Formal Proofs for Mobile Robot Swarms

Lionel Rieg

ENS de Lyon, 23 Oct. 2019

Outline

Inspiration: Swarms of Mobile Robots

- Swarms?

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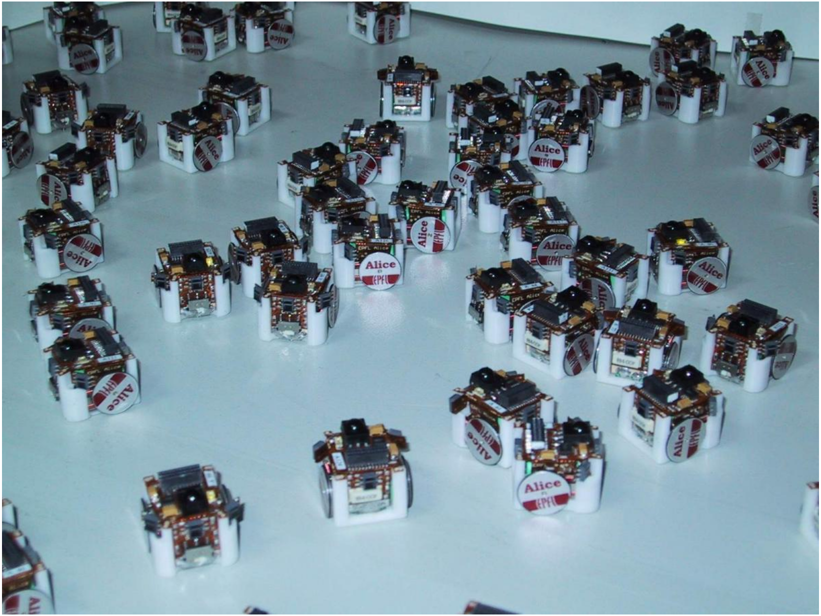

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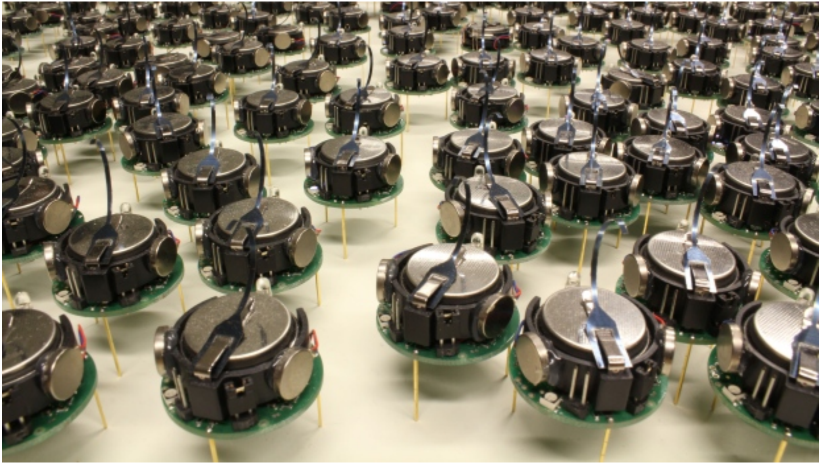

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  - Resilience

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- Opportunities?
  - Cooperative behavior (swarm intelligence)
  - Resilience
- Main challenge?
  - Understand what happens!

Why Formal Methods for Mobile Robots Swarms?

Many mobile robots network models:

- Space — discrete/continuous, bounded/unbounded, topology, ...
- Sensors — multiplicity, range, accuracy, orientation, ...
- Faults — none, crash, Byzantine, ...
- Execution — synchronous/asynchronous, fairness, interruption, ...

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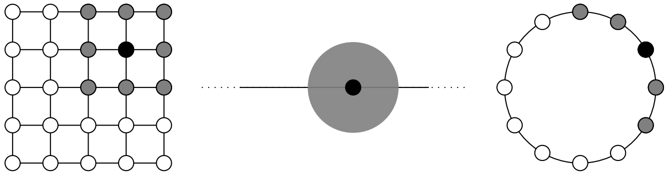

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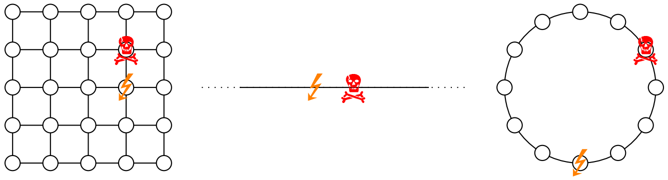

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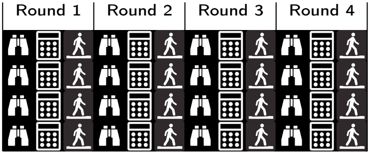

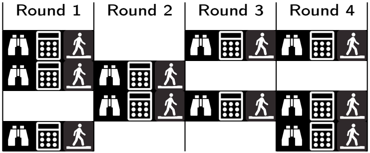

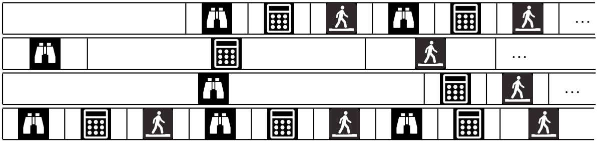

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- Subtle differences in models $\rightsquigarrow$ very error-prone
- Careful of mismatch spec/proof!
- Lots of proof cases — geometry

$\Rightarrow$ Formal methods can help

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Which model? (process algebra, TLA, ...)
Which tool? (model checking, proof assistant, ...)

We need a suitable framework — What are the requirements?

Outline

Robot Model

[Suzuki, Yamashita 99]

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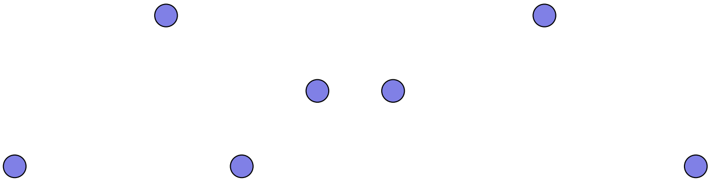
- Points



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[Suzuki, Yamashita 99]

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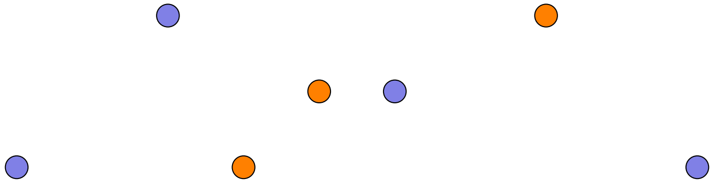

Robot Model [Suzuki, Yamashita 99]

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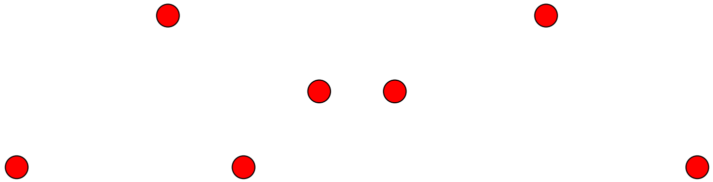

Robot Model

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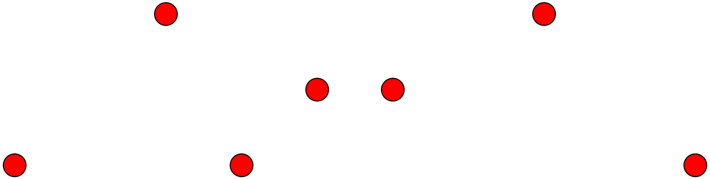

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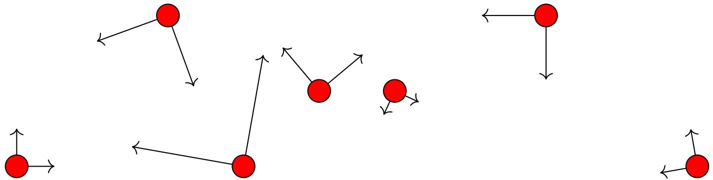

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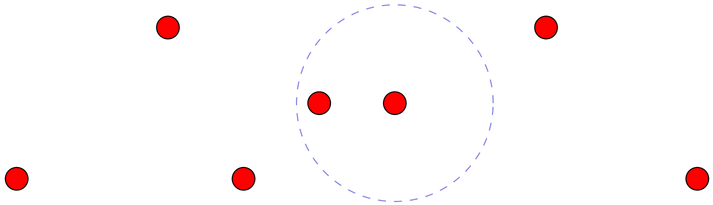

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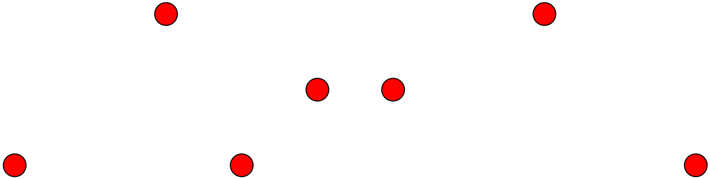
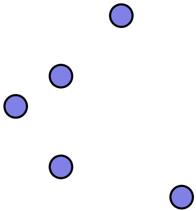
- Limited/unlimited vision? multiplicity?
- Same (deterministic) program everywhere



Example: Gathering

- Setting: $\mathbb{R}^2$, no Byzantine
- Objective: Have all robots reach in finite time the same location (unknown ahead of time) and then stay there



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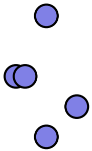

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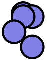

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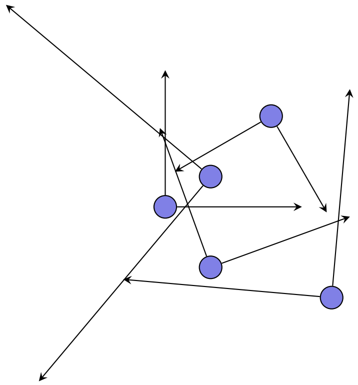

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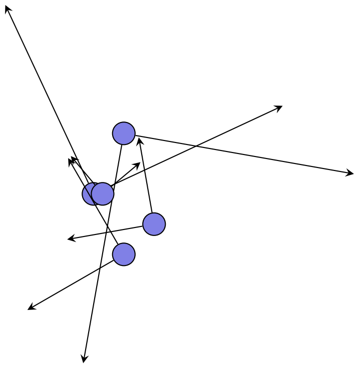

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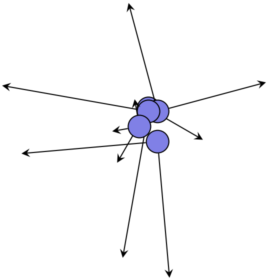

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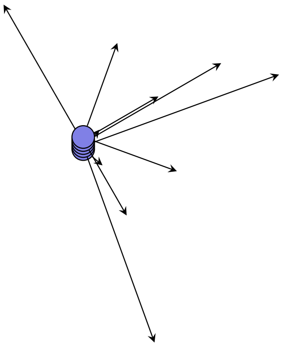

Execution Models (1) [Suzuki, Yamashita 99]

3 phases for each robot:

1. Look: observe its surrounding
   - Indirect communication
   - Depends on sensor capabilities
2. Compute: choose what to do
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3. Move: do it (or try to)
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and repeat

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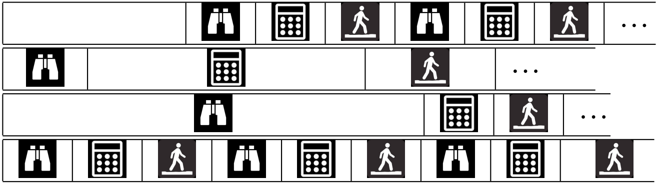

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- Time split into rounds

Round 1	Round 2	Round 3	Round 4	Round 5

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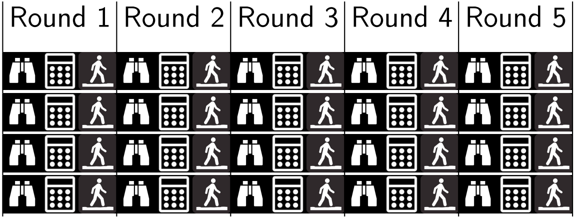

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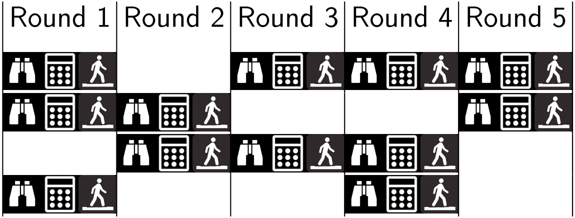
- FSYNC: all robots are activated each round
- SSYNC: only a subset is activated
  ⇝ Fairness assumptions on the scheduling (demon)



The Rest of the Vocabulary

- Robogram: robot program
- Demon/scheduler: environment (adversary part)
  a sequence of demonic actions (one for each round)
- Configuration: the states of all robots
  (includes locations and ids)
  $\rightsquigarrow$ full snapshot of the system
- Observation: information available to robograms
  (depends on sensors, no identifers)
  $\rightsquigarrow$ degraded form of configuration
- Execution: a sequence of configurations
  (usually given by the robogram and the demon)

Outline

Coq: a Proof Assistant

Definition (Proof Assistant (Wikipedia))

In computer science and mathematical logic, a proof assistant or interactive theorem prover is a software tool to assist with the development of formal proofs by human-machine collaboration. This involves some sort of interactive proof editor, or other interface, with which a human can guide the search for proofs, the details of which are stored in, and some steps provided by, a computer.

The Coq proof assistant:

- 4-color theorem, Feit-Thomson theorem, CompCert compiler
- Functional programming language
- Proof = program — Curry-Howard
- Build programs/proofs with tactics (reasoning steps)
- ... or just program them!

Pactole: a Coq Framework for Mobile Robots

Very Parametric (but still useful):

- Space
- State of Robots (memory, battery level, etc.)
- Sensors
- Environment (adversary)
- How states are updated during the move phase

Main Ingredients:

- Robogram
- Demon (scheduler)
- Round
- Execution
- Properties and Proofs

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A robogram is simply a function:

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Definition robogram := observation → location .
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- We can use all the expressiveness of Coq to define robograms.
- They can be extracted to OCaml/Haskell.
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+ a technical detail: compatibility with equivalence (Proper)

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```
(** Select which robots are activated *)
activate : ident → bool;
(** Update the state of inactive robots *)
choose_inactive : configuration → ident → info;
(** Update the state of (activated) byzantine robots *)
relocate_byz : configuration → B → info;
(** Local referential for (activated) good robots in the compute phase *)
change_frame : configuration → G → bijection location;
(** Update the state of (activated) good robots in the move phase *)
choose_update : configuration → G → location → info;
```

+ compatibility properties (Proper)

The Core of Pactole: the Round Function

```
Definition round (r : robogram) (da : demonic_action) (config : configuration )
  : configuration :=
  fun id ⇒                    (* for a given robot, we compute the new state *)
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        (* change the frame of reference *)
        let frame_choice := da.(change_frame) config g in
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          (* compute the observation *)
          let obs := obs_from_config local_config local_state in
          (* apply r on observation *)
          let decision := r obs in
          (* the demon chooses how to perform the state update *)
          let choice := da.(choose_update) local_config g decision in




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```
Definition round (r : robogram) (da : demonic_action) (config : configuration )
 : configuration :=
 fun id ⇒                         (* for a given robot, we compute the new state *)
   if da.( activate ) id              (* first see whether the robot is activated *)
   then
     match id with
       | Byz b ⇒ da.(relocate_byz) config b (* byzantine robots *)
       | Good g ⇒
         (* change the frame of reference *)
         let frame_choice := da.(change_frame) config g in
         let new_frame := frame_choice_bijection frame_choice in
         let local_config := map_config (lift new_frame ( ... )) config in
         let local_state := local_config (Good g) in
         (* compute the observation *)
         let obs := obs_from_config local_config local_state in
         (* apply r on observation *)
         let decision := r obs in
         (* the demon chooses how to perform the state update *)
         let choice := da.(choose_update) local_config g decision in
         (* the actual update of the robot state is performed by the update function *)
         let new_local_state := update local_config g frame_choice decision choice in


     end
   else inactive config id (da.(choose_inactive) config id ).
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        (* return to the global frame of reference *)
        lift (new_frame ^-1) (...) new_local_state
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How to Constrain the Demon to Follow the Model?

```
(** Update the state of good robots in the move phase *)
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This is too powerful: the demon could do anything!

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Instead, the demon just give orders as abstract datatypes.
Then an update function performs the update.

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Example

When active, the demon chooses half/full move: `Tactive := bool`

Nothing happens when inactive: `Tinactive := unit`

```
update := fun _ _ target choice ⇒
   if choice then target ratio_1 else target (1 /r 2);
 inactive := fun config id _ ⇒ config id;
```

Demonic action

Demonic action: what does the demon decide in each round?

1. Pick which robots are activated
2. Decide how to update inactive robots
3. Update Byzantine robots as it wishes
4. Select the new frame of reference for non-Byzantine robots
5. Decide how to update them depending on their destination

```
(** Select which robots are activated *)
activate : ident → bool;
(** Update the state of inactive robots *)
choose_inactive : configuration → ident → info;
(** Update the state of (activated) byzantine robots *)
relocate_byz : configuration → B → info;
(** Local referential for (activated) good robots in the compute phase *)
change_frame : configuration → G → bijection location;
(** Update the state of (activated) good robots in the move phase *)
choose_update : configuration → G → location → info;
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+ compatibility properties (Proper)

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Generalizing the Robogram

We can use the same trick for the robogram!

Change robogam : observation $\rightarrow$ location

to robogam : observation $\rightarrow$ Trobot

This way, we can model:

- curved trajectories
- direction of movement
- changes of orientation/color/memory
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- ...

Don't forget to also change

```
choose_update : configuration → G → Trobot → Tactive;
update : configuration → G → Trobot → Tactive → info;
```

Pactole: a Coq Framework for Mobile Robots

Main Ingredients:

- Robogram
- Demon (scheduler)
- Round
- Execution
- Properties and Proofs

observation → Trobot

Execution

Execution = infinite sequence of configurations
⇝ a stream of configurations

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Definition execution := Stream.t configuration .
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```

How to build streams?

- Coinductive type
- Constructors

```
Definition Stream.cons : A → Stream.t A → Stream.t A.
Definition Stream.constant : A → Stream.t A.
Definition Stream.alternate : A → A → Stream.t A.
```

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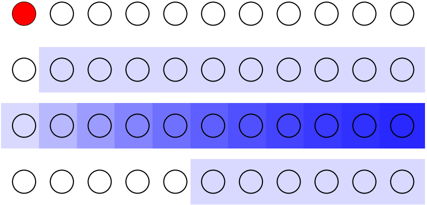
For streams (demon/execution), we define stream operators:

- P : configuration → Prop
- P : execution → Prop

- Stream.instant P:
- Stream.next P:
- Stream.forever P:
- Stream.eventually P:



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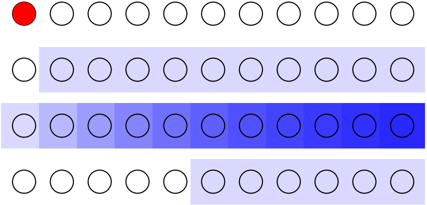

Also useful for defining fairness conditions over demons

(cf. exercises)

Example of Properties: Gathering

Definition (Gathering Problem)

Robots gather if all (non byzantine) robots reach in finite time the same location (unknown ahead of time) and then stay there.

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Robots gather if all (non byzantine) robots reach in finite time the same location (unknown ahead of time) and then stay there.

```
(* All good robots are at the same location pt (exactly). *)
Definition gathered_at (pt : loc) (config : configuration) :=
  ∀ g, get_location (config (Good g)) = pt.

(* At all rounds of the execution e, robots are gathered at pt. *)
Definition Gather (pt : loc) (e : execution) : Prop :=
  Stream.forever (Stream.instant (gathered_at pt)) e.

(* The (infinite) execution e is *eventually* Gathered. *)
Definition WillGather (pt : loc) (e : execution) : Prop :=
  Stream.eventually (Gather pt) e.
```

Proving in Pactole

0. Instantiate your setting

Correctness proof:

1. Formalize your problem
2. Write your algorithm

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Two different points of view

Global View (demon)	**Local View** (robots)
absolute location	local frame of reference
robots: Byzantine or not (B / G)	indistinguishable robots
identifiers ident = $G + B$	
configuration = ident → location	local configuration
	observation (abstract type)
	robogram
	: observation → Trobot
round r d : config → config	
execution = Stream.t configuration	

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   Lemma round_simplify :∀d config, round r d config == ...
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Impossibility proof:

1. Formalize your problem
2. Assume given a robogram (a variable) + its properties
3. Prove that the algorithm does not solve the problem

Universal Algorithms

We can easily formalize what a universal algorithm is:

- Under some conditions, the problem is unsolvable
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```
Lemma impossibility : ∀ r, ∃ d, ∀ config,
   invalid config → ¬good_execution (execute r d config).

Lemma correctness r : ∀ d, ∀ config,
  ¬invalid config → good_execution (execute r d config).
```

Conclusion About Pactole

- ⊕ Designed for mobile robot networks
- ⊕ Ease of use for specification
- ⊕ Broadly applicable

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- Very expressive
- Common base of definition (no more mismatches)

⇒ A junction point for several formal results

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$\Rightarrow$ A junction point for several formal results

- ⊖ Caveat
  - No fully automated procedure (yet)
  - Building proofs is a lot of work

Outline

Gathering

Objective

Have all (non byzantine) robots reach the same location in finite time and then stay there.

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Let's start simple:

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- FSYNC execution
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How to do it?

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What about SSYNC? Impossible!

Theorem [Suzuki & Yamashita 99]

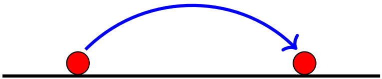
Gathering is impossible even for 2 robots only.

Proof

By symmetry, both robots act the same.

Two cases:

Proof



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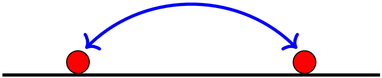

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   activate both: swap locations

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Generalizations:

- even number of robots
- type of line ($\mathbb{Q}$ ou $\mathbb{R}$)

Going Beyond Impossibility

Gathering is impossible in general

Why?

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Why? Because we cannot break symmetry

Here: two towers of the same size (bivalent config)

What about other configurations?

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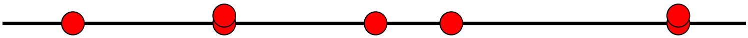
What about other configurations?

There is an algorithm! no byzantine, #robots ≥ 3

```
Definition solGathering (r : robogram) :=
  ∀ d : demon, SSYNC d → Fair d →
  ∀ config , ¬bivalent conf → ∃pt, WillGather pt (execute r d config).
```

Gathering Algo



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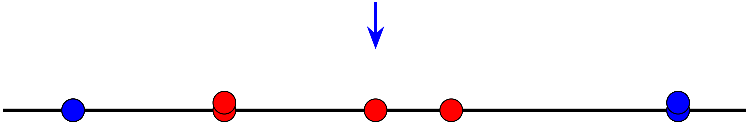

1. Find the middle of the extreme location.

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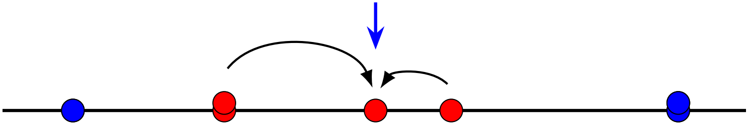

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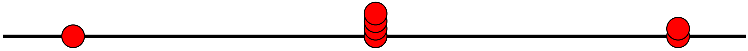

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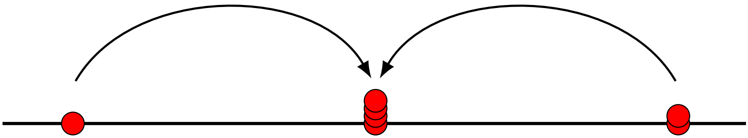

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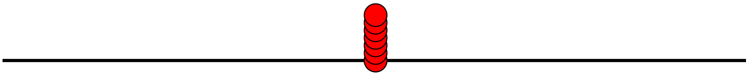

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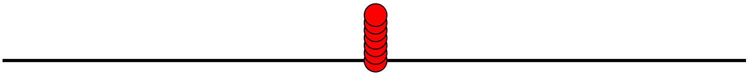

Careful to the bivalent config! SSYNC

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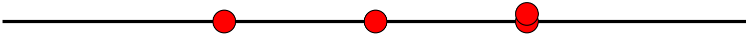

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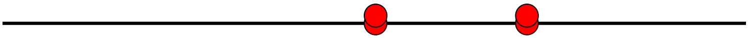

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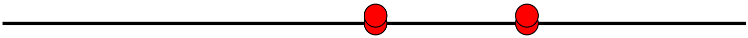

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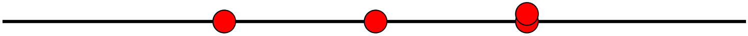

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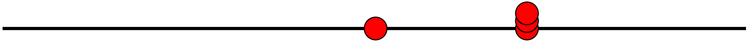

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3 configurations to consider:

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3. Otherwise, general case: ✓ in 2 rounds
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Idea: adapt FSYNC proofs

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And formally?
Separate proofs for correctness and termination

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(Partial) Correctness: if there is a result, it is correct easy!

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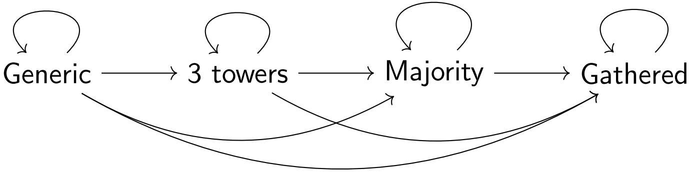
Termination: there is a result hard!

- lexicographic order on configurations
- movement $\implies$ smaller configuration



Main Steps of the Coq Formalization

1. Ability to detect majority towers   observation = multisets
2. Express the configuration after one round
   $\rightsquigarrow$ local/global frame of reference
   $\rightsquigarrow$ depends on the demon (which robots are activated?)
3. In a round, robots always move toward a single location
4. Bivalent configuration cannot appear
5. No backtrack to previous configurations (lexico order)
6. By fairness of the demon, a robot will eventually move

Going to 2D

Going to 2D

Switch « middle » for « center of the smallest enclosing circle ».

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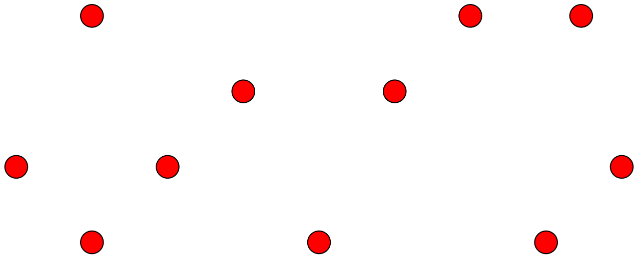

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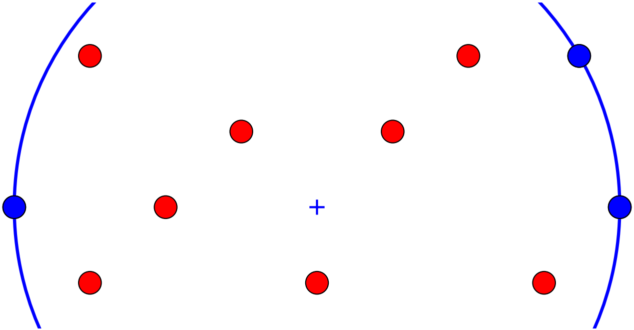

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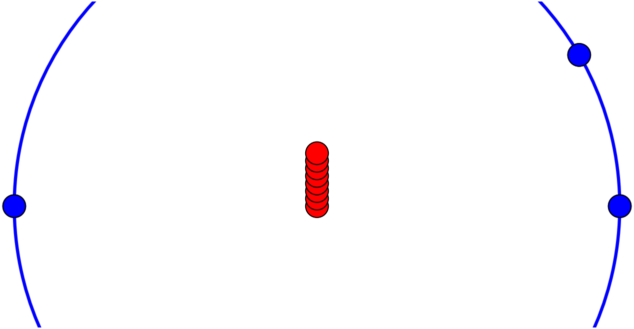

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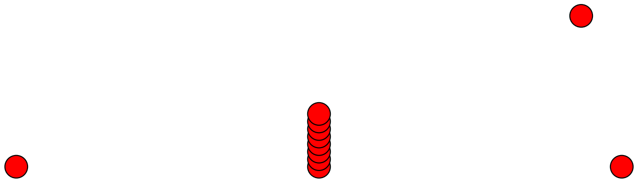

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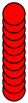

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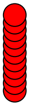

Problem in the last step!

Problem in 2D

It is possible to backtrack on the two phases of the algorithm.

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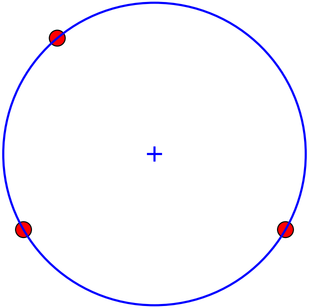

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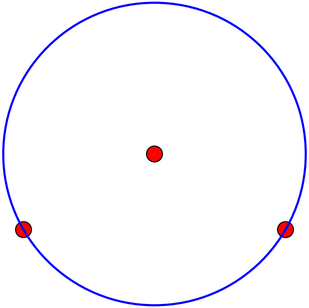

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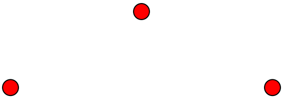

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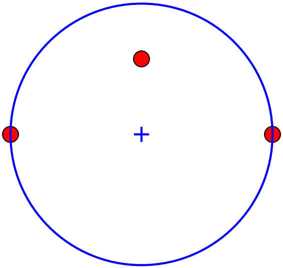

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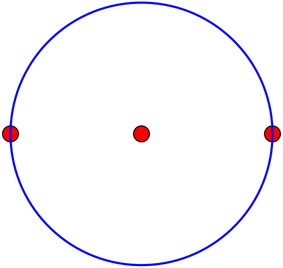

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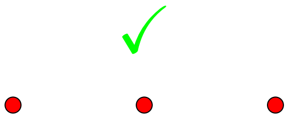

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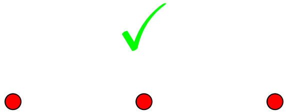

Does it work in general?

Problem in 2D

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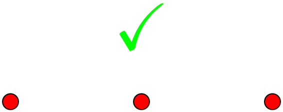

Does it work in general?

Yes, but we need to change the proof cases

Problem in 2D Solved

Key Ideas

- #robots on the circle decreases
  $\rightsquigarrow$ big cases of the proof

Problem in 2D Solved

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- #robots on the circle decreases
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n/a Majority

2. Diameter
3. Triangle
   - scalene, isosceles
   - equilateral

4+. Generic

Most cases = same as 1D

Problem in 2D Solved

Key Ideas

- #robots on the circle decreases $\rightsquigarrow$ big cases of the proof
- Clean/dirty config in each case
  - clean: config $\subseteq$ circle $\cup$ center
  - dirty: otherwise

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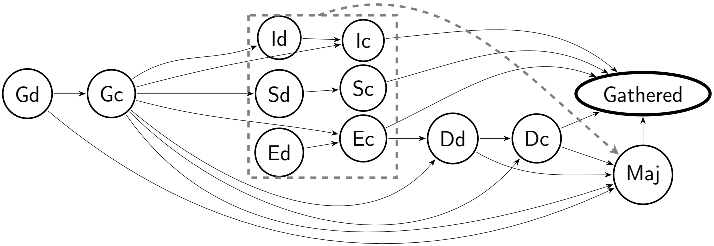

Overall, What Do We Need?

Lots of geometric properties:

- Invariance through frame change (similarity)
- Evolution of configurations along the algorithm (SEC)
- Some classical properties (barycenter)

A few other things:

- Permutations
- Except for termination, everything is easy (like in 1D)

Outline

Before the Practice Session

- Make sure you have downloaded and extracted the Pactole package from the course website

  `https://perso.liris.cnrs.fr/xavier.urbain/ens/m2ensl.html`

  or from the following link (from the pad of the class)

  `https://www-verimag.imag.fr/~riegl/teaching/package.tgz`

- Start compiling Pactole:
  Use `make` at the root of the package $\approx$ 5-6 minutes

- Possible internships
  - Topics (flexible): (non-euclidian) geometry
    certification for randomized algo
    automated proofs (graphs + swarms)
  - ANR project SAPPORO (France/Japan)
  - Collaboration between:
    Lyon 1 / Sorbonne univ. / CNAM Paris / Tokyo-Tech.

Structure of the libraries

Pactole:

- `Util/`
  Complements to Coq's libraries, external libraries
- `Core/`
  Core of the formalism
- `Spaces/`
  The spaces in which robots evolve
- `Observations/`
  The information available for robograms
- `Models/`
  Additional constraints on models
- `CaseStudies/`
  - `Gathering`
  - `Convergence`
  - `Exploration`

What do you need to set it up?

Parameters that need to be imported/instantiated:

- The core of the formalism
- A space in which robots evolve
- A type of observation
- (optional) Extra constraints on your setting

- The number of robots in your case (can be parameters)
- A type of state for robots (containing the space)
- The choices made by the demon:
  - the change of frame: frame_choice
  - the choices for update for active and inactive robots: update_choice and inactive_choice
- The `update` and `inactive` functions

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- The `update` and `inactive` functions

Everything is handled through typeclasses
(typeclasses = mechanism for modularity and overloading)

Why typeclasses?

Advantages:

- Glues everything together
- Does not require a specific order
- Better separation of concerns
- More flexibility for partial instances

But:

- Infinite loops if missing instances
- Unpredictable results if more than one instance
- Use About rather than Check
- Rather large and unpleasant unfoldings

How to do it? (1/6)

2 steps:

1. Require Import the files you need
2. Define the adequate Instances

How to do it? (1/6)

2 steps:

1. `Require Import` the files you need
2. Define the adequate `Instance`s

First step: Require Import

- Formalism core

```
Require Import Pactole.Setting.
```

- Space

```
Require Import Pactole.Spaces.XXX.
```

- Observations

```
Require Import Pactole.Observations.XXX.
```

- Extra constraints

```
Require Import Pactole.Models.XXX.
```

How to do it? (2/6)

Second step: Instances

- ~~Formalism core~~
- Space — type with decidable equivalence

```
Class Location := {
  location : Type;
  location_Setoid :> Setoid location ;
  location_EqDec :> EqDec location_Setoid }.
```

```
Instance XXX : Location := XXX.
```

Often: `Instance XXX : Location := make_Location XXX.`

- ~~Observation~~
- Extra contraints (depends on what you want)

```
Instance Update : RigidSetting .
```

How to do it? (3/6): Number of Robots

```
Instance XXX : Names := Robots nG nB.
```

nG = number of good robots
nB = number of Byzantine robots

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```
Instance XXX : Names := Robots nG nB.
```

nG = number of good robots
nB = number of Byzantine robots

nG and nB can be left as variables

```
Parameter n : nat.
Hypothesis n_non_0 : n ≠ 0.
Instance MyRobots : Names := Robots (2 * n) 0.
```

How to do it? (4/6): State of Robots

```
Instance XXX : State info := XXX.

Instance Info : State location := OnlyLocation _.
```

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Instance XXX : State info := XXX.

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```
Class State {Loc : Location} info := {
  get_location : info → location ;










    ... }.
```

How to do it? (4/6): State of Robots

```
Instance XXX : State info := XXX.

Instance Info : State location := OnlyLocation _.
```

```
Class State {Loc : Location} info := {
  get_location : info → location ;
  (** States are equipped with a decidable equality *)
  state_Setoid :> Setoid info ;
  state_EqDec :> EqDec state_Setoid;









    ... }.
```

How to do it? (4/6): State of Robots

```
Instance XXX : State info := XXX.

Instance Info : State location := OnlyLocation _.
```

```
Class State {Loc : Location} info := {
  get_location : info → location ;
  (** States are equipped with a decidable equality *)
  state_Setoid :> Setoid info ;
  state_EqDec :> EqDec state_Setoid;
  (** Lifting a change of frame from a location to a full state *)
  precondition : (location → location) → Prop;
  lift : forall f, precondition f → info → info ;




    ... }.
```

How to do it? (4/6): State of Robots

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  state_Setoid :> Setoid info ;
  state_EqDec :> EqDec state_Setoid;
  (** Lifting a change of frame from a location to a full state *)
  precondition : (location → location) → Prop;
  lift : forall f, precondition f → info → info ;
  (** Properties (compatibility, ...) *)
  lift_id : forall Pid, @lift id Pid == id;
  get_location_lift : forall f (Pf : precondition f) state,
    get_location (@lift f Pf state) == f (get_location state);
   ... }.
```

In Practice: Template.v (1/2)

```
Require Import Pactole.Setting.

(* Number of robots *)
Parameter n : nat.
Axiom n_non_0 : n <> 0.
Instance MyRobots : Names := Robots (2 * n) 0.

(* Space and state and robot choice *)
Require Import Pactole.Spaces.R2.
Close Scope R_scope.
Instance Loc : Location := make_Location R2.
Instance Info : State location := OnlyLocation (...).
Instance RC : robot_choice location := {
  robot_choice_Setoid := location_Setoid }.

(* Type of observations *)
Require Import Pactole.Observations.MultisetObservation.
```

How to do it? (5/6): Demon & Robot Choices

```
Instance XXX : robot_choice Trobot := XXX.
Instance XXX : frame_choice Tframe := XXX.
Instance XXX : update_choice Tactive := XXX.
Instance XXX : inactive_choice Tinactive := XXX.

Instance FC : frame_choice (Similarity.similarity location) :=
  FrameChoiceSimilarity.
Instance UC : update_choice unit := {
  update_choice_EqDec := unit_eqdec}.
```

```
Class frame_choice Tframe := {
  frame_choice_bijection : Tframe → bijection location;
  frame_choice_Setoid : Setoid Tframe;
  frame_choice_bijection_compat :
    Proper (equiv ==> equiv) frame_choice_bijection }.

Class update_choice Tactive := {
  update_choice_Setoid : Setoid Tactive;
  update_choice_EqDec : EqDec update_choice_Setoid }.
Class inactive_choice Tinactive := { ... }.
Class robot_choice Trobot := { ... }.
```

How to do it? (6/6): Update Functions

```
Instance  XXX : update_function Tactive := XXX.
Instance  XXX : inactive_function  Tinactive  := XXX.

Instance  UpdateFun : update_function bool := {
  update := fun _ _ target choice ⇒
     if  choice  then  target  ratio_1  else  target  (1 /r 2) }
Instance  UpdateFun : inactive_function  unit  := {
   inactive  := fun  config  id  _  ⇒ config  id  }.
```

```
Class  update_function ‘{robot_choice} ‘{frame_choice} ‘{update_choice} :=
  update :> configuration  → G → Tframe → Trobot →  Tactive  →  info ;
  update_compat :> Proper (equiv ==⇒ Logic.eq ==⇒ equiv ==⇒
                                  equiv  ==⇒ equiv ==⇒ equiv) update }.
Class  inactive_function   ‘{ inactive_choice} := {
   inactive   :> configuration  →  ident  →  Tinactive  →  info ;
  inactive_compat :> Proper (equiv ==⇒ Logic.eq ==⇒
                                    equiv  ==⇒ equiv)  inactive  }.
```

In Practice: `Template.v` (2/2)

```
(* Demon choices *)
Require Import Pactole.Models.Similarity.
Instance FC : frame_choice (Similarity.similarity location) :=
  FrameChoiceSimilarity.
Instance UC : update_choice unit := {update_choice_EqDec := unit_eqdec}.
Instance IC : inactive_choice unit := {inactive_choice_EqDec := unit_eqdec}.

(* Update functions *)
Instance UpdateFun : update_function unit := {
  update := fun _ _ _ pt _ ⇒pt }.
Proof. now repeat intro. Defined.

Instance InactiveFun : inactive_function unit := {
  inactive := fun config id _ ⇒ config id }.
Proof. now repeat intro; subst. Defined.

(* Properties about the framework *)
Require Import Pactole.Models.Rigid.
Instance Update : RigidSetting.
Proof. split. now intros. Qed.
```

Another Example: Gathering/Definitions.v

```
Require Export Pactole.Setting.
Require Export Pactole.Spaces.RealMetricSpace.
Require Pactole.Spaces.Similarity.

Section GatheringDefinitions.

(* We only required the space to be a real metric space.
   The actual number of robots is arbitrary. *)
Context {Tactive Tinactive : Type}.
Context {N : Names}.
Context {Loc : Location}.
Context {RMS : RealMetricSpace location}.

Global Instance Info : State location := OnlyLocation.

(** The observation and state updates are still arbitrary. *)
Context {Obs : Observation}.
Context {UC : update_choice Tactive}.
Context {IC : inactive_choice Tinactive}.
Context {UpdFun : update_functions Tactive Tinactive}.
```

Exercises

See file `exercises.v`**.**

Exercises

See file `exercises.v`**.**

- Modeling of problems
  - gathering
  - convergence
  - exploration with Stop
  - perpetual exploration
  - safety
- Properties of streams and demons
  - until / weak until
  - fully-synchronous / centralized demon
- State with lights
  - Internal: only visible by self (akin to memory)
  - External: only visible by others
  - Full: visible by all
- Rendezvous by Viglietta

Solution: Gathering

```
(* All good robots are at the same location [pt] (exactly). *)
Definition gathered_at (pt : location) (config : configuration) :=
  ∀ g, get_location (config (Good g)) == pt.

(* At all rounds of the execution [e], robots are gathered at [pt]. *)
Definition Gather (pt : location) (e : execution) : Prop :=
  Stream.forever (Stream.instant (gathered_at pt)) e.

(* The infinite execution [e] is *eventually* [Gather]ed. *)
Definition WillGather (pt : location) (e : execution) : Prop :=
  Stream.eventually (Gather pt) e.

Definition gathering (e : execution) := ∃ pt, WillGather pt e.
```

Solution: Convergence

```
(* All robots are contained in the disk defined by [center] and [radius]. *
Definition contained (center : location) (radius : R) config :=
  ∀ g, dist center (get_location (config (Good g))) ≤ radius.

(* All good robots stay confined in a small disk. *)
Definition imprisoned (center : location) (radius : R) (e : execution) :=
  Stream.forever (Stream.instant (contained center radius)) e.

(* The execution will end in a small disk. *)
Definition attracted (c : location) (r : R) (e : execution) : Prop :=
  Stream.eventually (imprisoned c r) e.

Definition convergence (e : execution) :=
  ∀ ε: R, 0 < ε → ∃ pt : location, attracted pt ε e

(* A solution ensures convergence for any demon and configuration. *)
Definition convergence_sol (r : robogram) : Prop :=
  ∀ d, Fair d → ∀ config, convergence (execute r d config).
```

Solution: Exploration with Stop

```
Definition visited pt config :=
  ∃ g, get_location (config (Good g)) == pt.
Definition will_be_visited pt e : Prop :=
  Stream.eventually (Stream.instant (visited pt)) e.

Definition stall (e : execution) :=
  Stream.hd e == Stream.hd (Stream.tl e).
Definition stopped (e : execution) : Prop :=
  Stream.forever stall e.
Definition will_stop (e : execution) : Prop :=
  Stream.eventually stopped e.

Definition Explore_and_Stop e :=
  (∀ pt, will_be_visited pt e) ∧
  will_stop e.
Definition is_solution (r : robogram) :=
∀ d config, Fair d → Explore_and_Stop (execute r d config).
```

Solution: Exploration with Stop & Safety

```
(** Perpetual exploration:
    each location is visited infinitely often. *)
Definition perpetual_exploration e :=
  forall pt, Stream.forever (Stream.eventually
               (Stream.instant (visited pt))) e.

(** Safety: stay outside of a given set [danger] of states. *)
Definition safe_config (danger : location → Prop) config :=
  forall g, ¬danger (config (Good g)).

Definition safe danger e :=
 Stream.forever (Stream.instant (safe_config danger)) e.
```

Solution: Until + FSYNC + Centralized

```
Inductive until (P Q : t A → Prop) (s : t A) : Prop :=
  | NotYet : P s → until P Q (tl s) → until P Q s
  | YesNow : Q s → until P Q s.
Definition weak_until P Q s := Stream.forever P s ∨ until P Q s.

Definition FSYNC_da da : Prop :=
 ∀ config g, activate da config g = true.
Definition FullySynchronous : demon → Prop :=
 Stream.forever (Stream.instant FSYNC_da).

Definition centralized_da da :=
 ∀ id1 id2, activate da id1 = true →
            activate da id2 = true → id1 = id2.
Definition centralized (d : demon) :=
 Stream.forever (Stream.instant centralized_da) d.
```

Solution: Lights

```
(* The simple version: only location and lights. *)
Context `{Location}.
Context {nbLights : nat}.
Definition lights := {k : nat | k < nbLights}.

Instance lights_Setoid : Setoid lights := @sig_Setoid _ _ _.
Instance lights_EqDec : EqDec lights_Setoid := sig_EqDec _ _.

Instance InfoWithLights : State (location * lights) := {|
  get_location := fst;
  state_Setoid := prod_Setoid location_Setoid lights_Setoid;
  precondition := fun _ => True;
  lift := fun f info => (projT1 f (fst info), snd info) |}.
Proof.
+ now intros [].
+ now intros f [].
+ now intros [] [] [].
+ intros f g Hfg [] [] []. simpl. split; trivial; []. now apply Hfg.
Defined.
```

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