#### Simulations numériques du champ magnétique terrestre

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### Foreword: Are planetary magnetic fields useful?

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### Foreword: Are planetary magnetic fields useful?

- Earth has life and a magnetic field.
- Mars has no life and no magnetic field.
- But a magnetic field does NOT shield from cosmic rays, the atmosphere does.
- A magnetic field **may** be important to keep an atmosphere from being eroded by solar/stellar wind, at least in case the surface gravity is not enough.

Lammer+ 2008, Space Science Reviews, Atmospheric escape and evolution of terrestrial planets and satellites.

# Outline

#### The core and magnetic field of the Earth

- Observations and facts
- The geomagnetic field
- A dynamic magnetic field
- 2 Numerical models
  - 20+ years of simulations
  - Recent advances

#### 3 Numerical method

#### 4 Reversals

- 20+ years of reversing simulations
- Geodynamo models vs Earth
- Reversals elsewhere
- Discussion & Outlook

### Structure of the Earth



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#### Geomagnetic field measurements

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Measures from satellites since 1979. Currently 3 dedicated satellites (SWARM).





#### Geomagnetic field models



Magnetic field at the Core-Mantle Boundary (CMB) in 2015. (CHAOS-6 model, from Finlay+ 2016).

### The magnetic north pole is moving



#### Polarity reversals



• A superchron with the same polarity for almost 40 Millions years.

 $\bullet$  Frequently reversing periods, where a given polarity stays for 1 Million year or less. from Hulot et al., 2010

### Paleointensity models: sint2000



• Typical intensity variations from 0.5 to 1.5 times the mean – factor 3 between min and max outside reversals.

from Valet, Meynadier, and Guyodo, 2005.

#### A very old magnetic field



from Landeau et al., 2022, Sustaining Earth's magnetic dynamo, Nature Review Earth & Environment.

#### Cool facts about the Earth's core

- A broad range of time-scales
  - from months (SV) to million years (reversals)
- Viscosity of water
  - Earth's spin (Coriolis force) dominates the dynamics  $\Leftrightarrow E \sim 10^{-15}$
  - Momentum diffuses much slower than magnetic field  $\Leftrightarrow Pm \sim 10^{-5}$
- Large scale motions at the top of the core have speeds around 10 km/year (0.3 mm/sec, turnover time is about 200 years)
  - Turbulent motion (very high Reynolds number  $Re \gtrsim 10^8$ ).
  - Magnetic Reynolds number  $Rm \gtrsim 1000$  (moderate compared to astrophysical objects)
  - Earth's spin (Coriolis force) dominates the dynamics  $\Leftrightarrow Ro \sim 3 \times 10^{-6}$ .

- Magnetic field at the surface is dominated by a tilted dipole.
- Magnetic energy dominates kinetic energy by a factor 10<sup>4</sup> (4 mT estimated in the core Gillet+ 2010, 0.5 mT or 5 gauss at the surface).
  - Earth's spin (Coriolis force) still dominates the fast dynamics  $\Leftrightarrow$   $Le \sim 10^{-4}$  (e.g. Jault 2008).
  - ▶ The magnetic field and the Coriolis force influence long-term dynamics  $\Leftrightarrow \Lambda \gtrsim 10$ .
- Heat flux extracted by the mantle ( $\sim 10$ TW, < 100mW/m<sup>2</sup>).
  - $\blacktriangleright$  Strong convection (very high Rayleigh number  $Ra \gg 10^{20}$  ? Probably many times critical.
  - A (thermo-chemical) convection-driven dynamo produces the Earth's magnetic field.

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- Yin-Yang grid, 2048 × 1024 × 511
- $E = 10^{-6}$ , Re=700, Pm=1
- convection sheets, zonal jets.



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#### • 2013 : Aubert+

- Chebychev, 768\_× 384 × 160
- $E = 2.5 \times 10^{-5}$ , Re=1000, Pm=0.2
- Coupled Earth, westward drift got right.





Earth:  $E = 10^{-15}$ ,  $Pm = 10^{-6}$ . From Schaeffer et al., 2017.

### Recent advances (I)

A series of recent studies at lower viscosities ( $E \le 10^{-6}$ , Yadav et al., 2016; Aubert, Gastine, and Fournier, 2017; Schaeffer et al., 2017) all point in the same direction:

- Stronger magnetic field, a Magneto-Buoyancy-Coriolis force balance replaces the inertia-driven regimes.
- The magnetic field changes the scale of convection to larger scales.



from Yadav et al., 2016

### Recent advances (II)

- Schaeffer et al., 2017 highlights the very large spatial and temporal fluctuations of the magnetic field.
- Intense polar vortices: thermal winds, magnetically shaped.



### Recent advances (III)

- Schaeffer et al., 2017 highlights the density segregation between polar and equatorial regions.
- Looking at the numbers, the density contrast is too low for being seen by seismology, except if composition variations contribute too.



Ostensibly lacking in these models are:

- slow MAC waves difficult to sort out?
- stably stratified layer & possible double-diffusive effects.
- reversals simulations are too short...

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#### 2 Numerical models

#### Numerical method



#### Equations of rotating MHD in planetary cores

Navier-Stokes equation

 $\partial_t \mathbf{u} + (2\Omega \, \mathbf{e}_z + \nabla \times \mathbf{u}) \times \mathbf{u} = -\nabla p + \nu \Delta \mathbf{u} + (\nabla \times \mathbf{b}) \times \mathbf{b} - \alpha g \, T \, \vec{r}$ 

Induction equation

$$\partial_t \mathbf{b} = \nabla \times (\mathbf{u} \times \mathbf{b}) + \eta \Delta \mathbf{b}$$

Temperature equation

$$\partial_t T + \mathbf{u} \cdot \nabla T = \kappa \Delta T$$

 $E = \nu/D^2 \Omega \sim 10^{-15}$  $Pm = \nu \mu_0 \sigma \sim 10^{-5}$ 

 $Ra = \Delta T \alpha g D^3 / \kappa \nu \gg 1 \quad (?)$  $Pr = \nu / \kappa \sim 1$ 

#### Equations of rotating MHD in planetary cores

Navier-Stokes equation

# Pressure gradient + Coriolis force

+ Magnetic force + Archimedes force + viscous drag

Induction equation

magnetic field variations = Induction + ohmic losses

Temperature equation

temperature variations = Advection + thermal conduction

 $E = \nu/D^2 \Omega \sim 10^{-15}$  $Pm = \nu \mu_0 \sigma \sim 10^{-5}$ 

 $Ra = \Delta T \alpha g D^3 / \kappa \nu \gg 1$  (?)  $Pr = \nu / \kappa \sim 1$ 

#### the standard framework to solve the problem

• Divergence-free vector fields are represented with two scalar fields

$$\mathbf{u} = \nabla \times (\mathbf{r}T) + \nabla \times \nabla \times (\mathbf{r}P)$$

• Spherical harmonic decomposition:

$$P(r, \theta, \phi) = \sum_{\ell} \sum_{m} P_{\ell,m}(r) Y_m^{\ell}(\theta, \phi)$$

• Finite difference discretization of  $P_{\ell,m}(r)$ .

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#### Pros:

- Two scalars instead of 3 components and zero-divergence constrain enforced.
- Spherical harmonics make the magnetic boundary condition easy;
- Spherical harmonics are concise: less data to describe the field;
- No problem at the poles.

#### • At least one order of magnitude faster than local methods

#### Cons:

• It is a spectral method: more difficult to parallelize.

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- Evaluate non-linear term *NL*(*u*)
  - in spatial space using spherical harmonic transforms  $\rightarrow$  costly, coupling in  $\theta, \phi$
  - FFT  $O(n^3 \log n)$  is bandwidth limited, Legendre transform  $O(n^4)$  not so much
- Solve for  $u_{t+dt}$  by "inverting" matrix M.
  - sparse matrix M, band-diagonal
  - Thomas algorithm (Gauss substitution)  $\rightarrow$  cheap, coupling in r, sequential in r
  - bandwidth limited

### the SHTns library: efficient spherical harmonic transforms

- blazingly fast.
- on-the-fly transforms (do not store matrix coefficients, compute them as needed).
- on-the-fly rotations (do not store Wigner-d matrices).
- use FFTW library for the Fourier transform, or VkFFT on GPU.
- hand vectorized routines (support AVX & AVX-512, VSX, Neon).
- Tested up to degree 65000+.



#### SHTns library is freely available

https://gricad-gitlab.univ-grenoble-alpes.fr/schaeffn/shtns

A high performance simulation code for rotating incompressible flows and magnetic fields in spherical shells.

- written in C++
- Free & Open-source software https://nschaeff.bitbucket.io/xshells
- Compilers: gnu or intel (OpenMP 4 support needed)
- Dependencies: FFTW (or MKL) and SHTns
- Parallelization: domain decomposition with MPI + OpenMP.
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### the XSHELLS code: key idea for parallel performance

#### limit communications as much as possible

- domain decomposition favors the SH transform: radial decomposition
- do not use "transpose" data (no MPI\_Alltoall)
- linear solve phase is tricky to parallelize efficiently



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# Porting of XSHELLS on GPU with CUDA

Except the Legendre transform, everything is "memory bound". GPU bring a lot more memory handwidth.

- Ongoing effort
- About 75% of the computations are ported.
- About 900 lines of additional GPU specific code so far.

XSHELLS code is freely available https://gricad-gitlab. univ-grenoble-alpes.fr/ schaeffn/xshells



#### XSHELLS strong scaling @ NR=512, Lmax=426

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  - tracking internal mechanism (+many other).



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- 2016 : Sheyko+
  - $E = 2.4 \times 10^{-6}$ , Pm = 0.04
  - Parker waves, quasi-periodic, low field



#### When do geodynamo models reverse?



Kutzner and Christensen, 2002

Olson, Glatzmaier, and Coe, 2011

### Examples of reversing dynamos

U.R. Christensen/Physics of the Earth and Planetary Interiors 187 (2011) 157-169



From Christensen, 2011:  $E = 3 \times 10^{-4}$ , Pm = 3; large fluctuations of intensity.

### Examples of reversing dynamos



- geodynamo run at Re=217, Pm=3,  $E_{mag}/E_{kin}=0.84$
- 3 reversals within 2 Myr
- same polarity for more than 17 Myr (superchron)

	all	reversing numerics		
	numerics	high viscosity	medium viscosity	Earth's core
Е	$\geq 10^{-7}$	$\geq 10^{-4}$	$\geq 2.4  imes 10^{-6}$	$\simeq 10^{-15}$
Pm	$\geq$ 0.05	$\geq$ 0.67	$\geq$ 0.05	$\simeq 10^{-6}$
$E_m/E_k$	$\leq$ 50	< 0.3	< 0.3	$\simeq 10000$
Λ	$\leq$ 40		< 0.4	$\simeq 10$
reversal mechanism		inertia driven	Parker waves	??

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#### Successes:

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Problems:

- Parker waves: periodic, weak-field: impossible in the Earth's core.
- inertia driven: impossible to happen in the Earth's core.

Inertia-driven reversing dynamos operate at  $Ro_{loc} = 0.1$  (e.g. Olson and Christensen, 2006). With  $Ro_{loc} = Ro R/\ell^*$ , where  $\ell^*$  is a typical scale of convection.

If polarity reversals were inertia-driven:

For the Earth,  $Ro = 3 \times 10^{-6} \Rightarrow \ell^{\star} = 100m$ . What do we know about  $\ell^{\star}$  in the core?

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- constraint #3: magnetic convection at ℓ ~ Ro<sup>1/4</sup> = 150km. Implies Ro<sub>loc</sub> ~ 2 × 10<sup>-5</sup> in the Earth (see Davidson, 2013; Aubert, Gastine, and Fournier, 2017).

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Ooops!

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#### Reversals...



#### Reversals in VKS2: a lab fluid dynamo (almost)

#### VKS2: Turbulent flow between discs

- VKS2 (2006): 0.15 m<sup>3</sup> Na, 300 kW
- dynamo only with soft iron blades (ie magnets)



#### Reversals are captured by a very simple model

A dynamical system below a saddlenode bifurcation can be driven into reversals by fluctuations (noise)  $\zeta(t)$ .

*Right:* 
$$\alpha_0 = -185 Myr^{-1}$$
,  $\alpha_0/\alpha_1 = -0.9$ ,  $\Delta/\sqrt{|\alpha_1|} = 0.2$ .

from Pétrélis et al., 2009.



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300

200 100 (D) B

0 -100 -200

-300<sup>1</sup>



VKS

100

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For a hope of obtain strong magnetic fields in reversing simulations:

- we need more realistic simulations (lower viscosity, strong forcing)
- but not too high definition, so we can run them long enough!

#### Magnetically-driven reversals?



### Zooming on a reversal



t=0.47962











#### What do we learn from these strong field reversals?

Joint distribution of total magnetic energy and surface axial dipole.



During reversal, magnetic field reduced by 15 to 20% in the whole core.

Strong field reversals exist in numerics. Study them to learn about Earth's ones.

- Influence of mantle geodynamics on reversals (input from N. Coltice)
- Statistical properties of reversal (need to simulate a lot of events)

#### Thank you for your attention



Core simulations made with XSHESLLS code: https://nschaeff.bitbucket.io/xshells