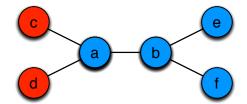
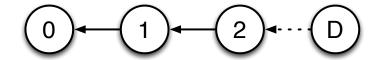
Scheduling

- **Scheduler** (a.k.a. **Daemon**): the daemon chooses among activatable processes those that will execute their actions
 - can be seen as an adversary whose role is to prevent stabilization

 $\begin{array}{c} \text{Spatial Scheduling} \\ \mathit{true} \rightarrow \mathit{color}_i := \mathit{Min} \big\{ \Delta \setminus \{\mathit{color}_j | j \in \mathit{Neighbors}_i \} \big\} \end{array}$ $\Delta = \{ \bigcirc \bigcirc \bigcirc \bigcirc \}$

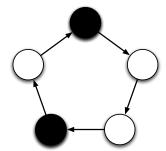


Spatial Scheduling



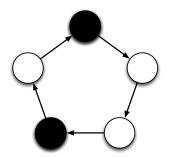
Temporal Scheduling

 $token \rightarrow pass \ token \ to \ left \ neighbor \ with \ probability \ \frac{1}{2}$ $token = \bigcirc$ no $token = \bigcirc$

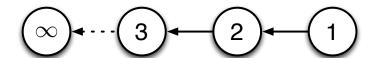


Temporal Scheduling

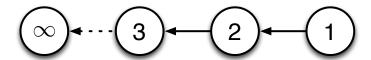
 $token
ightharpoonup pass token to left neighbor with probability <math>\frac{1}{2}$ $token = \bigcirc no \ token = \bigcirc$

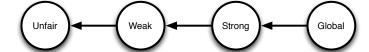


Temporal Scheduling

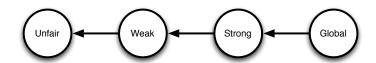


Temporal Scheduling

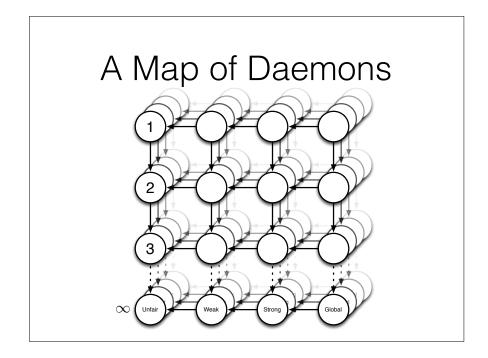


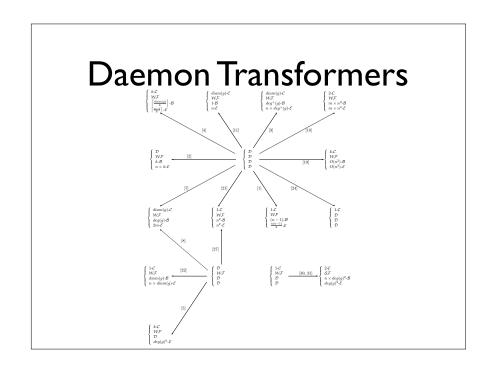


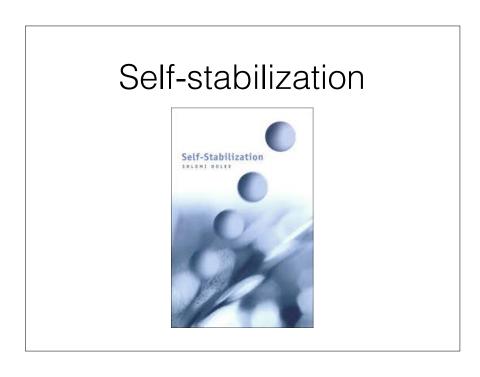
A Map of Daemons



A Map of Daemons 1 2 4 3 4 Weak Strong Global





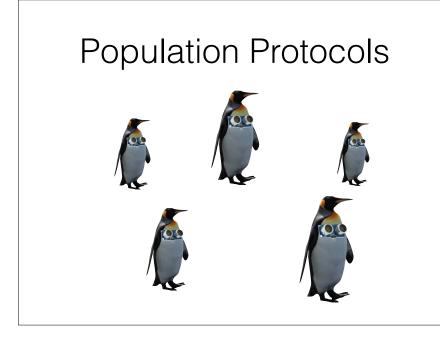


Population Protocols

Population Protocols



Dana Angluin, James Aspnes, Zoë Diamadi, Michael J. Fischer, René Peralta: Computation in networks of passively mobile finite-state sensors. Distributed Computing 18(4): 235-253 (2006)



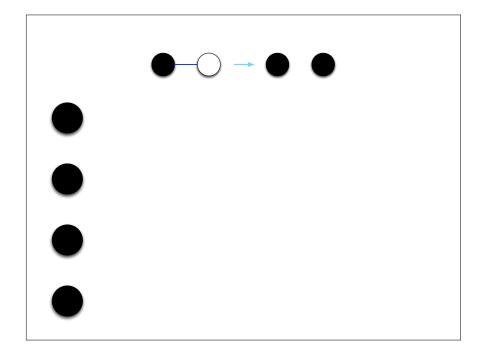
Population Protocols

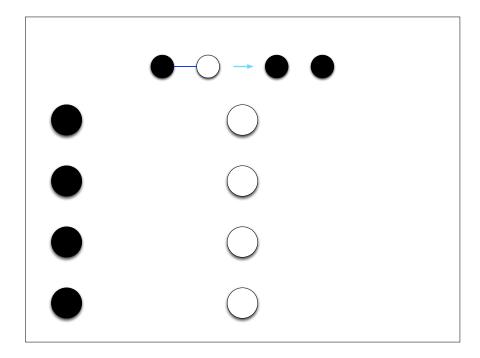
· Definition

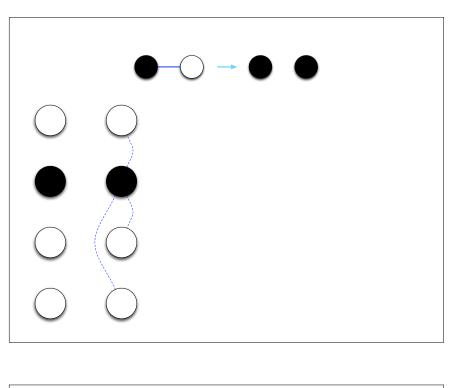
- A Population Protocol is a 6-tuple (X,Y,Q,I,O,T)
 - X: Set of inputs
 - Y: Set of outputs
 - Q: Set of states
 - I: Input mapping function, $X \longrightarrow Q$
 - **O**: Output mapping function, $Q \longrightarrow Y$
 - **T**: Transition function, $Q \times Q \longrightarrow Q \times Q$

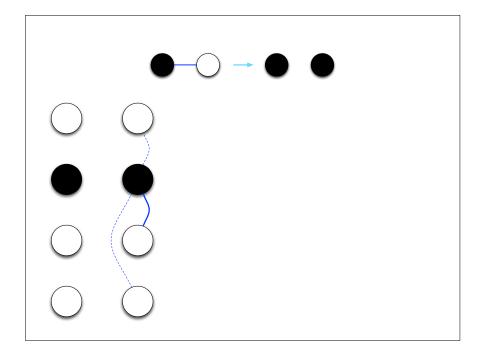


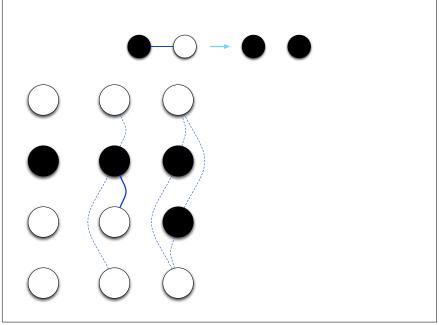


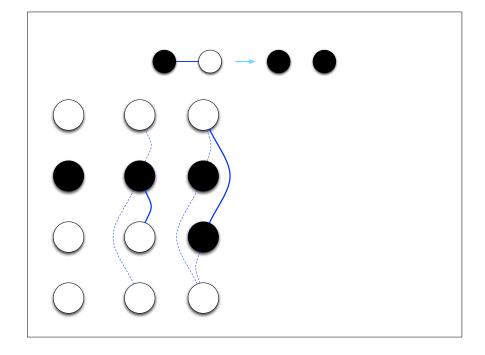


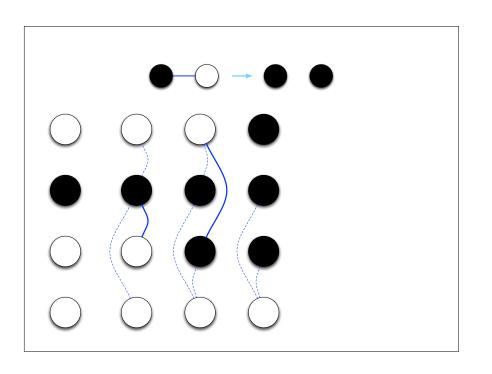


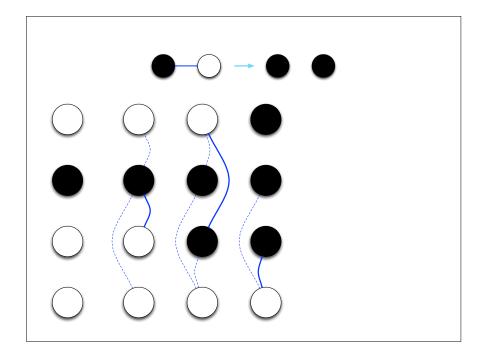


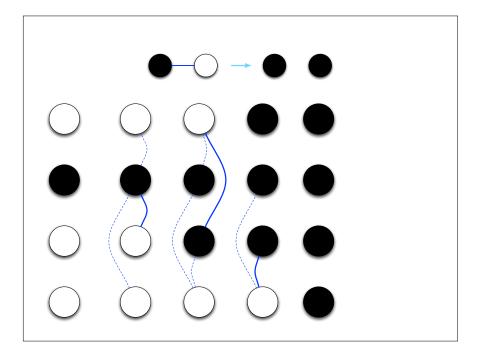


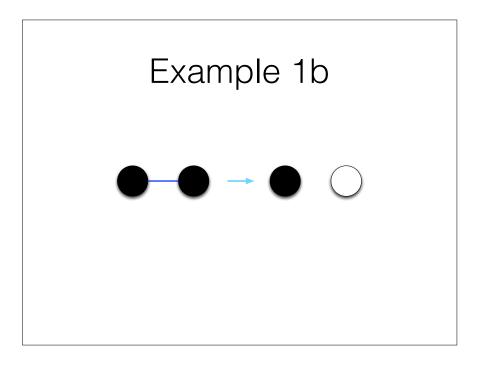


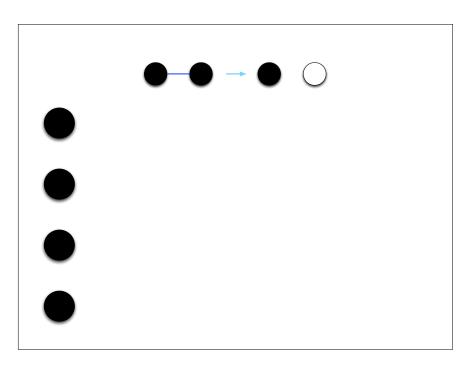


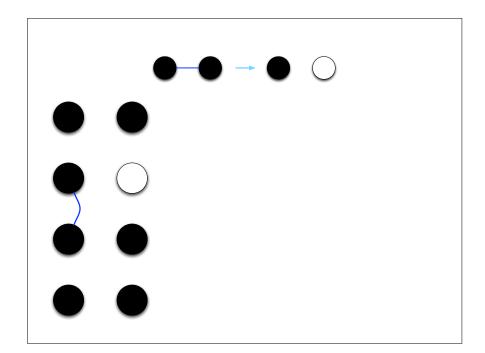


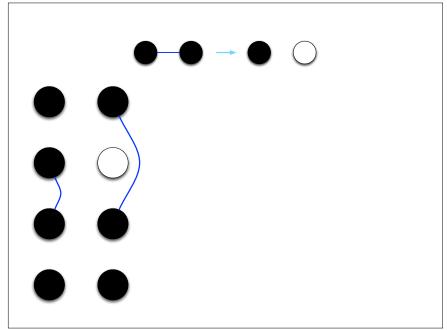


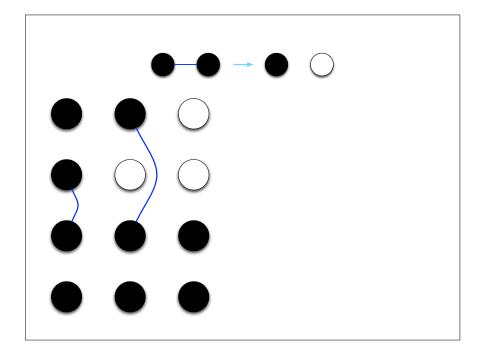


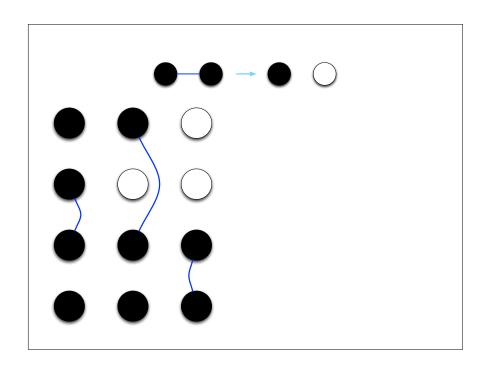


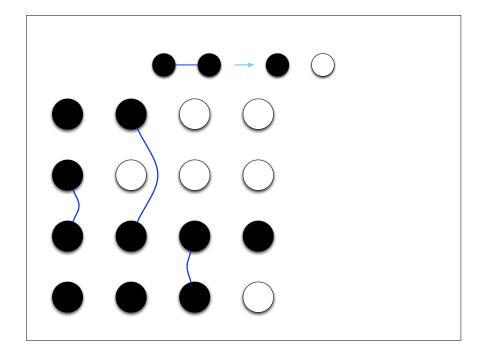


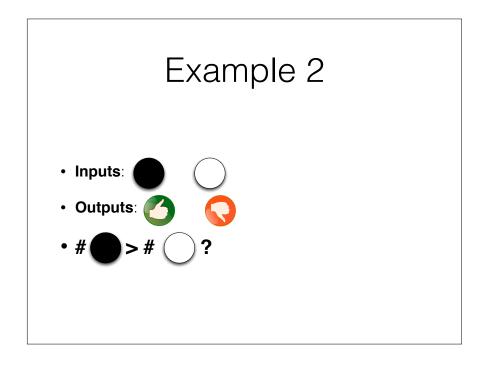


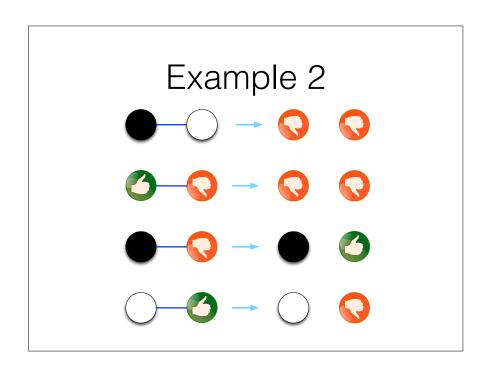


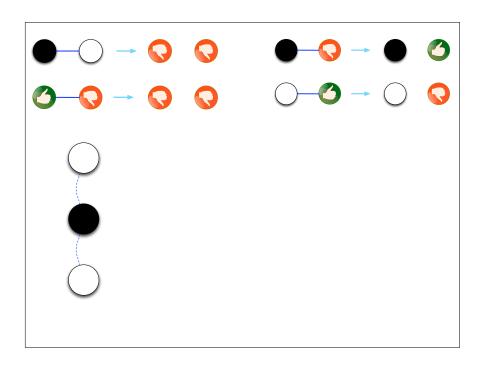


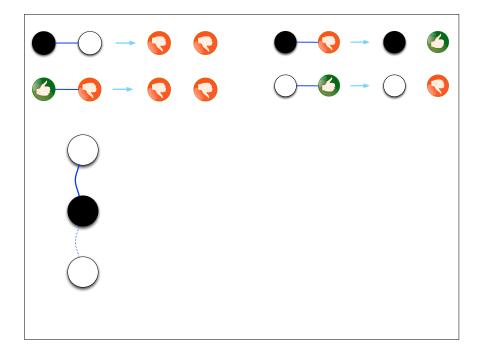


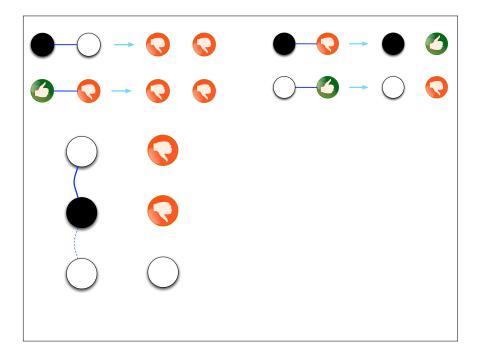


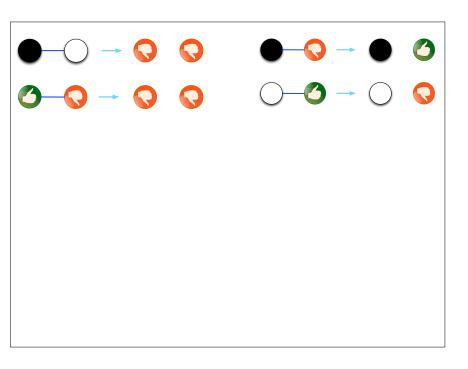


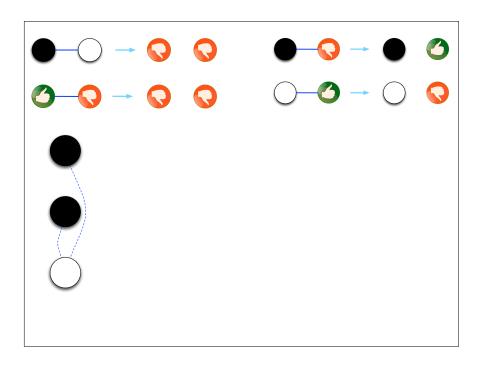


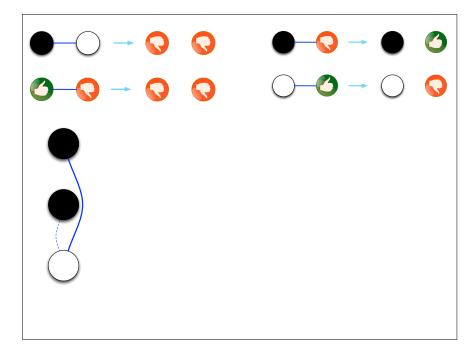


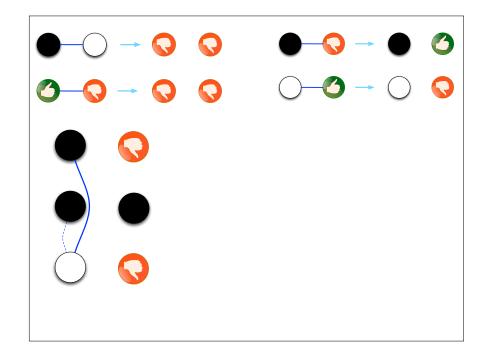


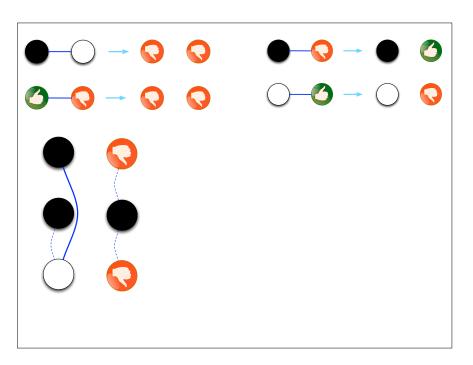


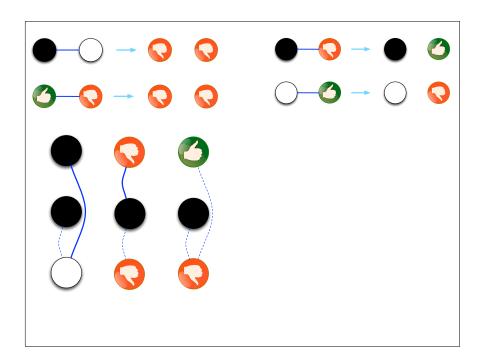


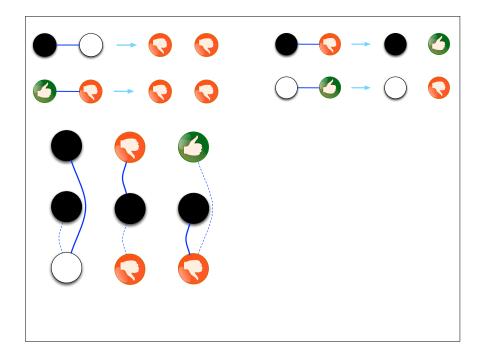


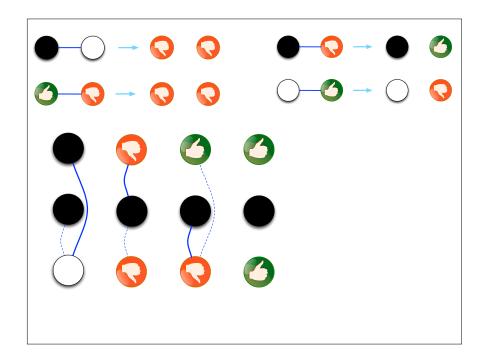


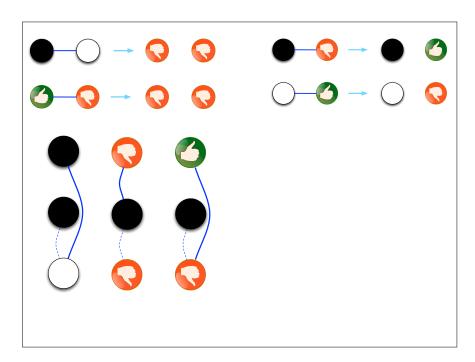


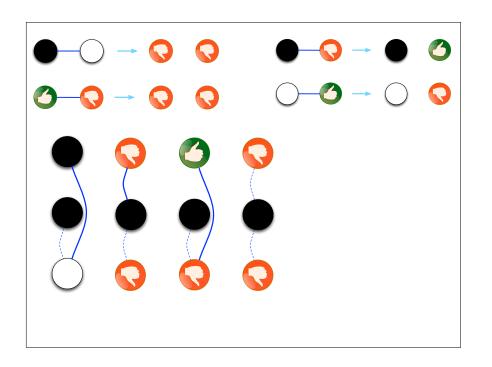


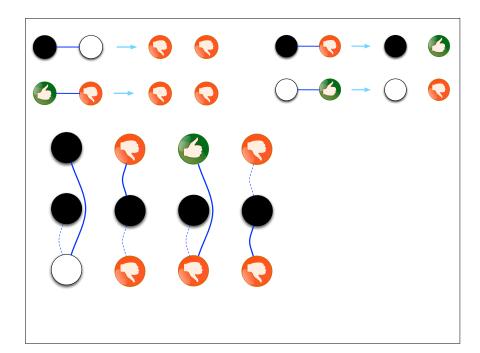


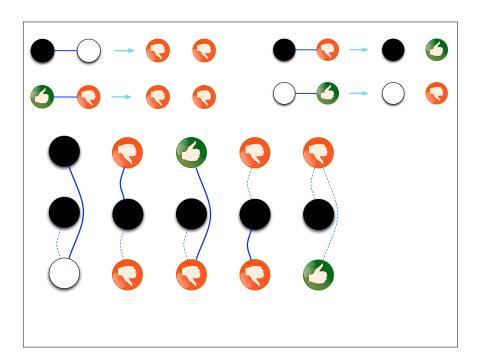


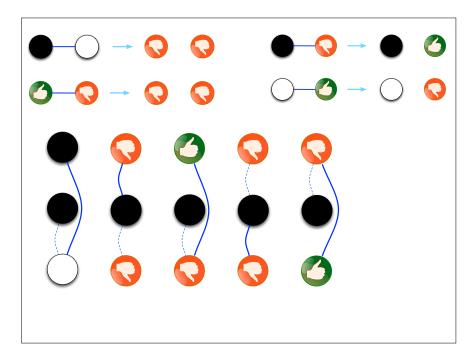


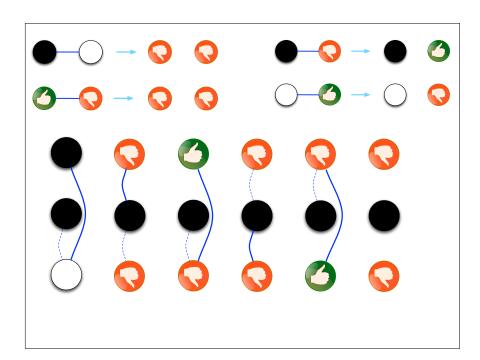






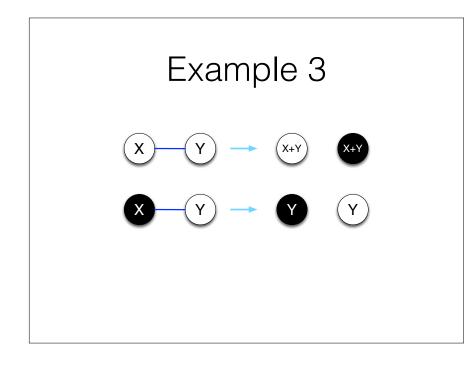


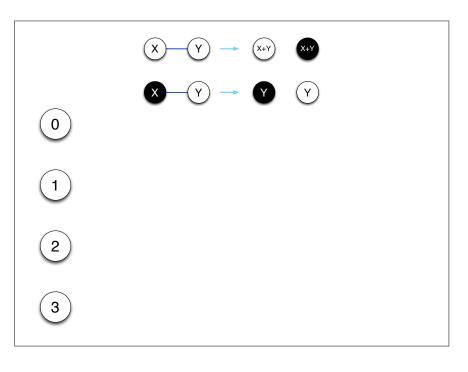


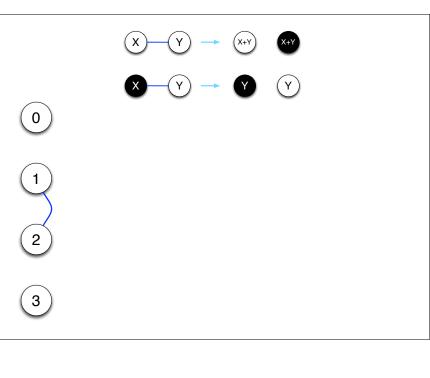


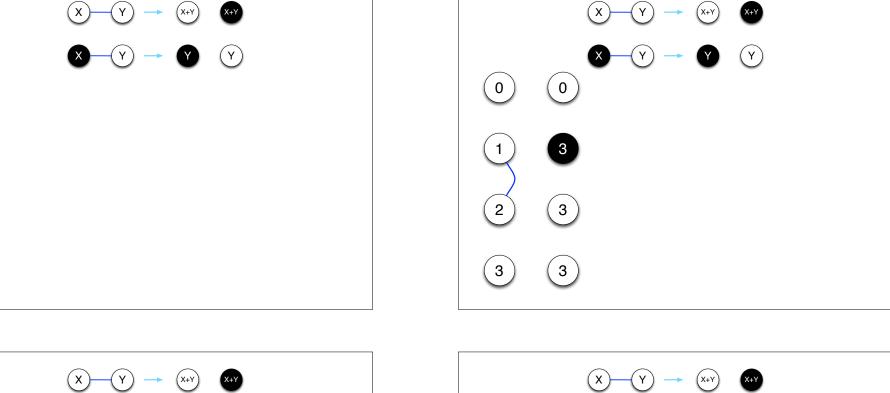


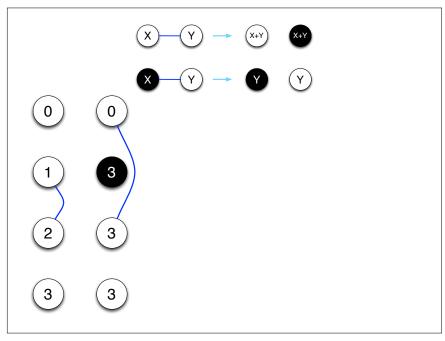
- Inputs: 0 1 2 3
 Outputs: 0 1 2 3 0 1 2 3
- · Sum mod 4?

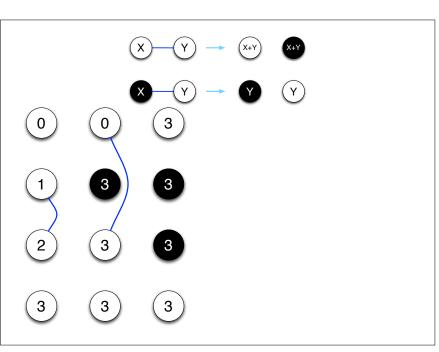


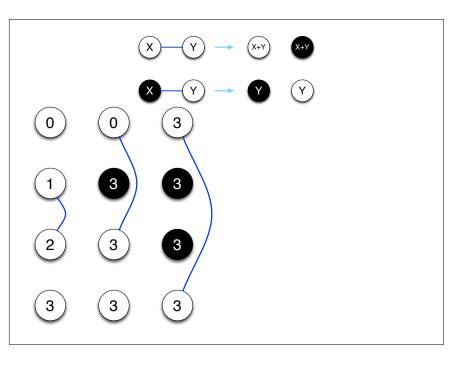


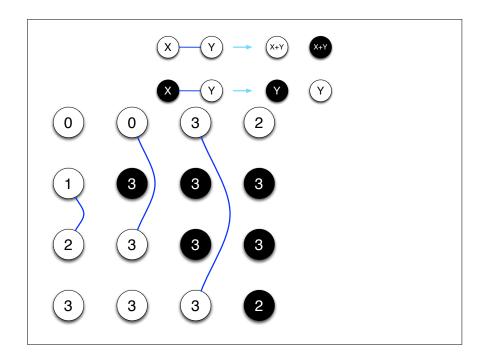


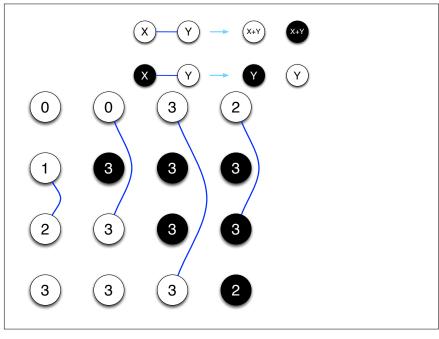


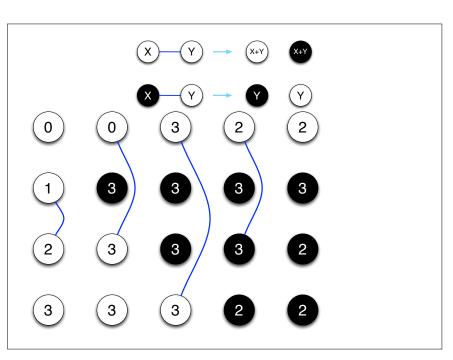


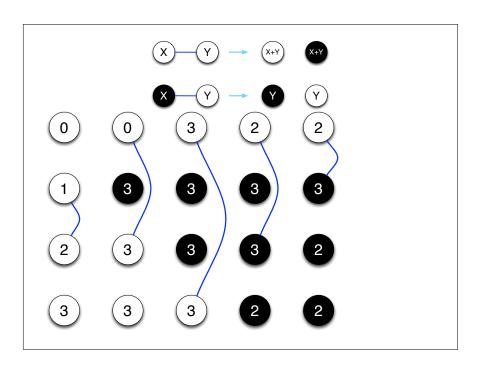


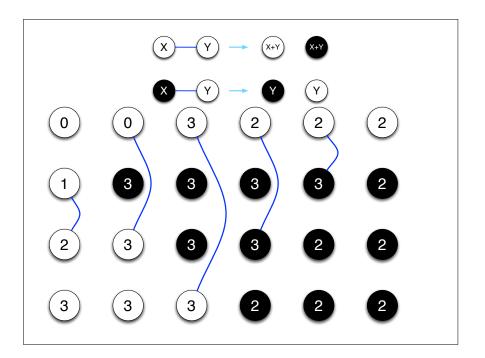


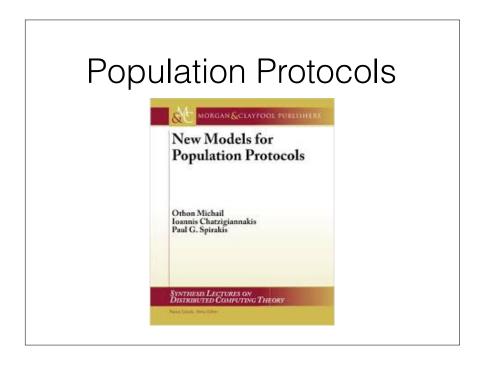












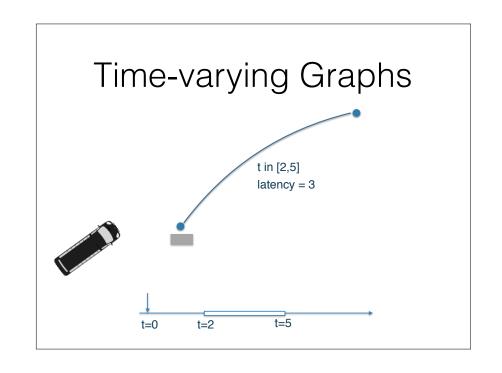
Dynamic Graphs

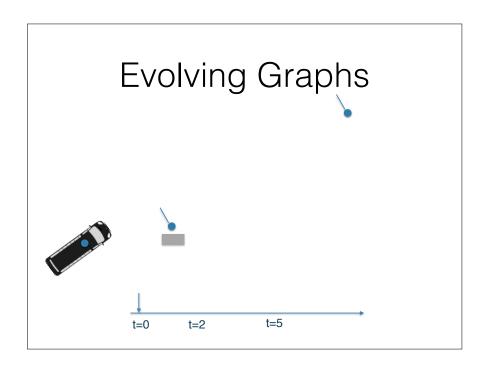
Time-varying Graphs

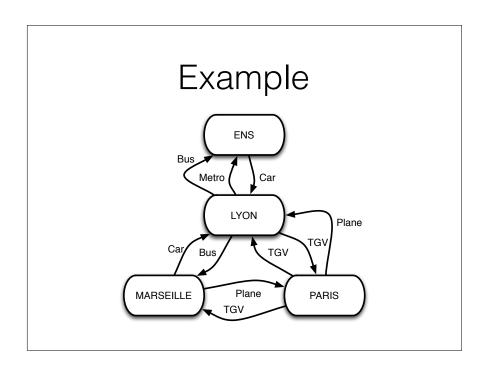
- A time-varying graph (TVG) is a 5-tuple (**V**,**E**,**T**,**p**,**I**)
 - V: set of nodes
 - E: (labelled) set of edges
 - **T**: lifetime, $\mathbf{T} \subseteq \mathcal{T}$
 - **p**: presence function, $\mathbf{E} \times \mathbf{T} \longrightarrow \{0,1\}$
 - I: latency function, $\mathbf{E} \times \mathbf{T} \longrightarrow \mathcal{T}$

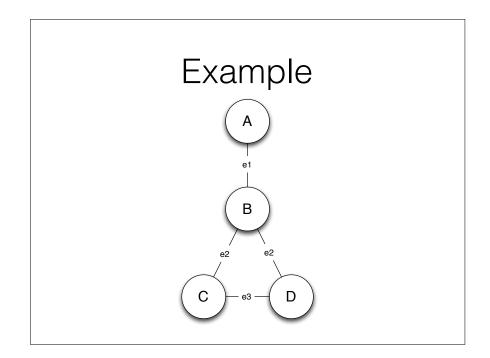
Time-varying Graphs

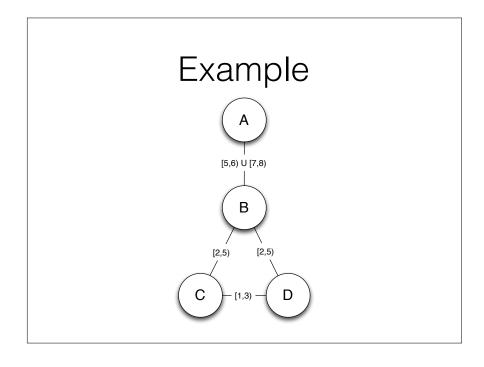
- A time-varying graph (TVG) is a 5-tuple (**V**,**E**,**T**,**p**',**I**')
 - V: set of nodes
 - E: (labelled) set of edges
 - **T**: lifetime, $\mathbf{T} \subseteq \mathcal{T}$
 - **p**': *node* presence function, $\mathbf{V} \times \mathbf{T} \longrightarrow \{0,1\}$
 - I': *node* latency function, $\mathbf{V} \times \mathbf{T} \longrightarrow \mathcal{T}$

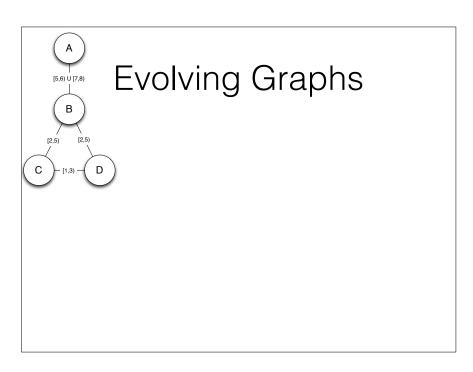


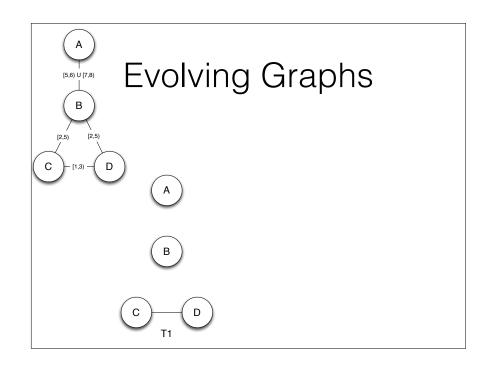


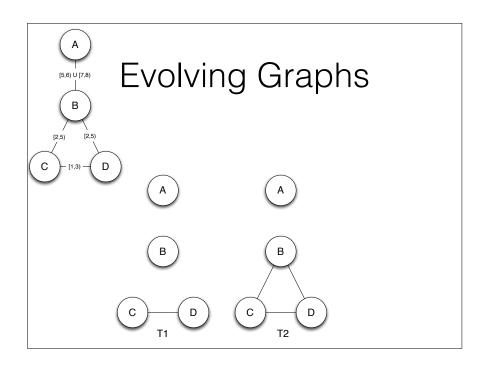


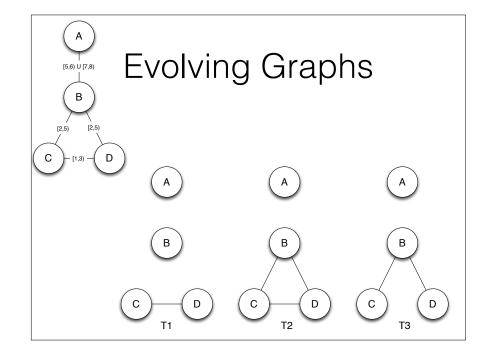


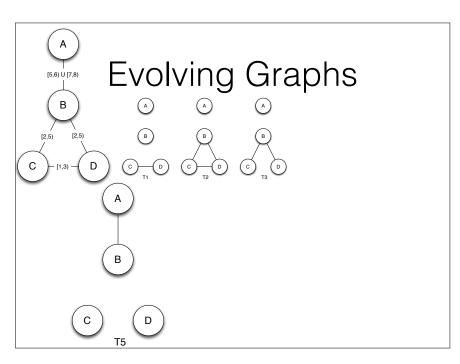


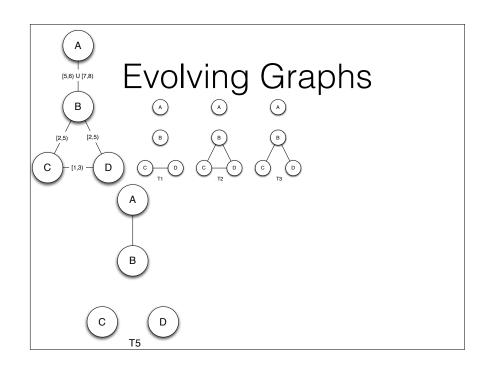


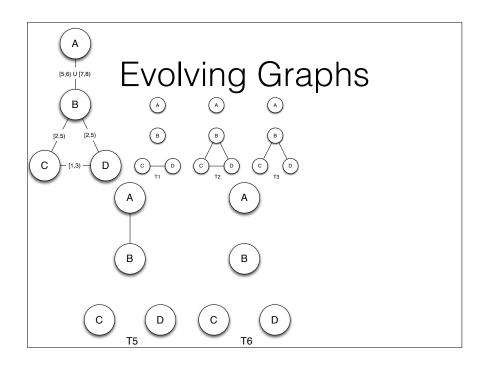


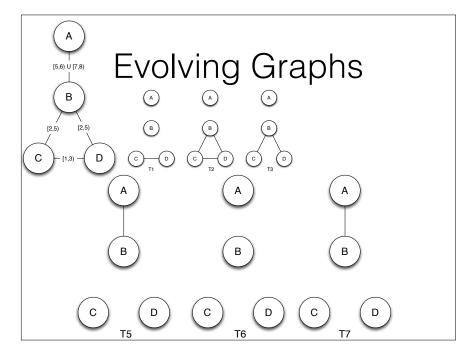


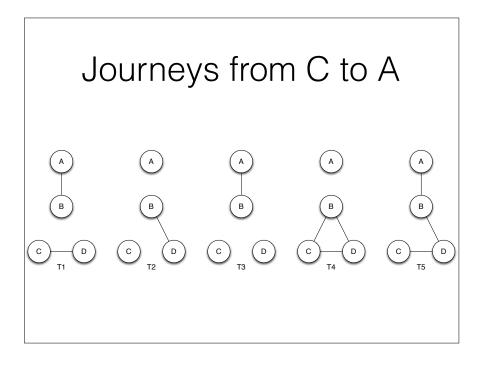


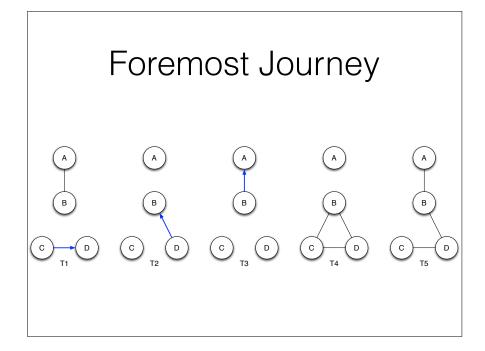


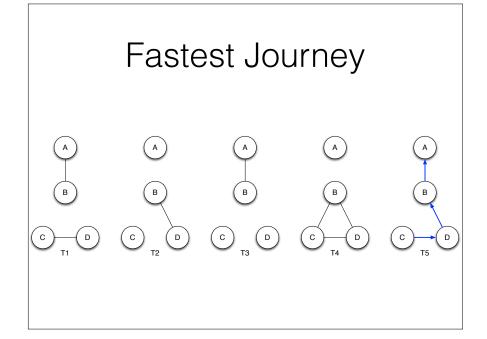


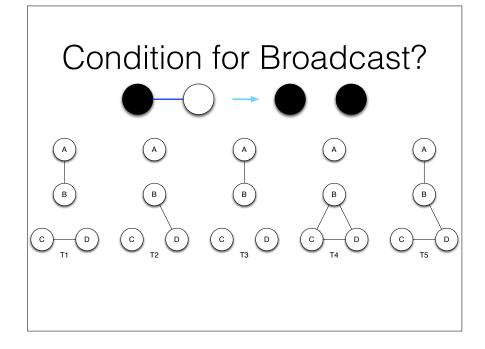


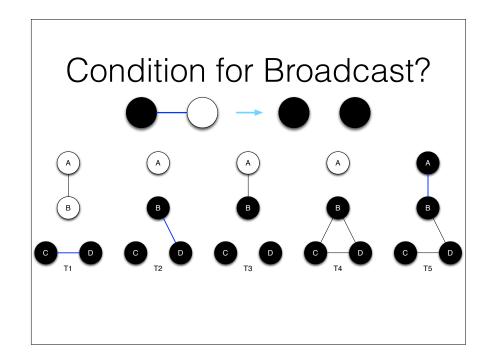


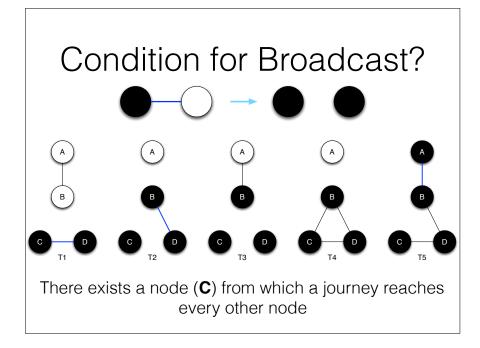


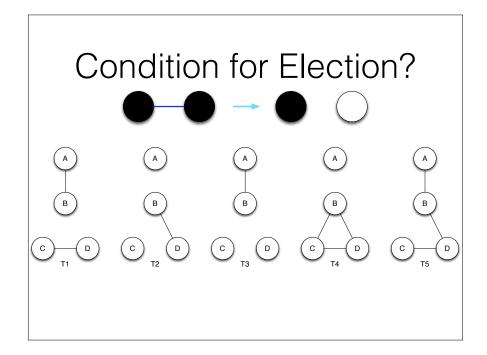


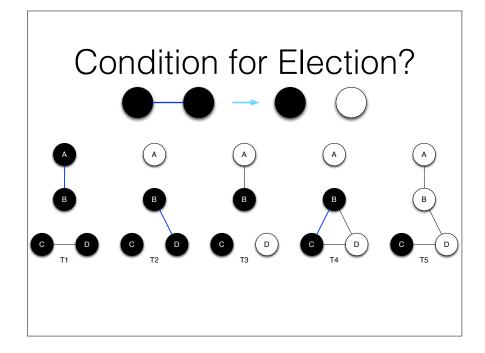


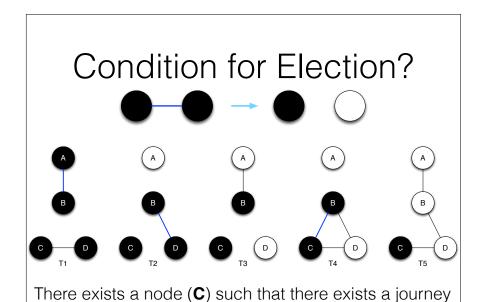




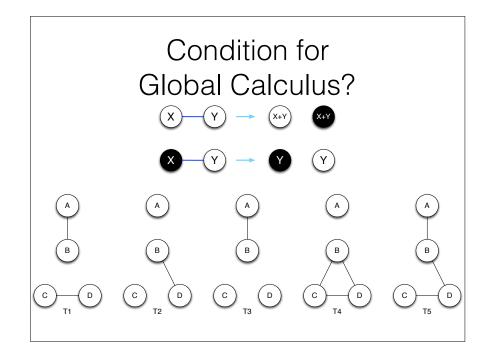


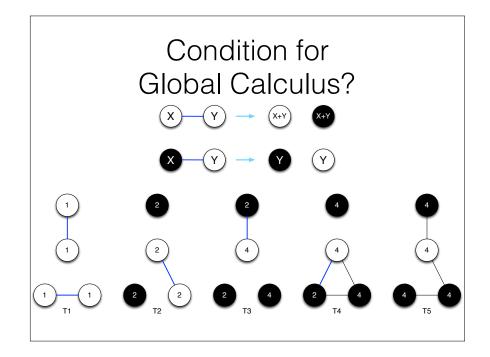


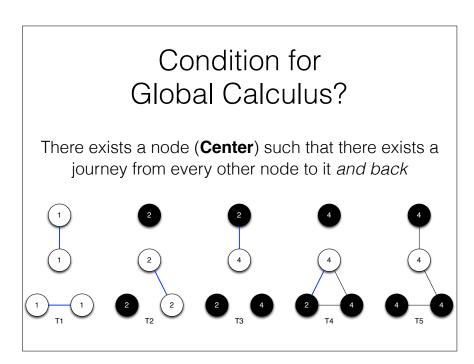




from every other node to it







Connectivity Classes

- There exists a node *r* from which a journey reaches every other node 1 → *
- There exists a node r such that there exists a journey from every other node to it * → 1
- There exists a node r such that there exists a
 journey from every other node to to and back
 1 ****

Arnaud Casteigts, Paola Flocchini, Walter Quattrociocchi, Nicola Santoro: Time-varying graphs and dynamic networks. IJPEDS 27(5): 387-408 (2012)

More Classes

- Every edge appears infinitely often, and there is an upper bound between between two occurrences
- Every edge appears infinitely often with some period p

Arnaud Casteigts, Paola Flocchini, Walter Quattrociocchi, Nicola Santoro: Time-varying graphs and dynamic networks. IJPEDS 27(5): 387-408 (2012)

More Classes

- There exists a journey between any two nodes * → *
- There exists a roundtrip journey between any two nodes *****
- There exists a journey between any two nodes infinitely often
- Every edge appears infinitely often $\frac{\mathcal{R}}{\bullet}$

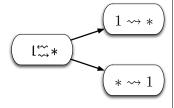
Arnaud Casteigts, Paola Flocchini, Walter Quattrociocchi, Nicola Santoro: Time-varying graphs and dynamic networks. IJPEDS 27(5): 387-408 (2012)

More Classes

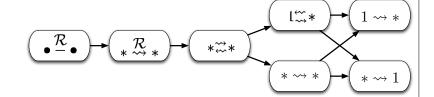
- · At any time, the graph is connected
- Every spanning subgraph lasts at least T time units
- Every edge appears infinitely often, and the underlying graph is a clique $\frac{\mathcal{R}}{\mathcal{L}}$

Arnaud Casteigts, Paola Flocchini, Walter Quattrociocchi, Nicola Santoro: Time-varying graphs and dynamic networks. IJPEDS 27(5): 387-408 (2012)

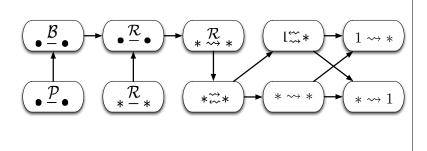
A Classification



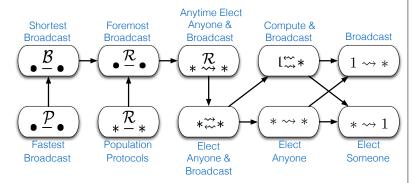
A Classification



A Classification



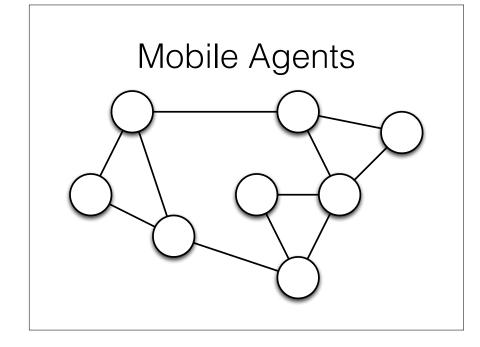
A Classification

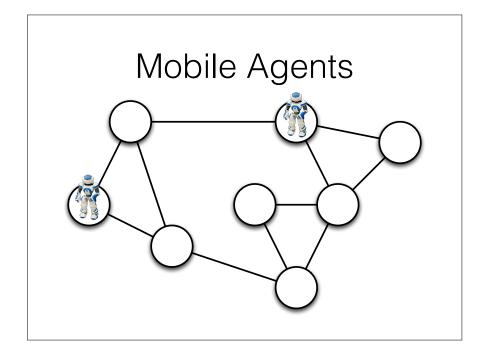


Arnaud Casteigts, Paola Flocchini, Bernard Mans, Nicola Santoro: Shortest, Fastest, and Foremost Broadcast in Dynamic Networks. Int. J. Found. Comput. Sci. 26(4): 499-522 (2015)

Actively Mobile Networks

Mobile Agents



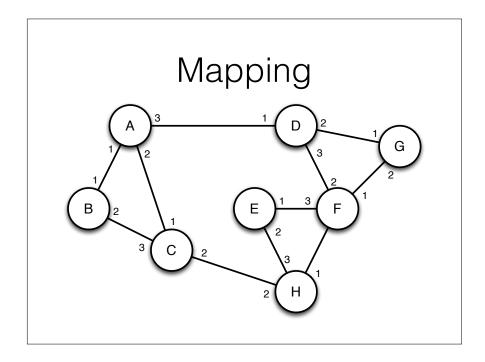


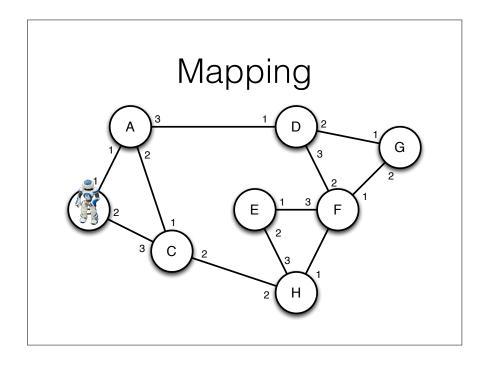
Problems to Solve

- **Exploration** (perpetual or with stop)
- · Mapping
- · Rendez-vous
- · Black hole search
- · Capturing an intruder

Models

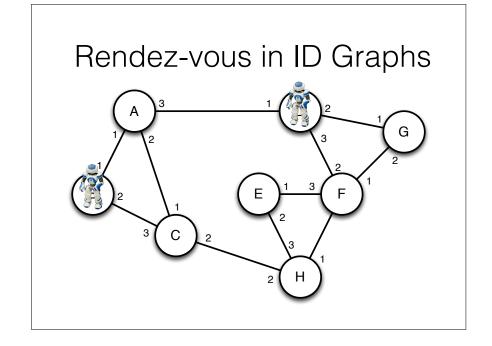
- **Network** (anonymous *vs.* ID based)
- **Agents** (anonymous *vs.* ID based)
- · Synchrony
- Initial (structural) knowledge
- **Communications** (none, peebles, whiteboards)
- Agent **memory** (infinite, bounded, constant)

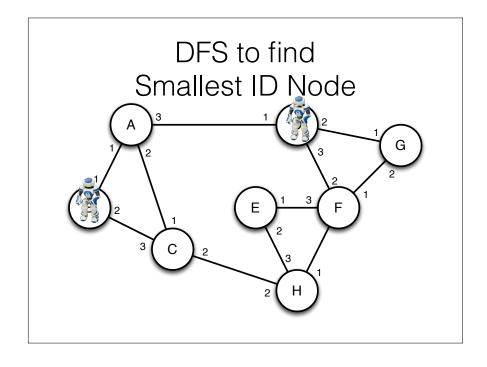


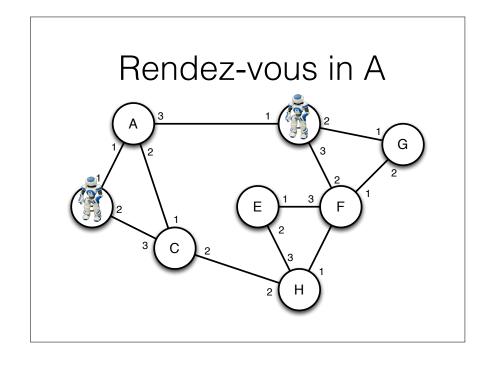


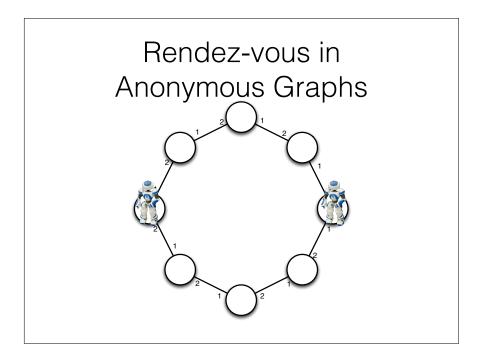
Rendez-vous

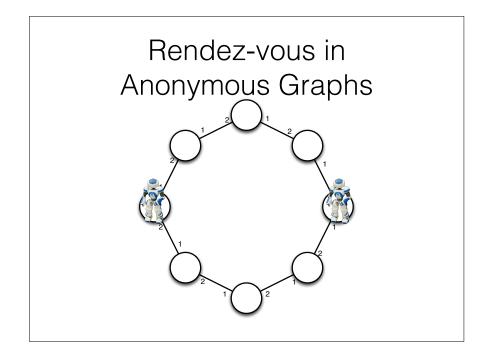
- Two (or more) mobile agents must meet in a graph
- They start on **distinct** locations
- Computability?
- Complexity?

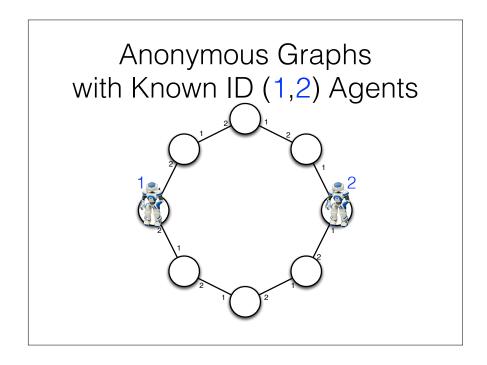


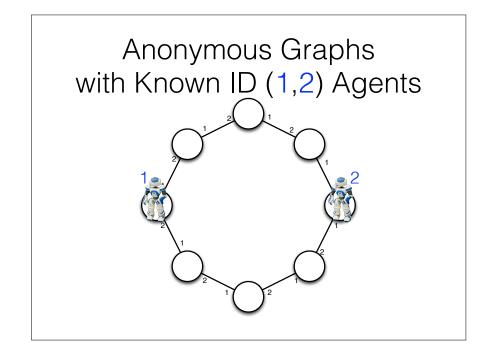


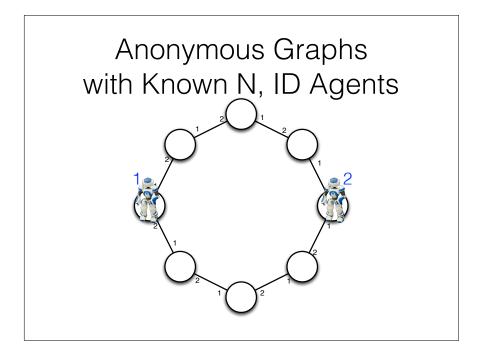


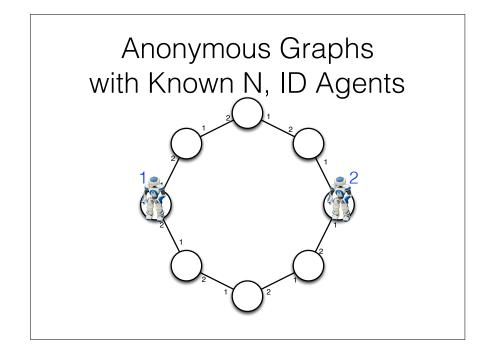


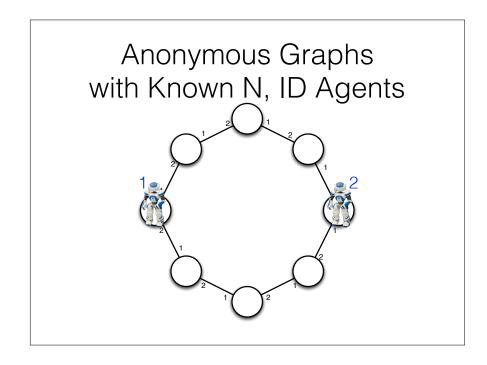


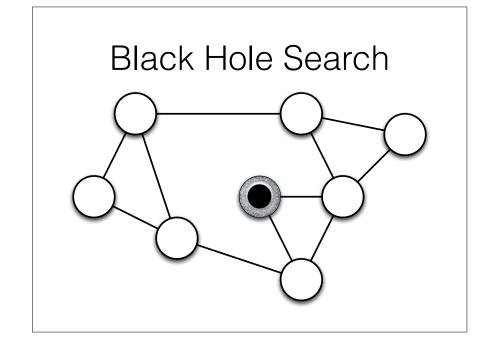








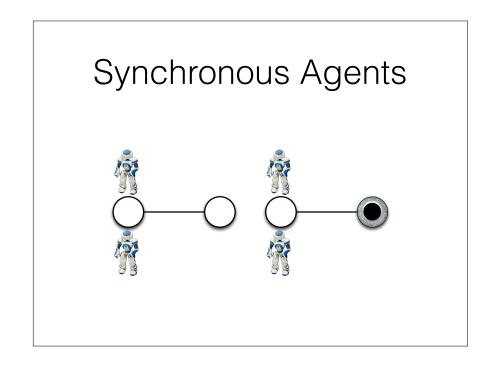


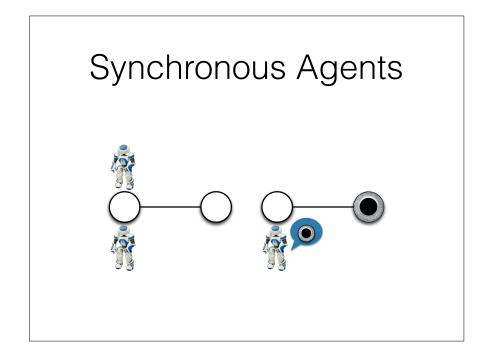


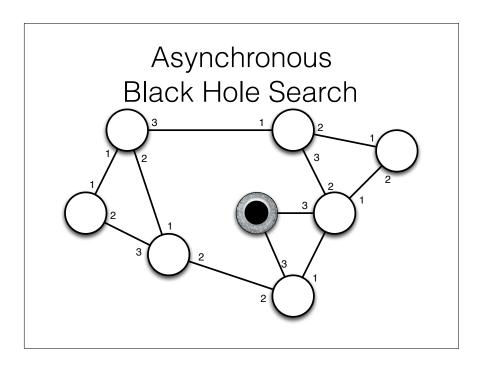
Black Hole Search

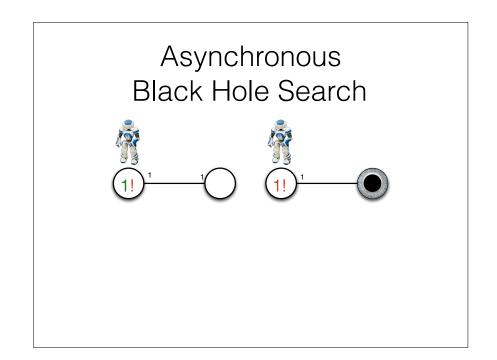
Black Hole Search

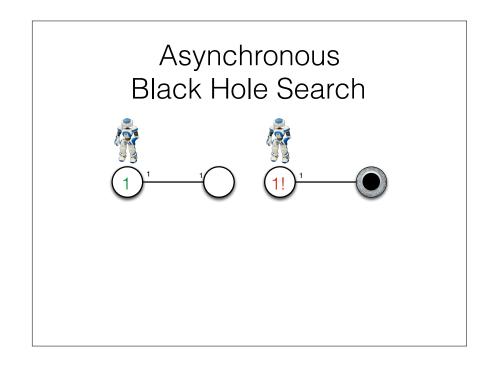
- A **single** black hole in the graph
- The black hole **does not disconnect** the graph
- Identify each adjacent edge
- Minimize #agents, #moves

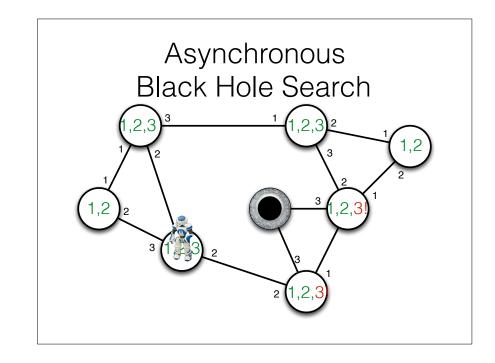




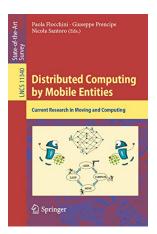




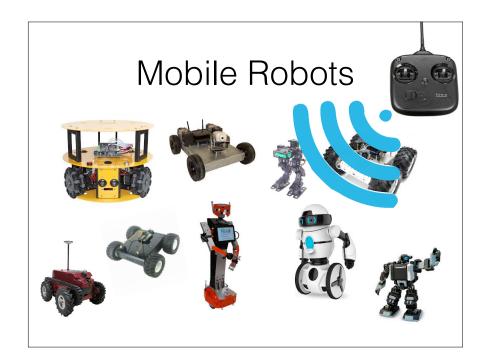




Mobile Agents



Mobile Robots

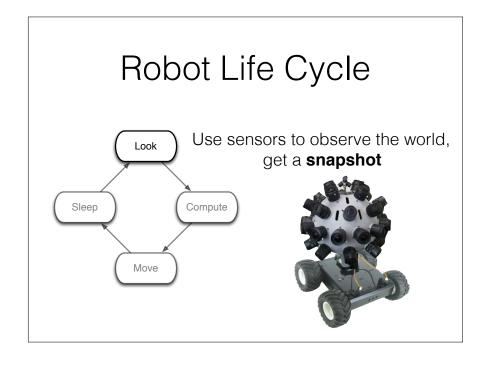


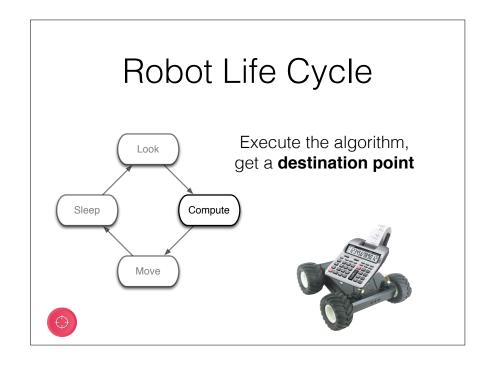


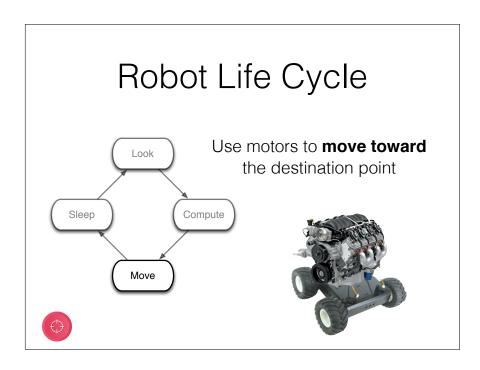


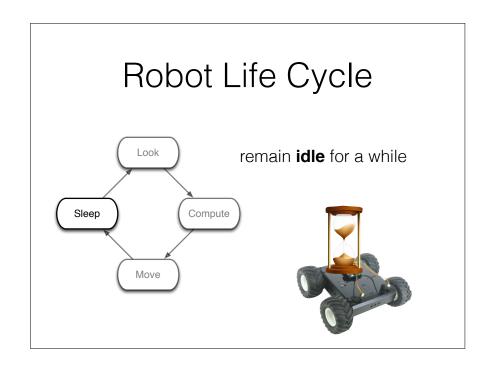
Mobile Robots

- Autonomous (no central control)
- **Homogeneous** (run same algorithm)
- Identical (indistinguishable)
- Silent (no explicit communication)

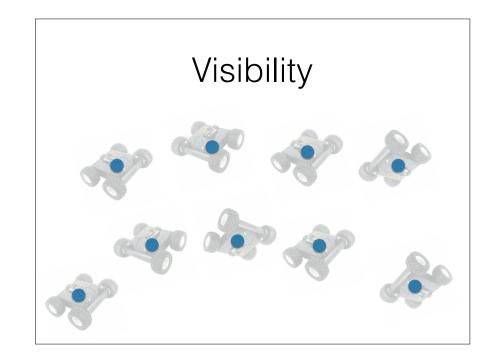


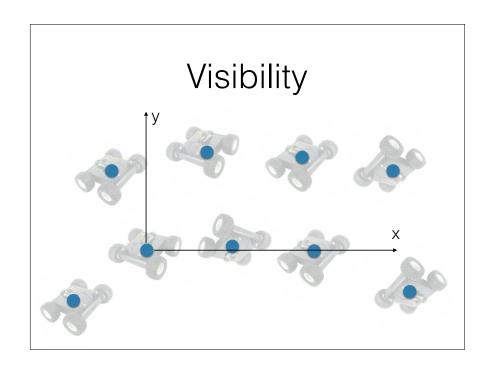


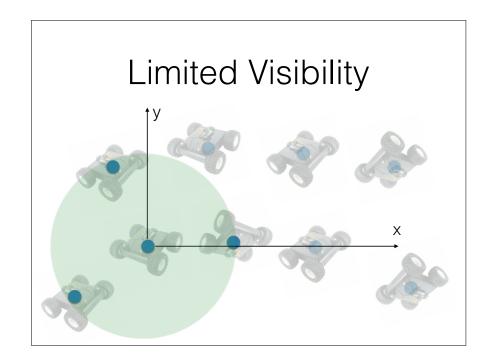


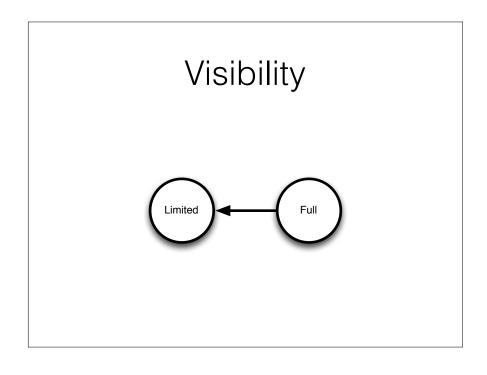








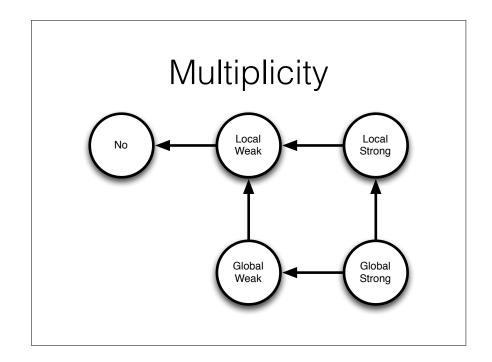




Multiplicity Detection How many robots do you see?

- No detection
- Weak multiplicity detection >1
- Strong multiplicity detection

Multiplicity No Weak Strong





Algorithm



Persistent Memory

Volatile Memory

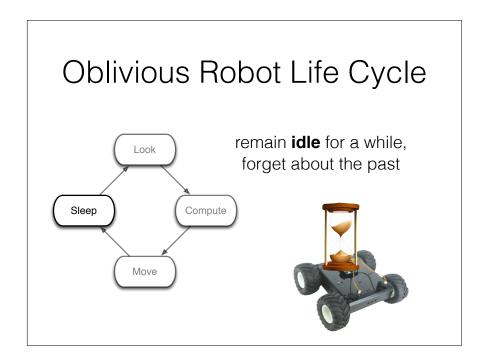
Oblivious Robot Memory

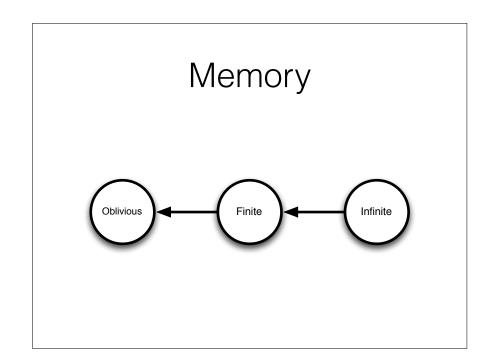
Algorithm

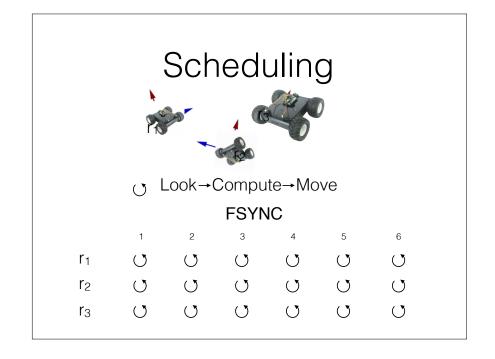


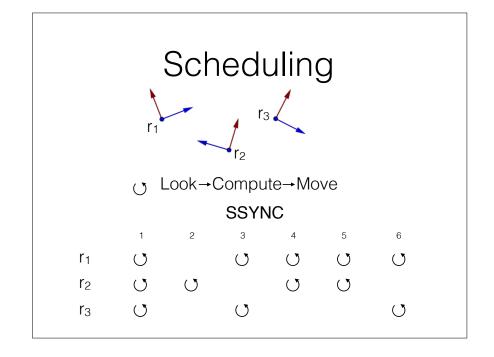


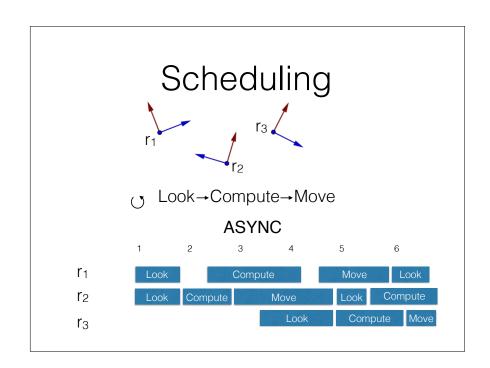
Volatile Memory

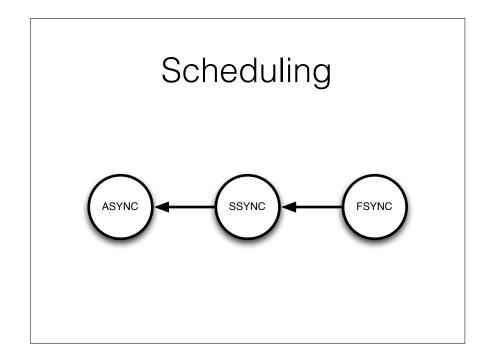


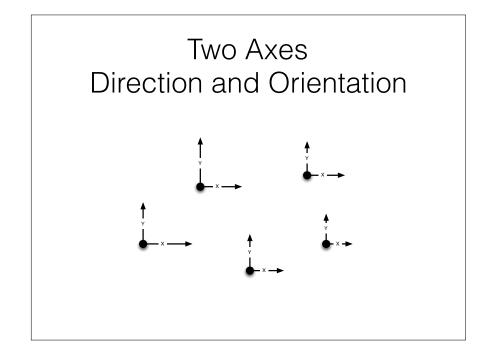


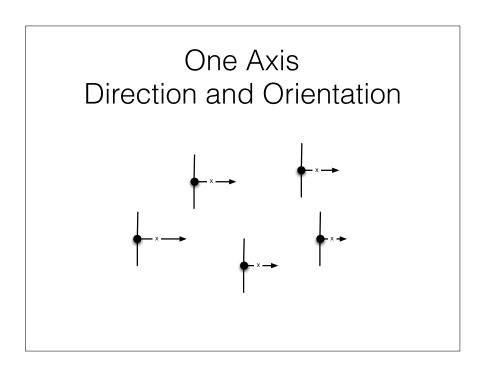


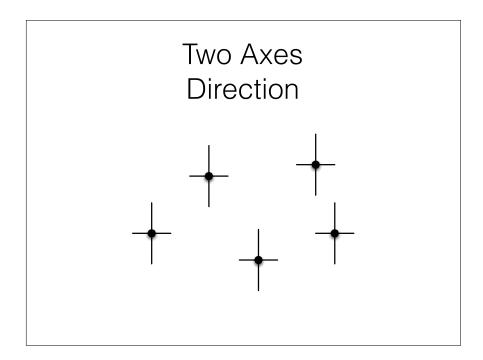


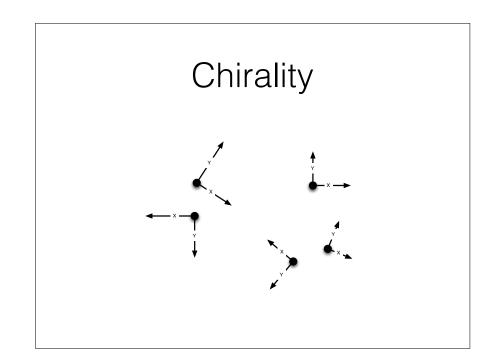


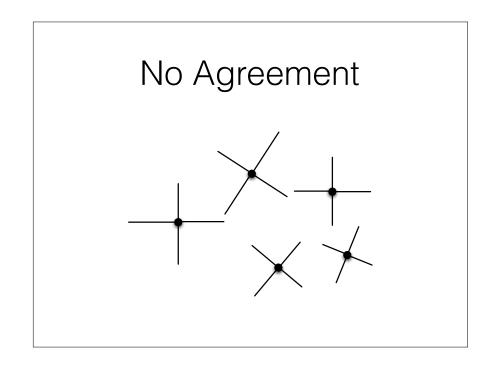


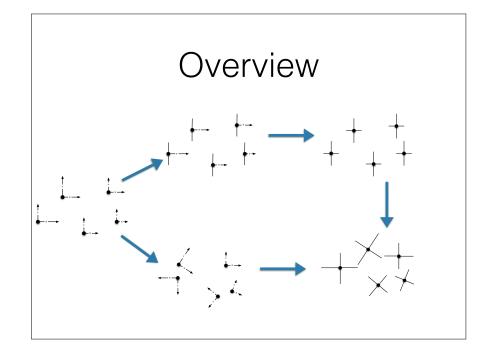












Scattering

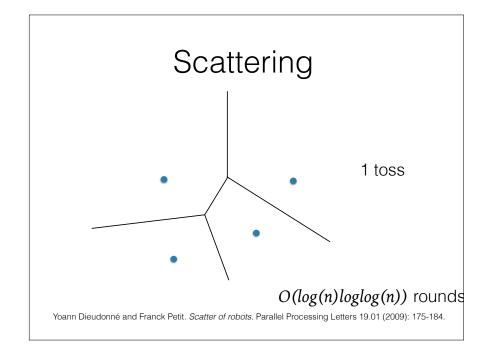
Scattering

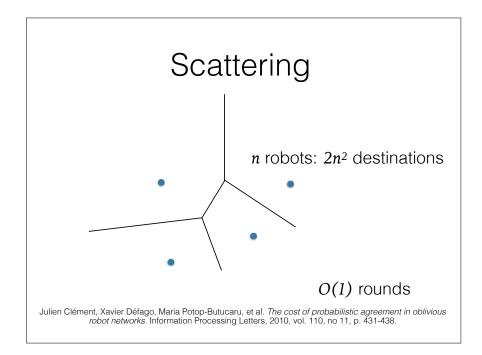
No two robots should occupy the same position

• No deterministic solution



• No termination without multiplicity detection





How Many Tosses?

Minimum?

Influence of multiplicity detection?

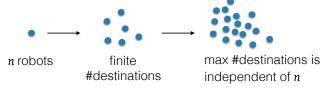
Relationship with scattering speed?

Optimal Speed

With strong multiplicity detection:

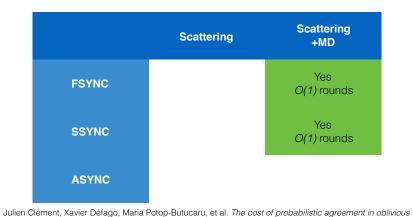
Algorithm with optimal #tosses terminates in O(1) rounds

Without strong multiplicity detection:



O(1) rounds scattering of n robots is impossible How fast can we go?

Scattering

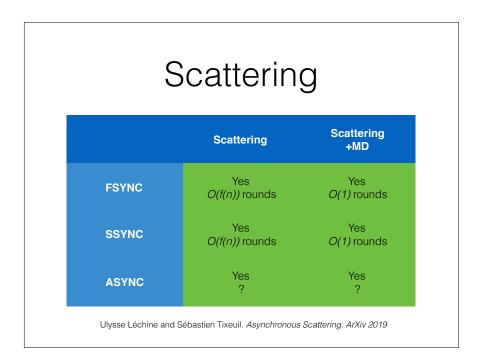


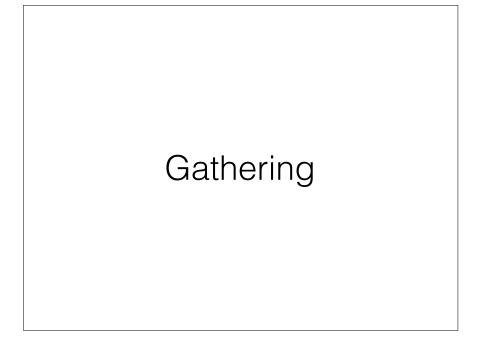
robot networks. Information Processing Letters, 2010, vol. 110, no 11, p. 431-438.

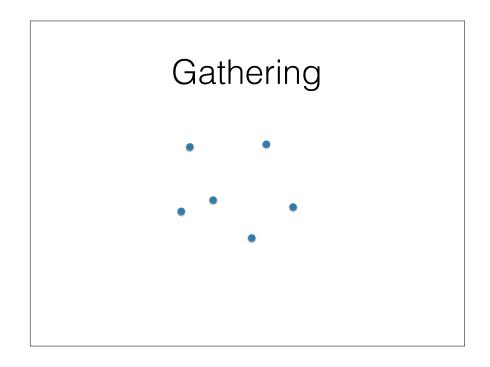
Scattering

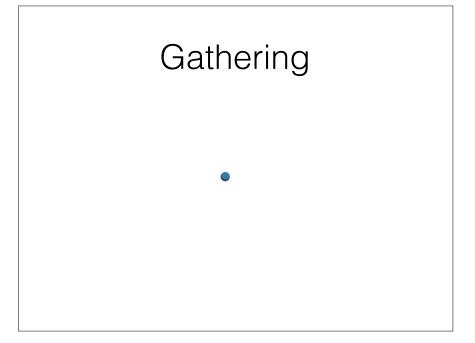
	Scattering	Scattering +MD
FSYNC	Yes <i>O(f(n))</i> rounds	Yes O(1) rounds
SSYNC	Yes <i>O(f(n))</i> rounds	Yes O(1) rounds
ASYNC		

Quentin Bramas and Sébastien Tixeuil. *The Ramdom Bit Complexity of Mobile Robot Scattering.* Int. J. Found. Comput. Sci. 28(2): 111-134 (2017)



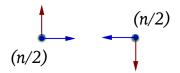






Gathering

Impossible for two robots



A bivalent configuration

Gathering vs. Convergence

- Gathering: robot must reach the same point in finite time
- Convergence: robots must get closer at time goes by

Center of Gravity

$$\vec{c}[t] = \frac{1}{n} \sum_{i=1}^{n} \vec{r_i}[t]$$















Center of Gravity

$$\vec{c}[t] = \frac{1}{n} \sum_{i=1}^{n} \vec{r_i}[t]$$













Center of Gravity

$$\vec{c}[t] = \frac{1}{n} \sum_{i=1}^{n} \vec{r_i}[t]$$











Center of Gravity of Positions

$$\vec{c}[t] = \frac{1}{p} \sum_{i=1}^{p} \vec{p_i}[t]$$











FSYNC Gathering

$$\vec{c}[t] = \frac{1}{p} \sum_{i=1}^{p} \vec{p_i}[t]$$











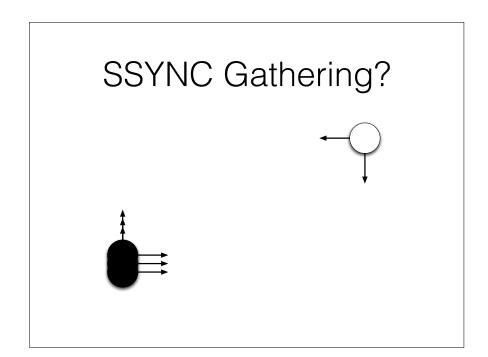


FSYNC Gathering

$$\vec{c}[t] = \frac{1}{p} \sum_{i=1}^{p} \vec{p_i}[t]$$



SSYNC Gathering?



Convergence & Gathering

	Convergence	2-Gathering	n-Gathering	n-Gathering +MD	n-Gathering +MD+WF
FSYNC	Yes	Yes	Yes	Yes	Yes
SSYNC	Yes	No	No	Yes	Yes
ASYNC	Yes	No	No	Yes	?

Convergence & Gathering

	Convergence	2-Gathering	n-Gathering	n-Gathering +MD	n-Gathering +MD+WF
FSYNC	Yes	Yes	Yes	Yes	Yes
SSYNC	Yes	No	No	Yes	Yes
ASYNC	Yes	No	No	Yes	?

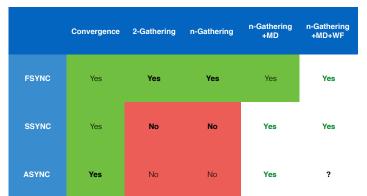
Reuven Cohen and David Peleg. Convergence Properties of the Gravitational Algorithm in Asynchronous Robot Systems. SIAM J. Comput. 34(6): 1516-1528 (2005)

Convergence & Gathering

	Convergence	2-Gathering	n-Gathering	n-Gathering +MD	n-Gathering +MD+WF
FSYNC	Yes	Yes	Yes	Yes	Yes
SSYNC	Yes	No	No	Yes	Yes
ASYNC	Yes	No	No	Yes	?

Ichiro Suzuki, Masafumi Yamashita: Distributed Anonymous Mobile Robots: Formation of Geometric Patterns. SIAM J. Comput. 28(4): 1347-1363 (1999)

Convergence & Gathering



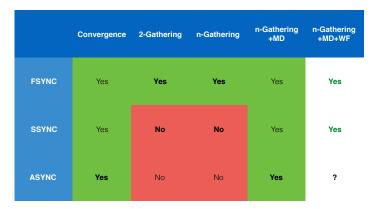
Thibaut Balabonski, Amélie Delga, Lionel Rieg, Sébastien Tixeuil, Xavier Urbain: Synchronous Gathering Without Multiplicity Detection: A Certified Algorithm. SSS 2016: 7-19

Convergence & Gathering



Guiseppe Prencipe. Impossibility of gathering by a set of autonomous mobile robots. Theor. Comput. Sci. 384(2-3): 222-231 (2007)

Convergence & Gathering



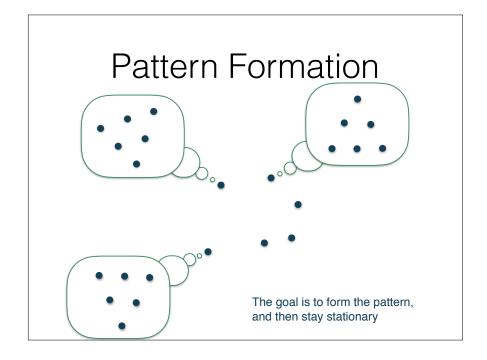
Mark Cieliebak, Paola Flocchini, Giuseppe Prencipe, Nicola Santoro. *Distributed Computing by Mobile Robots: Gathering.* SIAM J. Comput. 41(4): 829-879 (2012)

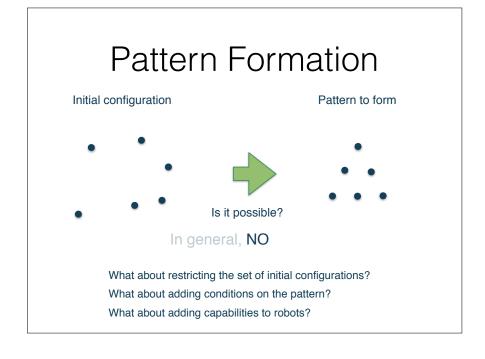
Convergence & Gathering

	Convergence	2-Gathering	n-Gathering	n-Gathering +MD	n-Gathering +MD+WF
FSYNC	Yes	Yes	Yes	Yes	Yes
SSYNC	Yes	No	No	Yes	Yes
ASYNC	Yes	No	No	Yes	?

Quentin Bramas, Sébastien Tixeuil. Wait-Free Gathering Without Chirality. SIROCCO 2015: 313-327

Pattern Formation





Pattern Formation

Initial configuration

All robots are here

Pattern to form

Is it possible?

No, so from now, we assume the initial configuration does not have points of multiplicity

Pattern Formation



Yes, if robots agree on a common North and a common Right Yes, if robots agree on a common North and n is odd

Paola Flocchini, Giuseppe Prencipe, Nicola Santoro, Peter Widmayer: Arbitrary pattern formation by asynchronous, anonymous, oblivious robots. Theor. Comput. Sci. 407(1-3): 412-447 (2008)

Pattern Formation



NO

Guiseppe Prencipe. Impossibility of gathering by a set of autonomous mobile robots. Theor. Comput. Sci. 384(2-3): 222-231 (2007)

Pattern Formation



...assuming a common chirality, and F does not have multiplicity points

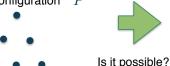
Yes, if $\ \rho(P) \ | \ \rho(F)$ where $\ \rho(P)$ is the symmetricity of $\ P$, the maximum integer such that the rotation by $\ 2\pi/\rho(P)$ is invariant for $\ P$

No, otherwise

Nao Fujinaga, Yukiko Yamauchi, Hirotaka Ono, Shuji Kijima, Masafumi Yamashita: Pattern Formation by Oblivious Asynchronous Mobile Robots. SIAM J. Comput. 44(3): 740-785 (2015)

Pattern Formation

Initial configuration P



Pattern to form F



... assuming a common chirality, and F does not have multiplicity points

Yes, if $\rho(P) \mid \rho(F)$

where $\rho(P)$ is the symmetricity of P,

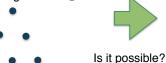
the maximum integer such that the rotation by $-2\pi/\rho(P)$ is invariant for $\,P\,$

No, otherwise

Nao Fujinaga, Yukiko Yamauchi, Hirotaka Ono, Shuji Kijima, Masafumi Yamashita: Pattern Formation by Oblivious Asynchronous Mobile Robots. SIAM J. Comput. 44(3): 740-785 (2015)

Pattern Formation

Initial configuration P



Pattern to form F



...assuming a common chirality, and F does not have multiplicity points

Yes, if $\rho(P) \mid \rho(F)$

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the maximum integer such that the rotation by $~~2\pi/\rho(P)$ is invariant for ~P

No, otherwise

Nao Fujinaga, Yukiko Yamauchi, Hirotaka Ono, Shuji Kijima, Masafumi Yamashita: Pattern Formation by Oblivious Asynchronous Mobile Robots. SIAM J. Comput. 44(3): 740-785 (2015)

Pattern Formation

Initial configuration P



Is it possible?

Pattern to form F

...assuming a common chirality, and F does not have multiplicity points

Yes, if $\rho(P) \mid \rho(F)$

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the maximum integer such that the rotation by $~~2\pi/\rho(P)$ is invariant for ~P

Nao Fujinaga, Yukiko Yamauchi, Hirotaka Ono, Shuji Kijima, Masafumi Yamashita: Pattern Formation by Oblivious Asynchronous Mobile Robots. SIAM J. Comput. 44(3): 740-785 (2015)

Pattern Formation

Initial configuration P



Is it possible?

Pattern to form F

• •

...assuming a common chirality, and F does not have multiplicity points

No, otherwise

Nao Fujinaga, Yukiko Yamauchi, Hirotaka Ono, Shuji Kijima, Masafumi Yamashita: Pattern Formation by Oblivious Asynchronous Mobile Robots. SIAM J. Comput. 44(3): 740-785 (2015)

Pattern Formation

Initial configuration $\ P$ Pattern to form $\ F$ Is it possible?

...assuming a common chirality, and F does not have multiplicity points

Yes, with a randomized algorithm

... assuming robots do not "pause" while moving

... and using infinitely many random bits per activation

Yukiko Yamauchi, Masafumi Yamashita: Randomized Pattern Formation Algorithm for Asynchronous Oblivious Mobile Robots. DISC 2014: 137-151

ASYNC Pattern Formation

Pattern	Agreement	Chirality	Randomization	
Point	Yes	No	?	
Divide Symmetricity	Yes	Yes	Yes	
No Multiplicity	Yes	No	Yes	
Not a Point	Not a Point Yes		Yes	
Arbitrary	Yes	No	?	

Pattern Formation



...assuming a common chirality, and F does not have multiplicity points

Yes, with a randomized algorithm

F is not a point

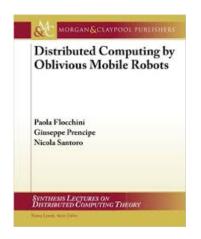
... assuming robots do not "pause" while moving really asynchronous

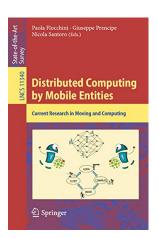
... and using infinitely many random bits per activation

only one random bit

Quentin Bramas, Sébastien Tixeuil: Brief Announcement: Probabilistic Asynchronous Arbitrary Pattern Formation. PODC 2016: 443-445

Mobile Robots





Conclusion

Static Networks

- Fundamental, well established model
 - Space-centric, complexity results
 - Time-centric, computability results

Mobility as an Adversary

- Can corrupt the distributed state of a network
- Can reduces communication capacity
- Can increase uncertainty
- Can increase protocol complexity

Mobility as a Friend

- Mobility can be the solution to the problem
- Mobility can improve efficiency
- Mobility can promote simplicity

Distributed Computing Complexity Problem

Thank You

FURETHERMORE: FUlly REliable THeorems in an ERa of MObility and REdundancy 1 Ox 2019 Warshame (France) PURETHERMORE Obtaining certified guarantees for distributed systems is a crucial issue, as this area of computer science is well-known reasoning, which potentially leads to disastrous errors. In practical cases, the number of slight variants of the original of the reasoning, which potentially leads to disastrous errors. In practical cases, the number of slight variants of the original of the reasoning, which potentially leads to disastrous errors. In practical cases, the number of slight variants of the original of the reasoning, which potentially leads to disastrous errors. In practical cases, the number of slight variants of the original of the reasoning of the property of the prope