Simulations numériques du champ magnétique terrestre

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Earth has life and a magnetic field.
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But a magnetic field does NOT shield from cosmic rays, the atmosphere does.
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But a magnetic field does NOT shield from cosmic rays, the atmosphere does.

A magnetic field **may** be important to keep an atmosphere from being eroded by solar/stellar wind, at least in case the surface gravity is not enough.

1 The core and magnetic field of the Earth
   • Observations and facts
   • The geomagnetic field
   • A dynamic magnetic field

2 Numerical models
   • 20+ years of simulations
   • Recent advances

3 Numerical method

4 Reversals
   • 20+ years of reversing simulations
   • Geodynamo models vs Earth
   • Reversals elsewhere
   • Discussion & Outlook
Structure of the Earth

- **Atmosphere**: 11 km
- **Crust**: 71 km
- **Mantle**: 6,300 km
- **Molten Outer Core**: 3,500 km
- **Solid Inner Core**: 1,200 km

**Graph**:
- **Density (relative to water)**
- **P-wave velocity (km/s)**
- **S-wave velocity (km/s)**

**Legend**:
- Black: density (relative to water)
- Blue: P-wave velocity (km/s)
- Orange: S-wave velocity (km/s)

**Axes**:
- **Radius (km)**: 0 to 6,371 km
- **Density**: 0 to 14 relative to water
- **P-wave velocity**: 0 to 14 km/s
- **S-wave velocity**: 0 to 14 km/s
Geomagnetic field measurements

Started by Gauss (and others) in 1836, today we have many magnetic ground observatories

(from Finlay+ 2016)
Geomagnetic field measurements

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Measures from satellites since 1979. Currently 3 dedicated satellites (SWARM).
Geomagnetic field models

Magnetic field at the Core-Mantle Boundary (CMB) in 2015. (CHAOS-6 model, from Finlay+ 2016).
The magnetic north pole is moving

https://maps.ngdc.noaa.gov/viewers/historical_declination/
Polarity reversals

- **A superchron** with the same polarity for almost 40 Millions years.
- **Frequently reversing** periods, where a given polarity stays for 1 Million year or less.

from Hulot et al., 2010
Paleointensity models: sint2000

Typical intensity variations from 0.5 to 1.5 times the mean – factor 3 between min and max outside reversals.

from Valet, Meynadier, and Guyodo, 2005.
A very old magnetic field

Cool facts about the Earth’s core

- A broad range of time-scales
  - from months (SV) to million years (reversals)

- Viscosity of water
  - Earth’s spin (Coriolis force) dominates the dynamics \( \Leftrightarrow E \sim 10^{-15} \)
  - Momentum diffuses much slower than magnetic field \( \Leftrightarrow Pm \sim 10^{-5} \)

- Large scale motions at the top of the core have speeds around 10 km/year (0.3 mm/sec, turnover time is about 200 years)
  - Turbulent motion (very high Reynolds number \( Re \gtrsim 10^8 \)).
  - Magnetic Reynolds number \( Rm \gtrsim 1000 \) (moderate compared to astrophysical objects)
  - Earth’s spin (Coriolis force) dominates the dynamics \( \Leftrightarrow Ro \sim 3 \times 10^{-6} \).
Cool facts about the Earth’s magnetic field

- Magnetic field at the surface is dominated by a tilted dipole.
- Magnetic energy dominates kinetic energy by a factor $10^4$ (4 mT estimated in the core – Gillet+ 2010, 0.5 mT or 5 gauss at the surface).
  - Earth’s spin (Coriolis force) still dominates the fast dynamics $\Leftrightarrow \text{Le} \sim 10^{-4}$ (e.g. Jault 2008).
  - The magnetic field and the Coriolis force influence long-term dynamics $\Leftrightarrow \Lambda \gtrsim 10$.
- Heat flux extracted by the mantle ($\sim 10\text{TW}, < 100\text{mW/m}^2$).
  - Strong convection (very high Rayleigh number $Ra \gg 10^{20}$? Probably many times critical.
  - A (thermo-chemical) convection-driven dynamo produces the Earth’s magnetic field.
Outline

1. The core and magnetic field of the Earth
2. Numerical models
   - 20+ years of simulations
   - Recent advances
3. Numerical method
4. Reversals
20+ years of geodynamo simulations: toward the Earth

1995: Glatzmaier & Roberts
   - Chebychev, 64 x 32 x 49
   - Hyperviscosity
   - Earth-like, reversals, and all the hype.
20+ years of geodynamo simulations: toward the Earth

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- **2006**: Christensen & Aubert
  - Chebychev, $168 \times 336 \times 97$
  - $E \geq 3 \times 10^{-6}$, $Pm \geq 0.06$
  - Extensive parameter study, scaling laws.

- **2008**: Kageyama+
  - Yin-Yang grid, $2048 \times 1024 \times 511$
  - $E = 10^{-6}$, $Re=700$, $Pm=1$
  - Convection sheets, zonal jets.

- **2013**: Aubert+
  - Chebychev, $768 \times 384 \times 160$
  - $E = 2.5 \times 10^{-5}$, $Re=1000$, $Pm=0.2$
  - Coupled Earth, westward drift got right.
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Earth: $E = 10^{-15}, Pm = 10^{-6}$. From Schaeffer et al., 2017.
Recent advances (I)

A series of recent studies at lower viscosities ($E \leq 10^{-6}$, Yadav et al., 2016; Aubert, Gastine, and Fournier, 2017; Schaeffer et al., 2017) all point in the same direction:

- Stronger magnetic field, a Magneto-Buoyancy-Coriolis force balance replaces the inertia-driven regimes.
- The magnetic field changes the scale of convection to larger scales.

from Yadav et al., 2016
Schaeffer et al., 2017 highlights the very large spatial and temporal fluctuations of the magnetic field.

Intense polar vortices: thermal winds, magnetically shaped.
Recent advances (III)

- Schaeffer et al., 2017 highlights the density segregation between polar and equatorial regions.
- Looking at the numbers, the density contrast is too low for being seen by seismology, except if composition variations contribute too.

![S2](image1.png)  

![C_tot](image2.png)

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Ostensibly lacking in these models are:

- slow MAC waves – difficult to sort out?
- stably stratified layer & possible double-diffusive effects.
- reversals – simulations are too short...
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Equations of rotating MHD in planetary cores

Navier-Stokes equation

\[ \partial_t \mathbf{u} + (2\Omega \mathbf{e}_z + \nabla \times \mathbf{u}) \times \mathbf{u} = -\nabla p + \nu \Delta \mathbf{u} + (\nabla \times \mathbf{b}) \times \mathbf{b} - \alpha g \mathbf{T} \cdot \mathbf{r} \]

Induction equation

\[ \partial_t \mathbf{b} = \nabla \times (\mathbf{u} \times \mathbf{b}) + \eta \Delta \mathbf{b} \]

Temperature equation

\[ \partial_t \mathbf{T} + \mathbf{u} \cdot \nabla \mathbf{T} = \kappa \Delta \mathbf{T} \]

\[ E = \frac{\nu}{D^2 \Omega} \sim 10^{-15} \]

\[ Pm = \frac{\nu \mu_0 \sigma}{\sim 10^{-5}} \]

\[ \frac{1}{Pr} = \frac{\nu}{\kappa} \sim 1 \]

\[ \frac{1}{Ra} = \frac{\Delta T \alpha g D^3}{\kappa \nu} \gg 1 \] (?)
Equations of rotating MHD in planetary cores

**Navier-Stokes equation**

$$\text{acceleration} = \text{advection}$$

\[ \text{Pressure gradient} + \text{Coriolis force} + \text{Magnetic force} + \text{Archimedes force} + \text{viscous drag} \]

**Induction equation**

\[ \text{magnetic field variations} = \text{Induction} + \text{ohmic losses} \]

**Temperature equation**

\[ \text{temperature variations} = \text{Advection} + \text{thermal conduction} \]

\[ E = \nu/D^2\Omega \sim 10^{-15} \]
\[ Pm = \nu\mu_0\sigma \sim 10^{-5} \]
\[ Ra = \Delta T\alpha g D^3/\kappa \nu \gg 1 \quad (?) \]
\[ Pr = \nu/\kappa \sim 1 \]

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Divergence-free vector fields are represented with two scalar fields

\[ \mathbf{u} = \nabla \times (rT) + \nabla \times \nabla \times (rP) \]

Spherical harmonic decomposition:

\[ P(r, \theta, \phi) = \sum_{\ell} \sum_{m} P_{\ell,m}(r) Y_{\ell}^{m}(\theta, \phi) \]

Finite difference discretization of \( P_{\ell,m}(r) \).
the standard framework to solve the problem

- Divergence-free vector fields are represented with two scalar fields
  
  \[ \mathbf{u} = \nabla \times (r \mathbf{T}) + \nabla \times \nabla \times (r \mathbf{P}) \]

- Spherical harmonic decomposition:
  
  \[ P(r, \theta, \phi) = \sum_{\ell} \sum_{m} P_{\ell,m}(r) Y_{m}^{\ell}(\theta, \phi) \]

- Finite difference discretization of \( P_{\ell,m}(r) \).

Pros:

- Two scalars instead of 3 components and zero-divergence constrain enforced.
- Spherical harmonics make the magnetic boundary condition easy;
- Spherical harmonics are concise: less data to describe the field;
- No problem at the poles.
- At least \textbf{one order of magnitude faster} than local methods

Cons:

- It is a spectral method: more difficult to parallelize.
Discretized equations

\[ Mu_{t+dt} = Lu_t + NL(u_t) \]
Discretized equations

\[ Mu_{t+dt} = Lu_t + NL(u_t) \]

- Evaluate linear term \( Lu \)
  - sparse matrix \( L \), band-diagonal \( \rightarrow \) easy, cheap, independent
  - bandwidth limited
Discretized equations

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- Evaluate non-linear term \( NL(u) \)
  - in spatial space using spherical harmonic transforms \( \rightarrow \) costly, coupling in \( \theta, \phi \)
  - FFT \( O(n^3 \log n) \) is bandwidth limited, Legendre transform \( O(n^4) \) not so much

- Solve for \( u_{t+dt} \) by "inverting" matrix \( M \).
  - sparse matrix \( M \), band-diagonal
  - Thomas algorithm (Gauss substitution) \( \rightarrow \) cheap, coupling in \( r \), sequential in \( r \)
  - bandwidth limited
the SHTns library: efficient spherical harmonic transforms

- blazingly fast.
- on-the-fly transforms (do not store matrix coefficients, compute them as needed).
- on-the-fly rotations (do not store Wigner-d matrices).
- use FFTW library for the Fourier transform, or VkFFT on GPU.
- hand vectorized routines (support AVX & AVX-512, VSX, Neon).
- Tested up to degree 65000+.

SHTns library is freely available
https://gricad-gitlab.univ-grenoble-alpes.fr/schaeffn/shtns

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A high performance simulation code for rotating incompressible flows and magnetic fields in spherical shells.

- written in C++
- Free & Open-source software  https://nschaeff.bitbucket.io/xshells
- Compilers: gnu or intel (OpenMP 4 support needed)
- Dependencies: FFTW (or MKL) and SHTns
- Parallelization: domain decomposition with MPI + OpenMP.
- ongoing port to GPU (cuda).
the XSHELLS code: resistive MHD in the sphere

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the X SHELLS code: key idea for parallel performance

- limit communications as much as possible
- domain decomposition favors the SH transform: radial decomposition
- do not use "transpose" data (no MPI_Alltoall)
- linear solve phase is tricky to parallelize efficiently
the XSHELLS code: key idea for parallel performance

- limit communications as much as possible
  - domain decomposition favors the SH transform: radial decomposition
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  - linear solve phase is tricky to parallelize efficiently
Except the Legendre transform, everything is “memory bound”. GPU bring a lot more memory bandwidth.

- Ongoing effort
- About 75% of the computations are ported.
- About 900 lines of additional GPU specific code so far.

**XSHELLS code is freely available**
https://gricad-gitlab.univ-grenoble-alpes.fr/schaeffn/xshells

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**XSHELLS strong scaling @ NR=512, Lmax=426**

- irene-amd
- irene-skl
- Jean Zay CPU
- Jean Zay GPU (V100)
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  - $E \geq 3 \times 10^{-4}$, $Pm \geq 3$
  - tracking internal mechanism (+many other).

- **2016**: Sheyko+
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  - Parker waves, quasi-periodic, low field
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geodynamo simulations

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When do geodynamo models reverse?

Kutzner and Christensen, 2002

Olson, Glatzmaier, and Coe, 2011
Examples of reversing dynamos

From Christensen, 2011: $E = 3 \times 10^{-4}$, $Pm = 3$; large fluctuations of intensity.
Examples of reversing dynamos

- geodynamo run at $Re = 217$, $Pm = 3$, $E_{mag}/E_{kin} = 0.84$
- 3 reversals within 2 Myr
- same polarity for more than 17 Myr (superchron)
20+ years of geodynamo simulations: reversals

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Successes: these models reverse polarity

20+ years of geodynamo simulations: reversals

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Successes:
- these models reverse polarity

Problems:
- Parker waves: periodic, weak-field: impossible in the Earth’s core.
- inertia driven: impossible to happen in the Earth’s core.
Inertia-driven reversals are impossible in the Earth’s core

Inertia-driven reversing dynamos operate at $Ro_{loc} = 0.1$ (e.g. Olson and Christensen, 2006). With $Ro_{loc} = Ro \frac{R}{\ell^*}$, where $\ell^*$ is a typical scale of convection.

**If polarity reversals were inertia-driven:**

For the Earth, $Ro = 3 \times 10^{-6} \Rightarrow \ell^* = 100\text{m}$. What do we know about $\ell^*$ in the core?
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- **constraint #2**: turbulent non-magnetic convection at $\ell \sim Ro^{1/2} = 30km$. Implies $Ro_{loc} \sim 10^{-4}$ in the Earth (Guervilly+ 2019).
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- **constraint #3**: magnetic convection at $\ell \sim \text{Ro}^{1/4} = 150\text{km}$. Implies $\text{Ro}_{\text{loc}} \sim 2 \times 10^{-5}$ in the Earth (see Davidson, 2013; Aubert, Gastine, and Fournier, 2017).
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Ooops!
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Reversals...

from Berhanu et al., 2007
Reversals in VKS2: a lab fluid dynamo (almost)

**VKS2: Turbulent flow between discs**

- **VKS2** (2006): 0.15 m³ Na, 300 kW
- Dynamo only with soft iron blades (ie magnets)
Reversals are captured by a very simple model

A dynamical system below a saddle-node bifurcation can be driven into reversals by fluctuations (noise) $\zeta(t)$.

**Right:** $\alpha_0 = -185 \text{Myr}^{-1}$, $\alpha_0/\alpha_1 = -0.9$, $\Delta/\sqrt{|\alpha_1|} = 0.2$.

from Pétrélis et al., 2009.
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Outline

1 The core and magnetic field of the Earth
2 Numerical models
3 Numerical method
4 Reversals
   - 20+ years of reversing simulations
   - Geodynamo models vs Earth
   - Reversals elsewhere
   - Discussion & Outlook
Can we get reversals with strong magnetic fields?

For a hope of obtain strong magnetic fields in reversing simulations:
- we need more realistic simulations (lower viscosity, strong forcing)
- but not too high definition, so we can run them long enough!
Magnetically-driven reversals?

\[ E = 10^{-5}, \ Pm = 2, \ Ro_{loc} = 0.053, \ f_{dip} = 0.33, \ \Lambda = 19, \ E_{mag}/E_{kin} = 2.8 \]
Zooming on a reversal
What do we learn from these strong field reversals?

Joint distribution of total magnetic energy and surface axial dipole.

During reversal, magnetic field reduced by 15 to 20% in the whole core.
Strong field reversals exist in numerics. Study them to learn about Earth’s ones.

- Influence of mantle geodynamics on reversals (input from N. Coltice)
- Statistical properties of reversal (need to simulate a lot of events)
Thank you for your attention

Core simulations made with XSHESLLS code: https://nschaeff.bitbucket.io/xshells