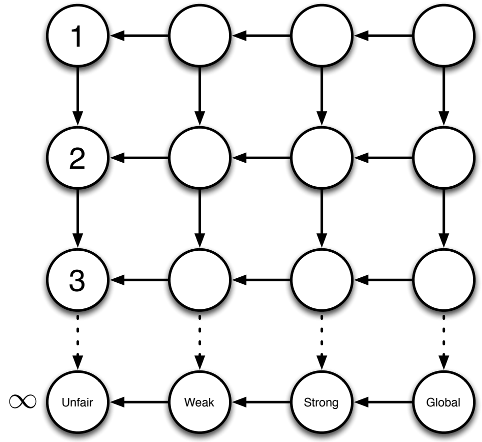

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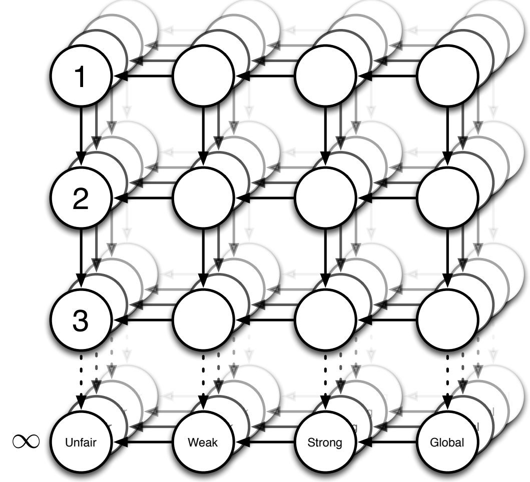
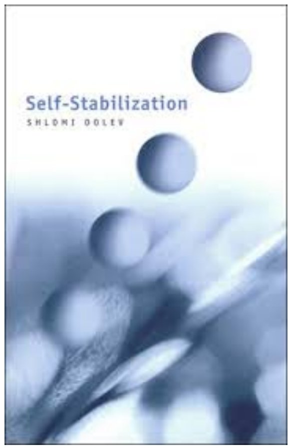


Self-stabilization

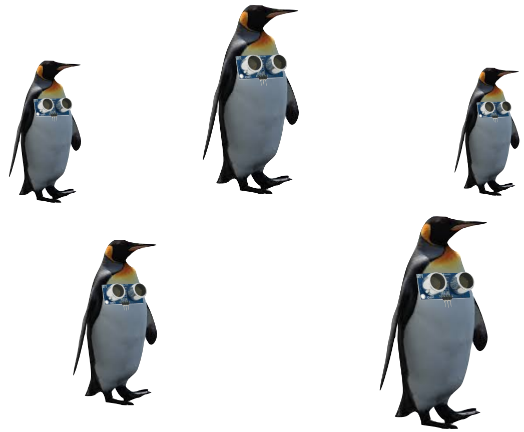


Population Protocols

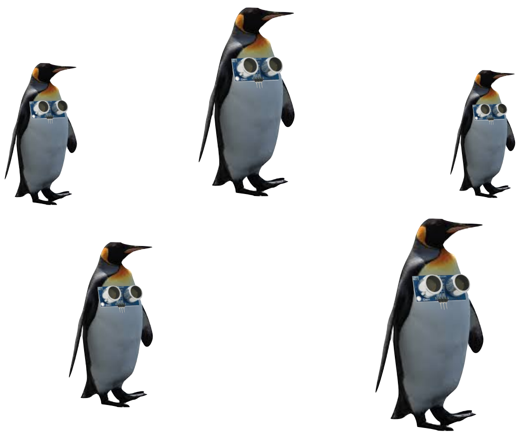
Population Protocols

Dana Angluin, James Aspnes, Zoë Diamadi, Michael J. Fischer, René Peralta: Computation in networks of passively mobile finite-state sensors. Distributed Computing 18(4): 235-253 (2006)

Population Protocols



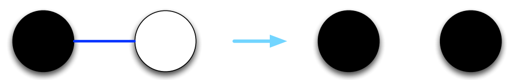
Population Protocols

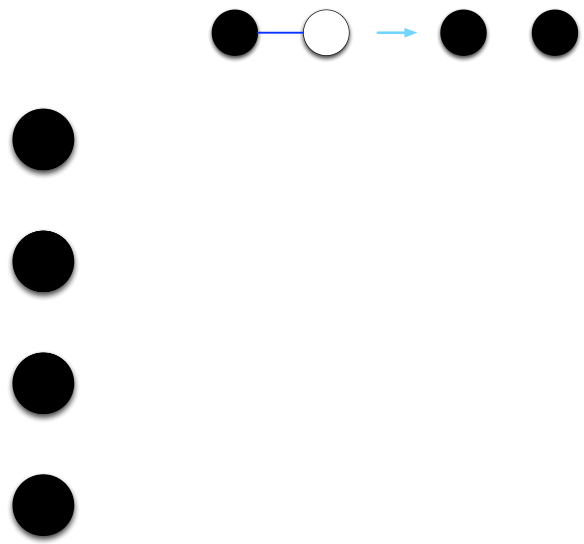


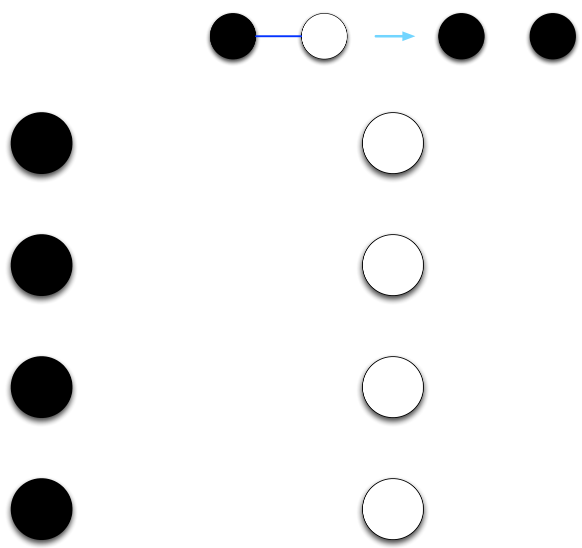
Population Protocols

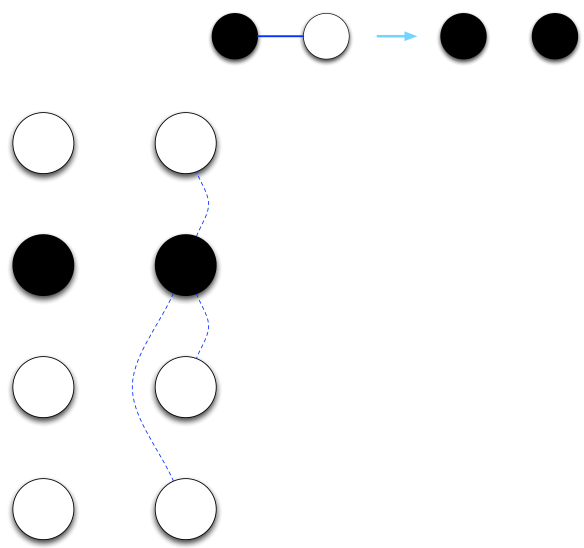
- **Definition**
    - A *Population Protocol* is a 6-tuple **(X,Y,Q,I,O,T)**
        - **X**: Set of inputs
        - **Y**: Set of outputs
        - **Q**: Set of states
        - **I**: Input mapping function, $X \longrightarrow Q$
        - **O**: Output mapping function, $Q \longrightarrow O$
        - **T**: Transition function, $Q \times Q \longrightarrow Q \times Q$

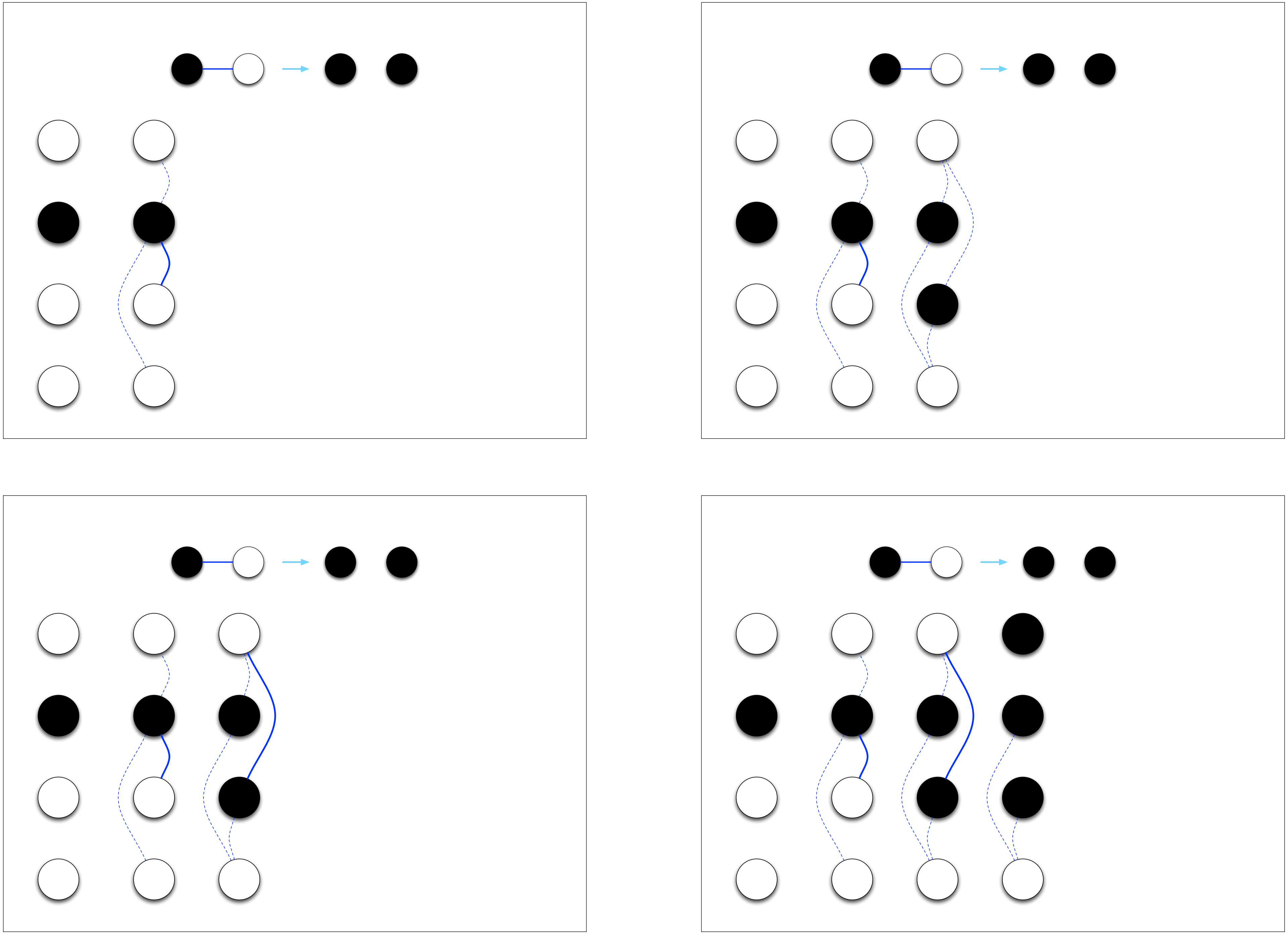
Example 1

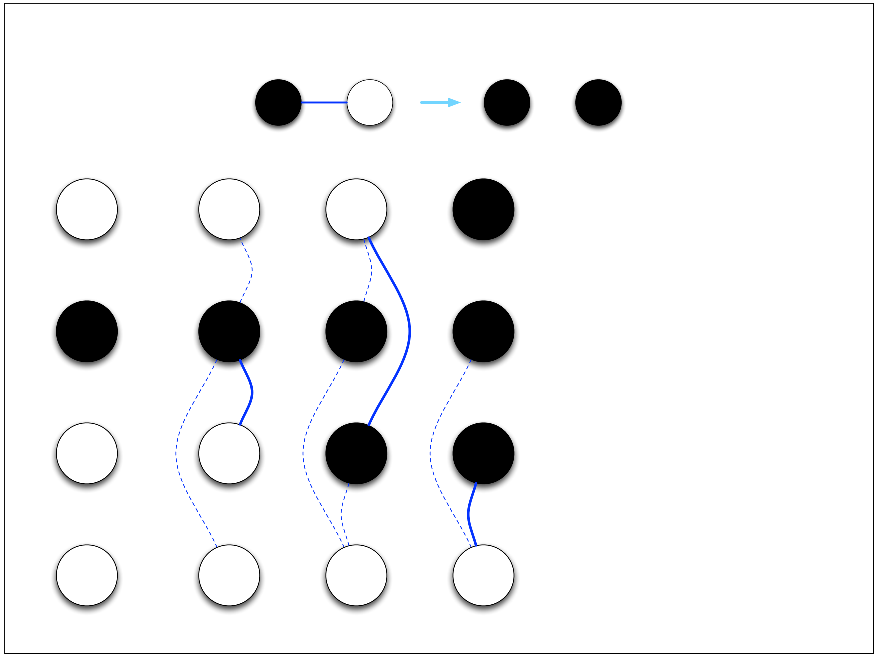


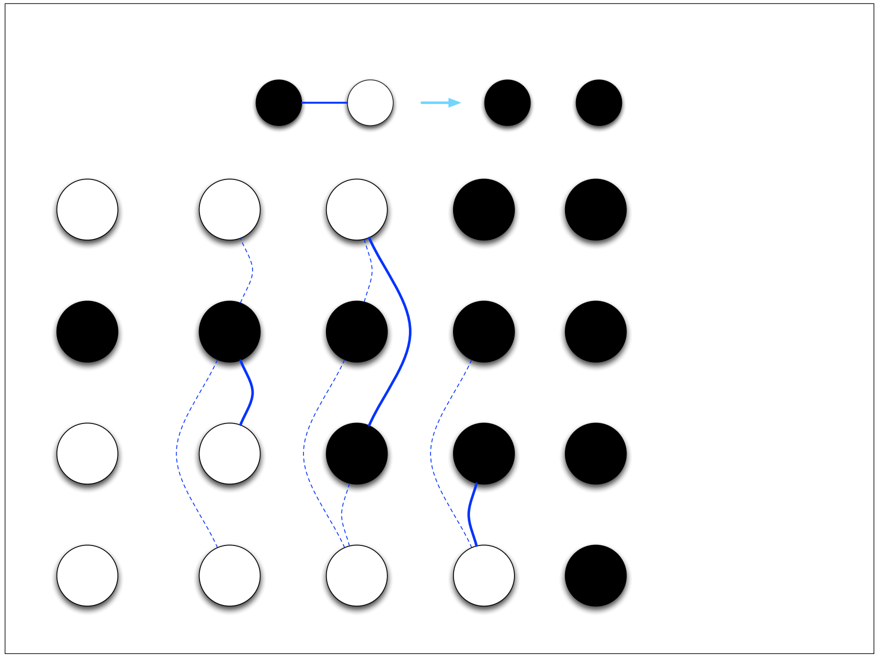




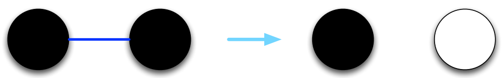


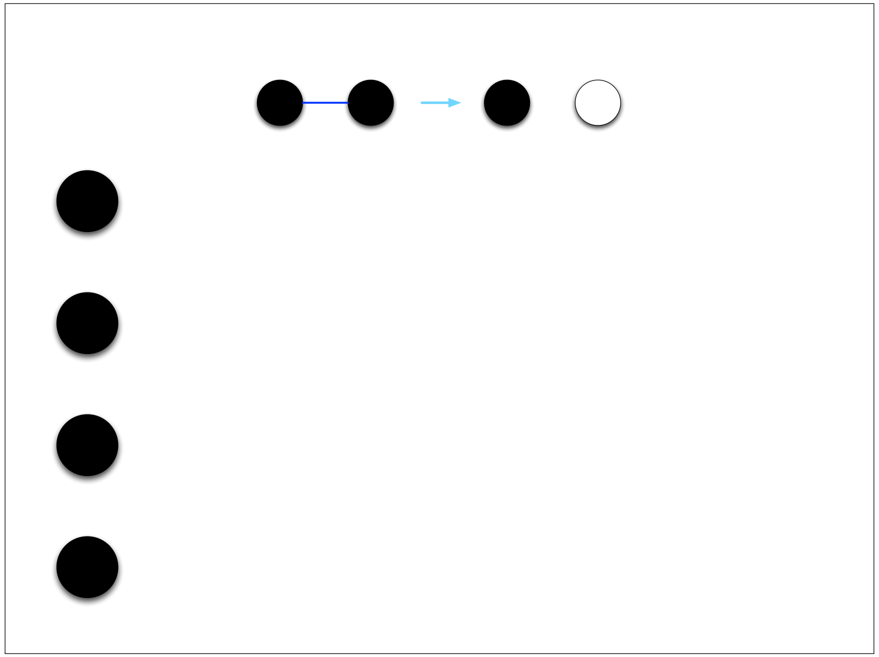


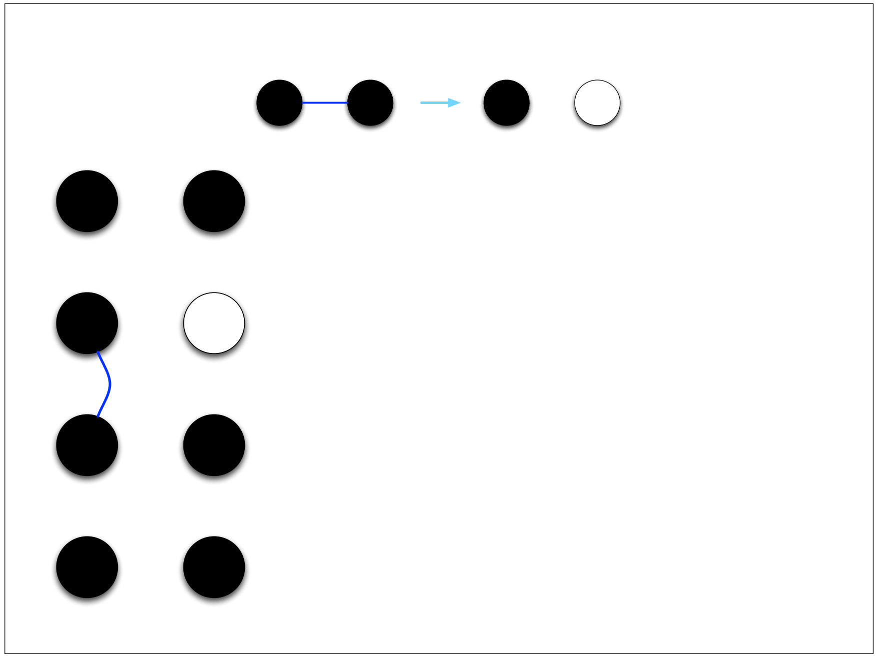
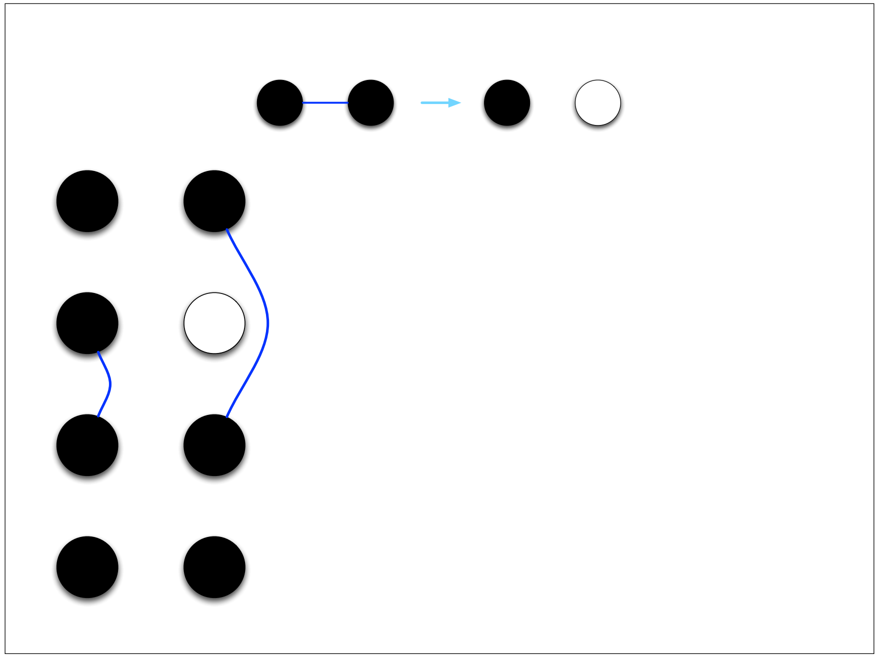
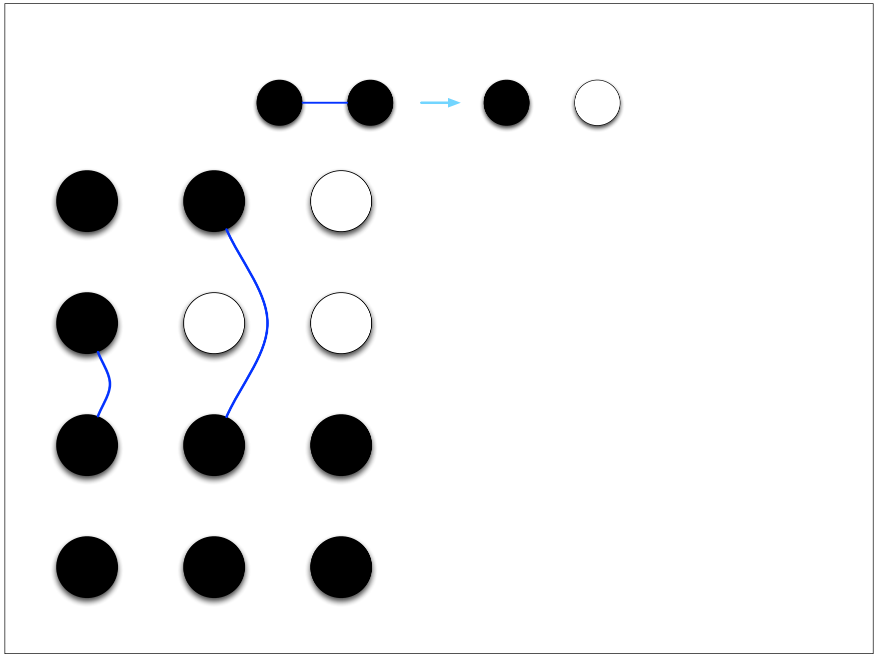
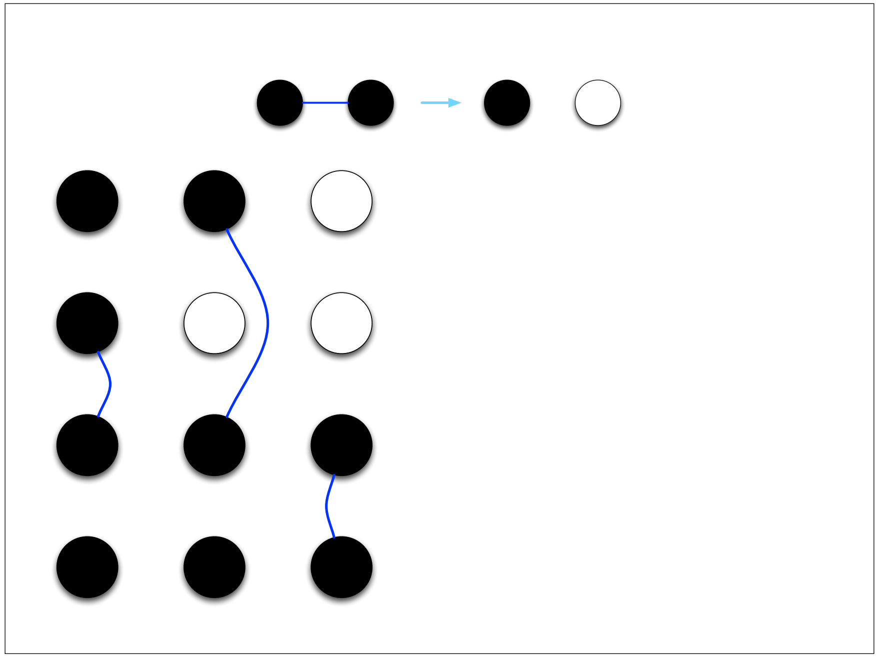


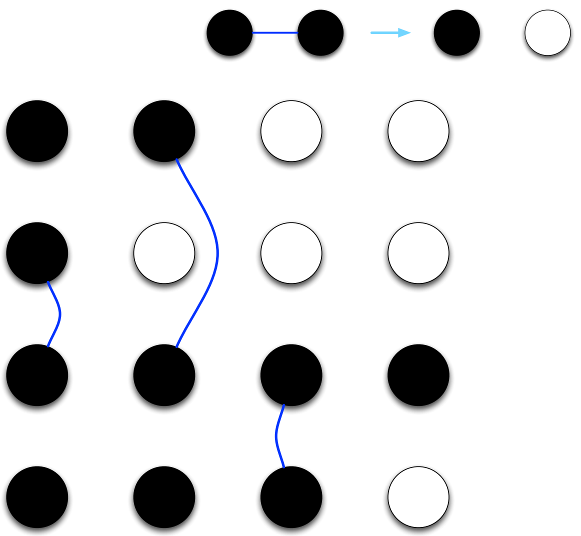


Example 1b

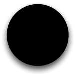
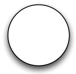




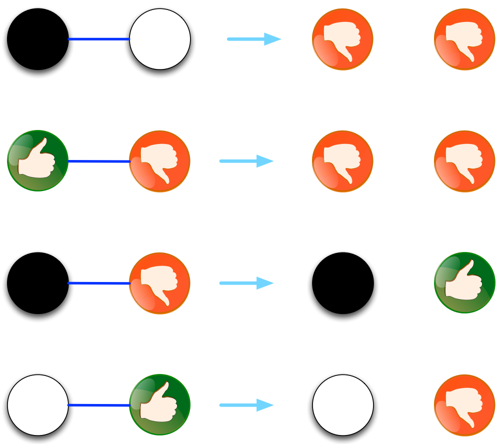





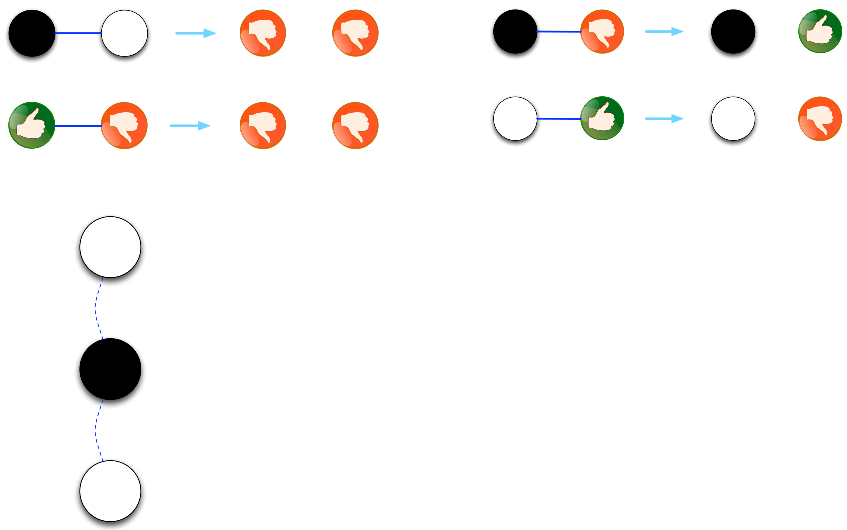


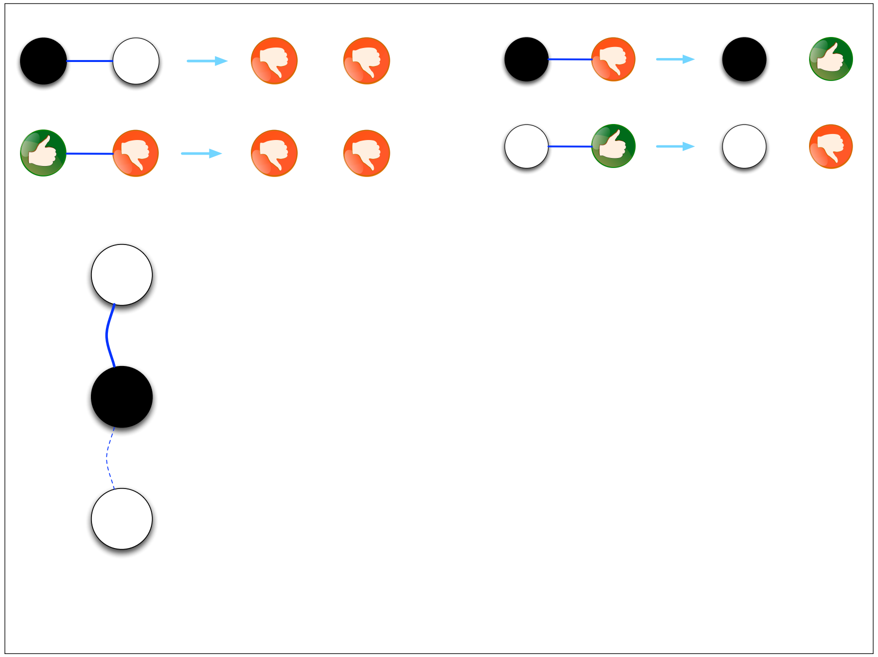
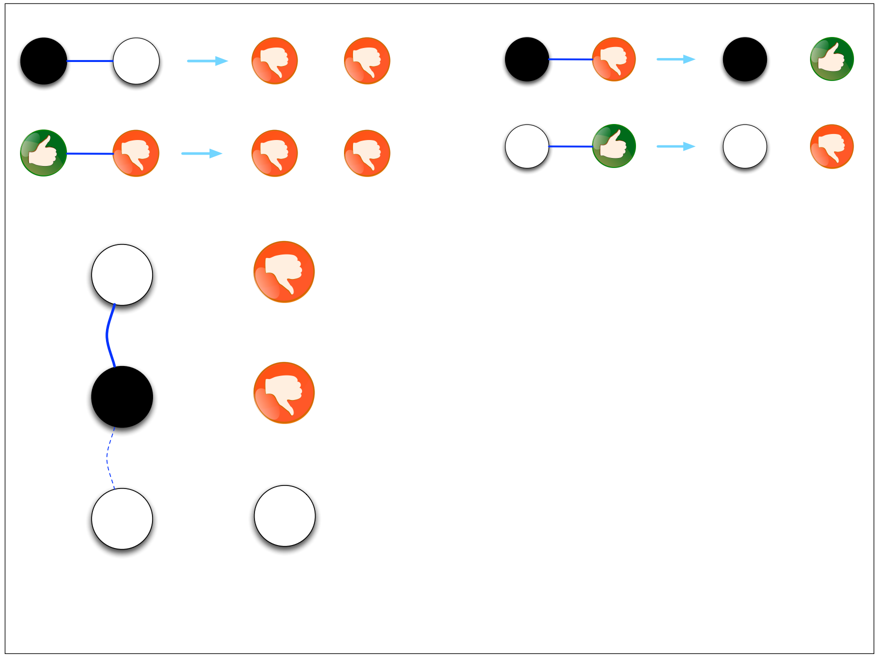
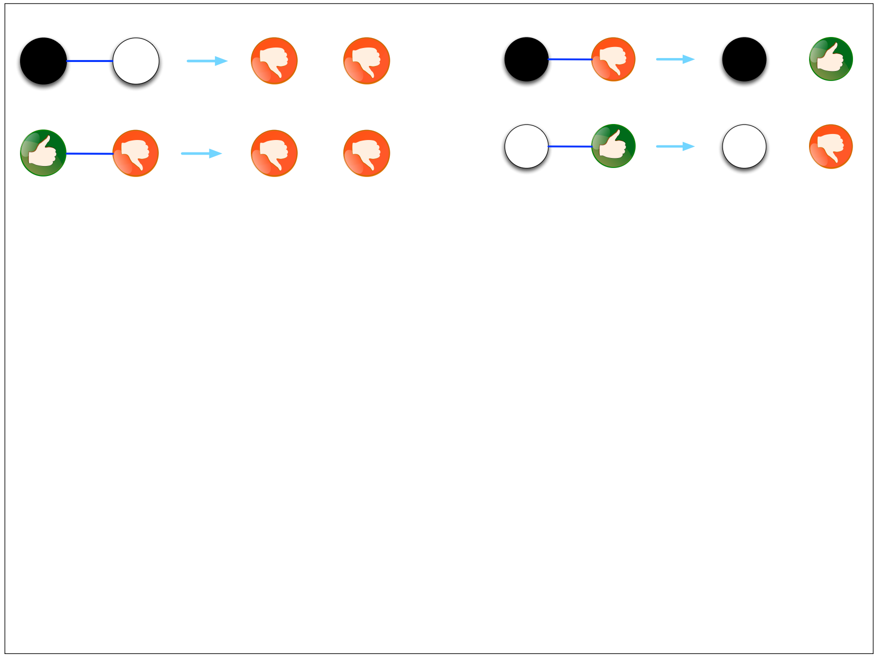
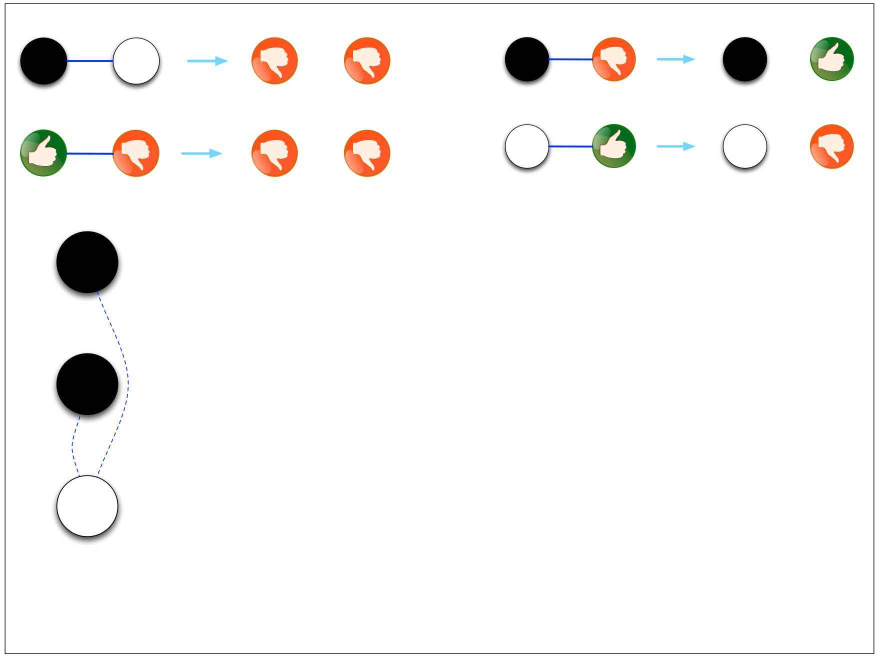
Example 2

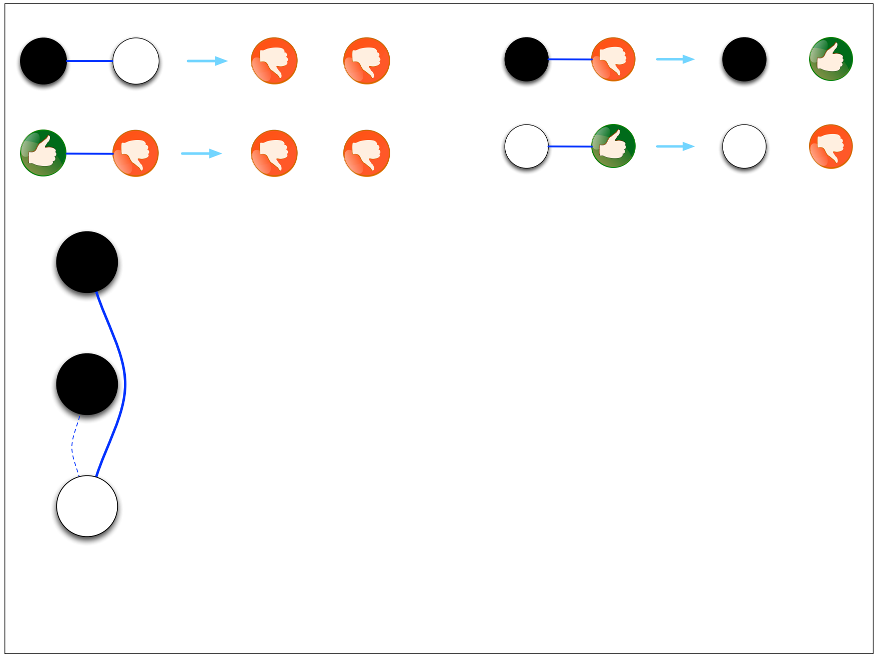
- **Inputs**: 
- **Outputs**: 
- **#**  **> #**  **?**

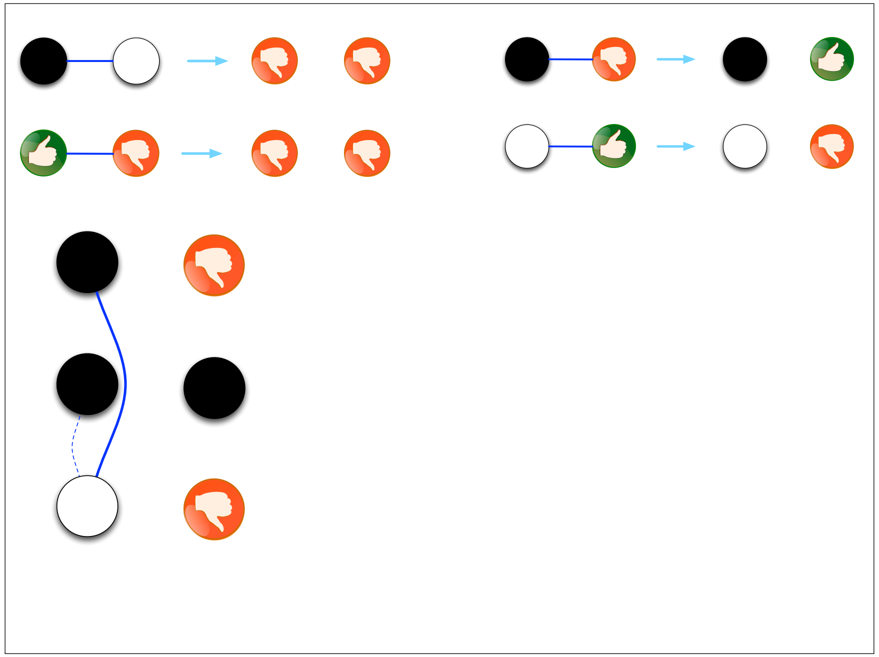
Example 2

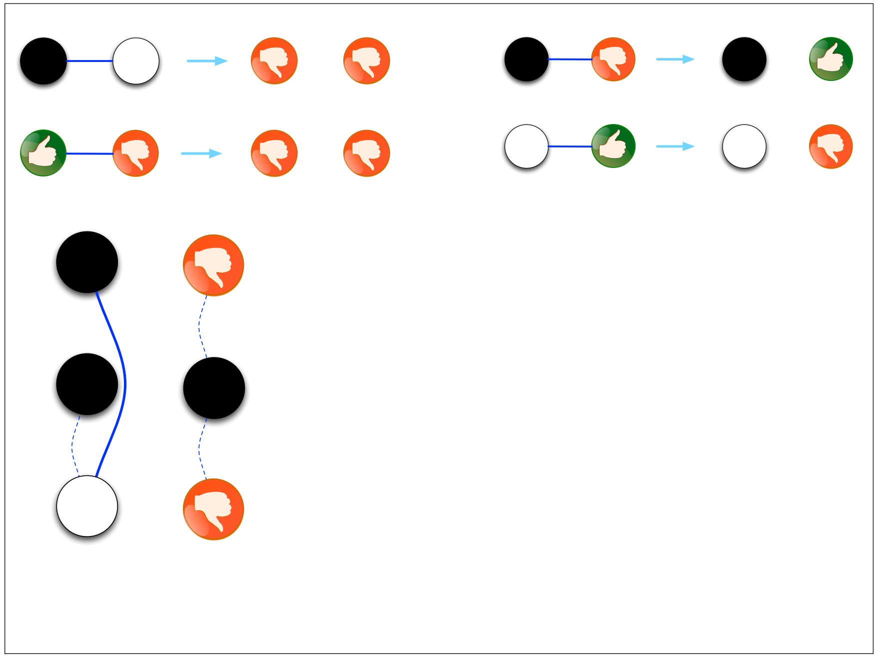


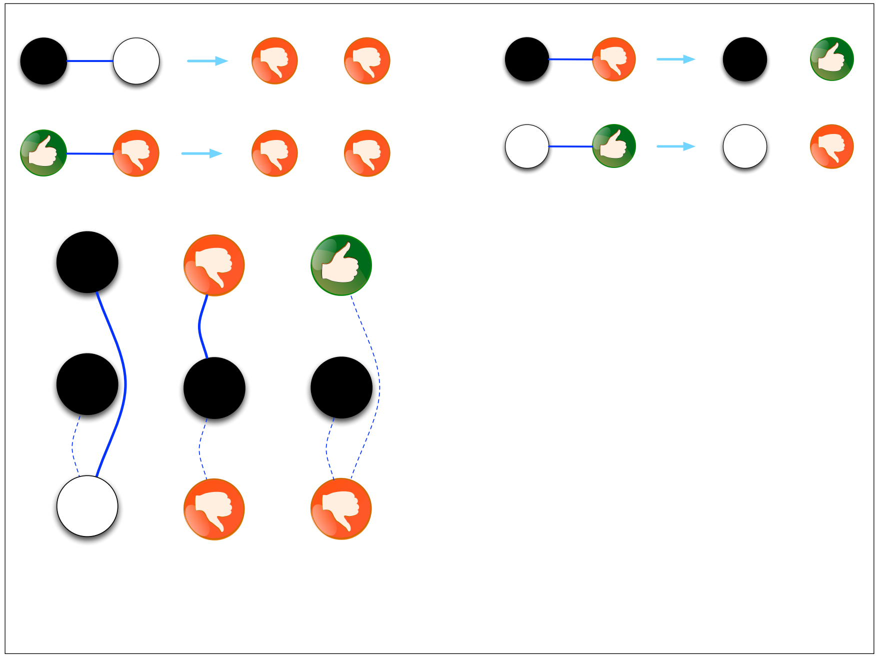


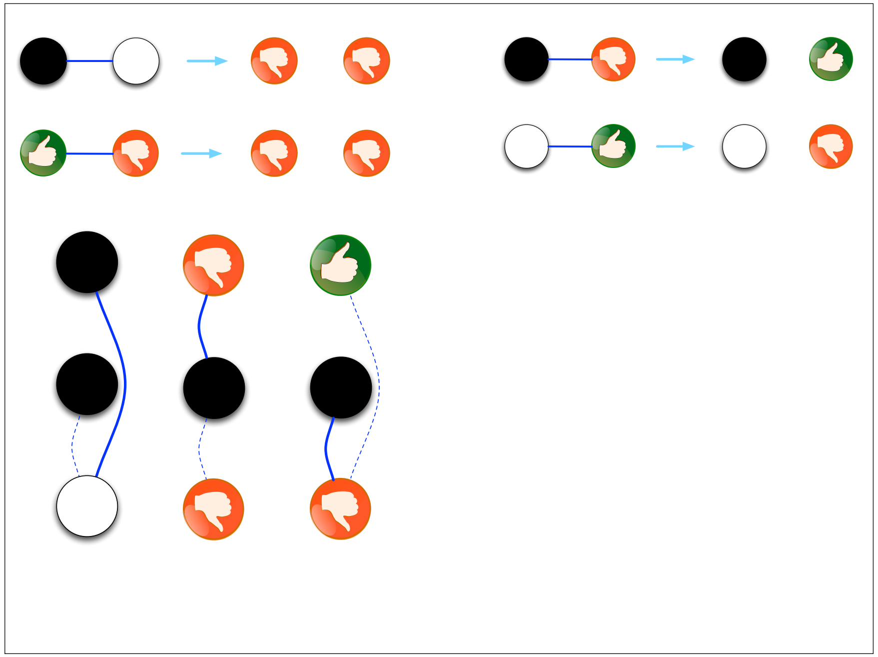





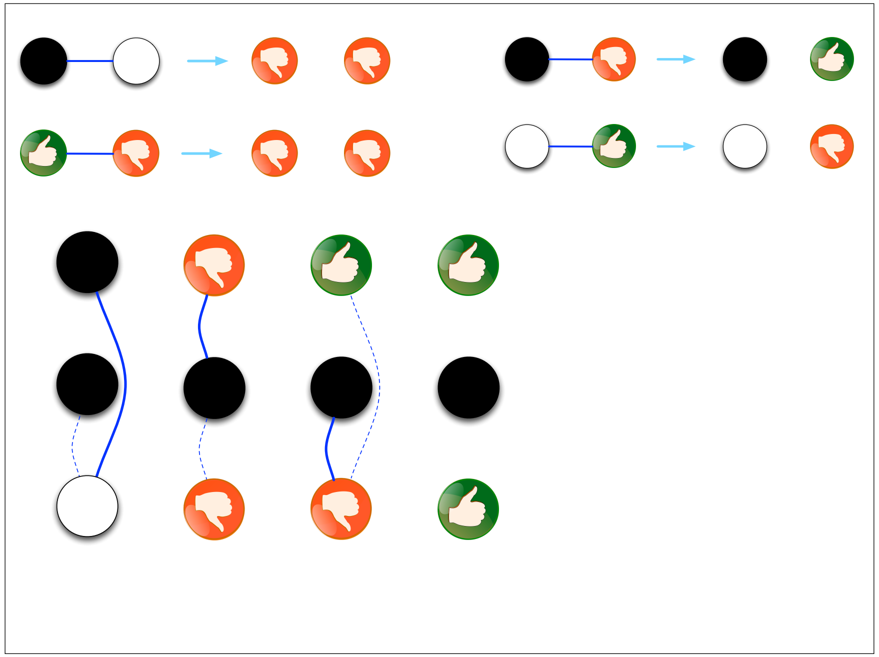


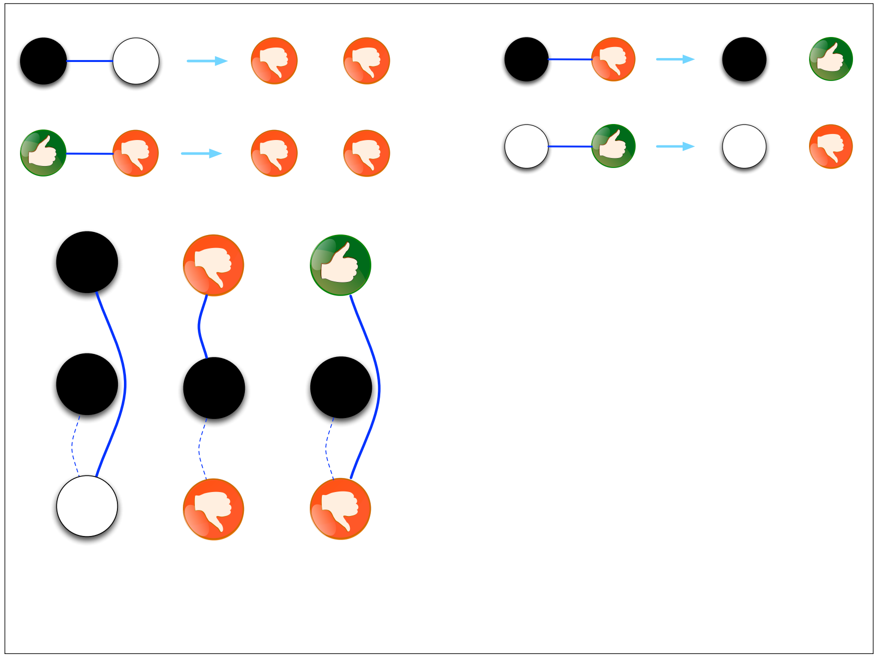


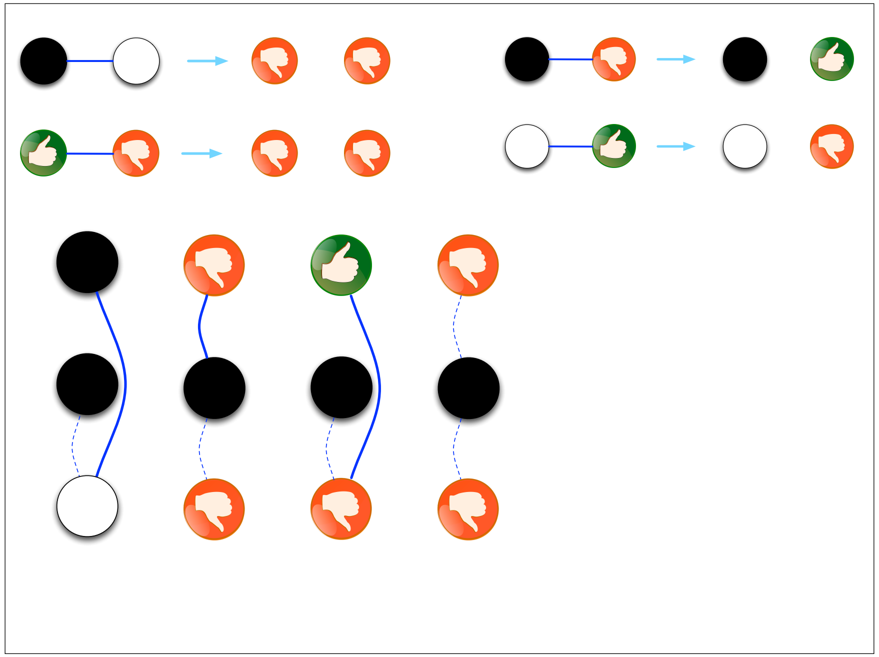


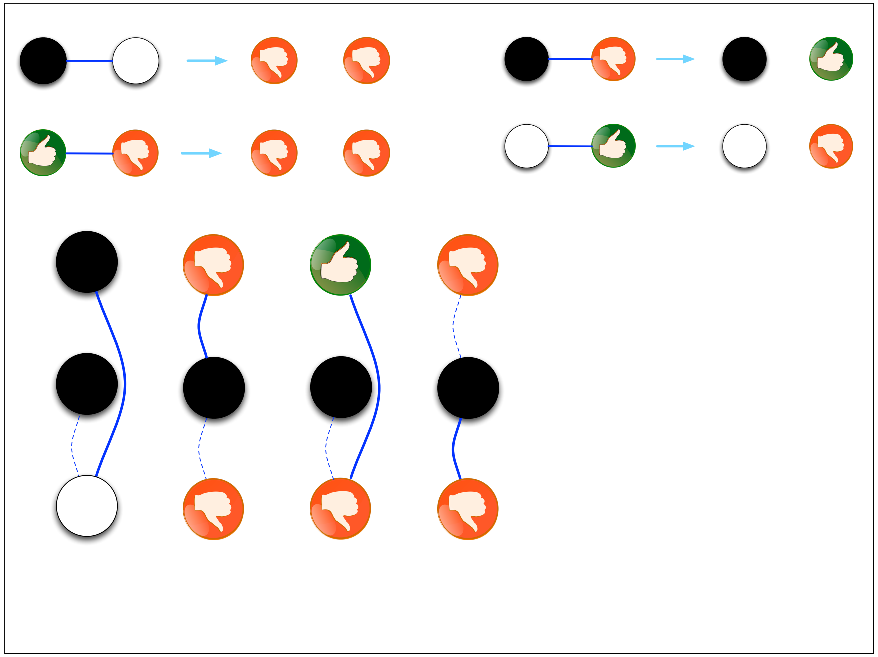


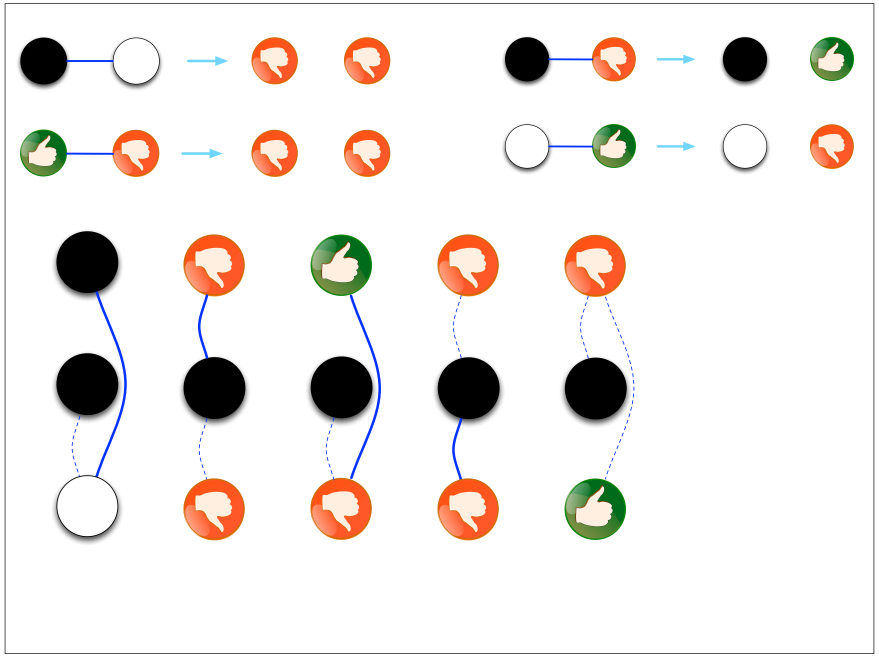


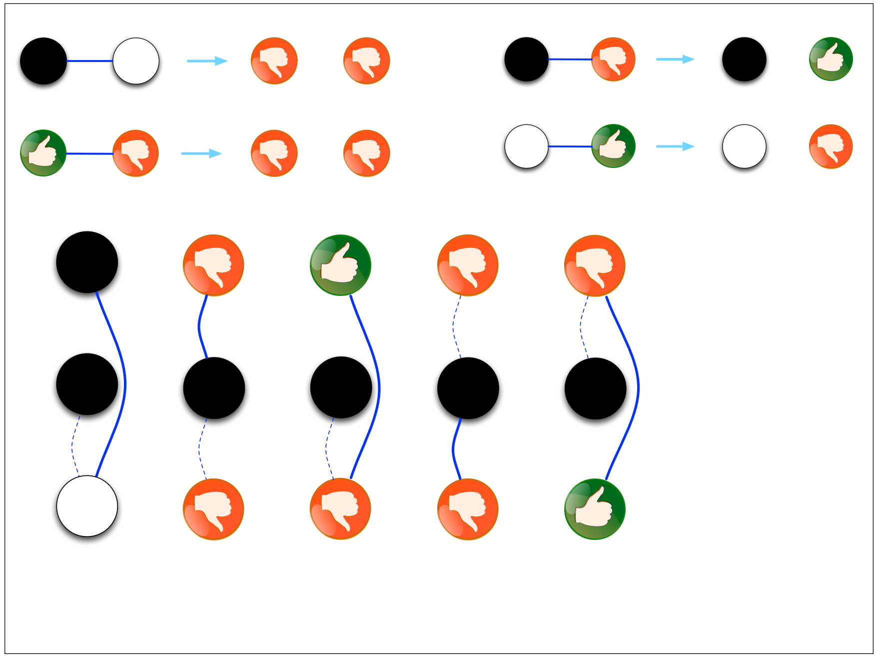


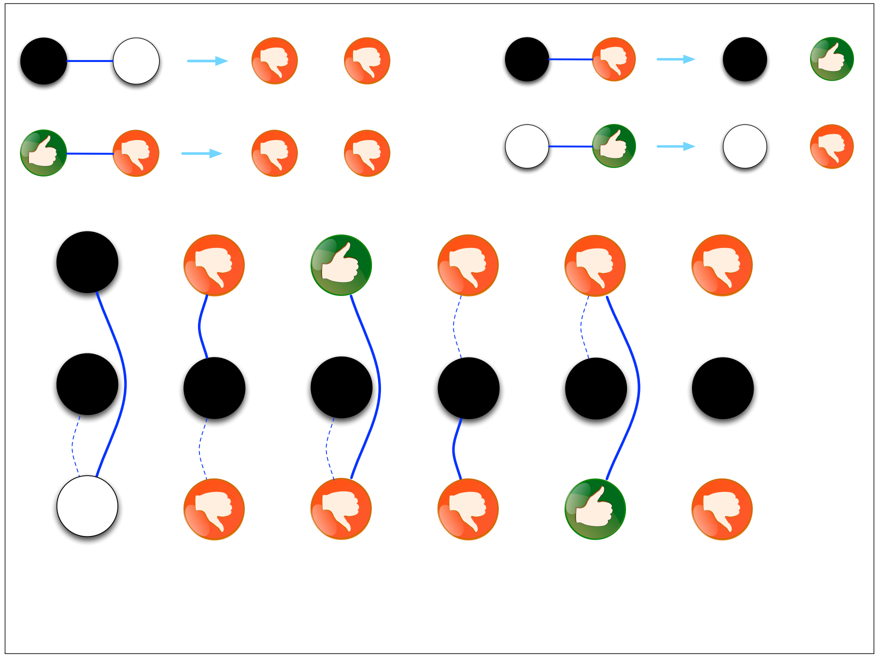








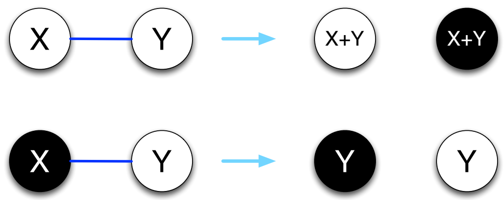


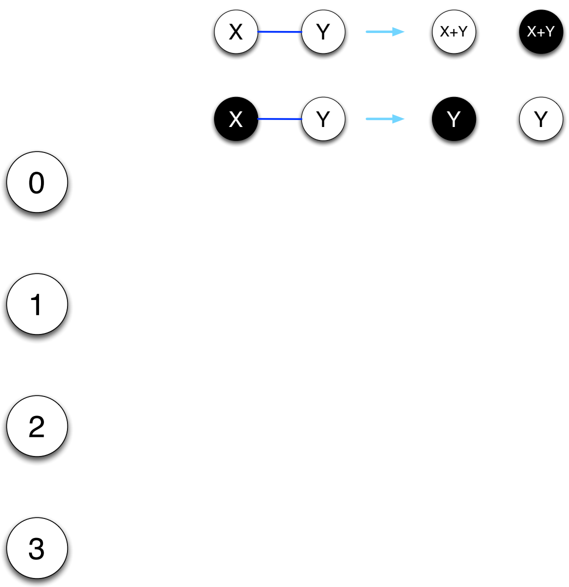


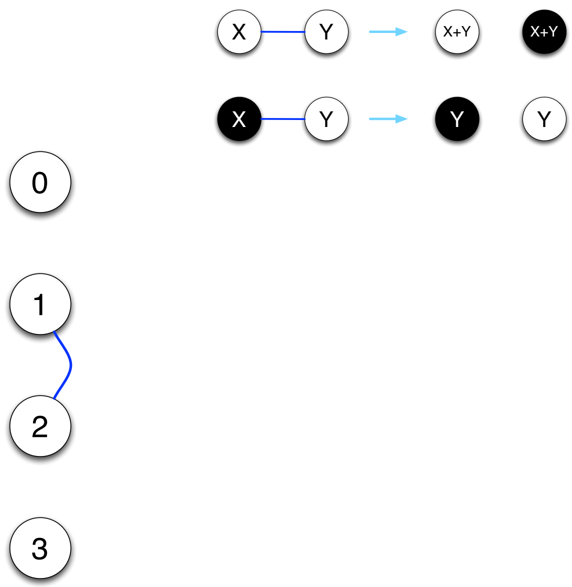
Example 3

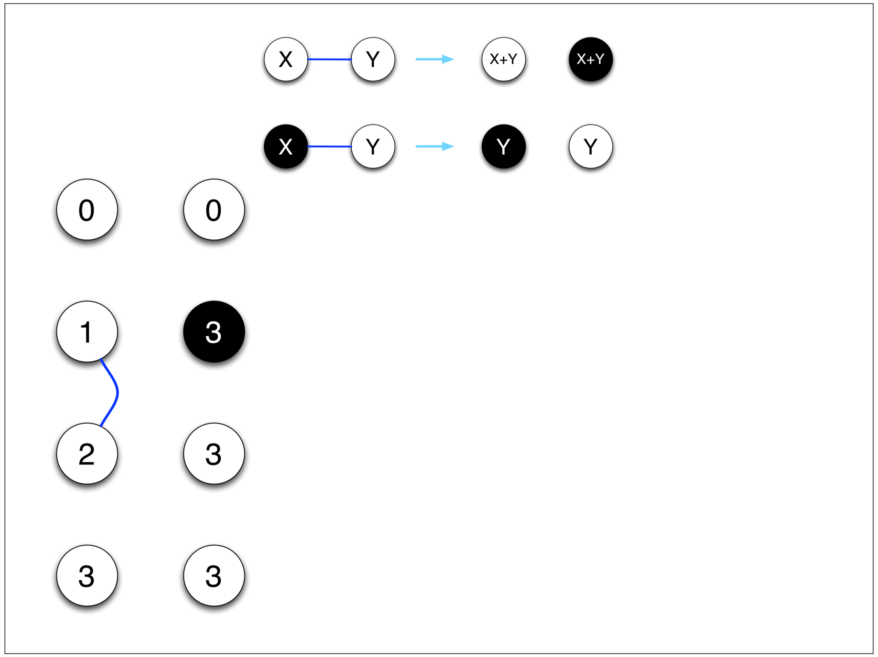
- **Inputs**: 0 1 2 3
- **Outputs**: 0 1 2 3 0 1 2 3
- **Sum mod 4 ?**

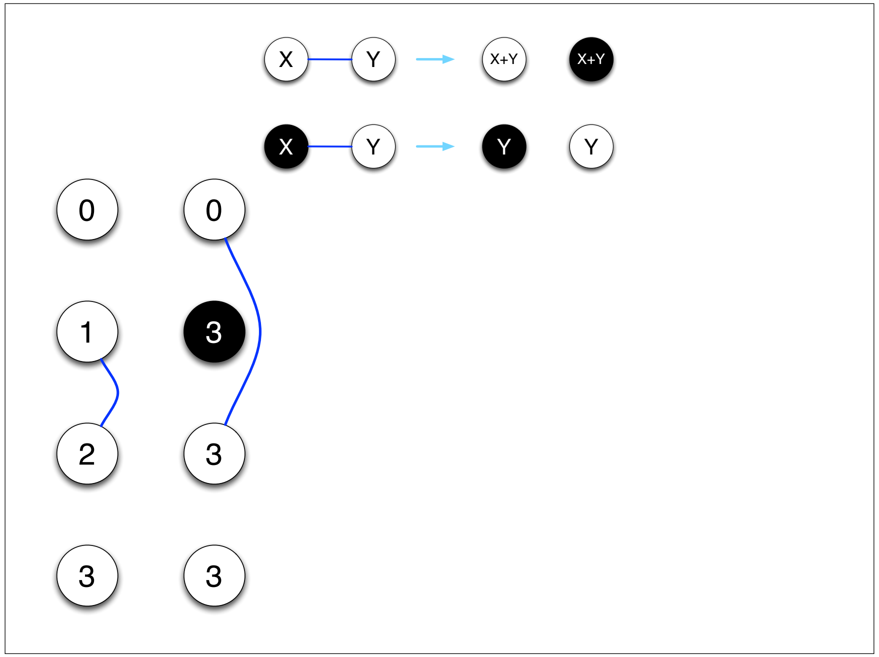
Example 3

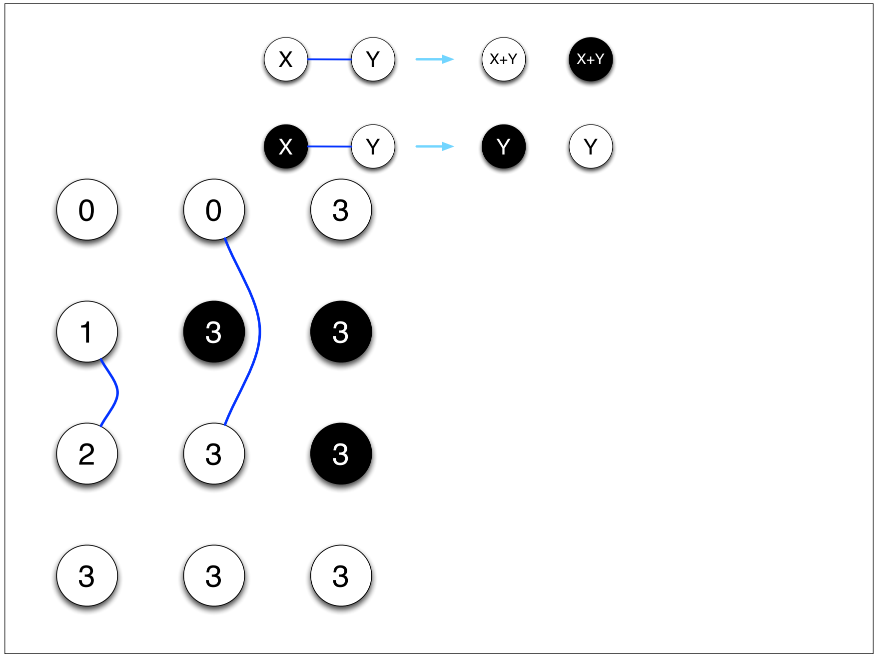


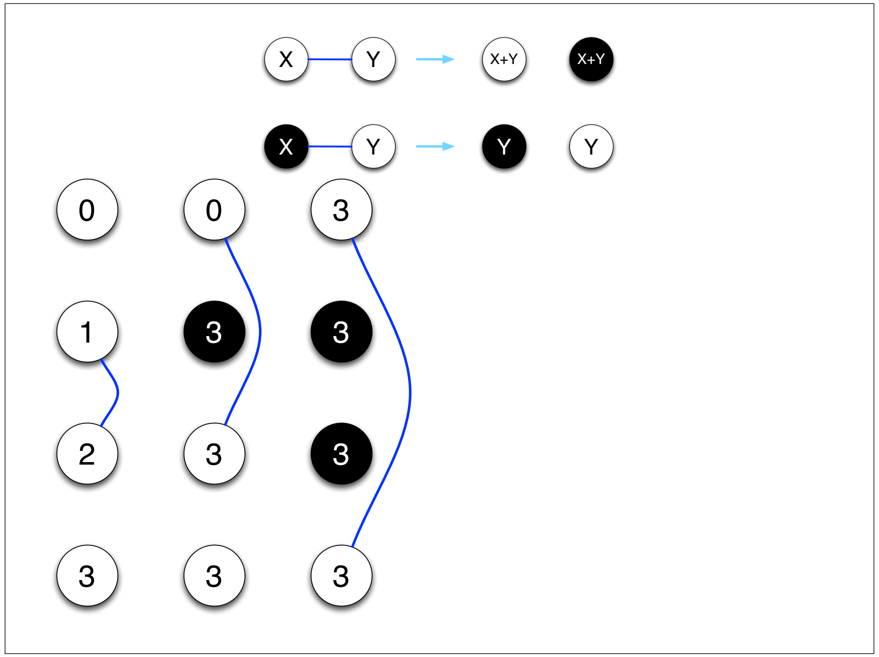


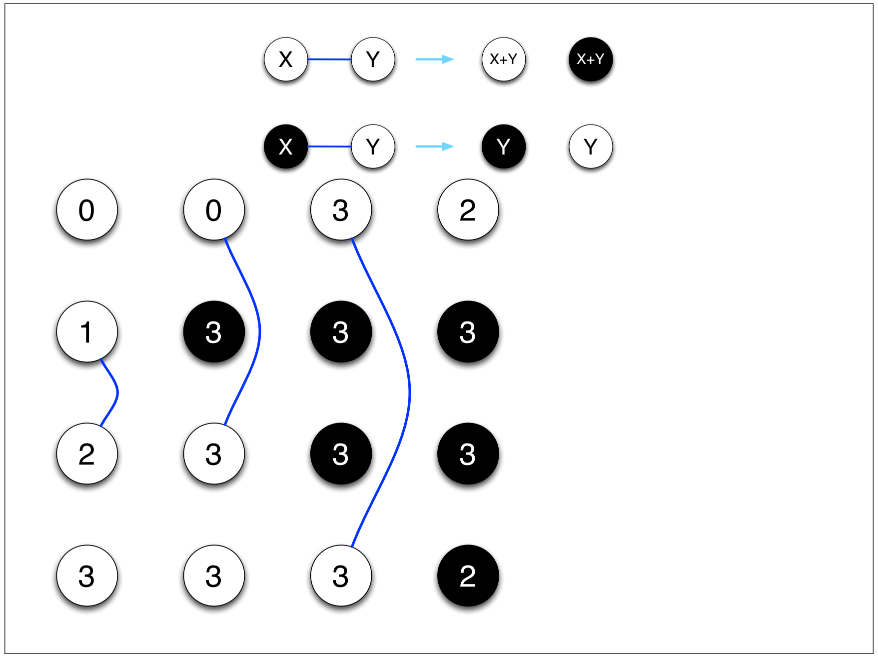


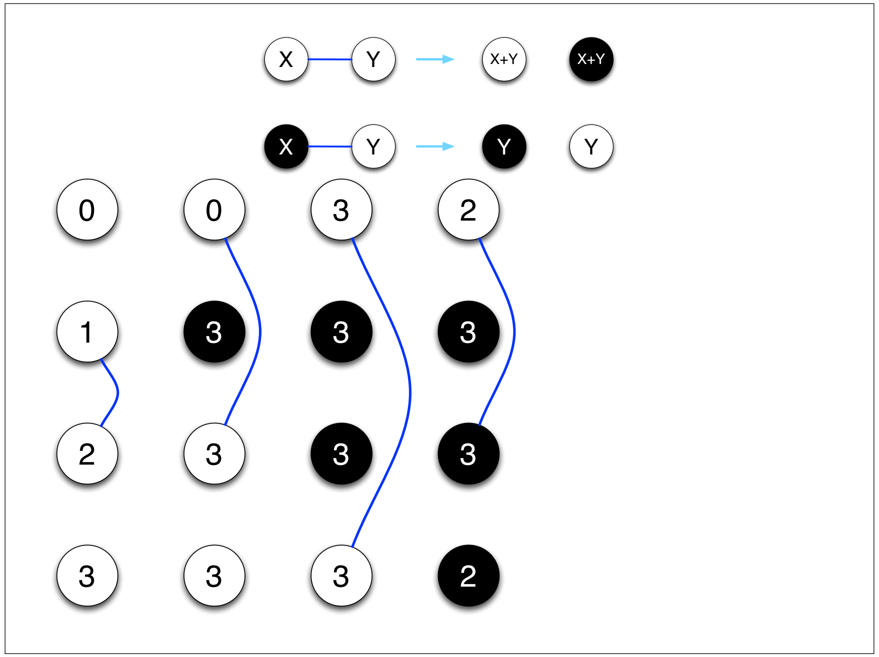

X Y X+Y X+Y
X Y Y Y
0 0
1 3
2 3
3 3


X Y X+Y X+Y
X Y Y Y
0 0
1 3
2 3
3 3


X Y X+Y X+Y
X Y Y Y
0 0 3
1 3 3
2 3 3
3 3 3


X Y X+Y X+Y
X Y Y Y
0 0 3
1 3 3
2 3 3
3 3 3


X Y X+Y X+Y
X Y Y Y
0 0 3 2
1 3 3 3
2 3 3 3
3 3 3 2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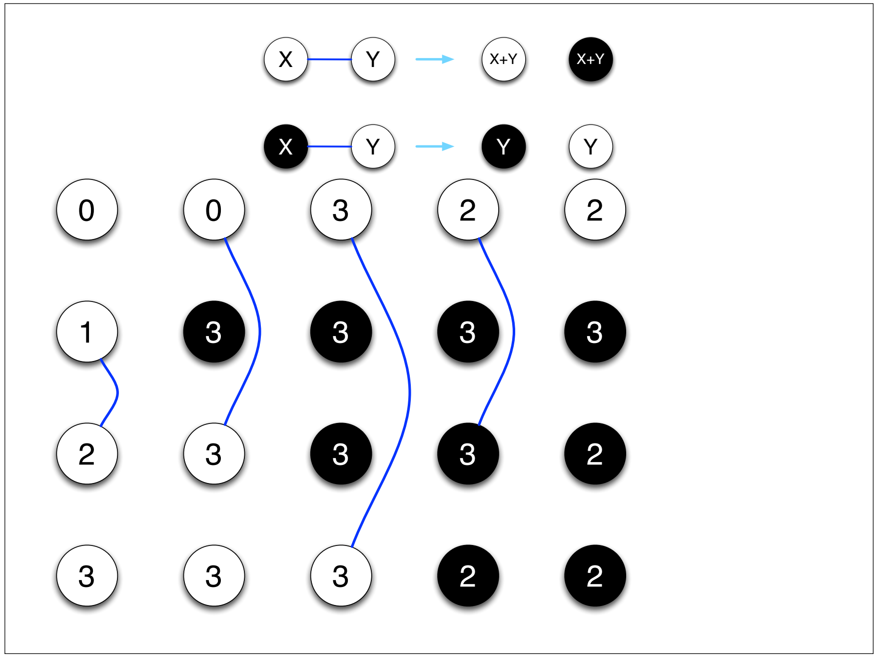
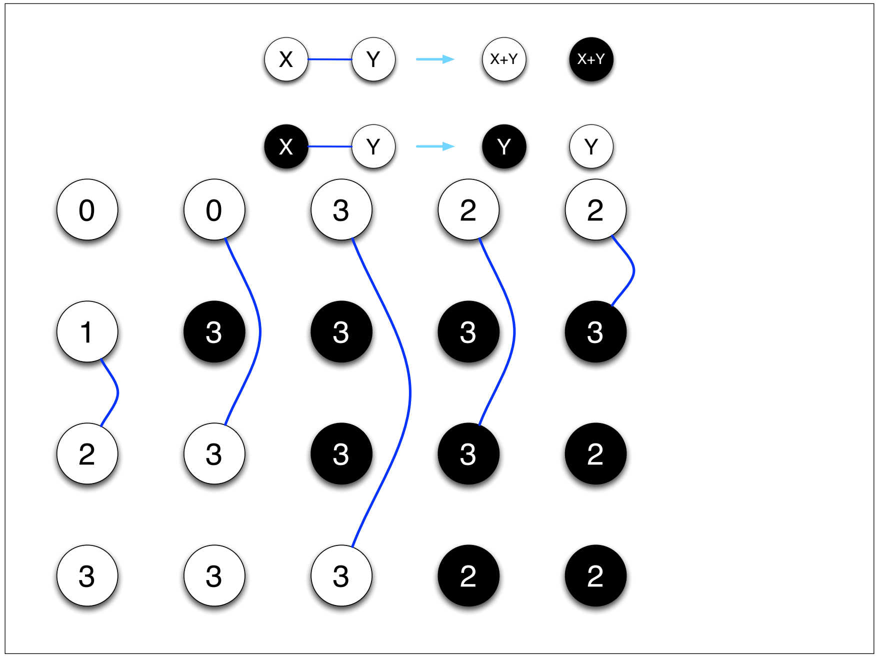
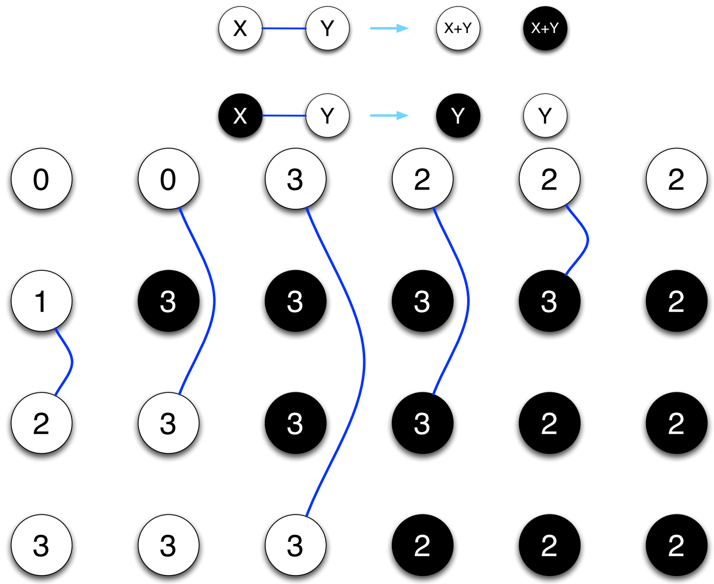
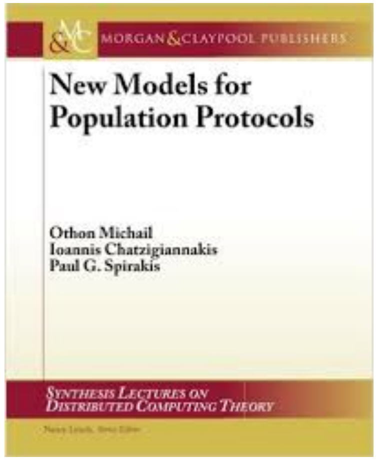
X Y Y Y
0 0 3 2
1 3 3 3
2 3 3 3
3 3 3 2


X Y X+Y X+Y
X Y Y Y
0 0 3 2 2
1 3 3 3 3
2 3 3 3 2
3 3 3 2 2


X Y X+Y X+Y
X Y Y Y
0 0 3 2 2
1 3 3 3 3
2 3 3 3 2
3 3 3 2 2



Population Protocols



Dynamic Graphs

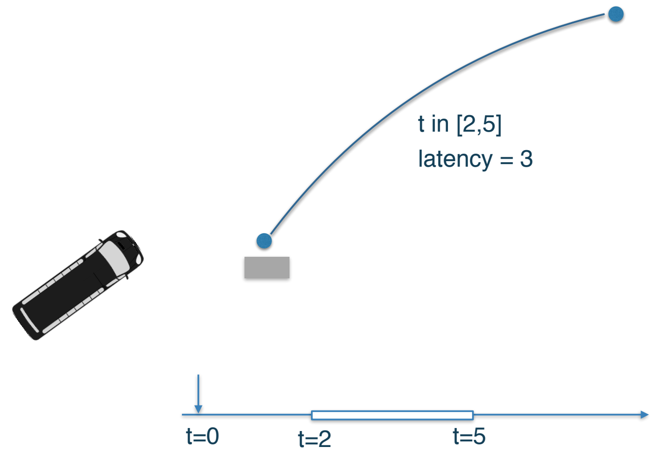
Time-varying Graphs

- A *time-varying graph* (TVG) is a 5-tuple (**V**,**E**,**T**,**p**,**l**)
  - **V**: set of nodes
  - **E**: (labelled) set of edges
  - **T**: lifetime, $\mathbf{T} \subseteq \mathcal{T}$
  - **p**: presence function, $\mathbf{E} \times \mathbf{T} \longrightarrow \{0,1\}$
  - **l**: latency function, $\mathbf{E} \times \mathbf{T} \longrightarrow \mathcal{T}$

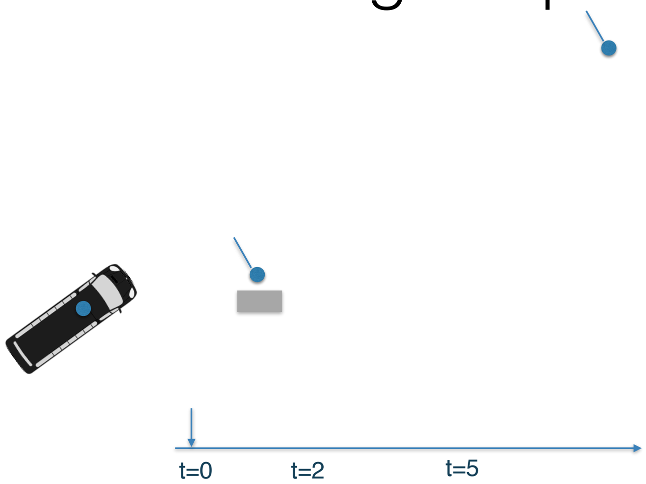
Time-varying Graphs

- A *time-varying graph* (TVG) is a 5-tuple (**V**,**E**,**T**,**p'**,**l'**)
  - **V**: set of nodes
  - **E**: (labelled) set of edges
  - **T**: lifetime, $\mathbf{T} \subseteq \mathcal{T}$
  - **p'**: *node* presence function, $\mathbf{V} \times \mathbf{T} \longrightarrow \{0,1\}$
  - **l'**: *node* latency function, $\mathbf{V} \times \mathbf{T} \longrightarrow \mathcal{T}$

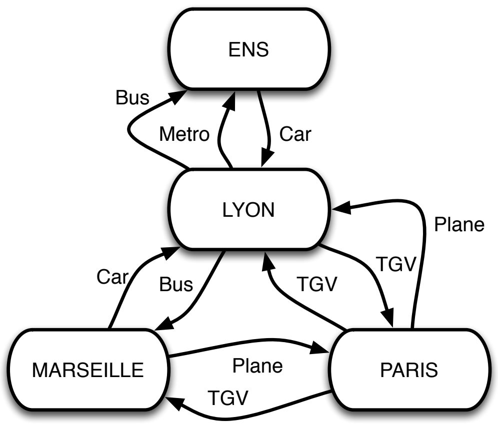
Time-varying Graphs



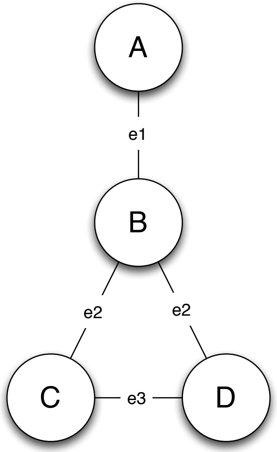
Evolving Graphs



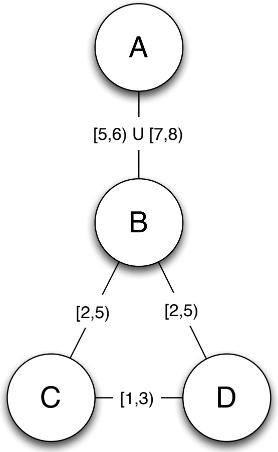
Example



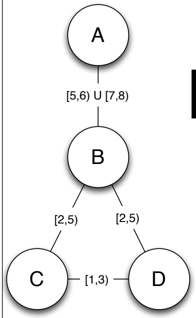
Example



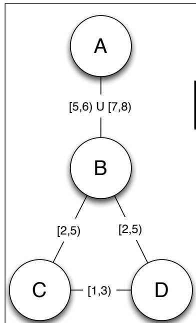
Example

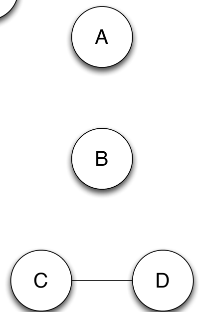


Evolving Graphs



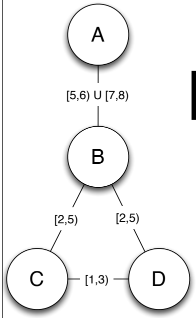
Evolving Graphs

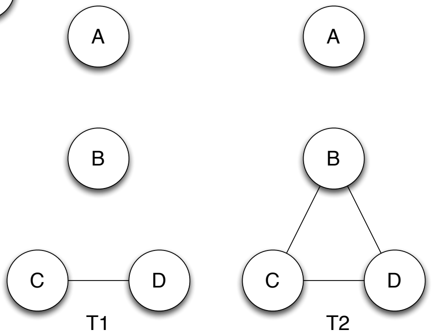




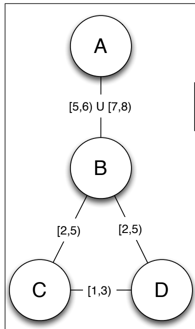
T1

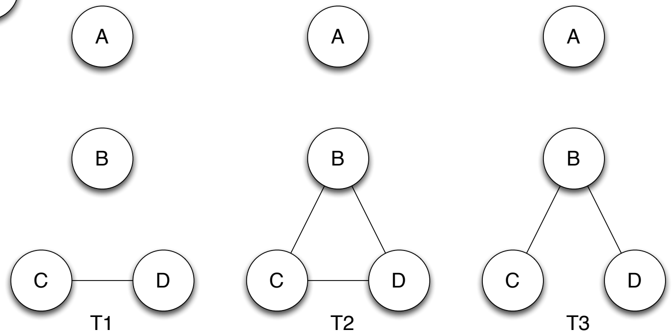
Evolving Graphs





Evolving Graphs





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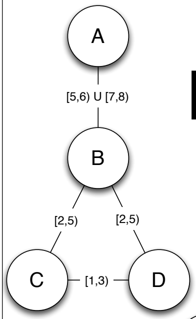
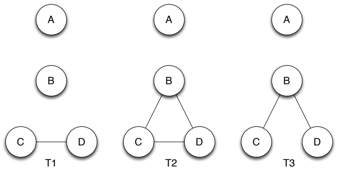
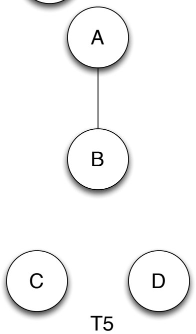
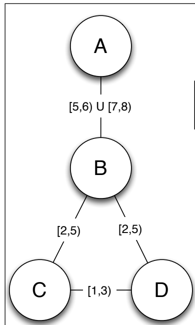
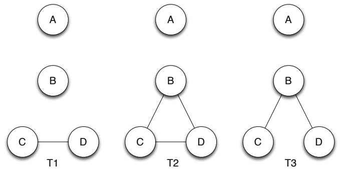


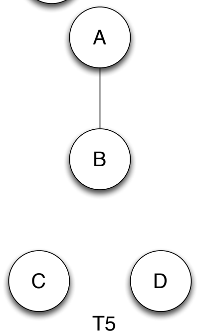




Evolving Graphs







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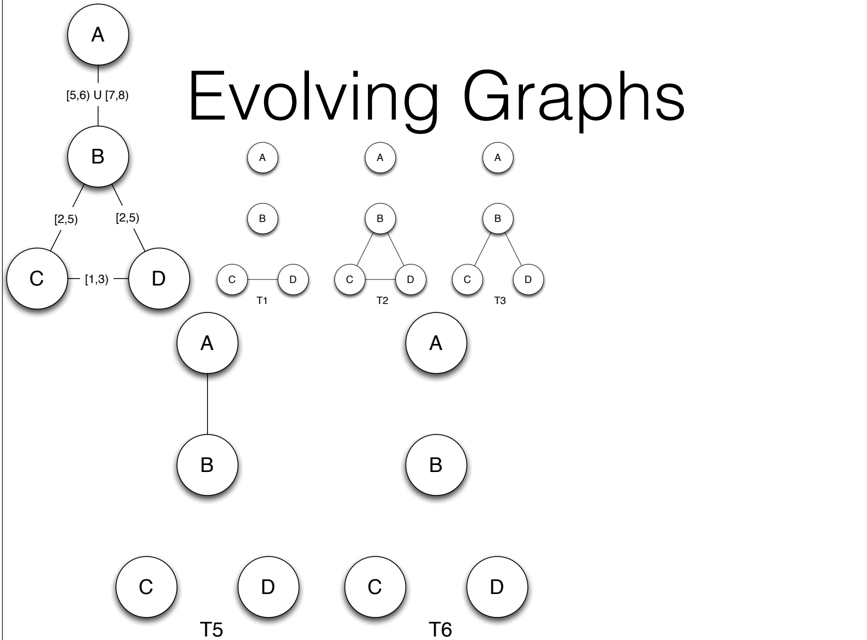
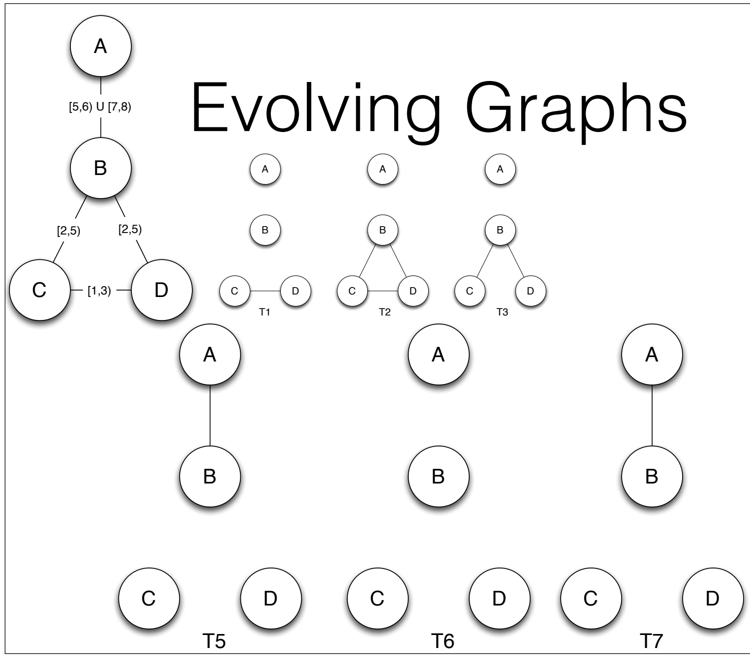
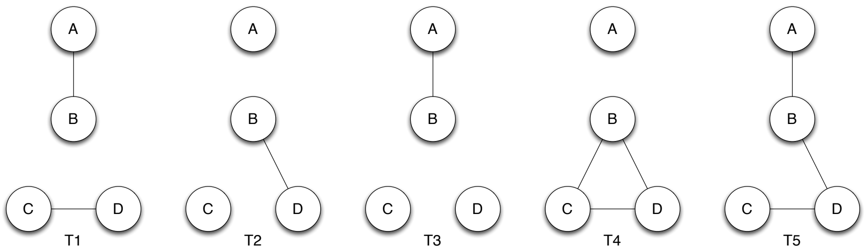


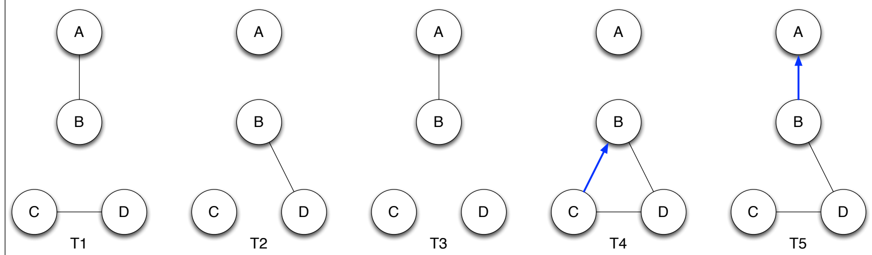
Evolving Graphs



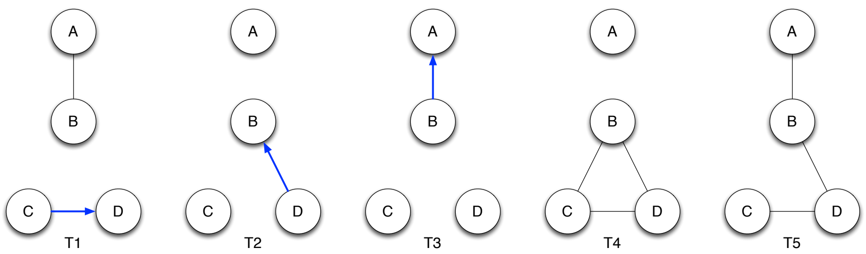
Journeys from C to A



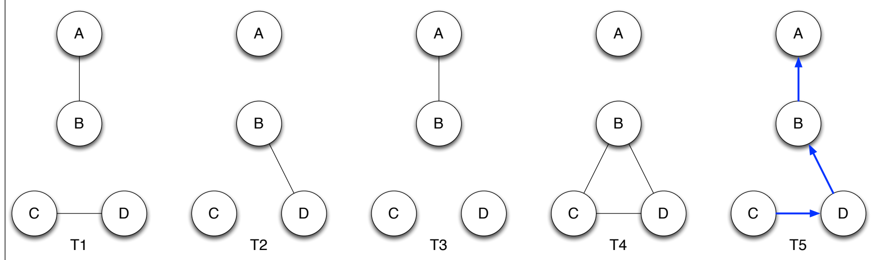
Shortest Journey



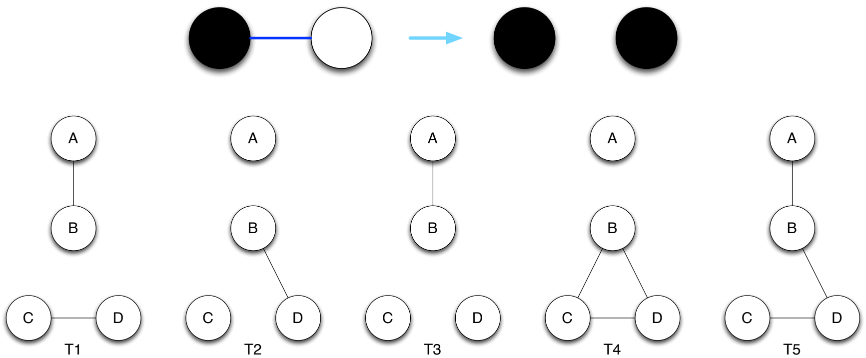
Foremost Journey



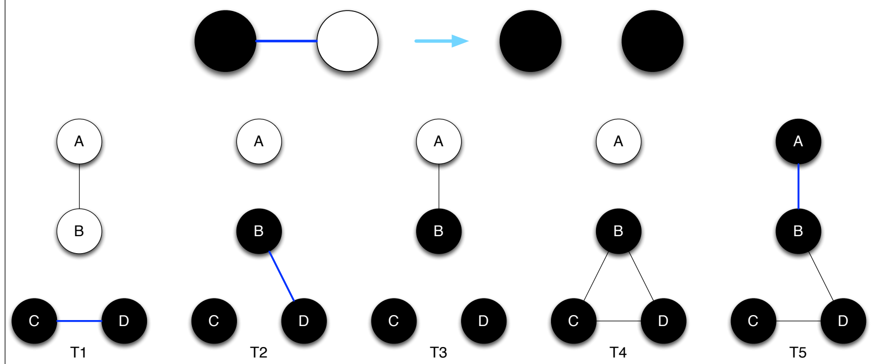
Fastest Journey



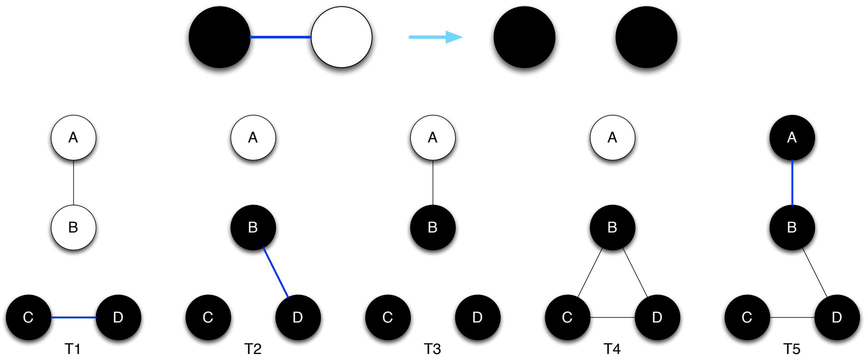
Condition for Broadcast?



Condition for Broadcast?

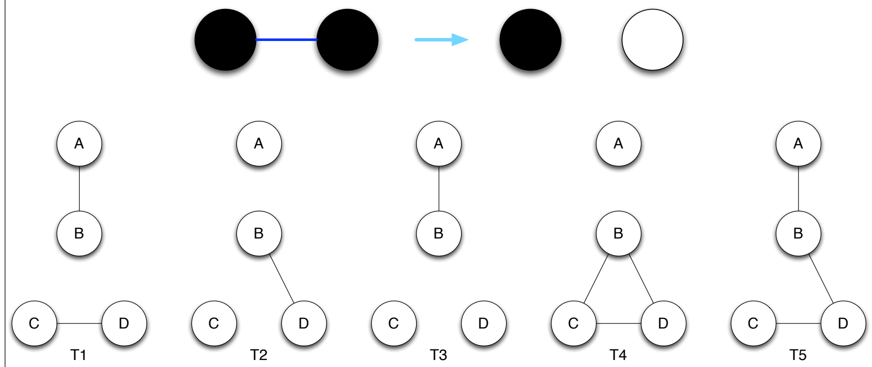


Condition for Broadcast?

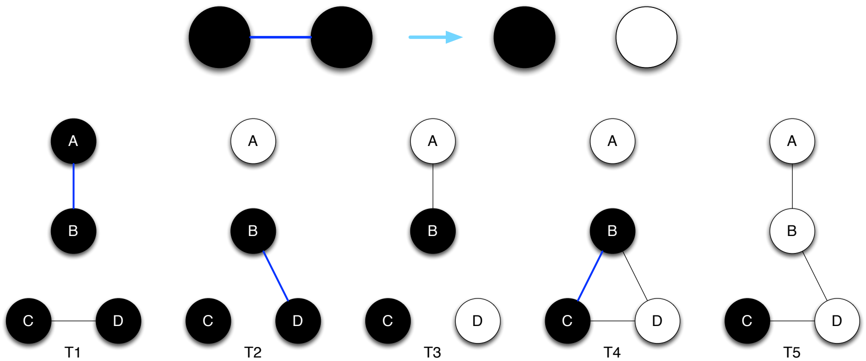


There exists a node (**C**) from which a journey reaches every other node

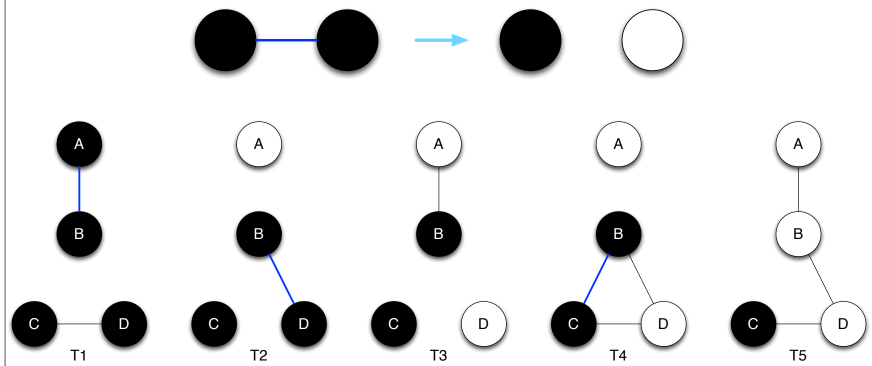
Condition for Election?



Condition for Election?

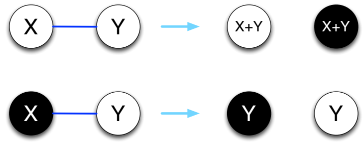


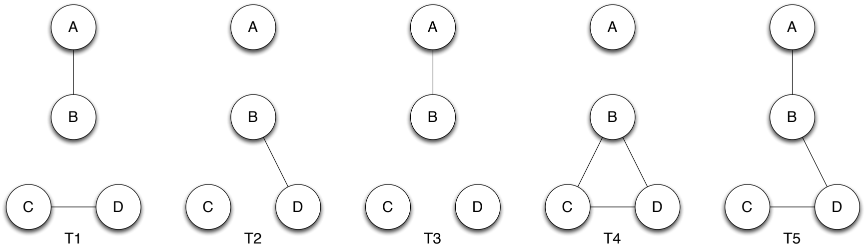
Condition for Election?



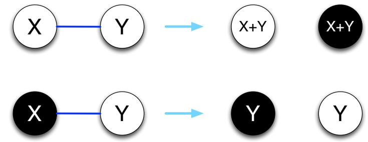
There exists a node (**C**) such that there exists a journey from every other node to it

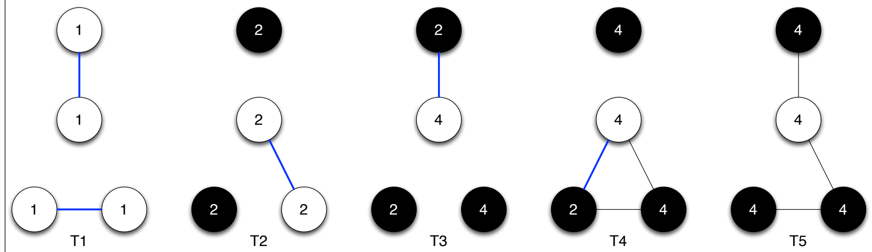
Condition for Global Calculus?





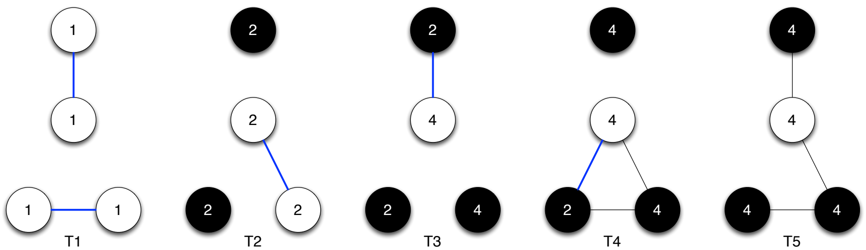
Condition for Global Calculus?





Condition for Global Calculus?

There exists a node (**Center**) such that there exists a journey from every other node to it *and back*



Connectivity Classes

- There exists a node *r* from which a journey reaches every other node $1 \rightsquigarrow *$
- There exists a node *r* such that there exists a journey from every other node to it $* \rightsquigarrow 1$
- There exists a node *r* such that there exists a journey from every other node to to and back $1 \leftrightsquigarrow *$

Arnaud Casteigts, Paola Flocchini, Walter Quattrociocchi, Nicola Santoro: Time-varying graphs and dynamic networks. IJPEDS 27(5): 387-408 (2012)

More Classes

- There exists a journey between any two nodes $* \rightsquigarrow *$
- There exists a roundtrip journey between any two nodes $* \overset{\rightsquigarrow}{\leftsquigarrow} *$
- There exists a journey between any two nodes infinitely often $* \overset{\mathcal{R}}{\rightsquigarrow} *$
- Every edge appears infinitely often $\bullet \overset{\mathcal{R}}{-} \bullet$

Arnaud Casteigts, Paola Flocchini, Walter Quattrociocchi, Nicola Santoro: Time-varying graphs and dynamic networks. IJPEDS 27(5): 387-408 (2012)

More Classes

- Every edge appears infinitely often, and there is an upper bound between between two occurrences $\bullet \overset{\mathcal{B}}{-} \bullet$
- Every edge appears infinitely often with some period *p* $\bullet \overset{\mathcal{P}}{-} \bullet$

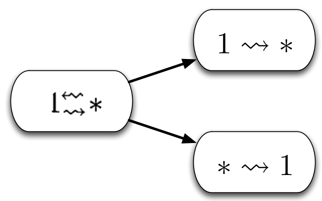
Arnaud Casteigts, Paola Flocchini, Walter Quattrociocchi, Nicola Santoro: Time-varying graphs and dynamic networks. IJPEDS 27(5): 387-408 (2012)

More Classes

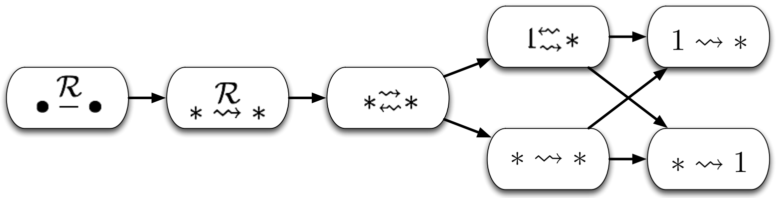
- At any time, the graph is connected
- Every spanning subgraph lasts at least T time units
- Every edge appears infinitely often, and the underlying graph is a clique $* \overset{\mathcal{R}}{-} *$

Arnaud Casteigts, Paola Flocchini, Walter Quattrociocchi, Nicola Santoro: Time-varying graphs and dynamic networks. IJPEDS 27(5): 387-408 (2012)

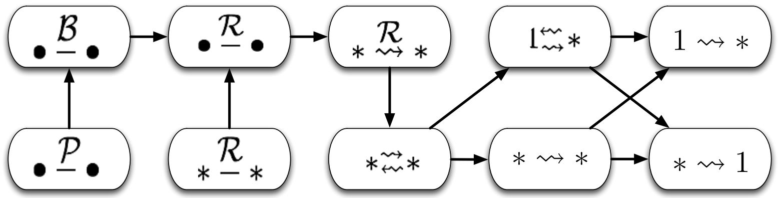
A Classification



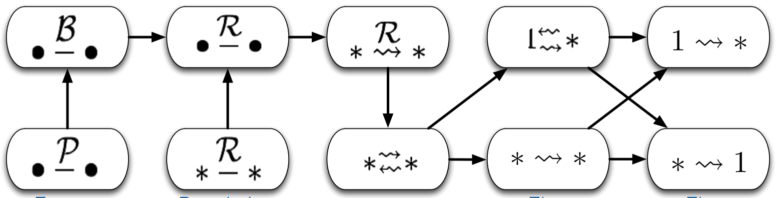
A Classification



A Classification



A Classification

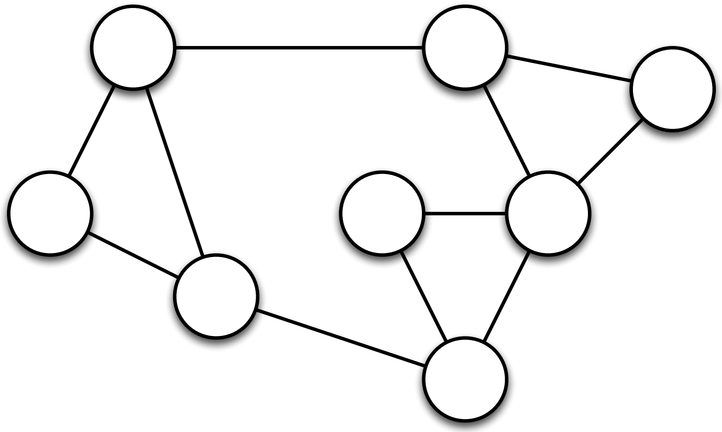


Arnaud Casteigts, Paola Flocchini, Bernard Mans, Nicola Santoro: Shortest, Fastest, and Foremost Broadcast in Dynamic Networks. Int. J. Found. Comput. Sci. 26(4): 499-522 (2015)

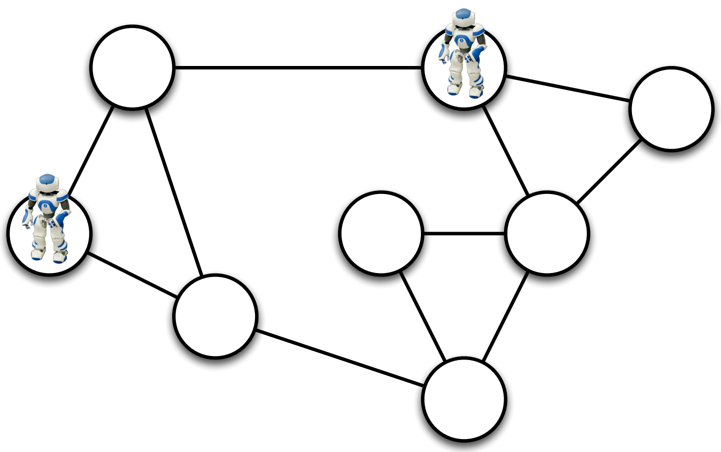
Actively Mobile Networks

Mobile Agents

Mobile Agents



Mobile Agents



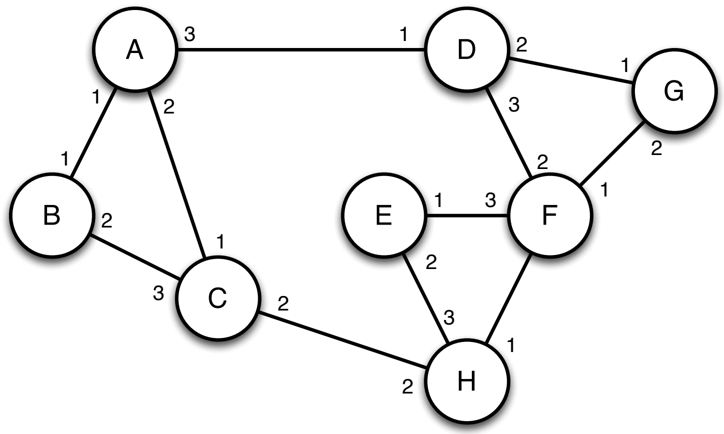
Problems to Solve

- **Exploration** (perpetual or with stop)
- **Mapping**
- **Rendez-vous**
- **Black hole search**
- **Capturing an intruder**

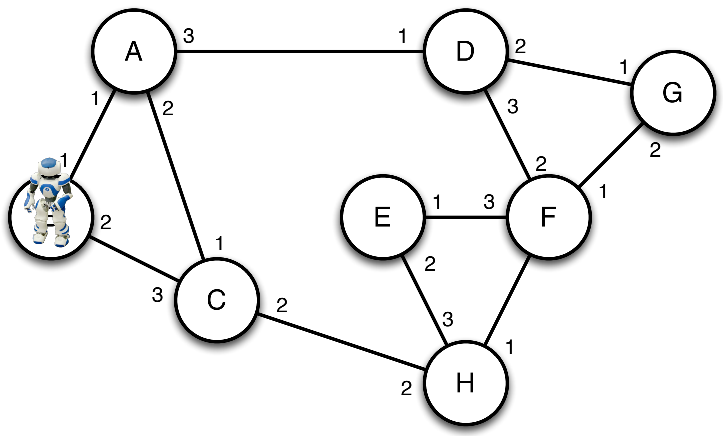
Models

- **Network** (anonymous *vs.* ID based)
- **Agents** (anonymous *vs.* ID based)
- **Synchrony**
- **Initial** (structural) **knowledge**
- **Communications** (none, peebles, whiteboards)
- Agent **memory** (infinite, bounded, constant)

Mapping



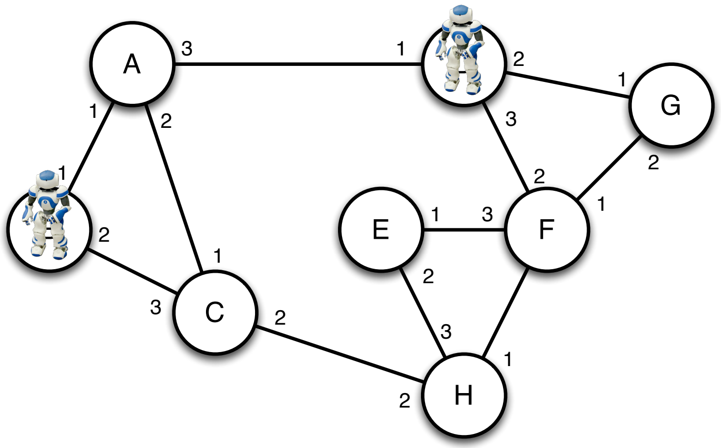
Mapping



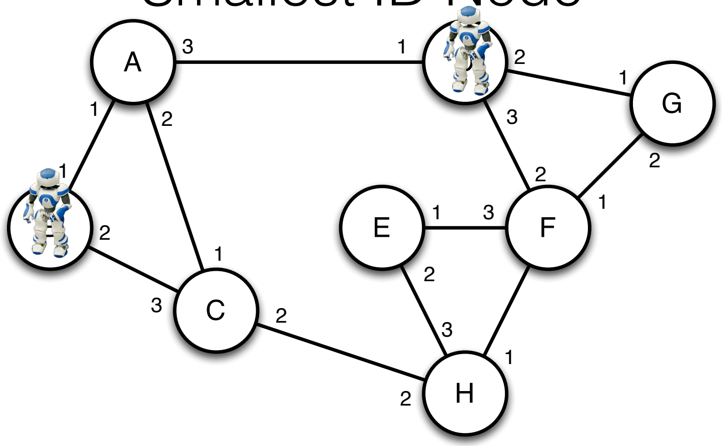
Rendez-vous

- **Two** (or more) **mobile agents must meet** in a graph
- They start on **distinct** locations
- **Computability**?
- **Complexity**?

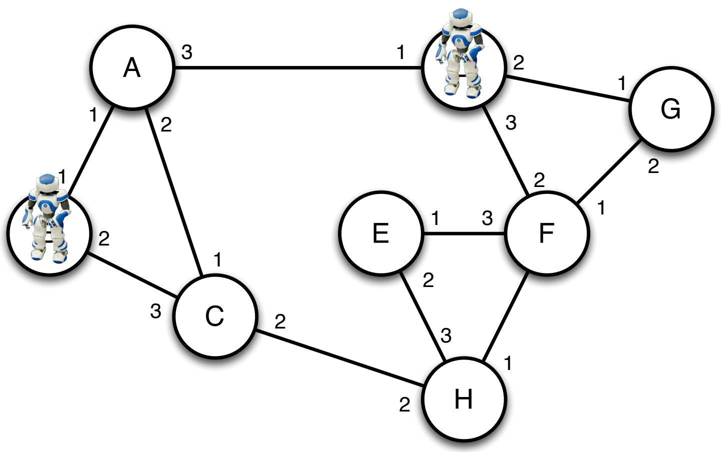
Rendez-vous in ID Graphs



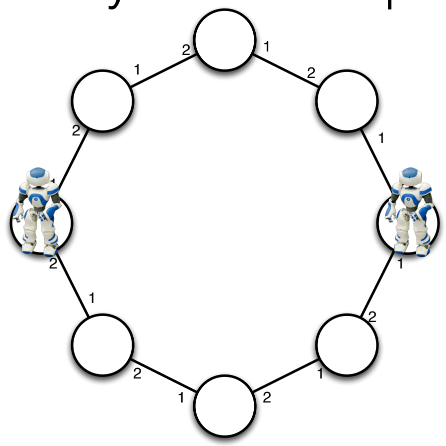
DFS to find Smallest ID Node



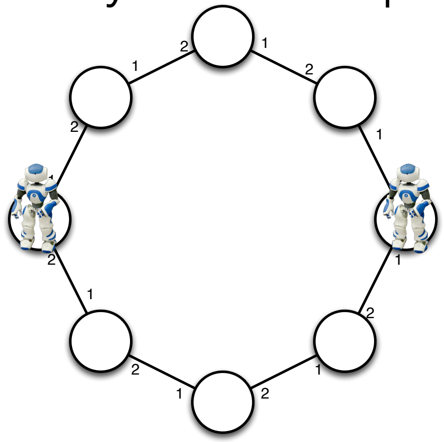
Rendez-vous in A



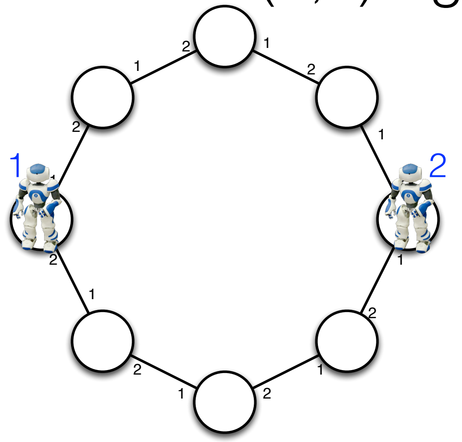
Rendez-vous in Anonymous Graphs



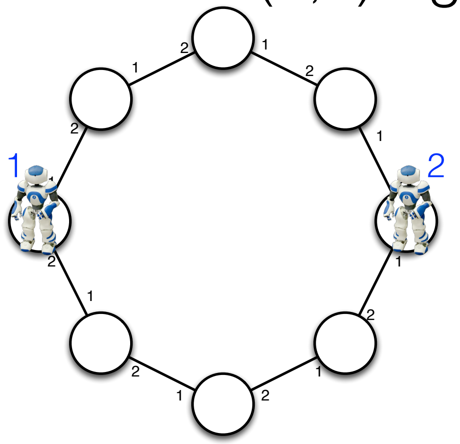
Rendez-vous in
Anonymous Graphs



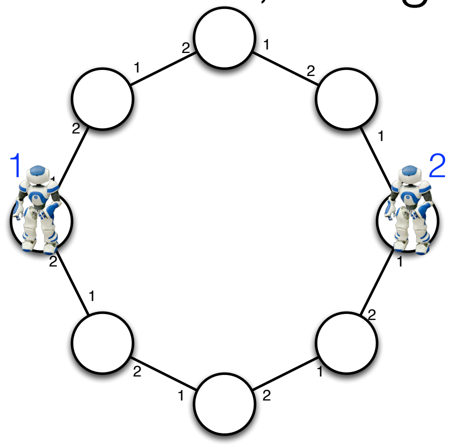
Anonymous Graphs
with Known ID (1,2) Agents



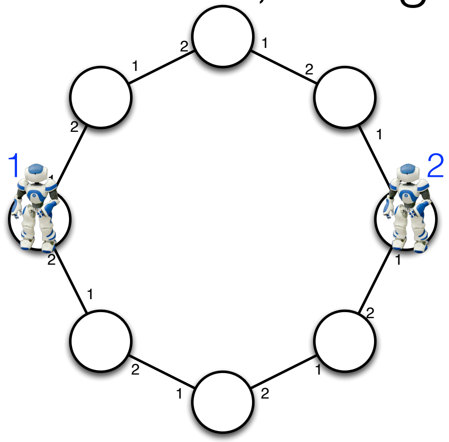
Anonymous Graphs
with Known ID (1,2) Agents



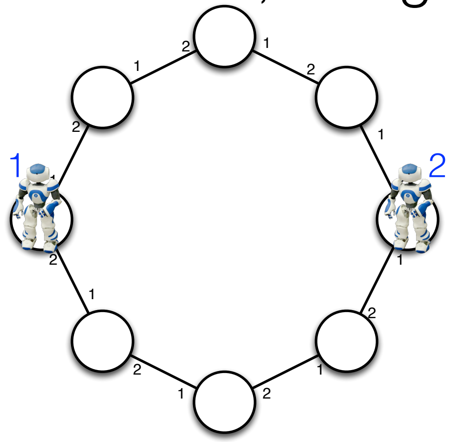
Anonymous Graphs
with Known N, ID Agents



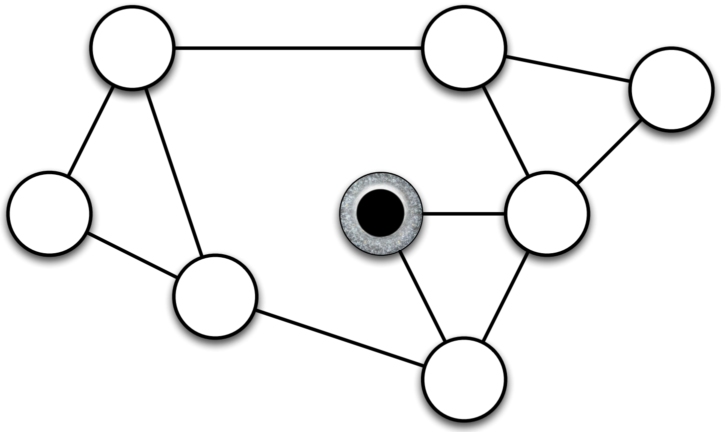
Anonymous Graphs with Known N, ID Agents



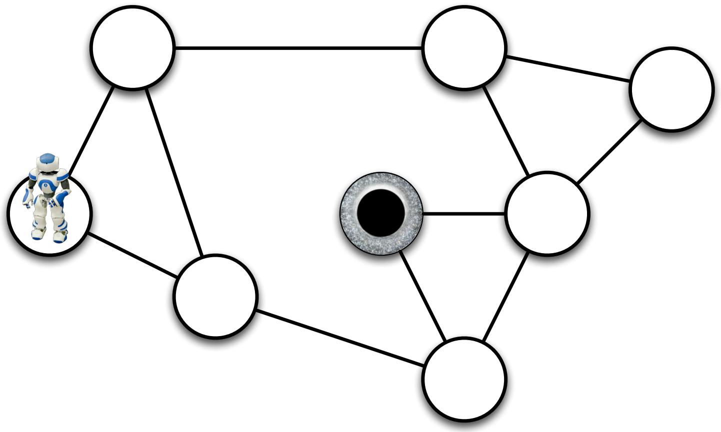
Anonymous Graphs with Known N, ID Agents



Black Hole Search



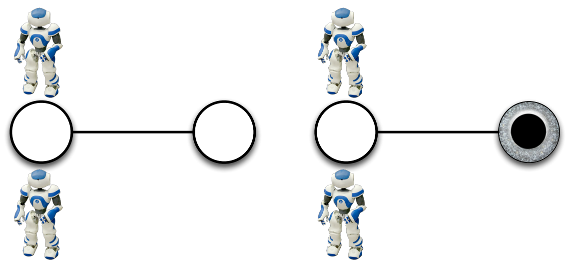
Black Hole Search



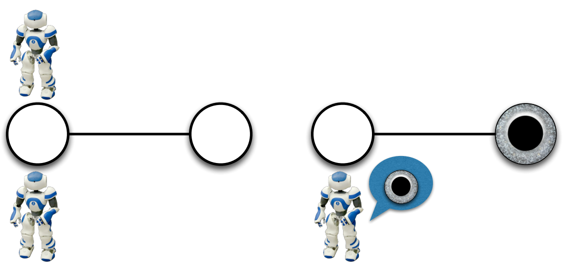
Black Hole Search

- A **single** black hole in the graph
- The black hole **does not disconnect** the graph
- Identify each **adjacent edge**
- **Minimize** #agents, #moves

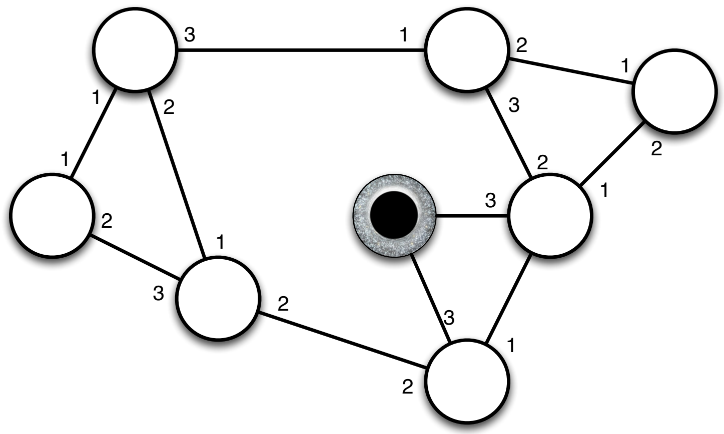
Synchronous Agents



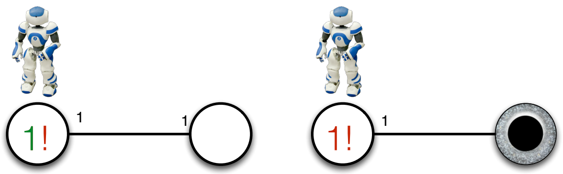
Synchronous Agents



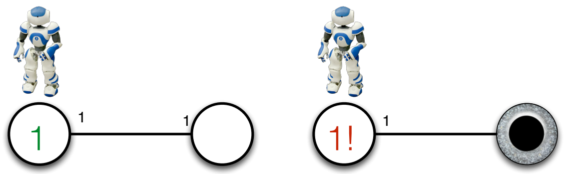
Asynchronous Black Hole Search



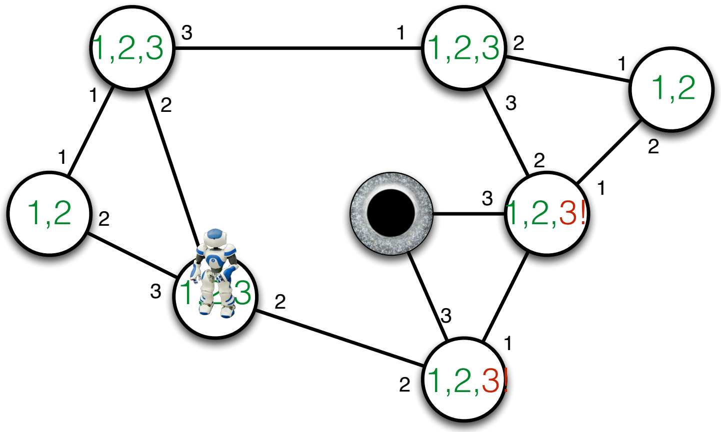
Asynchronous Black Hole Search



Asynchronous Black Hole Search

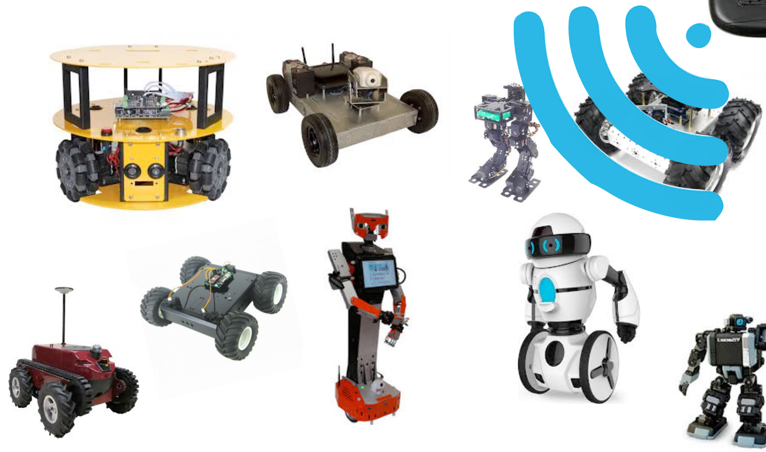


Asynchronous Black Hole Search



Mobile Robots

Mobile Robots



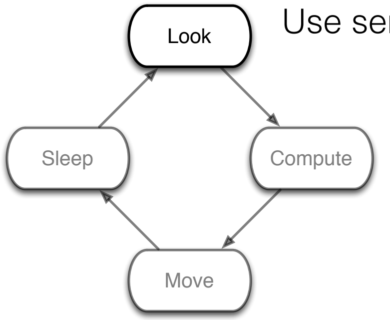
Mobile Robots

Mobile Robots

Mobile Robots

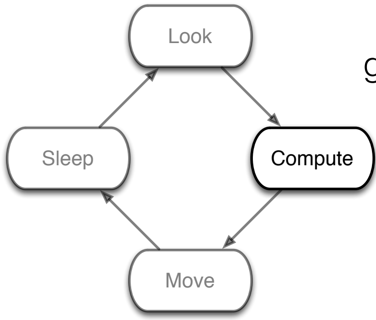
- **Autonomous** (no central control)
- **Homogeneous** (run same algorithm)
- **Identical** (indistinguishable)
- **Silent** (no explicit communication)

Robot Life Cycle



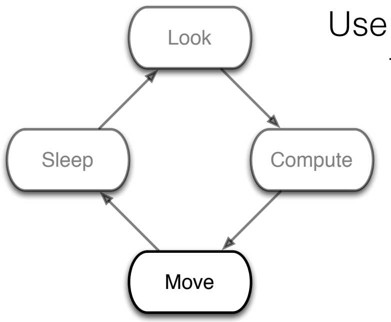
Use sensors to observe the world,
get a **snapshot**

Robot Life Cycle



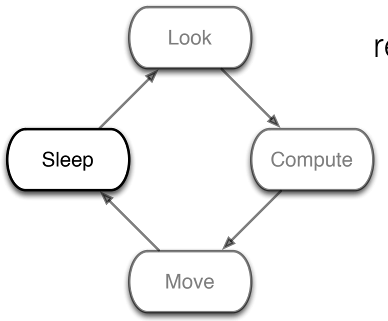
Execute the algorithm,
get a **destination point**

Robot Life Cycle

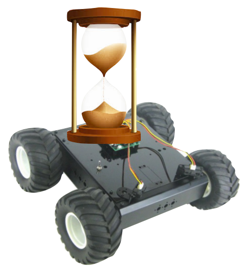


Use motors to **move toward**
the destination point

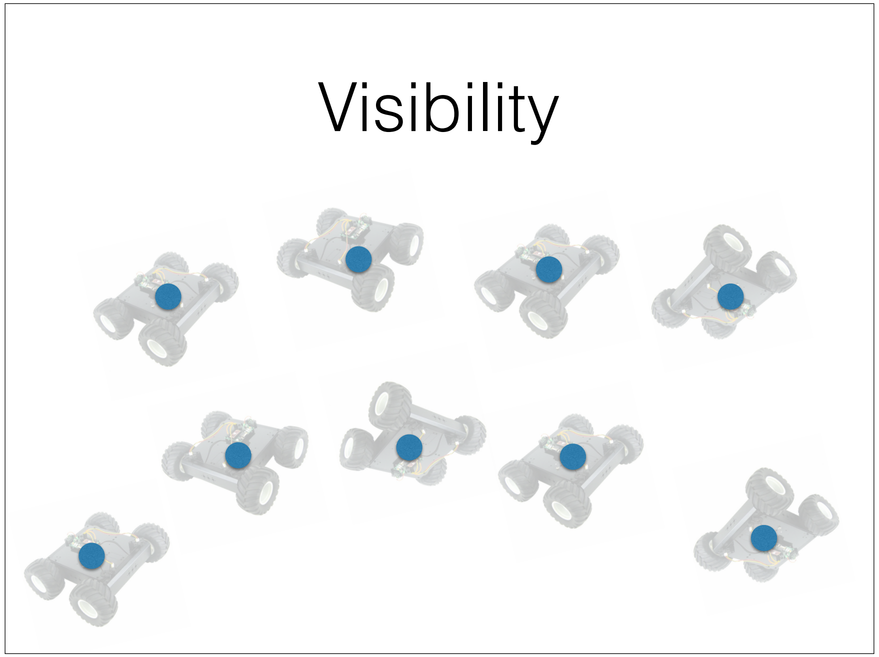
Robot Life Cycle

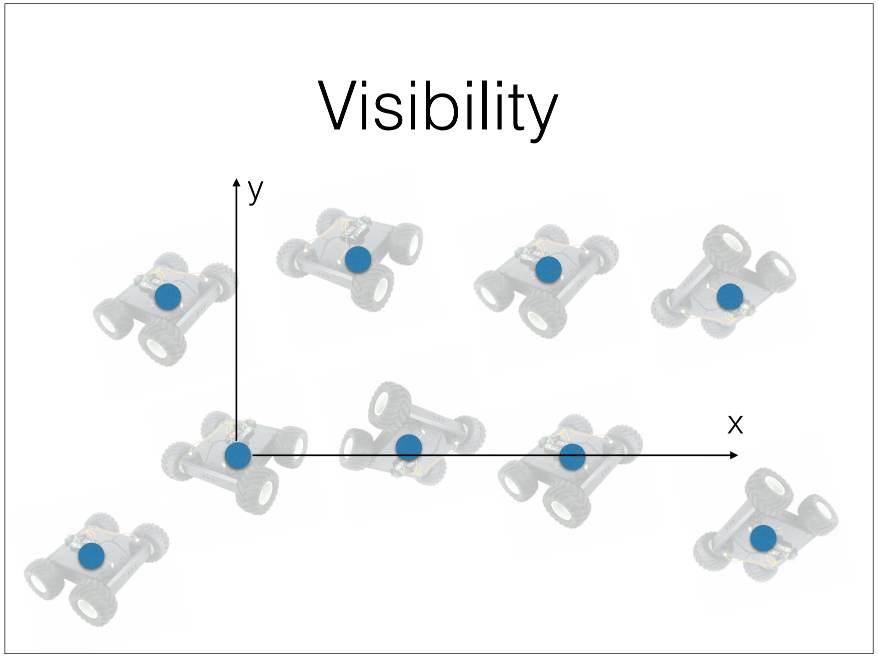


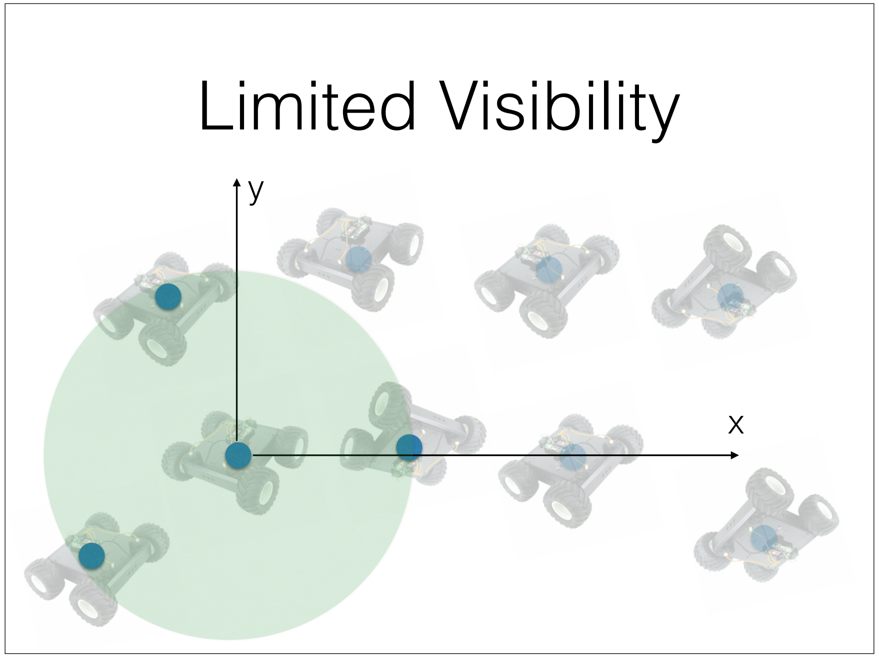
remain **idle** for a while



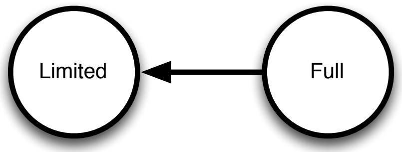

Visibility


Visibility


Visibility
y
x


Limited Visibility
y
x

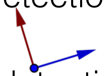
Visibility



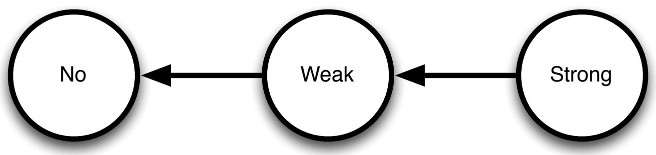
Multiplicity Detection

How many robots do you see?

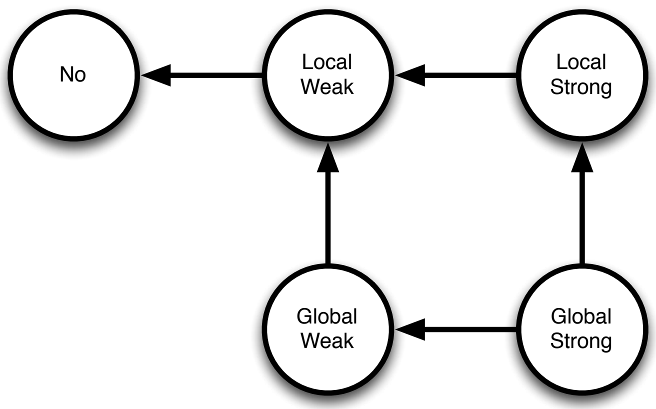
- No detection 1
- Weak multiplicity detection >1
- Strong multiplicity detection



Multiplicity



Multiplicity



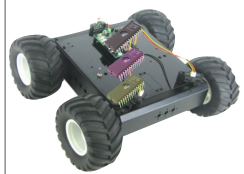
Memory

Algorithm

Persistent Memory

Volatile Memory

Oblivious Robot Memory

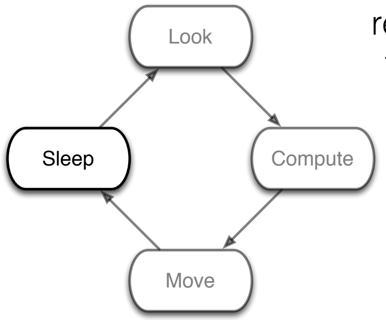


Algorithm

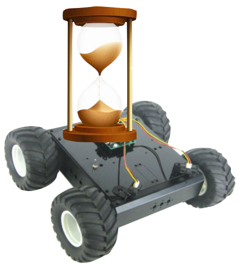
~~Persistent Memory~~

Volatile Memory

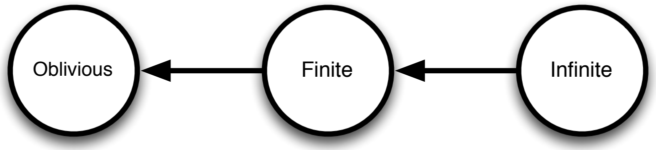
Oblivious Robot Life Cycle



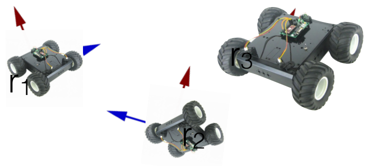
remain **idle** for a while,
forget about the past



Memory



Scheduling

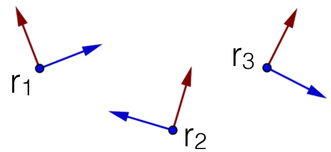


↻ Look→Compute→Move

FSYNC

	1	2	3	4	5	6
r_1	↻	↻	↻	↻	↻	↻
r_2	↻	↻	↻	↻	↻	↻
r_3	↻	↻	↻	↻	↻	↻

Scheduling

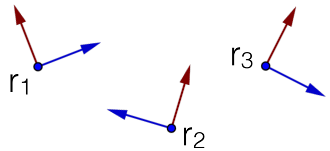


↻ Look→Compute→Move

SSYNC

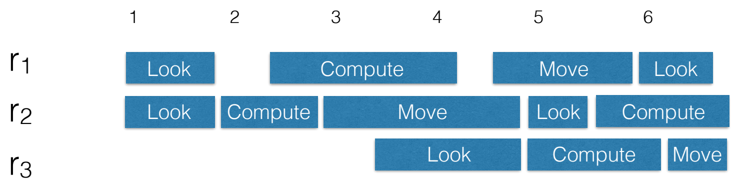
	1	2	3	4	5	6
r_1	↻		↻	↻	↻	↻
r_2	↻	↻		↻	↻	
r_3	↻		↻			↻

Scheduling

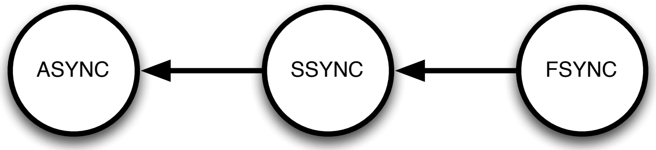


↻ Look→Compute→Move

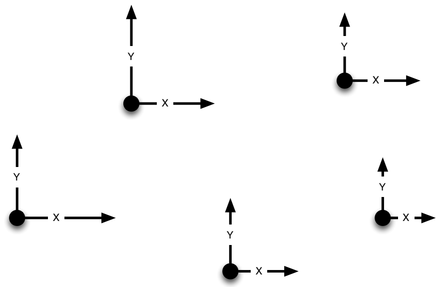
ASYNC



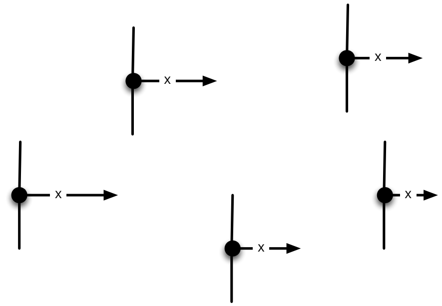
Scheduling



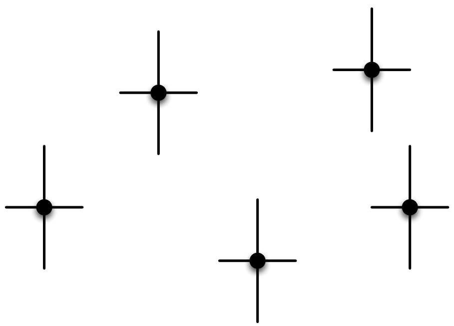
Two Axes
Direction and Orientation



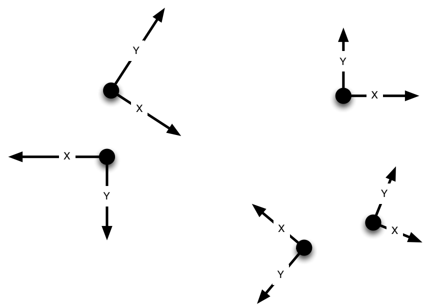
One Axis
Direction and Orientation



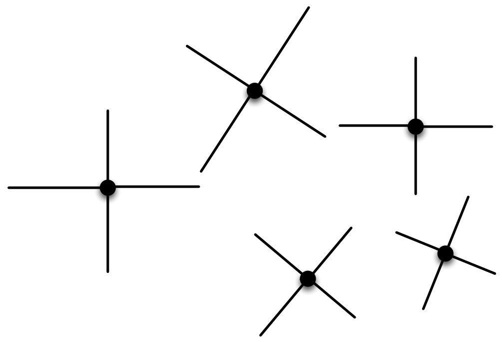
Two Axes
Direction



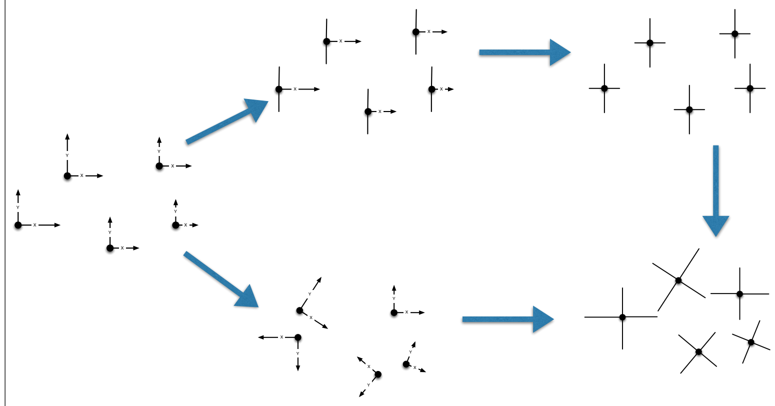
Chirality



No Agreement



Overview

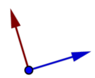


Scattering

Scattering

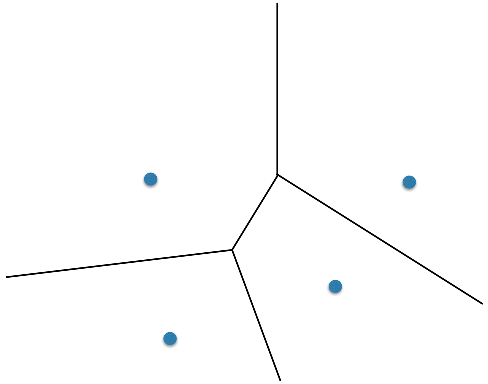
No two robots should occupy the same position

- No deterministic solution



- No termination without multiplicity detection

Scattering

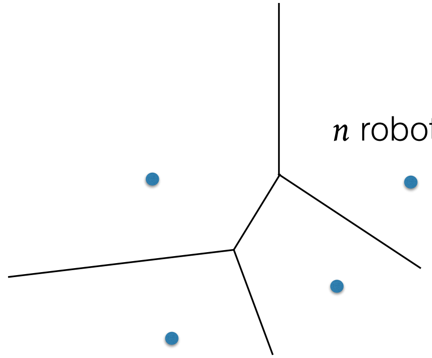


1 toss

$O(log(n)loglog(n))$ rounds

Yoann Dieudonné and Franck Petit. *Scatter of robots.* Parallel Processing Letters 19.01 (2009): 175-184.

Scattering



n robots: $2n^2$ destinations

$O(1)$ rounds

Julien Clément, Xavier Défago, Maria Potop-Butucaru, et al. *The cost of probabilistic agreement in oblivious robot networks.* Information Processing Letters, 2010, vol. 110, no 11, p. 431-438.

How Many Tosses?

Minimum?

Influence of multiplicity detection?

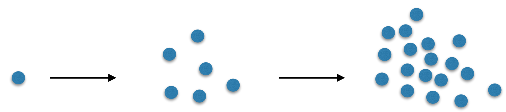
Relationship with scattering speed?

Optimal Speed

With strong multiplicity detection:

Algorithm with optimal #tosses terminates in $O(1)$ rounds

Without strong multiplicity detection:



n robots

finite #destinations

max #destinations is independent of n

$O(1)$ rounds scattering of n robots is impossible

How fast can we go?

Scattering

	Scattering	Scattering +MD
FSYNC		Yes $O(1)$ rounds
SSYNC		Yes $O(1)$ rounds
ASYNC		

Julien Clément, Xavier Défago, Maria Potop-Butucaru, et al. *The cost of probabilistic agreement in oblivious robot networks.* Information Processing Letters, 2010, vol. 110, no 11, p. 431-438.

Scattering

	Scattering	Scattering +MD
FSYNC	Yes $O(f(n))$ rounds	Yes $O(1)$ rounds
SSYNC	Yes $O(f(n))$ rounds	Yes $O(1)$ rounds
ASYNC		

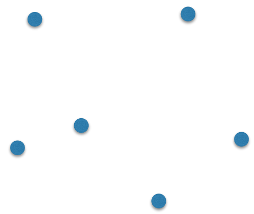
Quentin Bramas and Sébastien Tixeuil. *The Ramdom Bit Complexity of Mobile Robot Scattering.* Int. J. Found. Comput. Sci. 28(2): 111-134 (2017)

Scattering

	Scattering	Scattering +MD
FSYNC	Yes $O(f(n))$ rounds	Yes $O(1)$ rounds
SSYNC	Yes $O(f(n))$ rounds	Yes $O(1)$ rounds
ASYNC	?	?

Gathering

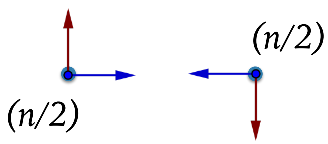
Gathering



Gathering

Gathering

Impossible for two robots



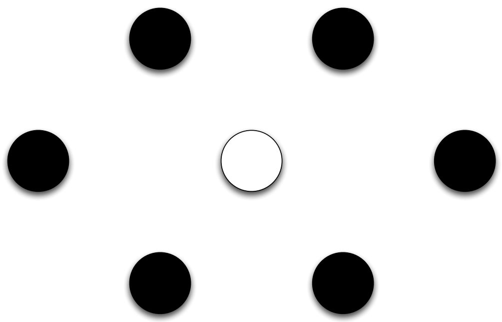
A bivalent configuration

Gathering vs. Convergence

- **Gathering**: robot must **reach** the same point in finite time
- **Convergence**: robots must get **closer** at time goes by

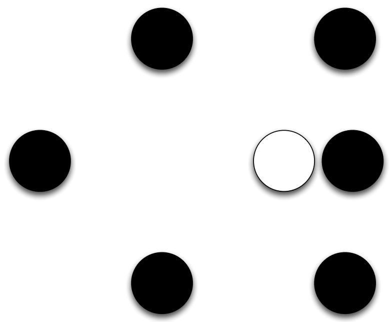
Center of Gravity

$$\vec{c}[t] = \frac{1}{n}\sum_{i=1}^{n}\vec{r_i}[t]$$



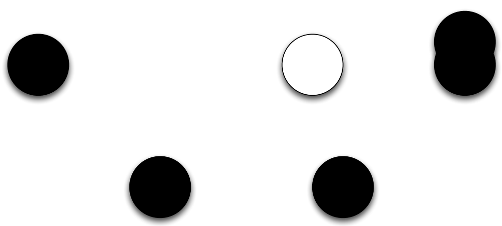
Center of Gravity

$$\vec{c}[t] = \frac{1}{n}\sum_{i=1}^{n}\vec{r_i}[t]$$



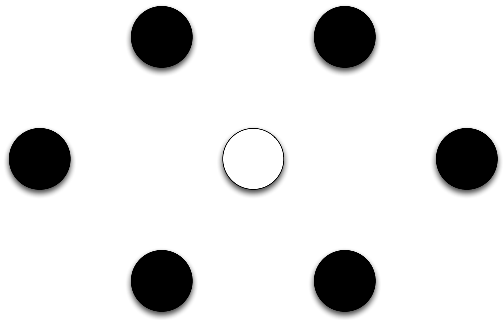
Center of Gravity

$$\vec{c}[t] = \frac{1}{n}\sum_{i=1}^{n}\vec{r_i}[t]$$



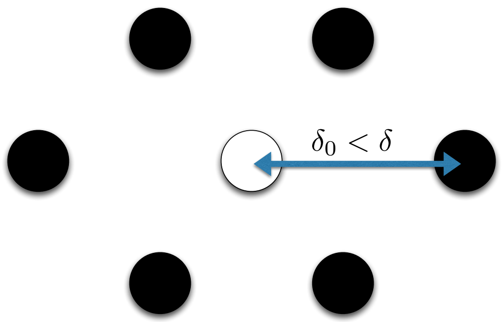
Center of Gravity of Positions

$$\vec{c}[t] = \frac{1}{p}\sum_{i=1}^{p}\vec{p_i}[t]$$



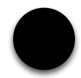
FSYNC Gathering

$$\vec{c}[t] = \frac{1}{p}\sum_{i=1}^{p}\vec{p_i}[t]$$

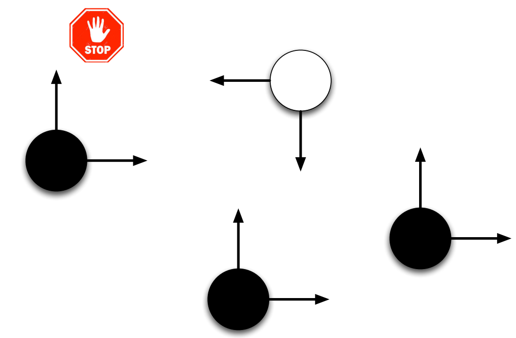


FSYNC Gathering

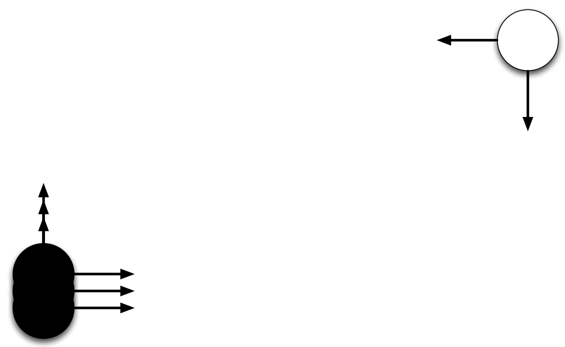
$$\vec{c}[t] = \frac{1}{p}\sum_{i=1}^{p}\vec{p_i}[t]$$



SSYNC Gathering?



SSYNC Gathering?



Convergence & Gathering

	Convergence	2-Gathering	n-Gathering	n-Gathering +MD	n-Gathering +MD+WF
FSYNC	Yes	Yes	Yes	Yes	Yes
SSYNC	Yes	No	No	Yes	Yes
ASYNC	Yes	No	No	Yes	?

Convergence & Gathering

	Convergence	2-Gathering	n-Gathering	n-Gathering +MD	n-Gathering +MD+WF
FSYNC	Yes	Yes	Yes	Yes	Yes
SSYNC	Yes	No	No	Yes	Yes
ASYNC	**Yes**	No	No	Yes	?

Reuven Cohen and David Peleg. *Convergence Properties of the Gravitational Algorithm in Asynchronous Robot Systems*. SIAM J. Comput. 34(6): 1516-1528 (2005)

Convergence & Gathering

	Convergence	2-Gathering	n-Gathering	n-Gathering +MD	n-Gathering +MD+WF
FSYNC	Yes	**Yes**	Yes	Yes	Yes
SSYNC	Yes	**No**	No	Yes	Yes
ASYNC	**Yes**	No	No	Yes	?

Ichiro Suzuki, Masafumi Yamashita: Distributed Anonymous Mobile Robots: Formation of Geometric Patterns. SIAM J. Comput. 28(4): 1347-1363 (1999)

Convergence & Gathering

	Convergence	2-Gathering	n-Gathering	n-Gathering +MD	n-Gathering +MD+WF
FSYNC	Yes	**Yes**	Yes	Yes	Yes
SSYNC	Yes	**No**	**No**	Yes	Yes
ASYNC	**Yes**	No	No	Yes	?

Guiseppe Prencipe. *Impossibility of gathering by a set of autonomous mobile robots. Theor. Comput. Sci. 384(2-3): 222-231 (2007)*

Convergence & Gathering

	Convergence	2-Gathering	n-Gathering	n-Gathering +MD	n-Gathering +MD+WF
FSYNC	Yes	**Yes**	**Yes**	Yes	**Yes**
SSYNC	Yes	**No**	**No**	**Yes**	**Yes**
ASYNC	**Yes**	No	No	**Yes**	**?**

Thibaut Balabonski, Amélie Delga, Lionel Rieg, Sébastien Tixeuil, Xavier Urbain: Synchronous Gathering Without Multiplicity Detection: A Certified Algorithm. SSS 2016: 7-19

Convergence & Gathering

	Convergence	2-Gathering	n-Gathering	n-Gathering +MD	n-Gathering +MD+WF
FSYNC	Yes	**Yes**	**Yes**	Yes	**Yes**
SSYNC	Yes	**No**	**No**	Yes	**Yes**
ASYNC	**Yes**	No	No	**Yes**	**?**

Mark Cieliebak, Paola Flocchini, Giuseppe Prencipe, Nicola Santoro. *Distributed Computing by Mobile Robots: Gathering.* SIAM J. Comput. 41(4): 829-879 (2012)

Convergence & Gathering

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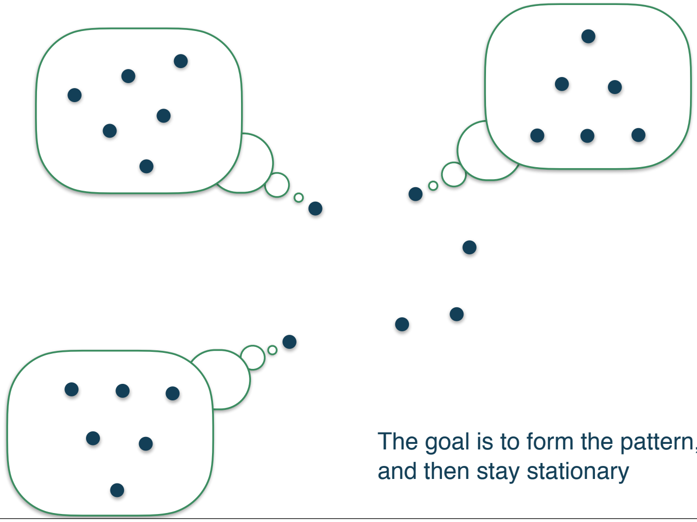
Quentin Bramas, Sébastien Tixeuil. *Wait-Free Gathering Without Chirality.* SIROCCO 2015: 313-327

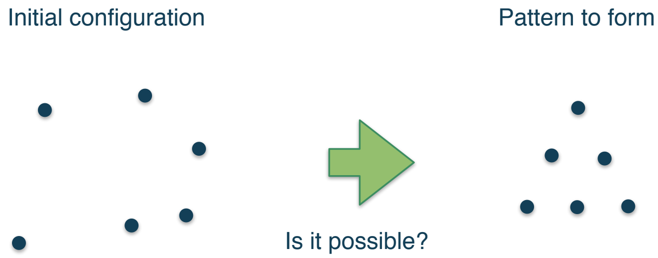
Pattern Formation

Pattern Formation



The goal is to form the pattern,
and then stay stationary

Pattern Formation



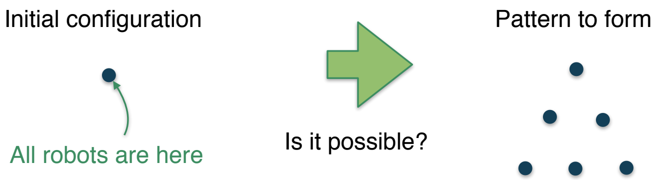
In general, NO

What about restricting the set of initial configurations?

What about adding conditions on the pattern?

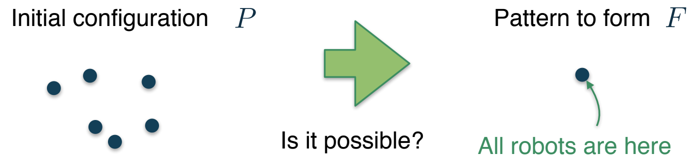
What about adding capabilities to robots?

Pattern Formation



No, so from now, we assume the initial configuration does not have points of multiplicity

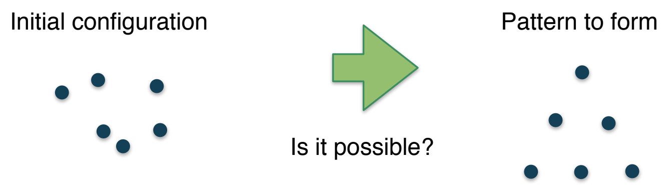
Pattern Formation



NO

Guiseppe Prencipe. *Impossibility of gathering by a set of autonomous mobile robots. Theor. Comput. Sci. 384(2-3): 222-231 (2007)*

Pattern Formation

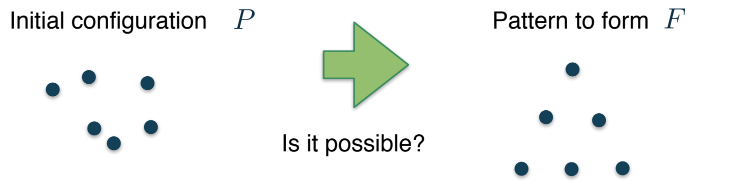


Yes, if robots agree on a common North and a common Right

Yes, if robots agree on a common North and n is odd

Paola Flocchini, Giuseppe Prencipe, Nicola Santoro, Peter Widmayer: Arbitrary pattern formation by asynchronous, anonymous, oblivious robots. Theor. Comput. Sci. 407(1-3): 412-447 (2008)

Pattern Formation



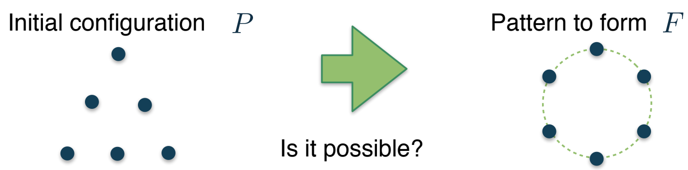
…assuming a common chirality, and F does not have multiplicity points

Yes, if $\rho(P) \mid \rho(F)$ where $\rho(P)$ is the symmetricity of P, the maximum integer such that the rotation by $2\pi/\rho(P)$ is invariant for P

No, otherwise

Nao Fujinaga, Yukiko Yamauchi, Hirotaka Ono, Shuji Kijima, Masafumi Yamashita: Pattern Formation by Oblivious Asynchronous Mobile Robots. SIAM J. Comput. 44(3): 740-785 (2015)

Pattern Formation



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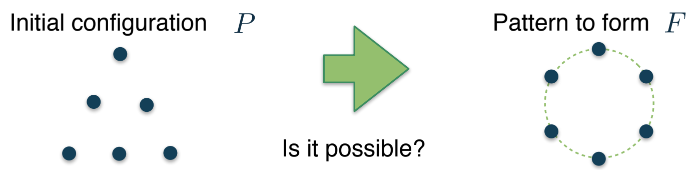

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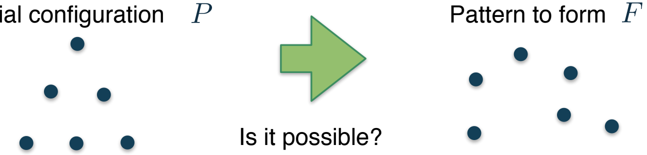
Pattern Formation

Initial configuration P Pattern to form F



Is it possible?

…assuming a common chirality, and F does not have multiplicity points

Yes, if $\rho(P) \mid \rho(F)$ where $\rho(P)$ is the symmetricity of P,
the maximum integer such that the rotation by $2\pi/\rho(P)$ is invariant for P

No, otherwise

Nao Fujinaga, Yukiko Yamauchi, Hirotaka Ono, Shuji Kijima, Masafumi Yamashita:
Pattern Formation by Oblivious Asynchronous Mobile Robots. SIAM J. Comput. 44(3): 740-785 (2015)

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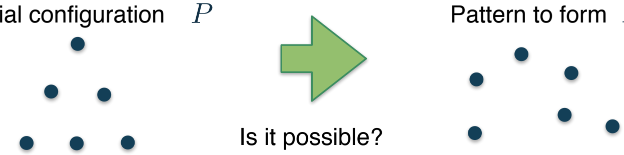

Is it possible?

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Nao Fujinaga, Yukiko Yamauchi, Hirotaka Ono, Shuji Kijima, Masafumi Yamashita:
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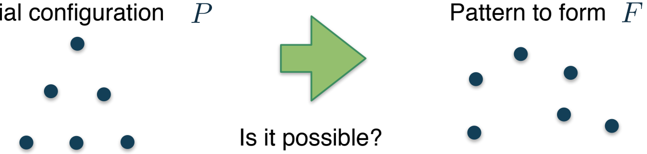


Is it possible?

…assuming a common chirality, and F does not have multiplicity points

Yes, with a randomized algorithm

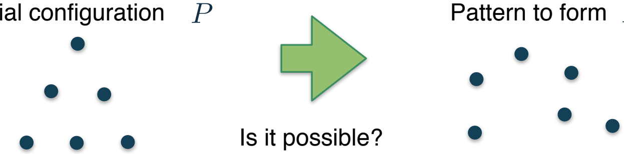
… assuming robots do not "pause" while moving

… and using infinitely many random bits per activation

Yukiko Yamauchi, Masafumi Yamashita: Randomized Pattern Formation Algorithm
for Asynchronous Oblivious Mobile Robots. DISC 2014: 137-151

Pattern Formation

Initial configuration P Pattern to form F



Is it possible?

~~…assuming a common chirality, and F does not have multiplicity points~~

Yes, with a randomized algorithm F is not a point

… ~~assuming robots do not "pause" while moving~~ really asynchronous

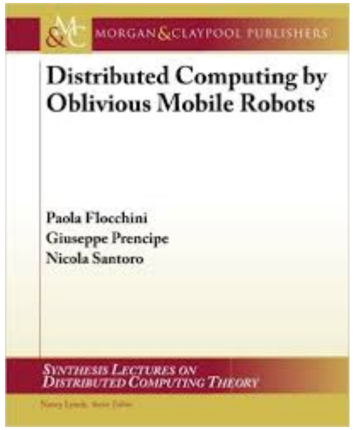
… ~~and using infinitely many random bits per activation~~ only one random bit

Quentin Bramas, Sébastien Tixeuil: Brief Announcement: Probabilistic Asynchronous
Arbitrary Pattern Formation. PODC 2016: 443-445

ASYNC Pattern Formation

Pattern	Agreement	Chirality	Randomization
Point	Yes	No	?
Divide Symmetricity	Yes	Yes	Yes
No Multiplicity	Yes	No	Yes
Not a Point	Yes	No	Yes
Arbitrary	Yes	No	?

Mobile Robots



Conclusion

Static Networks

- **Fundamental**, well established model
  - **Space-centric**, *complexity* results
  - **Time-centric**, *computability* results

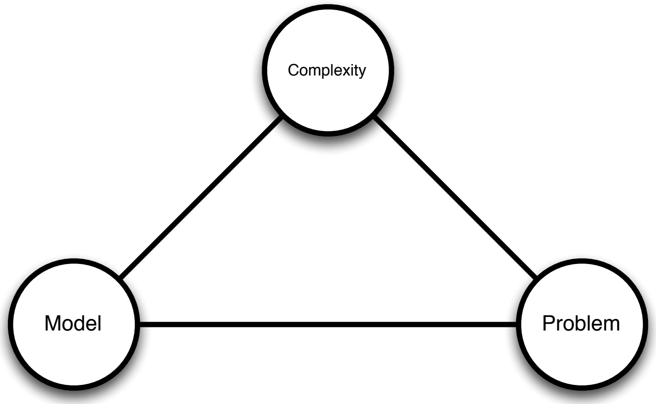
Mobility as an Adversary

- Can corrupt the distributed state of a network
- Can reduces communication capacity
- Can increase uncertainty
- Can increase protocol complexity

Mobility as a Friend

- Mobility can be the solution to the problem
- Mobility can improve efficiency
- Mobility can promote simplicity

Distributed Computing



Thank You